Wave Digital Approach - A Different Procedures for Modeling of Microstrip Step Discontinuities

Biljana P. Stošić, Miodrag V. Gmitrović

Abstract - A theoretical model for the modeling of the microstrip structure as well as one type of regular discontinuity (step) is described. A microstrip structure, divided into cascade connection of uniform sections, can be efficiently modelled by wave digital networks. A wave digital network is a model of the microstrip structure modeled by wave digital elements. Appropriate choice of a minimal section number in that model is very important because of the direct influence on the sampling frequency of that digital model, and on accuracy of the desired response. Also, it is very important to achieve a good compensation of the effects of identified step discontinuities. In this paper, a choice of a minimal number of sections based on the given relative error and four different procedures for modeling the equivalent network of step discontinuity are presented. First of all, a wave digital element is formed for the equivalent *T*-network of the step discontinuity. In other procedures, the equivalent *L*-network of discontinuity is modeled by one equivalent transmission line, by cascade-connected two transmission lines, and by increasing the lengths of the lines in the junction. Verification of the results obtained is done in one example of bandpass filter.

Keywords - Wave digital approach, Microstrip lines, Microwave structure, Step discontinuity, Transmission lines.

I. Introduction

The modeling of the planar structures by wave digital elements, based on the well known theory of wave digital filters [1]-[3], can be efficiently used for analysis of these structures in both the time and the frequency domains. The microwave planar structures can be modeled by one-dimensional [4]-[7] and by two-dimensional [4], [8] wave digital elements.

A nonuniform structure has to be divided into a cascade connection of uniform transmission lines (UTL) where each UTL is modeled by unit wave digital elements [6]. A lossless uniform transmission line is modeled by a two-port digital element with a delay occurring in the forward path. This wave digital two-port network is called the unit element (UE) [2]. The port resistances of the UE are equal and correspond to the characteristic impedance of UTL. The connection of two UE with different port resistances is achieved by two-port series/parallel adaptors [3].

In the complex microstrip structures, the delays of transmission lines vary from one another, and because of this each transmission line has to be represented as a cascade connection of a certain number of UE. This paper presents a way of determining a minimal number of sections in the wave digital network (WDN) of the complex microstrip structure.

In the papers [5]-[6] nonuniform microstrip structures are observed as cascaded UTL segments, but the effects of step discontinuities have not been taken into consideration. But, once the step discontinuities have been identified in the structure, they must be corrected. In this paper, a wave digital model of an asymmetrical equivalent *T*-network of this discontinuity (WDE_Step) is given. Also, three new procedures of modeling the equivalent *L*-network of this discontinuity are given. The modeling is based on the possible approximation of reactive elements by transmission lines, which is well known from the electric circuit theory.

2 Minimal Section Numbers

A. Determination of Section Numbers

A real delay T_Σ of a complex microstrip structure differs from the delay T_t of the WDN. In complex microstrip structures, delays of transmission lines are not multiple integers of the minimum delay. The number of sections n_k used for modeling an individual transmission line is found as the nearest integer of the ratio

$$T_k / T_{\min}, \tag{1}$$

where T_k is a delay on the k^{th} transmission line, $T_{\min} = \min\{T_1, T_2, ..., T_M\}$ is a minimum delay, $k = 1,2,..., M$ and M is the number of transmission lines in the microstrip structure, [6]. The delay of the individual UE in WDN is found as

$$T = T_\Sigma / n_t, \tag{2}$$

where

$$T_\Sigma = \sum_{k=1}^{M} T_k \tag{3}$$

is the sum of all transmission line delays, i.e. the total real delay of the structure, and

$$n_t = \sum_{i=1}^{M} n_k \tag{4}$$

is the total number of UE in the WDN.

B. A Choice of Minimal Section Numbers for a Known Error

If the number of cascaded UE for each transmission line is found in the above described manner, then the relative error of the total delay is great, which means that response accuracy is

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less. In order to find a lesser relative error, an extra segmentation of the transmission lines has to be done (the multiple factor $q \geq 1$ has to be used). The number of section n_k used for modeling an individual transmission line is found as the nearest integer of the ratio

$$q \cdot T_k / T_{\min} . \quad (5)$$

According to these data, the total delay for the digital model of the structure is

$$T_t = n_t \cdot T_{\min} / q . \quad (6)$$

The relative error of a total delay in percentages is found as

$$er[\%] = \frac{T_\Sigma - T_t}{T_\Sigma} \cdot 100 , \quad (7)$$

where T_Σ is given by (3) and T_t is given by (6).

The minimal number of sections needed for the modeling of the observed structure is found by using the relative error $n\_er[\%]$, which is already known. The procedure for determining the minimal number of sections with the error less than the given one can be done in a few steps. At the beginning, errors $er[\%]$ are found for different values of the multiple factor $q = 1,2,...,q_{\max}$, where $q_{\max}$ is an arbitrary chosen value. Then, the first relative error with an absolute value less than the previously given error $|er[\%]| \leq n\_er[\%]$ is chosen. The number of sections for the k^{th} transmission line n_k, $k = 1,2,...,M$, the total number of sections n_t, and the total delay of a digital model of the structure T_t, corresponding to that error are then used for modeling, [9]-[10].

III. Sampling Frequency

The sampling frequency of the digital model of the planar structure is found for the chosen minimal number of sections, and is

$$F_s = 1/T_s , \quad (8)$$

where

$$T_s = T_t / n_t \quad (9)$$

is the sampling period of the planar structure modeled by n_t sections. In order to match the response of the digital model with a real response, a new frequency is defined

$$F_{sm} = F_s / 2 . \quad (10)$$

IV. Approximate Equivalent Relationships

From the electric circuit theory it is well known that an inductance in a series branch and a capacitance in a parallel branch can be approximated by two-port transmission lines, [11]-[12]. For high frequencies, an inductance is replaced by a transmission line of high characteristic impedance (Fig.1), and a capacitance by a transmission line of low characteristic impedance (Fig.2). The electrical lengths of the transmission lines are $\theta < 45^o$.

