Algebraic Control of Integrating Processes with Dead Time by Two Feedback Controllers in the Ring R_{MS}

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***Abstract*—** The objective of this contribution is to demonstrate the utilization of algebraic controller design in an unconventional ring while control integrating processes with time delay. In contrast to many other methods, the proposed approach is not based on the time delay approximation. A control structure combining a simple feedback loop and a two-degrees-of-freedom control structures is considered. This structure can be also conceived as a simple feedback loop with inner stabilizing loop. The control design is performed in the ring of retarded quasipolynomial (RQ) meromorphic functions (R_{MS}) - an algebraic method based on the solution of the Bézout equation with the Youla-Kučera parameterization is presented. Final controllers may be of so-called anisochronic type and they ensure feedback loop stability, tracking of the step reference and load disturbance attenuation. Among many possible tuning methods, the dominant pole assignment method is adopted. This approach is compared with the conventional polynomial LQ method using an illustrative simulation example..

***Keywords*—** Time delay systems, algebraic control, Bézout identity, Youla-Kučera parameterization.

I. Introduction

INTEGRATING models appear while modeling mass or energy accumulation, a rotation of machineries, etc. and they contain undesirable pole which need to be shifted by suitable design of the feedback loop. As well, the great deal of technological and other processes, such as distributed networks, long transmission lines in pneumatic systems or neural networks [1], to name a few, own an input-output time delay. The presence of a delay entails problems with controllers design due to the fact that the delay significantly influences the dynamic properties of a feedback control system. The combination of integrating behavior of the system and delays makes controller design more difficult and it requires utilization of some advanced procedures.

There have been recently investigated various principles for control of integrating processes with time delay. A group of methods utilizes standard PI or PID controllers in an effort to get a certain optimization and robustness, see e.g. [2]-[3]. In [4] allowable PI and PD controller gains have been investigated. One type of a pole assignment approach in [5] was developed. Some ideas are based on generalized Smith predictor, e.g. [6]-[7] or even more general control loops [8], designed mainly in order to obtained the satisfactory disturbance response. Last but not least, authors present predictive approaches mainly incorporating state-space description e.g. in [9].

One of the most significant approaches in modern control theory is a family of algebraic methods. Unlike some traditional state-space models, algebraic tools are based on fractional description of systems where a transfer function can be expressed as a ratio of two elements in an appropriate ring. From the historical point of view and the natural correspondence between time-domain description and the transformation for discrete-time systems, traditional transfer functions are represented by polynomial fractions. This idea was adopted for continuous-time systems as well. This description is employed for algebraic control strategy for integrating delayed systems in [10] where the control structure with two feedback controllers (Fig. 1) is considered. However, a transfer function can be written as a fractional field of more general algebraic structures – rings. One of basic requirements on a control system is that both a plant and a controller are proper and a control system is internally stable, which brings the possibility of introduction of another frequently used ring, R_{PS} (the ring of Hurwitz stable and proper rational functions) [11], [12]. Algebraic control philosophy in this ring then exploits Bézout identity (Diophantine equation) along with the Youla-Kučera parameterization to obtain stable and proper controllers. Nevertheless, utilization of this ring is rather restrictive while dealing with time delay systems since requires rational approximation of exponentials expressing delays, usually via the first order Padé approximation.

This contribution presents transfer function description avoiding any time delay approximation. The ring of stable and proper retarded quasipolynomial meromorphic functions (R_{MS}) for this purpose is utilized. A term of this ring is a ratio of two so-called quasipolynomials where the denominator

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quasipolynomial is stable and the whole ratio is proper with respect to highest s-powers. The only effort to design controllers in this ring for integration delayed systems is in [13] where a simple 1DOF control structure was utilized.

In this paper, an algebraic approach based on Bézout identity and the Youla-Kučera parameterization using control system with two controllers is considered. The presented feedback system can be comprehend and solved in double meaning; first, one can take the system as a whole, which means that the overall input-output transfer functions for controllers design are utilized, and on the basis of this knowledge the appropriate controller structures are determined; second, the control system can be viewed as a simple control feedback with the inner (stabilizing) feedback loop. In this case, the inner loop is solved first and the main loop follows. Final controllers of the so-called anisochronic type ensure (in both cases) feedback loop stability, step reference tracking and load disturbance attenuation, and they are tuned by the pole assignment method described e.g. in [14]. Both approaches are tested and verified using an illustrative simulation example and they are compared with the linear quadratic (LQ) polynomial approach [10], which demonstrates the usefulness and applicability of the proposed method.

II. System Description in R_{MS} Ring

A. R_{MS} Ring

Algebraic control methods are based on input-output system formulation in the form of a transfer function. Conventional transfer functions in the form of a ratio of two polynomials are not directly applicable for models containing delays due to exponentials resulting from the Laplace transform of delays. In order to express the numerator and denominator in polynomials, the first order Padé approximation is then usually utilized; however, there it is also possible to use another way. Rational approximation can be avoided so that the transfer function can be performed in the ring of stable and proper RQ-meromorphic functions, R_{MS}.

