Estimating Wideband Polynomial Phase Signals in Sensor Arrays Using the Extended Kalman Filter

A. Ouldali, S. Sadoudi, and Z. Messaoudi

Abstract**—**In the present paper, we consider the problem of parameter estimation of wideband polynomial phase signals (PPS) impinging on a uniform linear array antenna. The parameters of interest are the polynomial phase coefficients and the direction of arrival of the signal. The principle of estimation is based on the introduction of an exact but unfortunately nonlinear state space modelization, of the wideband PPS, which compels us to use the extended Kalman filter (EKF) instead of the usual Kalman filter. Furthermore, we propose a solution to the problem of initialization of the EKF since the initial conditions are assumed to be unavailable. The proposed solution is based on the use of the high-order ambiguity function, generally used to estimate PPS, and the Cramer-Rao bounds. Under this solution, the numerical simulations show that the use of the EKF improves existing methods in terms of statistical performances since the EKF-based estimators exhibit high performances.

Keywords**—**Cramer-Rao bound, direction of arrival, Extended Kalman filter, high-order ambiguity function, parameter estimation, wideband polynomial phase signal.

I. Introduction

The problem of parameter estimation of polynomial phase signal (PPS) occurs in many engineering applications such as radar, communications and seismology. Moreover, PPS waveforms can be intentionally transmitted in multisensor systems. For example, frequency modulated (FM) signal waveforms are widely used for pulse-compression in radar and sonar [10] [12]. Both cases of constant-amplitude and time-varying amplitude PPS have attracted much attention in the literature [2-3] [11] [16]. Recently with the development of the technologies, there has been a growing interest in estimating wideband PPS impinging on a sensor array [5-[6] [8] [14-15].

In [6], a new form of the maximum likelihood (ML) estimator of signal parameters is introduced. However, since the proposed estimator is computationally intensive an approximate technique called the chirp beamformer is proposed. This approach requires solving a three-dimensional (3-D) optimization problem and therefore enjoys essentially simpler implementation than that entailed by the exact ML. In [8], the application of the high-order instantaneous moment (HIM) transforms the PPS array signal into stationary joint angle-frequency estimation (JAFE) problem which is based on the use of the ESPRIT algorithm [7-8]. In fact, it has been shown in [8] that it is possible to jointly estimate two parameters of the signal: the highest order phase coefficient (HOC) and the direction of arrival (DOA). In the following, we call this approach “joint angle highest order coefficient estimation” (JAHOCE). In [4-5] [14-15], the proposed approaches suffer from problems. In fact, in [4-5] the method is basically restricted by short sliding data window lengths. In [14], the approach assumes linear FM signals with known central frequency. Finally, in [15] the iterative approach may lead to strongly biased DOA estimates [4] and its convergence is not guaranteed [6].

In the present paper, we address the problem of parameter estimation of wideband PPS affected by additive noise and impinging on a sensor arrays. The parameters of interest are the polynomial phase coefficients and DOA of the signal. The principle of estimation is based on the introduction of an exact but unfortunately nonlinear state model, of the wideband PPS, which compels us to use, for the first time to the best of our knowledge, the extended Kalman filter (EKF) instead of the Kalman filter usually used in the linear case [9]. We should emphasize that the EKF has already been used to only estimate narrowband PPS as reported in [1]. Furthermore, since the initial conditions of the state model are assumed to be unavailable, we also propose a solution to the problem of initialization of the proposed filter. Under this solution, the numerical simulations show that the proposed EKF improves JAHOCE in terms of statistical performances. In fact the EKF-based estimators exhibit high performances since our proposed method exploits implicitly the double of the initial number of snapshots.

Our paper is organized as follows. The array signal model used in the underlying problem is presented in Section II. Based on this model, we introduce the state space modelization of the wideband PPS in section III. Then, section IV presents the EKF-based estimators. In section V, we provide the solution to the problem of initialization of the

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EKF. Simulation results are presented in Section VI. They show that the use of the EKF improves existing methods, in particular JAHOCE, in terms of statistical performances since the EKF-based estimators exhibit high performances. Finally, section VII concludes this paper.

II. Array Signal Model

Let a wideband PPS $s(n)$ impinges, from an unknown DOA θ, on a uniform linear array (ULA) antenna of L sensors. Then, the $L \times 1$ vector array outputs is given by [6] [8]

$$\mathbf{y}(n) = \mathbf{a}(\theta, n)\, s(n) + \mathbf{w}(n), \qquad n = 0, \ldots, Ne - 1 \tag{1}$$

where

- $\mathbf{a}(\theta, n)$ is the $L \times 1$ time-varying steering vector
- $\mathbf{w}(n)$ is the $L \times 1$ vector of complex circularly Gaussian zero-mean temporally and spatially white noise with known variance σ^2
- Ne is the number of snapshots
- $s(n)$ is the PPS waveform given by [6] [8]

$$s(n) = A \exp\left\{ j \sum_{i=0}^{N} a_i (n\Delta)^i \right\} \tag{2}$$

where

- A is the amplitude of the PPS ($A > 0$)
- $\{a_i\}_{i=0,\ldots,N}$ are the phase coefficients
- N is the degree of the polynomial phase assumed to be known in the following
- Δ is the sampling period