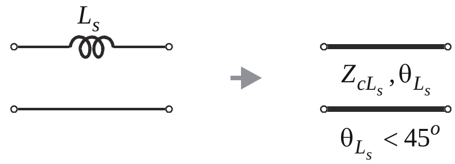

Fig.1 Approximation of an inductance in series branch by transmission line of high characteristic impedance Z_{cL_s}

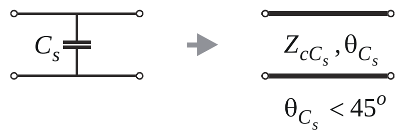

Fig.2 Approximation of a capacitance in parallel branch by transmission line of low characteristic impedance Z_{cC_s}

An electrical length of a transmission line corresponding to the capacitance C_s in the parallel branch can be found as

$$\theta_{C_s} = 2\pi f_c \cdot C_s \cdot Z_{cC_s} , \quad (11)$$

Also, an electrical length of a transmission line corresponding to the inductance L_s in the series branch can be found as

$$\theta_{L_s} = 2\pi f_c \cdot L_s / Z_{cL_s} , \quad (12)$$

The parameter f_c is the cutoff frequency for lowpass and highpass filters and the center frequency for bandpass and bandstop filters.

V. Modeling of Steps in Line Width

The discontinuity in the width of a microstrip line is very offten used in the microstrip circuits in order to change the characteristic impedance of the line. This is important for designing filters and impedance matching networks. In practice, the accuracy of the discontinuity models depends on their physical dimensions [11].

For a step discontinuity shown in Fig.3*a*, the equivalent asymmetrical *T*-network has the series inductance L_h placed near the narrow line, the series inductance L_l placed near the wide line, and the parallel capacitance C_s, Fig.3*b*.

The equivalent *L*-network of discontinuity has the parallel capacitance C_s placed on the side of the wide line, and the series inductance L_s placed on the side of the narrow line, as shown in Fig.3*c*.

This discontinuity is typically modeled by use of the quasistatic analysis, which enables their lumped parameters to be derived. Here, four different ways for modeling the equivalent circuit of the discontinuity are given. The modeling procedures are based on:

- the wave digital element,
- one transmission line,
- two-cascaded transmission lines, and
- increasing line lengths.

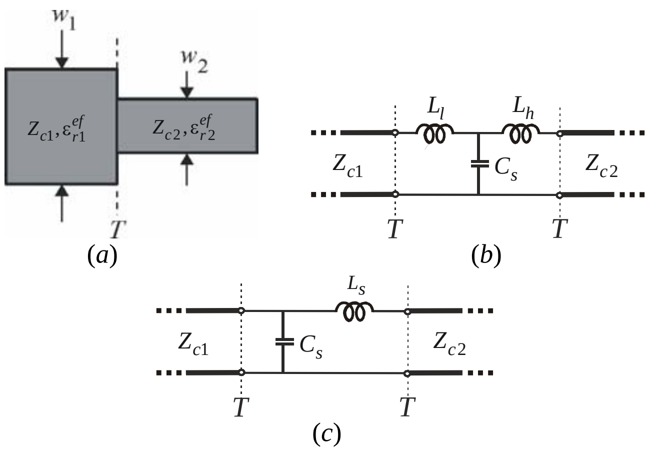

Fig.3 (*a*) Step discontinuity
(*b*) Equivalent *T*-network of the discontinuity
(*c*) Equivalent *L*-network of the discontinuity

C. Wave Digital Element

The analog *T*-section shown in Fig.3*b* is modeled by the wave digital element shown in Fig.4. The WDE_Step element is realized as a cascade connection of two three-port series adaptors with reflection free ports for inductances L_h and L_l, and one three-port parallel adaptor for capacitance C_s, [1]-[3]. Adaptors are memoryless devices whose task is to perform transformations between pairs of wave variables that are referred to different levels of port resistance.

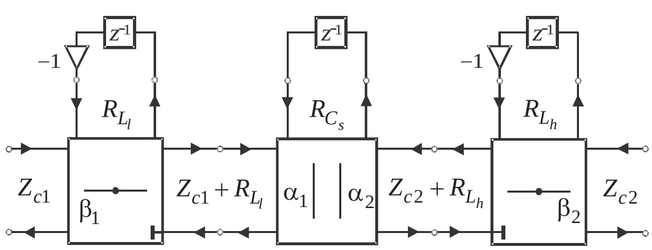

Fig.4 WDE for steps in line width (WDE_Step)

For the coefficients of the thee-port adaptors shown in Fig.4, can be written [1]-[3], [10], [13]: $\beta_1 = Z_{c1}/(Z_{c1}+R_{L_l})$, $\alpha_k = 2G_k/(G_1+G_2+G_3)$, $k=1,2$ and $\beta_2 = Z_{c2}/(Z_{c2}+R_{L_h})$, where port conductances are: $G_1 = 1/(R_{L_l}+Z_{c1})$, $G_2 = 1/(R_{L_h}+Z_{c2})$ and $G_3 = 1/R_{C_s}$. Port resistances are: $R_{L_h} = a\cdot L_h$, $R_{L_l} = a\cdot L_l$ and $R_{C_s} = 1/(a\cdot C_s)$, where $a = 2\pi f_c/\tan(\pi f_c/F_{sm})$ is a scaling parameter and F_{sm} is given by (10).

D. Modeling by One Transmission Line

The discontinuity *L*-network given in Fig.3*c* can be approximated with one equivalent transmission line [14]. The characteristic impedance of the transmision line is

$$Z_{cvs} = \sqrt{L_s/C_s}\,, \tag{13}$$

and the delay on the transmission line is

$$T_s = \sqrt{L_s\cdot C_s}\,, \tag{14}$$

where L_s is the inductivity of the series inductance and C_s is the capacitivity of the parallel capacitance. The electrical length of the transmission line is

$$\theta_s = 2\pi f_c\cdot T_s \tag{15}$$

and the physical length can be found as

$$d_s = c\cdot T_s/\sqrt{\varepsilon_r}\,, \tag{16}$$

where $c = 3\cdot 10^8 m/s$ is light velocity in free space and ε_r is relative dielectric constant of the substrate.

The planar microstrip structure with step discontinuity shown in Fig.3*a*, can be now approximated by cascade connection of three transmission lines as shown in Fig.5. Each *UTL* segment from Fig.3*a* is approximated by one transmission line (Z_{c1} and Z_{c2}). The equivalent network of discontinuity is aproximated by one transmission line given in the middle (Z_{cvs}).