The time-varying steering vector $\mathbf{a}(\theta, n)$ is given by [6] [8]

$$\mathbf{a}(\theta, n) = \begin{bmatrix} 1 \\ e^{jf(n)\psi} \\ \vdots \\ e^{j(L-1)f(n)\psi} \end{bmatrix} \tag{3}$$

where

$f(n)$ is the instantaneous frequency, of the signal, assumed to be constant during the time necessary for a wave to travel across the array aperture. It is given by

$$f(n) = \sum_{i=0}^{N-1} (i+1) a_{i+1} (n\Delta)^i \tag{4}$$

ψ is given by

$$\psi = \frac{d}{c} \sin\theta \tag{5}$$

where

- d is the spacing between two adjacent sensors
- c is the propagation speed in the medium.

III. State Space Modelization

Let $x(n)$ be the vector of the unknown parameters to be estimate

$$x(n) = \begin{bmatrix} A \\ \theta \\ a_0 \\ \vdots \\ a_N \end{bmatrix} \tag{6}$$

Thanks to this vector, we can represent the signal (1) by the following exact but nonlinear state model

$$\begin{cases} x(n+1) = x(n) \\ y_{1,1}(n) = g_{1,1}(x(n)) + \mathrm{Re}\{\mathbf{w}_1(n)\} \\ y_{1,2}(n) = g_{1,2}(x(n)) + \mathrm{Im}\{\mathbf{w}_1(n)\} \\ \quad\vdots \\ \quad\vdots \\ y_{L,1}(n) = g_{L,1}(x(n)) + \mathrm{Re}\{\mathbf{w}_L(n)\} \\ y_{L,2}(n) = g_{L,2}(x(n)) + \mathrm{Im}\{\mathbf{w}_L(n)\} \end{cases} \quad n = 0, \cdots, Ne - 1 \tag{7}$$

where

$y_{l,1}(n) = \mathrm{Re}\{\mathbf{y}_l(n)\}$

$y_{l,2}(n) = \mathrm{Im}\{\mathbf{y}_l(n)\}$

with

$y_l(n)$ is the l^{th} row of $\mathbf{y}(n)$

$\mathbf{w}_l(n)$ is the l^{th} row of $\mathbf{w}(n)$

and

$$g_{l,1}(x(n)) = x_1(n) \cos\{\Phi_l(x(n))\} \tag{8}$$

$$g_{l,2}(x(n)) = x_1(n) \sin\{\Phi_l(x(n))\} \tag{9}$$

$$\Phi_l(x(n)) = x_{N+3}(n)(n\Delta)^N + \Bigg[\sum_{i=0}^{N-1} x_{i+3}(n) + (l-1) \times (i+1) x_{i+4}(n) \left\{ \frac{d}{c} \sin(x_2(n)) \right\} (n\Delta)^i \Bigg] \tag{10}$$

In order to obtain the $(N + 3)$ unknown parameters $\{A, \theta, a_0, \ldots, a_N\}$, we should estimate the state vector $x(n)$ given by (6). This will be done in the present paper by the EKF. In fact, the use of the Kalman filter is impossible since the proposed state model presents a non linear observation equation.

However and fortunately, this model is characterized by an evolution matrix equals to identity. This property reduces significantly, in the process of implementation, the number of operations (multiplications and additions) in the equations of the EKF.

Furthermore, the EKF allows us to obtain at each time $(n\Delta)$ the unknown parameters at a time. This property is very useful for tracking problems.

IV. Extended Kalman Filter Based Estimators

The following algorithm summarizes the EKF and proposes the EKF-based estimators, of the (N + 3) unknown parameters, given Ne noisy observations y(n).

Algorithm 1

1) Initial Conditions (n = 0)

$$\hat{x}_p(0) = E[x(0)] \tag{11}$$

$$P_p(0) = E\left[(x(0) - \hat{x}_p(0))(x(0) - \hat{x}_p(0))^T\right] \tag{12}$$

2) Update equations

$$K(n) = P_p(n)\mathbf{J}^T(\hat{x}_p(n)) \times \left[\mathbf{J}(\hat{x}_p(n))P_p(n)\mathbf{J}^T(\hat{x}_p(n)) + \frac{\sigma^2}{2}\mathbf{I}\right]^{-1} \tag{13}$$

$$\begin{cases} \tilde{y}_1^k(n) = y_{1,k}(n) - g_{1,k}(\hat{x}_p(n)) \\ \vdots \\ \tilde{y}_L^k(n) = y_{L,k}(n) - g_{L,k}(\hat{x}_p(n)) \end{cases} \quad k=1.2 \tag{14}$$

$$\tilde{y}(n) = \left[\tilde{y}(n)_1^1, \tilde{y}(n)_2^1, \cdots, \tilde{y}(n)_L^1, \tilde{y}(n)_L^2\right]^T \tag{15}$$

$$\hat{x}(n) = \hat{x}_p(n) + K(n)\tilde{y}(n) \tag{16}$$

$$P(n) = P_p(n) - K(n)\mathbf{J}(\hat{x}_p(n))P_p(n) \tag{17}$$

3)Prediction equations

$$\hat{x}_p(n+1) = x(n) \tag{18}$$

$$P_p(n+1) = P(n) \tag{19}$$

4) $n = n + 1$. If $n \le Ne - 1$ go to step 2)

5) EKF-based estimators

$$\begin{bmatrix} \hat{A} \\ \hat{\theta} \\ \hat{a}_0 \\ \vdots \\ \hat{a}_N \end{bmatrix} = \hat{x}(Ne - 1) \tag{20}$$

where in particular

$\mathbf{I}$	is the ($2L \times 2L$) identity matrix
$\hat{x}(\bullet)$	is the estimated state vector
$K(n)$	is the Kalman gain
$\mathbf{J}(\bullet)$	is the Jacobean of $[g_{1,1}(\bullet), g_{1,2}(\bullet), \ldots, g_{L,1}(\bullet), g_{L,2}(\bullet)]^T$.

V. Solution to the Problem of Initialization

The EKF needs the initial conditions of the model. As, these ones are not available, we should propose a solution to solve this problem. However, before addressing this aspect we should

1) See the effect of demodulation of the output of each sensor after obtaining the DOA θ and the HOC a_N thanks to JAHOCE proposed in [8].
2) Recall the high-order ambiguity function (HAF) generally used to estimate PPS.
3) Present the Fisher information matrix (FIM) which is necessary to obtain the CRB of each parameter.

A. Effect of demodulation

Let $\mathbf{z}(n)$ be the following signal

$$\mathbf{z}(n) = \mathbf{y}(n) - \mathbf{w}(n).$$

The l^{th} row of $\mathbf{z}(n)$ is given by

$$z_l(n) = A\exp\left\{j\left[a_N(n\Delta)^N + \sum_{i=0}^{N} a_i + (i+1)a_{i+1}(l-1)\psi(n\Delta)^i\right]\right\} \tag{21}$$

The multiplication of $\mathbf{z}_l(n)$ by

$$\exp\left\{-j\left[n\Delta + N(l-1)\psi\right]a_n(n\Delta)^i\right\} \tag{22}$$

leads to the following PPS

$$u(n) = A\exp\left\{-j\left[\sum_{i=0}^{N-1} a_i(n\Delta)^i + \sum_{i=0}^{N-2}(i+1)a_{i+1}(n\Delta)^i(l-1)\psi(n\Delta)^i\right]\right\} \tag{23}$$

which is characterized by a polynomial phase of degree $N - 1$ and with a HOC a_{N-1}.

B. High-order ambiguity functions

The HAF is generally used to estimate constant amplitude PPS [12]. This tool can also be used to estimate time-varying amplitude PPS [2-3] [11] [16]. The HAF of order Q, of a complex signal $v(n)$ is defined as follows [12]

$$P_Q(v,\theta,\tau) = \sum_{n=(Q-1)\tau}^{Ne-1} \prod_{k=0}^{Q-1} I_k\{v(n-k\tau)\}^{b_{Q-1,k}} e^{-j(n\Delta)\theta} \tag{24}$$

where

$$I_{2k}\{v(n)\} = v(n)$$
$$I_{2k+1}\{v(n)\} = v^*(n)$$
$$b_{Q,k} = \binom{Q}{k}.$$

Main property of the HAF

For a complex signal $u(n)$ given by (23), we can easily show the following interesting result [12]

$$a_{N-1} = \frac{1}{(N-1)!(\tau\Delta)^{N-2}} \arg\max_{\theta} \left| P_{N-1}(u,\theta,\tau) \right| \tag{25}$$

Thus, we can see that the HOC a_{N-1}, of the PPS given by (23), can be easily obtained from the abscissa of the peak of the modulus of $P_{N-1}(u, \theta, \tau)$.

Generally, for a PPS (with degree M and Ne samples) affected by an additive white noise (which is complex, circularly, Gaussian and zero-mean) the authors have shown in [12] that the optimal value of the parameter τ (which ensures the lowest value for the variance of the estimation of the HOC a_M) is almost equals to Ne/M. This value is taken in all the following.