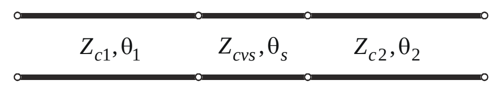

Fig.5 Cascade connection of three transmission lines

Each transmision line assigned as blocks TLine_1, TLine_LsCs and TLine_2 in Fig.6 is then modeled by a certain number of cascaded *UE*. The nonuniform structure is now modeled by a WDN composed only of two types of building blocks (TLines and adaptors) as shown in Fig.6. The coefficients of the two-port series adaptors (blocks ADP-S, ADP-T1TLsCs, ADP-TLsCsT2 and ADP-L) in the *WDN* are

$$\begin{aligned}\alpha_S &= (R_S - Z_{c1})/(R_S + Z_{c1})\,,\\ \alpha_{T1TLsCs} &= (Z_{c1} - Z_{cvs})/(Z_{c1} + Z_{cvs})\,,\\ \alpha_{TLsCsTL2} &= (Z_{cvs} - Z_{c2})/(Z_{c2} + Z_{cvs})\,,\\ \alpha_L &= (Z_{c2} - R_L)/(Z_{c2} + R_L)\,,\end{aligned} \tag{17}$$

respectively. In the symbolic representation of a two-port series/parallel adaptor [1]-[3] given in Fig.7, it is shown explicitly the parameter α next to the port 1.

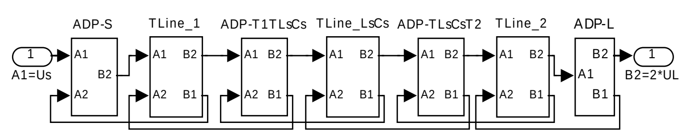

Fig.6 WDN of the structure with modeled discontinuity elements with one equivalent transmission line

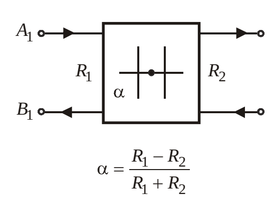

Fig.7 Representation of a two-port series/parallel adaptor

E. Modeling by Two Cascaded Transmission Lines

This approach of modeling structure with discontinuity is based on approximation of reactive elements by transmission lines. Each element of the equivalent discontinuity *L*-network, L_s and C_s, is approximated by one separate transmission line [14]. Thus the equivalent network of discontinuity is approximated by a cascade connection of two transmission lines.

The physical length of the transmisison line given in Fig.1 is

$$d_{C_s} = c \cdot C_s \cdot Z_{cC_s} / \sqrt{\varepsilon_{r1}^{ef}} \,, \qquad (18)$$

where ε_{r1}^{ef} is the effective dielectric constant of the wide line in the structure shown in Fig.3*a*.

The physical length of the transmisison line given in Fig.2 is

$$d_{L_s} = c \cdot L_s / \left(Z_{cL_s} \cdot \sqrt{\varepsilon_{r2}^{ef}} \right), \qquad (19)$$

where ε_{r2}^{ef} is the effective dielectric constant of the narrow line in the structure shown in Fig.3*a*.

These physical lengths depend on the characteristic impedances of the transmission lines. They are chosen in the next way: the characteristic impedance of the transmission line used for approximation of the series inductance is chosen to be $Z_{cL_s} = 150\,\Omega$, and the characteristic impedance of the transmission line used for approximation of the parallel capacitance is $Z_{cC_s} = 5\,\Omega$. This is the result of the well known fact that the typical values of these impedances are $\leq 10\,\Omega$ and $\geq 150\,\Omega$, respectively.

The equivalent representation of microstrip structure with step discontinuity can be approximated here by cascade connection of four transmission lines as shown in Fig.8. The equivalent *L*-network of the discontinuity is approximated by two cascaded transmission lines given in the middle (Z_{cC_s} and Z_{cL_s}).

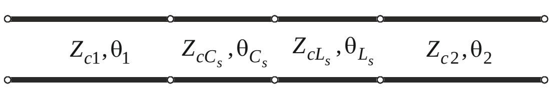

Fig.8 Cascade connection of four transmission lines

Cascade connection of four transmission lines shown in Fig.8 is modeled by WDN as shown in Fig.9. The adaptor coefficients for the blocks ADP-T1TCs, ADP-TCsTLs and ADP-TLsT2 are

$$\alpha_{TL1TCs} = (Z_{c1} - Z_{cC_s})/(Z_{c1} + Z_{cC_s}) \,,$$
$$\alpha_{TCsTLs} = (Z_{cC_s} - Z_{cL_s})/(Z_{cL_s} + Z_{cC_s}) \,, \qquad (20)$$
$$\alpha_{TLsTL2} = (Z_{cL_s} - Z_{c2})/(Z_{c2} + Z_{cL_s}) \,,$$

respectively.

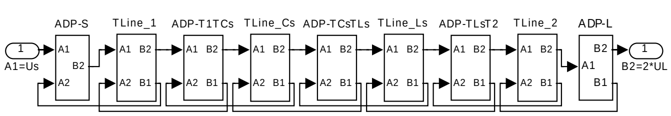

Fig.9 *WDN* of the structure with modeled discontinuity elements with separate transmission lines

F. Modeling by Increasing Line Lengths

The procedure of modeling discontinuity by increasing lengths of the lines in the junction is based on the procedure described in the previous section. Compensation of the discontinuity effects is done as follows: the physical length of the transmisison line used for approximation of the narrow line is increased by the value d_{Ls}, and the length of the transmission line corresponding to the wide line is increased by the value d_{Cs}. It means that the planar microstrip structure given in Fig.3*a* can be approximated by a cascade connection of two transmission lines as shown in Fig.10. The equivalent network of the discontinuity is included through increasing the lengths of the transmission lines (Z_{c1} and Z_{c2}).

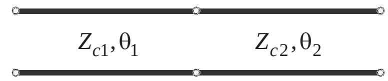

Fig.10 Two cascade connected transmission lines

Finally, WDN of a nonuniform planar microstrip structure is shown in Fig.11. The adaptor coefficient in the block ADP_TL1TL2 is

$$\alpha_{TL1TL2} = (Z_{c1} - Z_{c2})/(Z_{c2} + Z_{c1}) \,. \qquad (21)$$

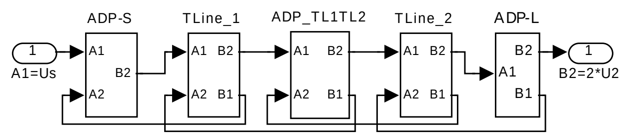

Fig.11 WDN of the structure shown in Fig.3 with modeled discontinuity elements

G. Analysis Procedures

In the modeling procedure described in Section *A*, the WDN is composed of three types of building blocks: TLines, WDE_Step and two-port adaptors. A very simple method of analysis of that WDN is a block-diagram method. WDN is formed directly in the Simulink toolbox of the MATLAB environment. Programs *dlinmod.m*, *dimpulse.m* and *fft.m* are employed to find the response.