C. Cramer-Rao Bounds

Using the properties of the additive noise $\mathbf{w}(n)$, we can easily show that the FIM, noted F, of the set $\mathbf{P}$ of the (N + 3) unknown parameters $\mathbf{P} = [A, \theta, a_0, \ldots, a_N]^T$, is given by the following expression

$$F = \begin{bmatrix} \frac{2NeL}{\sigma^2} & \underline{\mathbf{0}} \\ \underline{\mathbf{0}}^T & \frac{2A^2}{\sigma^2}\begin{pmatrix} R & S_0 & \cdots & S_N \\ S_0 & T_{0,0} & \cdots & T_{0,N} \\ \vdots & \vdots & \ddots & \vdots \\ S_N & T_{N,0} & \cdots & T_{N,N} \end{pmatrix} \end{bmatrix} \tag{26}$$

where $\underline{\mathbf{0}} = [0, 0, \ldots, 0]$ (dim($\underline{\mathbf{0}}$) = $1 \times (N+2)$) and

$$R = \sum_{n=0}^{Ne-1} G_2 \left\{ f(n) \frac{d\psi}{d\theta} \right\}^2 \tag{27}$$

$$S_j = \sum_{n=0}^{Ne-1} (n\Delta)^{j-1} \{(n\Delta)G_1 + jG_2\psi\} f(n) \frac{d\psi}{d\theta} \tag{28}$$

$$T_{i,j} = \sum_{n=0}^{Ne-1} \left\{ ijG_2\psi^2 + (i+j)(n\Delta)G_1\psi + L(n\Delta)^2 \right\} \times (n\Delta)^{i+j-2} \tag{29}$$

where $f(n)$ is given by (4), ψ is given by (5) and

$$G_k = \sum_{m=0}^{L-1} m^k \tag{30}$$

The CRB's of $\mathbf{P}$ are given by

$$CRB(\mathbf{P}_j) = (F^{-1})_{j,j} \qquad j = 1, \cdots, N+3 \tag{31}$$

D. Solution to the problem of initialization

The following algorithm proposed the solution of the problem of initialization of the EKF.

Algorithm 2

1) Estimation step

1.a) Estimate a_N and θ thanks to JAHOCE of [8]

1.b) Let l= 1 and

$$\hat{\psi} = d \sin \frac{\hat{\theta}}{c}$$

1.c) Let

$$\hat{a}_{l,N} = \hat{a}_N, \quad m = N, \quad y_l^{(m)}(n) = y_l(n)$$

and do

$$y_l^{(m-1)}(n) = y_l^{(m)}(n) \times \exp\left\{ -j[n\Delta + m(l-1)\hat{\psi}]\hat{a}_{l,m}(n\Delta)^{m-1} \right\} \tag{32}$$

1.d) Choose $\tau = Ne/(m-1)$ [12] and do

$$\hat{a}_{l,m-1} = \frac{1}{(m-1)!(\tau\Delta)^{m-2}} \arg\max_{\theta} \left| P_{m-1}(y_l^{m-1}, \theta, \tau) \right| \tag{33}$$

$$y_l^{(m-2)}(n) = y_l^{(m-1)}(n) \times \exp\left\{ -j[n\Delta + m(l-1)\hat{\psi}]\hat{a}_{l,m}(n\Delta)^{m-1} \right\} \tag{34}$$

1.e) $m = m-1$. If $m > 1$ go to step 1.d) else do

$$\hat{a}_{l,0} = \arg\left\{ \frac{1}{Ne} \sum_{n=0}^{Ne-1} y_l^{(0)}(n) \right\} \tag{35}$$

$$\hat{A}_l = \mathrm{Re}\left\{ \frac{1}{Ne} \sum_{n=0}^{Ne-1} y_l^{(0)}(n) \exp\{-j\hat{a}_{l,0}\} \right\} \tag{36}$$

1.f) $l = l+1$. If $l \le L$ go to step 1.c) else do

$$\hat{a}_i = \frac{1}{L} \sum_{l=1}^{L} \hat{a}_{l,i} \qquad i = 0, \cdots, N\text{-}1 \tag{37}$$

$$\hat{A} = \frac{1}{L} \sum_{l=1}^{L} \hat{A}_l \tag{38}$$

2) Initialization step

$$\hat{x}_p(0) = \begin{bmatrix} \hat{A} & \hat{\theta} & \hat{a}_0 & \cdots & \hat{a}_N \end{bmatrix}^T \tag{39}$$

$$P_p(0) = \begin{bmatrix} \lambda_1 \operatorname{var}\{\hat{A}\} & 0 & \cdots & 0 \\ 0 & \lambda_2 \operatorname{var}\{\hat{\theta}\} & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \cdots & 0 & \lambda_{N+3} \operatorname{var}\{\hat{a}_N\} \end{bmatrix} \tag{40}$$

where var{•} is the theoretical variance (TV) of the corresponding estimator and the parameters $\{\lambda_i\}_{i=1,\ldots,N+3}$ are positive numbers ≥ 1.

From (40), we see clearly that the initial matrix $P_p(0)$ needs the TV of each estimator. However, those TV are too much difficult to derive and this point is beyond the scope of this paper. To overcome this problem, we can choose $P_p(0)$ according to the following matrix

$$P_p(0)=\begin{bmatrix} \mu_1\mathbf{crb}\{A\} & 0 & \cdots & 0 \\ 0 & \mu_2\mathbf{crb}\{\theta\} & 0 & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & \mu_{N+3}\mathbf{crb}\{a_N\} \end{bmatrix} \quad (41)$$

where crb{•} denotes the CRB of the corresponding parameter and $\{\mu_i\}_{i=1,\ldots,N+3} \geq 1$. $\mu_i = 1$ (respectively $\mu_i >> 1$) means that the i^{th} component of $x_i(0)$ is efficient (respectively is far from the true value to be estimate).

In order to get an idea about the coefficient μ_i, we can evaluate the ratio κ_i of the empirical variance (EV), of the i^{th} estimator, to its corresponding CRB then we can choose $\mu_i = \alpha_i\kappa_i$ with $\alpha_i \geq 1$ since the EV is not necessary equal to the TV and also with $\alpha_i \leq$ B (B is a small positive number) to guarantee that

$$\left[x(0)-\hat{x}_p(0)\right]\left[x(0)-\hat{x}_p(0)\right]^T \quad (42)$$

is of the same order than

$$E\left\{\left[x(0)-\hat{x}_p(0)\right]\left[x(0)-\hat{x}_p(0)\right]^T\right\} \quad (43)$$

Remark 1: In algorithm 2, the total number, of available temporal snapshots, has essentially doubled from *Ne* to 2 *Ne*. In fact, the estimation step uses *Ne* snapshots, to obtain the initial state vector $\hat{x}_p(0)$, and the EKF operates on the same *Ne* snapshots. We should emphasize that this data extension (off-line method) will give a significant improvement of the estimation of the $(N + 3)$ unknown parameters $\{A, \theta, a_0, \ldots, a_N\}$ as it will be seen in the next section.

Remark 2: From (26), we see that crb{A} is independent from the unknown parameters (the variance σ^2, of the additive noise, is assumed to be known), whereas the others crb{•} can depend on the unknown values of the parameters. To overcome this problem we can replace these unknown values by their estimate in the CRB expressions.

Remark 3: For on-line estimation, we apply the estimation step of algorithm 2 on the first *Nr* (*Nr* < *Ne*) snapshots, to obtain the initial state vector $\hat{x}_p(Nr)$, then we start the EKF from $n = Nr$ with $P_p(Nr) = \text{diag}\{\mu_1\text{cb}\{A\}, \mu_2\text{cb}\{\theta\}, \mu_3\text{cb}\{a_0\},\ldots, \mu_{N+3}\text{cb}\{a_N\}\}$ where crb{•} is the CRB evaluated on the first *Nr* snapshots.

VI. Numerical Example

In the present section, we first begin by the evaluation of the statistical performances of the estimation step of the algorithm 2 in order to obtain the ratios $\{\kappa_i\}_{i=1,\ldots,N+3}$. Then, we present the statistical performances of the EKF-based estimators. The SNR is defined as SNR = A^2 / σ^2.

For the numerical simulations, we consider a quadratic phase signal (QPS) impinges, from a DOA θ assumed to be unknown, on ULA antenna of L sensors with the following parameters

A = 1
N = 2,
a_0 = 0.2π, a_1 = 400π, a_2 = 200π,
Ne = 256
Δ = 0,004s
L = 10
θ = 40°
d = 1.5, c = 1500

Remark 4: In the estimation step (algorithm 2), we incorporate in the algorithm JAHOCE only the temporal smoothing with the *r*-factor temporal smoothing $r = 32$ instead of $r = 64$, without incorporating the spatial smoothing and the forward-backward averaging. We should emphasize that with $r = 32$ or $r = 64$, we obtain practically the same performances.

A. *Statistical performances of estimation step (algorithm2)*

In order to evaluate the ratios $\{\kappa_i\}_{i=1,\ldots,5}$ we evaluate the EV of the estimators $\hat{\theta}$ and $\hat{a}_2$ (obtained by JAHOCE of [8]) and the estimators (37) and (38) by carrying out 1000 independent realizations of Monte-Carlo.

The following table summarizes some values of $\{\kappa_i\}_{i=1,\ldots,5}$. It shows that the estimator (38), of the amplitude A, is efficient from SNR ≥ 0 dB whereas the other estimators, in particular $\hat{\theta}$, are not efficient even for high SNR.

SNR (dB)	κ_1	κ_2	κ_3	κ_4	κ_5
0	1	156.4	79.4	2.9	2.23
2	1.03	121.6	18.1	2.47	1.77
4	1.05	116.4	14.6	2.19	1.61
6	1.01	93.2	10.8	1.94	1.55
8	1.01	103.9	11.9	2.08	1.54
10	1.03	98.3	11.7	1.91	1.36
12	1.07	94.6	10.6	2.1	1.45
14	1.03	88.6	10.4	1.78	1.23
16	1.03	91.5	10.8	1.73	1.19
20	1.04	85.5	12.3	1.94	1.3
25	1.01	93.2	19.3	1.77	1.29

Table I. Ratio of the empirical variance to CRB

B. Statistical performances of the EKF-based estimators

In order to evaluate the statistical performances of the proposed EKF-based estimators, we vary the SNR from 0 dB to 25 dB and carry out 1000 independent realizations of Monte-Carlo. For each SNR we choose, in equation (41), $\{\mu_i = 1.5\kappa_i\}_{i=1,\ldots,5}$.