The WDN obtained by the other modeling approaches of step discontinuity described in Sections *B*-*D* are analyzed by use of wave transfer matrices. Efficient and very simple algorithms for calculating transmission and input reflection coefficients of the microstrip structure modeled by wave digital elements are described in papers [6]-[7]. The algorithms are very easily implemented in the *MATLAB* environment. The analysis of the WDN is efficiently automated, which is inevitable when structures with large numbers of building blocks are to be dealt with. In the case of cascaded microstrip lines, the proposed method reduces drastically the computation time, while giving acceptable accuracy.

VI. Analysis Example

A microstrip stepped-impedance bandpass filter with a passband ripple of $0.0137\,dB$ and a central bandpass frequency of $5.6\,GHz$ [12] is used for verification of the proposed method. The layout is shown in Fig.12. The substrate dielectric constant is $\varepsilon_r = 6.0$, and the board thickness is $h = 635\,\mu m$. Metalisation is cooper and the metal thickness is $t = 18.034\,\mu m$.

The microstrip lowpass filter is observed as a cascade connection of nine UTL segments whose lengths and widths are: $d_1 = d_9 = 1.1709\,mm$, $d_2 = d_8 = 1.3970\,mm$, $d_3 = d_7 = 14.6380\,mm$, $d_4 = d_6 = 3.8405\,mm$, $d_5 = 13.6779\,mm$, $w_1 = w_3 = w_5 = w_7 = w_9 = 0.3175\,mm$ and $w_2 = w_4 = w_6 = w_8 = 5.0800\,mm$. The lines at the ends are the $50\,\Omega$ leader lines.

Fig.12 Layout of bandpass filter

Here, the $50\ \Omega$ leader lines at the ends of the structure are not included during the analysis of the structure modeled by UE. These leader lines affect the total delay in the WDN and have the effect of shifting response characteristics. Thus, the microstrip filter is observed as a cascade connection of nine UTL segments. Their delays vary from one another because of their dependance on the effective dielectric constant, and are: $T_1 = T_9 = 7.8520\ ps$, $T_2 = T_8 = 10.7111\ ps$, $T_3 = T_7 = 98.1587\ ps$, $T_4 = T_6 = 29.4458\ ps$ and $T_5 = 91.7204\ ps$. In order to have delays in wave digital models as possible equal to these delays, each transmission line has to be represented as a cascade connection of a certain number of UE.

A. Appropriate Choice of Minimal Section Numbers for Known Error

In order to find the minimal number of sections for the given error, testing is done for the next values of the multiple factor $q = 1,2,...,q_{max}$, and $q_{max} = 100$. The total number of sections n_t and the counted errors $er\,[\%]$, for $q = 1,2,...,24$, are given in the Table 1.

Table 1
Total Number of Sections and Relative Error

q	n_t	$er\,[\%]$	q	n_t	$er\,[\%]$
1	**50**	**-2.2250**	13	638	-0.3378
2	99	-1.2027	14	686	-0.1805
3	147	-0.1805	15	733	0.0921
4	195	0.3306	**16**	**783**	**-0.0527**
5	246	-0.5894	17	833	-0.1805
6	294	-0.1805	18	882	-0.1805
7	344	-0.4726	19	930	-0.0729
8	**391**	**0.0751**	20	978	0.0240
9	441	-0.1805	21	1029	-0.1805
10	491	-0.3850	22	1077	-0.0876
11	538	0.0054	23	1125	-0.0027
12	586	0.1602	24	1174	-0.0101

As shown in Table 1, the relative error of the total delay does not have a convergence effect with increasing the total number of sections. These errors vary from one another, and because of that the procedure of determination of the minimal number of sections for the given error is presented in this paper. Also, extra segmentation of the transmission lines leads to the best solution in the modeling procedure, i.e. to the minimal chosen relative error.

Further, in order to better explain the determination of a minimal number of sections for a given error, a few cases from Table 1 will be extracted and explained.

For $n_t = 50$, the agreement between the results obtainted by WDN and one obtained in ADS (Advanced Design Software) is very poor in the whole frequency band, see Fig.13.

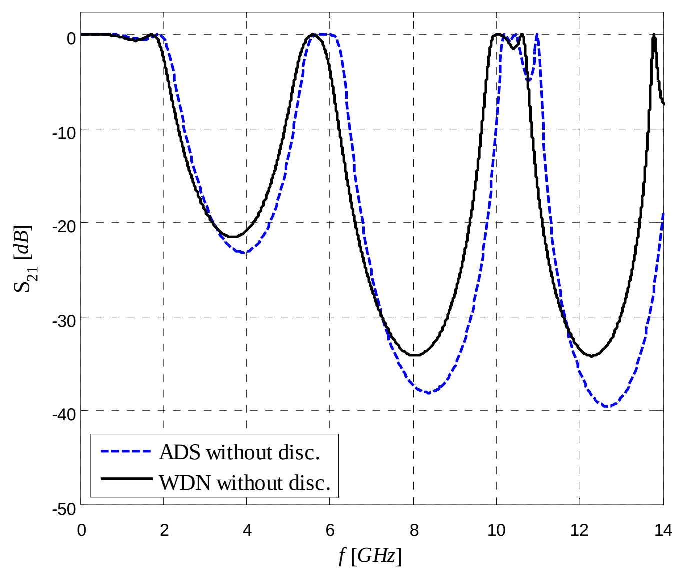

Fig.13 Response comparison for $n_t = 50$

From Table 1 for the given error $n\_er = 0.1\%$, the first positive error is for $q = 8$, and the first negative error is for $q = 16$.

For $q = 8$, the total section number in the WDN is $n_t = 391$. The total delay of the structure digital model is $T_t = 383.7671\ ps$ and the sum of all transmission line delays is $T_\Sigma = 384.0554\ ps$. According to relation (8), a sampling frequency is $F_s = 1018.8471\,GHz$. The relative error of delay found by (7) is $er = 0.0751\,\%$.

For $q = 16$, the total section number in the WDN is $n_t = 783$. The sampling frequency is $F_s = 2037.6941\,GHz$. The total delay of the structure digital model is $T_t = 384.2579\ ps$. The relative error of the delay is $er = -0.0527\,\%$. The response comparisons are shown in Figs.14 and 15.

The standard values for accuarcy control, ATE (*Average Test Error*), WCE (*Worst Case Error*), and *Pearson-Producd Moment* correlation coeficient r of the response $S_{21}[dB]$ obtained by the wave digital approach and the one obtained in ADS, are shown in Table 2. It is clear that the agreement between the responses is very good in both cases, for $T_\Sigma \le T_t$ ($n_t = 783$) and for $T_\Sigma \ge T_t$ ($n_t = 391$).

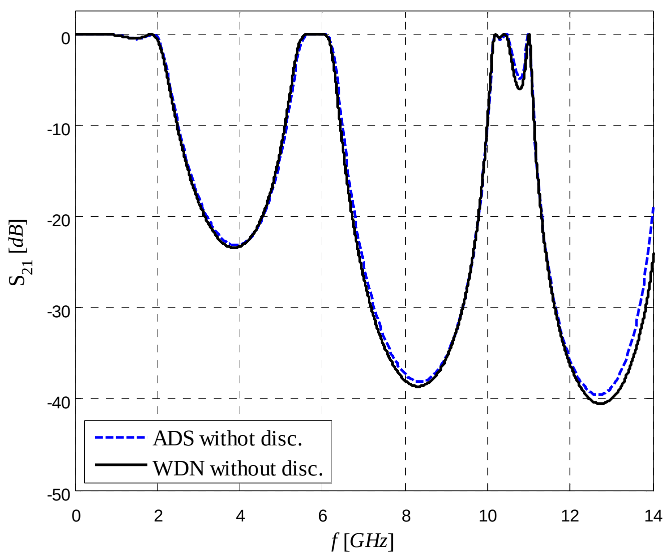

Fig.14 Response comparison for $n_t = 391$

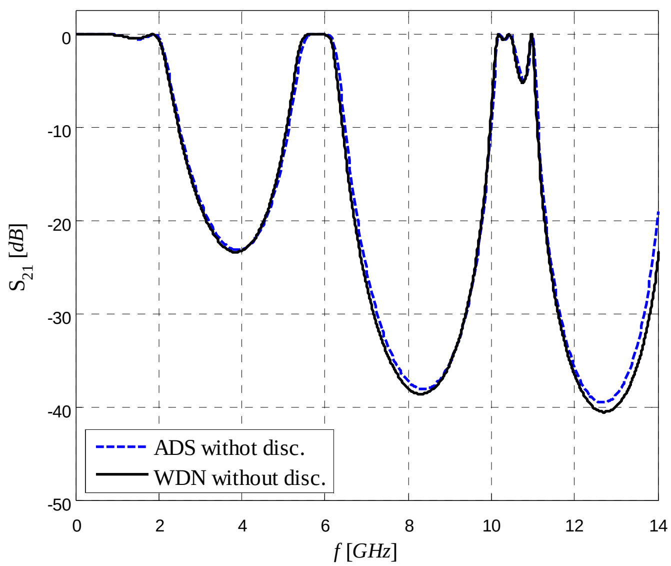

Fig.15 Response comparison for $n_t = 783$

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Table 2
Standard Values for Accuracy Control in Case of Given Error $n\_er = 0.1\%$

	$n_t = 391$		$n_t = 783$	
	$\lvert S_{21} \rvert$	$S_{21}[dB]$	$\lvert S_{21} \rvert$	$S_{21}[dB]$
ATE [%]	1.3251	0.0167	1.6811	0.0198
WCE [%]	11.6380	0.1548	15.9792	0.1402
r	0.9978	0.9982	0.9964	0.9979

B. Wave Digital Element

The parameters of elements of the equivalent discontinuity circuit according to relation from [11], are: $L_s = 0.3570\, nH$, $C_s = 0.1906\, pF$, $L_h = 0.3360\, nH$ and $L_l = 0.0210\, nH$.

Table 3
Total Number of Sections and Relative Error

q	n_t	er [%]
1	50	-2.2250
2	99	-1.2027
3	**147**	**-0.1805**
4	195	0.3306
5	246	-0.5894

If the number of sections is found for $q = 3$, $n\_er = 1\%$, the total number of UE in the WDN is $n_t = 147$. UTL segments assigned as UTL1 and UTL9 are modeled with three cascaded UE (blocks 3T_1 and 3T_2), segments assigned as UTL2 and UTL8 with four cascaded UE (blocks 4T_1 and 4T_2), segments assigned as UTL3 and UTL7 with 38 cascaded UE (blocks 38T_1 and 38T_2), segments assigned as UTL4 and UTL6 with 11 cascaded UE (blocks 11T_1 and 11T_2), and segment assigned as UTL5 with 35 cascaded UE (block 35T_1). The formed WDN is depicted in Fig.16 and a block for cascade connection of 4 UE is shown in Fig.17. Fig.18 shows block WDE_Step formed in Simulink toolbox in the MATLAB enviroment (blocks Step_1 to Step_8 in Fig.16). The analysis parameters are: $T_{\Sigma} = 384.0554\, ps$, $F_s = 382.0676\, GHz$, and $T_t = 384.7486\, ps$.

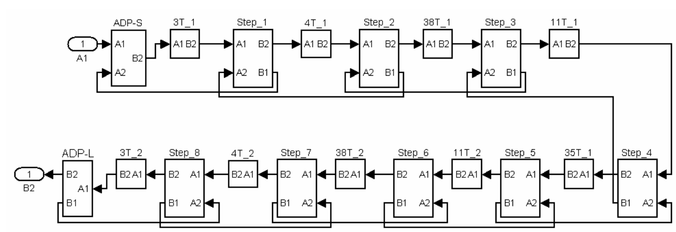

Fig.16 Wave digital network in Simulink toolbox for $n_t = 147$

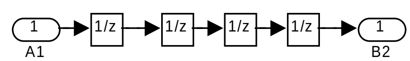

Fig.17 Block 4T_1

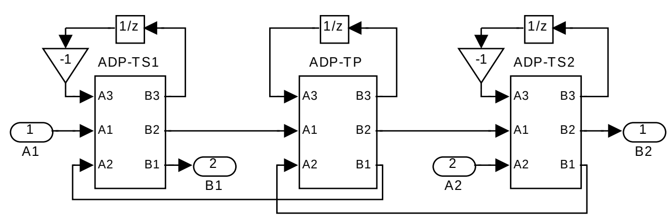

Fig.18 Block WDE_Step in Simulink toolbox

Response comparison of the results obtainted by WDN to the one obtained in ADS is shown in Fig.19. In the region below $3.5\, GHz$, the agreement is good and the results are acceptable. For the region above the frequency of $3.5\, GHz$ the WDN curve with modeled discontinuity is shifted to the left. A curve for WDN without modeled discontinuities is shifted to the right in the whole frequency band.