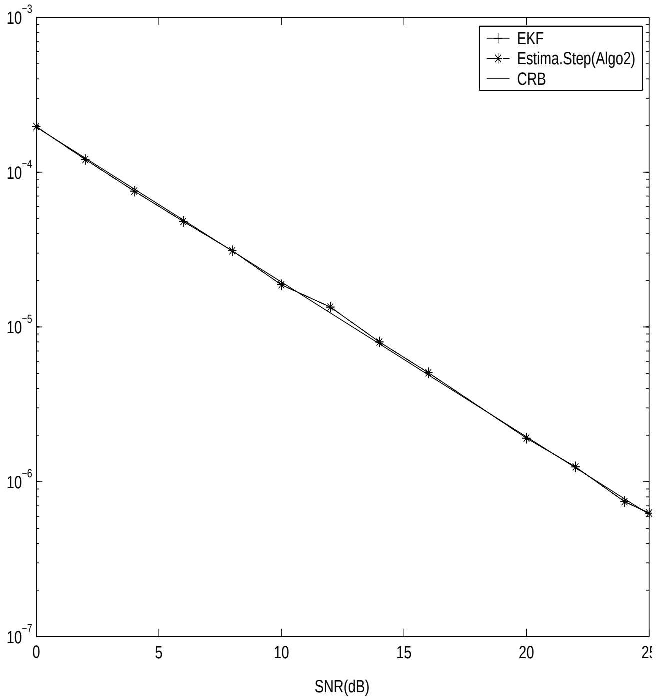

Fig. 1 statistical performances of the EKF-based Estimators: CR and EV of $\hat{A}$ *vs SNR*

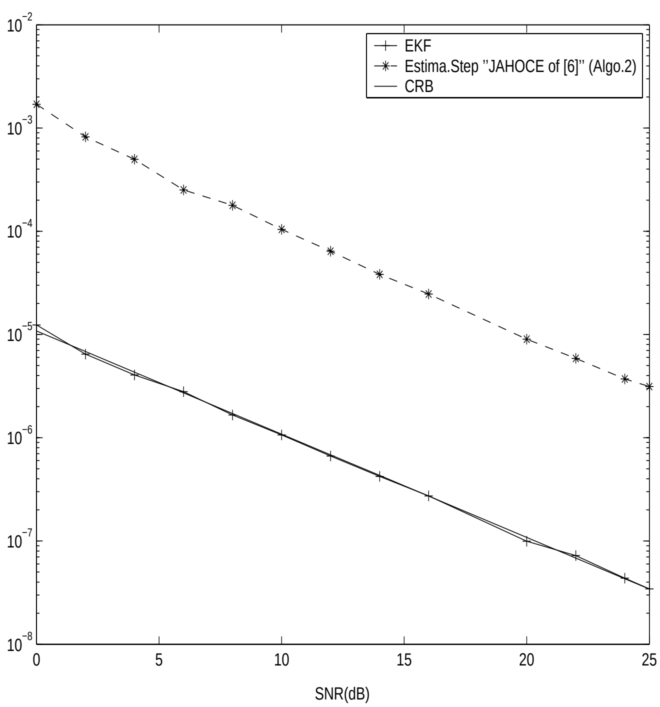

Fig. 2 statistical performances of the EKF-based Estimators: CRB and EV of $\hat{\theta}$ *vs SNR*

It should be noted that in all the figures, the CRB's appear in solid lines, whereas the EV's, of the EKF-based estimators, are represented by (- +) and the EV's, of the estimators obtained in the estimation step (algorithm 2), are represented by (- -*).

From all the figures, the obtained results show that the proposed EKF-based estimators exhibit performances which reach the CRB for low enough SNR. Therefore, our method outperforms, in terms of statistical performances, the JAHOCE method proposed in [8] for only estimating the DOA θ of the wideband PPS and the HOC a_N of the polynomial phase with degree *N*.

Furthermore, from Fig. 1 we should emphasize that the EKF-based estimator, of the amplitude *A*, performs as well as the estimator (38) from SNR ≥ 0 dB. In fact, since this last estimator is efficient from SNR ≥ 0 dB (see Table 1) then the use of the EKF-based estimator, for estimating *A*, will not improve the performances in comparison with the proposed method in [8].

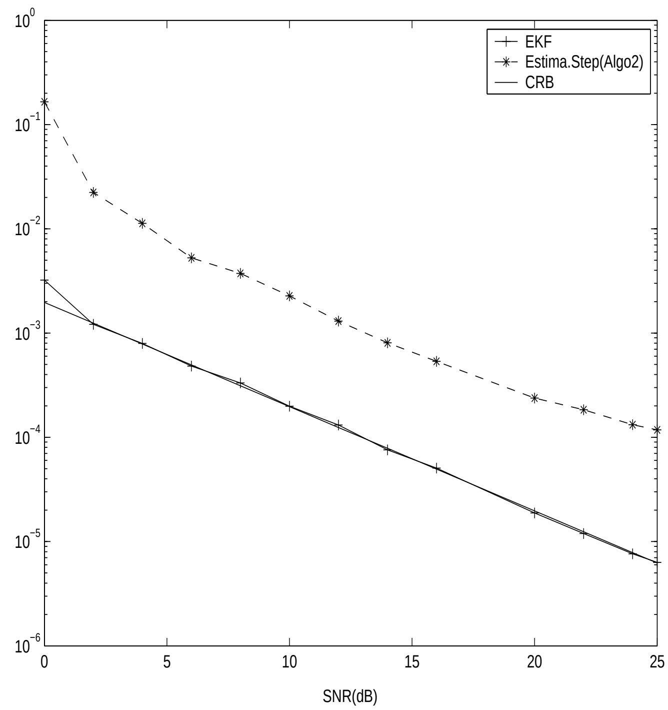

Fig. 3 statistical performances of the EKF-based Estimators: CRB and EV of $\hat{a}_0$ *vs SNR*

The obtained results, which illustrate the high and very interesting performances of the EKF-based estimators proposed in this work, are due to the fact that the proposed method exploits implicitly the double of the initial number of snapshots. In fact, firstly the proposed step of initialization operates on *Ne* snapshots and provides an initial state vector $x_p(0)$, nearest to the true values of the unknown parameters $\{A, \theta, a_0, \ldots, a_N\}$ for low enough SNR, and secondly the EKF uses the same temporal snapshots to improve the performances of the estimation step proposed in the algorithm 2.

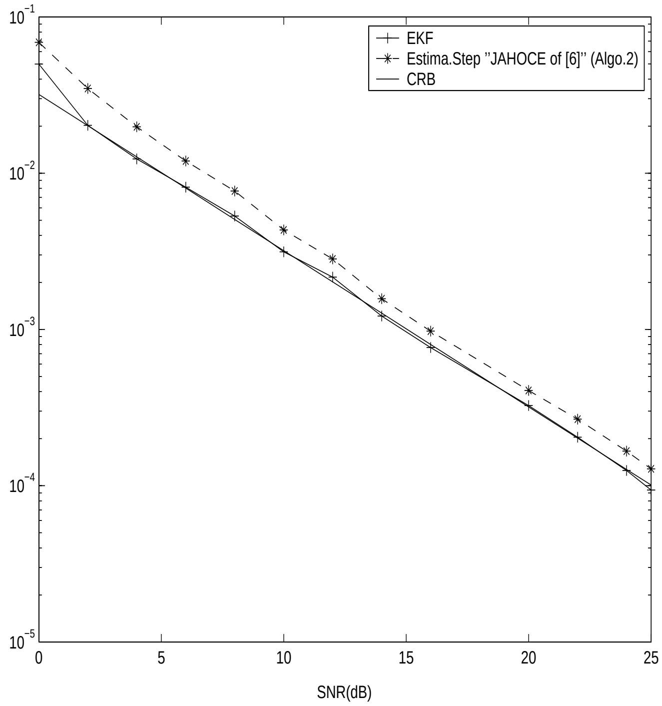

Fig. 4 statistical performances of the EKF-based Estimators: CRB and EV of $\hat{a}_2$ *vs SNR*

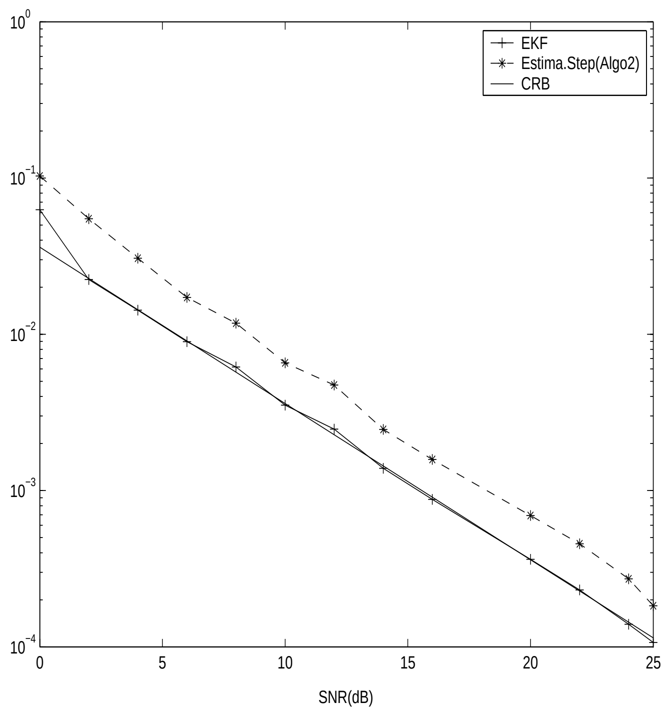

Fig. 4 statistical performances of the EKF-based Estimators: CRB and EV of $\hat{a}_1$ *vs SNR*