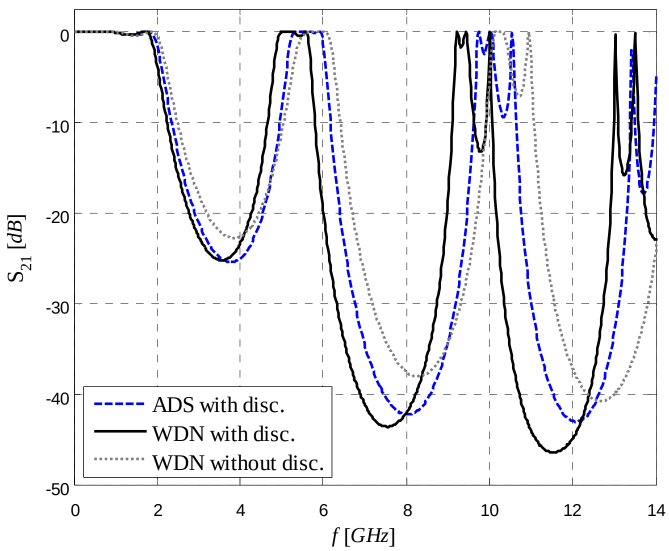

Fig.19 Response comparison for $n_t = 147$

C. Modeling of the Discontinuity Equivalent Network by One Transmission Line

Cascade connection of the equivalent *L*-network elements L_s and C_s is approximated by one transmission line with a characteristic impedance $Z_{cvs} = 43.2815\,\Omega$ according to (13), and the physical length $d_{vs} = 1.0102\,mm$ according to (16).

The microstrip structure given in Fig.12 is approximated by cascade connection of 17 transmission lines with the parameters given in Table 4. WDN has the structure given in Fig.6, where the number of blocks corresponding to transmission lines is 17 and the number of blocks corrsponding to two-port adaptors is 18.

Table 4
Transmission Line Parameters

nv	d [mm]	Zc [Ohm]	Tv [ps]
1	1.1709	79.6365	7.8520
2	1.0102	43.2815	8.2481
3	1.3970	14.9620	10.7111
4	1.0102	43.2815	8.2481
5	14.6380	79.6365	98.1587
6	1.0102	43.2815	8.2481
7	3.8405	14.9620	29.4458
8	1.0102	43.2815	8.2481
9	13.6779	79.6365	91.7204
10	1.0102	43.2815	8.2481
11	3.8405	14.9620	29.4458
12	1.0102	43.2815	8.2481
13	14.6380	79.6365	98.1587
14	1.0102	43.2815	8.2481
15	1.3970	14.9620	10.7111
16	1.0102	43.2815	8.2481
17	1.1709	79.6365	7.8520

The total number of sections n_t and the counted errors $er\,[\%]$, for $q = 1,2,...,22$, are given in Table 5. For a given error $n\_er = 0.01\%$, the first relative error of delay with an absolute value less than the given error is for $q = 22$. Then the total minimal number of sections in WDN is $n_t = 1261$. The total delay for the digital model of the structure is $T_t = 450.0631\,ps$, and the sampling frequency of the digital model of the planar structure for the chosen minimal number of sections is $F_s = 2801.8294\,GHz$. The total real delay of the structure is $T_\Sigma = 450.0404\,ps$.

Table 5
Total Number of Sections and Relative Error

q	n_t	$er\,[\%]$
1	58	-1.9463
2	115	-0.2226
3	171	0.5010
4	227	0.8629
5	286	0.0116
6	342	0.5010
7	400	0.0086
8	455	0.6820
9	513	0.5010
10	579	-1.2016
11	634	-0.6018
12	690	-0.2226
13	750	-0.5779
14	806	-0.4689
15	861	-0.4779
16	919	-0.1322
17	977	-0.7095
18	1034	-0.2533
19	1090	-0.9269
20	1146	0.2668
21	1205	-0.1456
22	**1261**	**-0.0050**

Response comparison of the results obtainted by WDN to the one obtained in ADS is depicted in Fig.20. In the region below $3.5\,GHz$, the agreement is good and the results are acceptable. For the region above the frequency of $3.5\,GHz$ the WDN curve (1 transmission line) is shifted to the left.

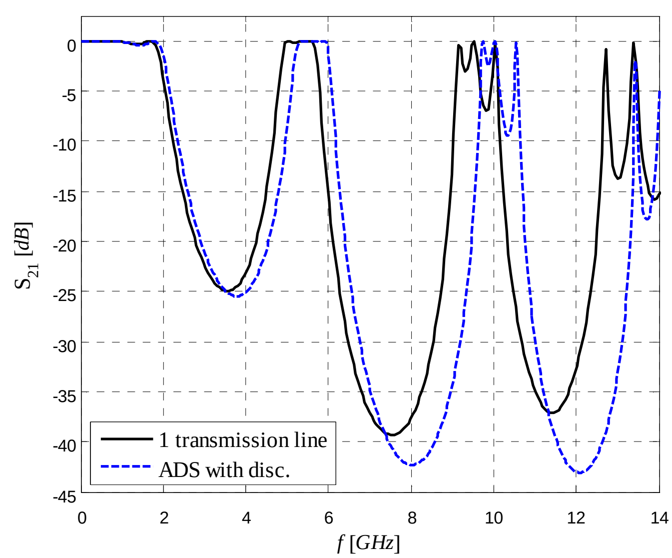

Fig.20 Response comparison

D. Modeling of the Discontinuity Equivalent Network by Two Cascaded Transmission Lines

Each element of the equivalent circuit of the discontinuity, L_s and C_s, is aproximated by one transmission line with characteristic impedances $Z_{cL_s} = 150\,\Omega$ and $Z_{cC_s} = 5\,\Omega$, respectively. The physical lengths of transmission lines according to relations (18) and (19) are: $d_{Cs} = 0.1243\,mm$ and $d_{Ls} = 0.3549\,mm$. The transmission line parameters are given in Table 6. *WDN* has the structure given in Fig.9, where the number of blocks corresponding to transmission lines is 25 and the number of blocks corrsponding to two-port adaptors is 26. The total number of sections n_t and the counted errors $er\,[\%]$, for $q_{\max} = 10$, are given in the Table 7.