VII. Conclusion

In the present paper, we have considered the problem of estimating the parameters of wideband polynomial phase signals (PPS) impinging on a uniform linear array antenna and affected by additive noise. The unknown parameters of interest are the polynomial phase coefficients and the direction of arrival of the signal. The principle of estimation is based on the introduction of an exact but nonlinear state model of the wideband PPS. However and fortunately, this model is characterized by an evolution matrix equals to identity. This state space modelization compels us to use the extended Kalman filter (EKF) instead of the usual Kalman filter. To the best of our knowledge, the EKF has never been used to solve such kind of problems. Furthermore, since the EKF needs the initial conditions, of the state model, which are not available, we have proposed a solution based on the use of the JAHOCE, the HAF and the CR bounds. Under this initialization, which provides an initial state vector nearest to the true values of the unknown parameters, the numerical simulations show, for wideband QPS, that the proposed EKF-based estimators exhibit high performances and outperform in terms of statistical performances the proposed method in [8]. These results are due to the fact that our proposed method exploits implicitly the double of the initial number of snapshots.

References

[1] M. Adjrad, A. Belouchrani and A. Ouldali, “Estimation of chirp signal parameters using state space modelization by incorporating spatial information, ” in *Proc. ISSPA*, Paris, France, July 2003.

[2] M. Benidir and A. Ouldali, ‘‘Statistical analysis of a time-varying amplitude polynomial phase signal,’’ in *Proc. ISSPA 2001*, Kuala lampur, Malaysia, August13-16, 2001.

[3] M. Benidir, A. Ouldali and M. Sahmoudi, ‘‘Performances analysis for the HAF-estimator for a time-varying amplitude phase-modulated signals,’’ in *Proc. of CA 2002, IASTED International Conference in Control and Applications,* Cancun, Mexico, May 20-22, 2002.

[4] A. Gershman and M. Amin, “Coherent wideband DOA estimation of multiple FM signals using spatial time-frequency distributions,” in *Proc. ICASSP*, Istanbul, Turkey, June 2000.

[5] A. Gershman and M. Amin, “Wideband direction-of-arrival estimation of multiple chirp signals using spatial time-frequency distributions,” *IEEE Signal processing Letters*, Vol. 7, pp. 152-155, June 2000.

[6] A. Gershman, M. Pesavento and M. Amin, “Estimating parameters of multiple wideband polynomial-phase sources in sensor arrays, *IEEE Trans. Signal processing*, Vol. 49, pp. 2924-2934, Dec. 2001.

[7] A. Lemma, A.-J. Van Der Veen and E. Deprettere, Analysis of joint angle-frequency estimation using ESPRIT,” *IEEE Trans. Signal processing*, Vol. 51, pp. 1264-1283, May 2003.

[8] S. Lie, R. Leyman and Y. Chew, “Wideband polynomial-phase parameter estimation in sensor array,” in *Proc. ISSPIT*, Darmstadt, Germany, Dec. 2003.

[9] F. Muzi, “A filtering procedure based on least squares and Kalman algorithm for parameter estimate in distance protection,” *International Journal of Circuit, System, and Signal processing*, Issue 1. Vol. 1, pp. 16-21, 2007.

[10] F. Nishiyama and H. Murakami, ‘‘A method of improving SCR for millimeter wave FM-CW radar without knowledge of target and clutter statistics,’’ *International journal of communications*, Issue 3, Vol. 1, pp. 100-106, 2007.

[11] A. Ouldali and M. Benidir, ‘‘Statistical analysis of polynomial phase signals affected by multiplicative and additive noise,’’ *Signal processing*, Vol. 78, pp. 19-42, July 1999.

[12] S. Peleg and B. Porat, “Estimation and classification of signals with polynomial phase, ” *IEEE Trans. Information theory*, Vol. 37, pp. 422–430, March 1991.

[13] Y. Takizawa, S. Yatano and A. Fukasawa, "Wideband Spread Spectrum Modulation for Higher-Data-Rate Mobile Communications," *International Journal of Circuit, System, and Signal processing*, Issue 1. Vol. 1, pp. 48-53, 2007.
[14] Völcker and B. Ottersten, "Linear chirp parameter estimation from multichannel data, " *in Proc. ACSSC*, Pacific Grove, USA, Nov. 1998.
[15] G. Wang and X.-G. Xia, "Iterative algorithm for direction of arrival estimation with wideband chirp signals, " in *Proc. IEE Radar, Sonar, Navig.,* Vol. 147, pp. 233-238, Oct. 2000.
[16] G. Zhou, G. Giannakis and A. Swami, "On polynomial phase signals with time-varying amplitudes,'' *IEEE Trans. Signal processing*, Vol. 44, No. 4, pp. 848-861, April 1996.

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