Table 6
Transmission Line Parameters

nv	d [mm]	Zc [Ohm]	Tv [ps]
1	1.1709	79.6365	7.8520
2	0.3549	150.0000	2.3799
3	0.1243	5.0000	0.9528
4	1.3970	14.9620	10.7111
5	0.1243	5.0000	0.9528
6	0.3549	150.0000	2.3799
7	14.6380	79.6365	98.1587
8	0.3549	150.0000	2.3799
9	0.1243	5.0000	0.9528
10	3.8405	14.9620	29.4458
11	0.1243	5.0000	0.9528
12	0.3549	150.0000	2.3799
13	13.6779	79.6365	91.7204
14	0.3549	150.0000	2.3799
15	0.1243	5.0000	0.9528
16	3.8405	14.9620	29.4458
17	0.1243	5.0000	0.9528
18	0.3549	150.0000	2.3799
19	14.6380	79.6365	98.1587
20	0.3549	150.0000	2.3799
21	0.1243	5.0000	0.9528
22	1.3970	14.9620	10.7111
23	0.1243	5.0000	0.9528
24	0.3549	150.0000	2.3799
25	1.1709	79.6365	7.8520

Table 7
Total Number of Sections and Relative Error

q	n_t	$er\,[\%]$
1	426	1.1700
2	861	0.1260
3	1291	0.1646
4	1725	-0.0480
5	2151	0.1956
6	2584	0.0873
7	3014	0.1094
8	**3448**	**0.0099**
9	3874	0.1389
10	4309	0.0332

For a given error $n\_er = 0.01\%$, the total minimal number of sections in *WDN* is $n_t = 3448$. The other analysis parameters are: $F_s = 8395.8976\,GHz$, $T_t = 410.6768\,ps$, and $T_\Sigma = 410.7177\,ps$.

Response comparison of the results obtainted by WDN (with and without modeled discontinuities) and the one obtained in ADS is shown in Figs. 21 and 22. The curve for WDN without modeled discontinuities is shifted to the right in the whole frequency band. The curve for WDN with modeled discontinuities is shifted to the left.

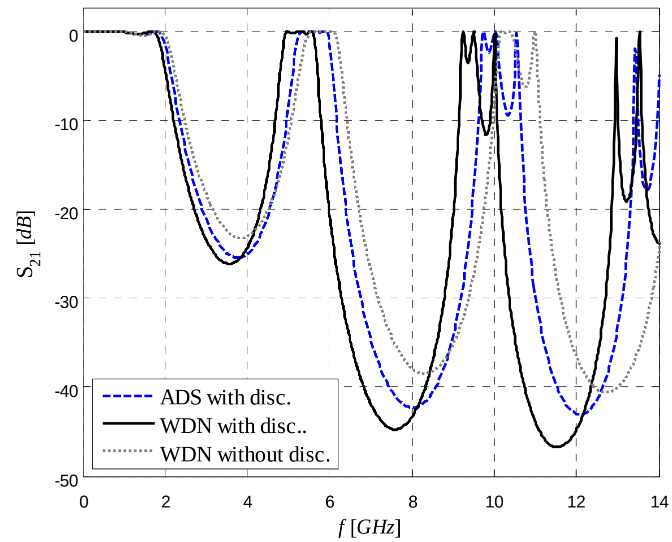

Fig.21 Response comparison

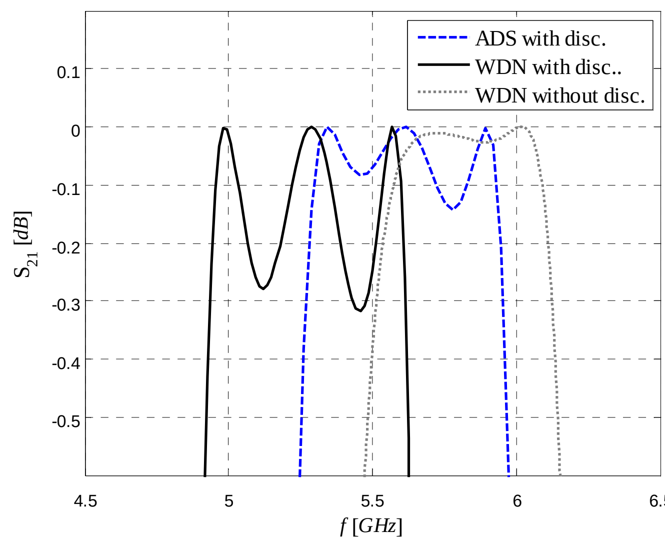

Fig.22 Response comparison in the bandpass region

E. Modeling of the Discontinuity Equivalent Network by Increasing Line Lengths

In Table 8 transmission line parameters without modeled discontinuities are shown.

Table 8
Transmission Line Parameters without Modeled Discontinuites

nv	d [mm]	Zc [Ohm]	Tv [ps]
1	1.1709	79.6365	7.8520
2	1.3970	14.9620	10.7111
3	14.6380	79.6365	98.1587
4	3.8405	14.9620	29.4457
5	13.6779	79.6365	91.7204
6	3.8405	14.9620	29.4457
7	14.6380	79.6365	98.1587
8	1.3970	14.9620	10.7111
9	1.1709	79.6365	7.8520

The characteristic impedances are chosen to be: $Z_{cL_s} = 150\,\Omega$ and $Z_{cC_s} = 5\,\Omega$. The physical lengths of transmission lines according to relations (18) and (19) are: $d_{Cs} = 0.1243\,mm$ and $d_{Ls} = 0.3549\,mm$.

Transmission line parameters for this case are given in Table 9. According to the parameters given in Tables 8 and 2, it can be concluded that the number of transmission lines is the same in both cases, but their physical lengths differ because their delays differ also.

Table 9
Transmission Line Parameters with Modeled Discontinuites

nv	d [mm]	Zc [Ohm]	Tv [ps]
1	1.5259	79.6365	10.2320
2	1.6456	14.9620	12.6168
3	15.3478	79.6365	102.9186
4	4.0890	14.9620	31.3514
5	14.3877	79.6365	96.4802
6	4.0890	14.9620	31.3514
7	15.3478	79.6365	102.9186
8	1.6456	14.9620	12.6168
9	1.5259	79.6365	10.2320

The total number of sections n_t and the counted errors $er\,[\%]$ for $q_{\max} = 14$ are given in Table 10. For a given error $n\_er = 0.01\,\%$, the total minimal number of sections in *WDN* is $n_t = 562$. The other analysis parameters are: $F_s = 1368.2625\,GHz$, $T_t = 410.7399\,ps$ and $T_\Sigma = 410.7177\,ps$.

Response comparisons for the proposed approach of modeling step discontinuity are shown in Figures 23, 24 and 25. It is clear that the curve corresponding to the result obtained by modeling structure and its discontinuities by increasing line lenghts are very close to the one corresponding to ADS results.

Table 10
Total Number of Sections and Relative Error

q	n_t	$er\,[\%]$
1	39	2.8417
2	79	1.5961
3	120	0.3505
4	160	0.3505
5	199	0.8487
6	239	0.7657
7	280	0.3505
8	321	0.0391
9	363	-0.4799
10	402	-0.1478
11	444	-0.5554
12	483	-0.2723
13	523	-0.2244
14	**562**	**-0.0054**

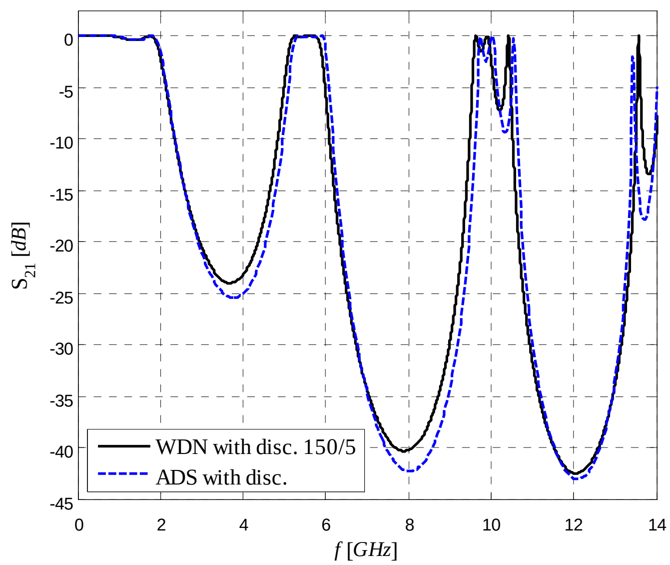

Fig.23 Response comparison

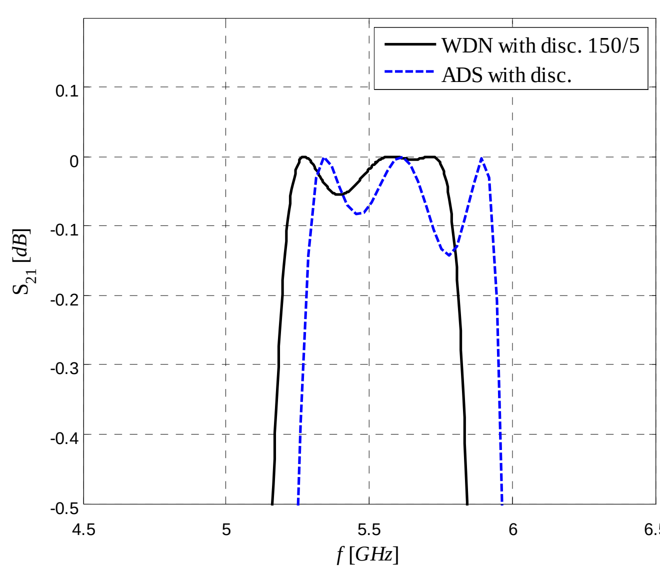

Fig.24 Comparison of the bandpass responses

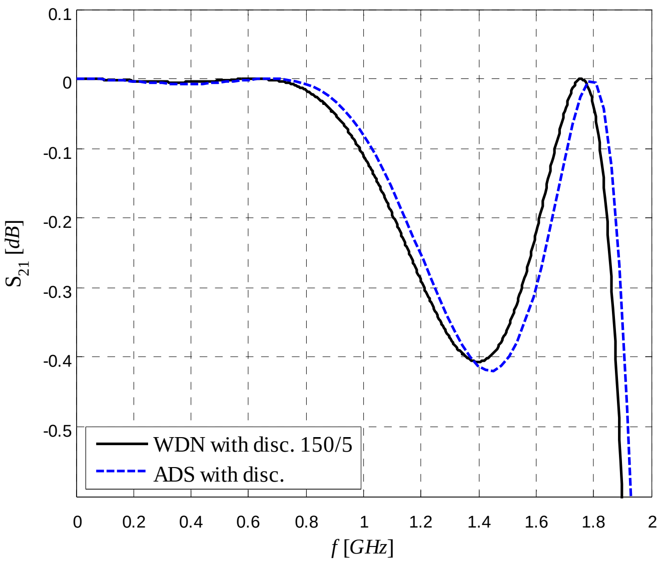

Fig.25 Comparison of the responses in frequency range $[0 \div 2]\,GHz$

VII. CONCLUSION

Planar microwave structures with their step discontinuities can be completely analyzed and modeled by wave digital elements. Each transmision line is modeled by a certain number of UE, i.e. by cascade connection of several sections. Improved accuracy of the analysis results is obtained by choosing an appropriate minimal number of sections. A very simple algorithm for determining the minimal number of sections for a given error is described here.

To prove the accuracy of the proposed modeling of step discontinuity, computer simulated results obtained by WDN where discontinuities are modeled in four different ways are compared to those obtained in ADS (Advanced Design Systems). If these procedures are compared, it can be seen that the procedure of modeling discontinuities with increasing line lengths is the best when structures with a great discontinuity influence are to be dealt with. That procedure requires less UE and a simpler digital WDN.

The proposed approach can be used by microwave engineers because of the associated computational efficiency and accuracy. It can be used for analyzing the transmission lines of various nonuniform shapes present in practice. The new approach can be easily implemented in the MATLAB environment.

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