Any function in this ring is a ratio of two retarded quasipolynomials $y(s)/x(s)$, in general, where a denominator quasipolynomial is Hurwitz stable and the ratio is proper. Quasipolynomials, in contrast to polynomials, are formed not only by weighted sums of s-power but also by exponentials relating to delays. A denominator quasipolynomial $x(s)$ of degree n means

$$x(s)=s^n+\sum_{i=0}^{n-1}\sum_{j=1}^{h}x_{ij}s^i\exp\left(-\vartheta_{ij}s\right) \tag{1}$$

where "retarded" refers to the fact that the highest s-power is not affected by exponentials. Quasipolynomial (1) is stable iff it owns no finite zero s_0 such that Re $\{s_0\}\geq 0$, i.e., a term in R_{MS} ring is analytic in the right half complex plane. Stability can be verified by the Michailov stability criterion, which can be used due to the validity of argument principle, see details e.g. in [15]. The numerator $y(s)$ of an element in R_{MS} can be factorized in the form $y(s)=\tilde{y}(s)\exp(-\tau s)$, where $\tau>0$ and $\tilde{y}(s)$ is a retarded quasipolynomial of degree l

$$\tilde{y}(s)=s^l+\sum_{i=0}^{l-1}\sum_{j=1}^{h}\tilde{y}_{ij}s^i\exp\left(-\tau_{ij}s\right) \tag{2}$$

A quasipolynomial fraction is called proper iff $l\leq n$.

B. Integrating Delayed Plant in R_{MS}

R_{MS} ring can be naturally utilized for description of systems with delays in both left and right sides of an appropriate differential equation. The transfer function of the plant or the controller is then expressed as a ratio of two elements in R_{MS} ring. This contribution deals with integrating time delay systems inscribed with the transfer function

$$G(s)=\frac{K}{s}\exp(-\tau s)=\frac{\dfrac{K\exp(-\tau s)}{m_0(s)}}{\dfrac{s}{m_0(s)}}=\frac{B(s)}{A(s)} \tag{3}$$
$$A(s),B(s)\in R_{MS}$$

where $m_0(s)$ is an appropriate stable quasipolynomial of degree one. This quasipolynomial can not be of order higher than one because the transfer function factorization then would not be coprime; details about coprimeness for R_{MS} are in [13]. The suitable form of $m_0(s)$ is discussed in the Section 4 where algebraic controllers design is described.

III. Control System

Up to this day, the algebraic controller design principle in R_{MS} (described further in Section 4) was employed for the simple feedback loop with one degree of freedom (1DOF) which is pictured in Fig. 1, and for the control system with two degrees of freedom (2DOF), see Fig. 2., and for the internal model structure (IMC) only, [13], [15]-[16]. However, this contribution deals with the control system with two controllers combining 1DOF and 2DOF structures, see Fig. 3.

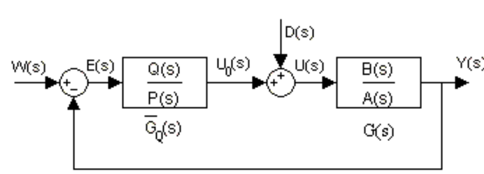

Fig. 1 1DOF control system structure.

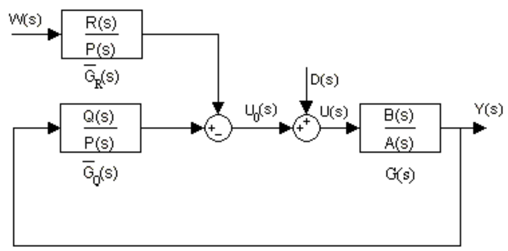

Fig. 2 2DOF control system structure.

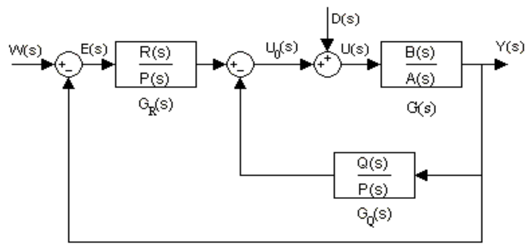

Fig. 3 Proposed control structure with two controllers.

In schemes, $W(s)$ is the reference signal, $D(s)$ is the load disturbance, $E(s)$ refers to the control error, $U_0(s)$ is the controller output, $U(s)$ is the plant input, and Y(s) means the plant output (controlled value) in the Laplace transform. The plant transfer function is depicted as $G(s)$, next, $\overline{G}_Q(s)$ and $G_Q(s)$ are "feedback" controller transfer functions for 1DOF and 2DOF structures and for the control system with two controllers, respectively, and $\overline{G}_R(s)$ or $G_R(s)$ represents "feedforward" controller in the appropriate scheme.

Using this description, the following correspondence between the structures can be written:

For 1DOF scheme holds:

$$\overline{G}_Q(s) = G_R(s),\ G_Q(s) = 0 \tag{4}$$

For 2DOF scheme holds:

$$\overline{G}_R(s) = G_R(s),\ \overline{G}_Q(s) = G_Q(s) + G_R(s) \tag{5}$$

The advantage of the structure with two controllers rests in the possibility to satisfy decoupling of reference tracking and load disturbance rejection.

The following transfer functions can be derived in the control system in general:

$$\begin{aligned}
G_{WY}(s) &= \frac{Y(s)}{W(s)} = \frac{B(s)R(s)}{M(s)},\ G_{DY}(s) = \frac{Y(s)}{D(s)} = \frac{B(s)P(s)}{M(s)} \\
G_{WE}(s) &= \frac{E(s)}{W(s)} = \frac{A(s)P(s) + B(s)Q(s)}{M(s)} \\
G_{DE}(s) &= \frac{E(s)}{D(s)} = -\frac{B(s)P(s)}{M(s)}
\end{aligned} \tag{6}$$

where controllers are

$$G_R(s) = \frac{R(s)}{P(s)},\quad G_Q(s) = \frac{Q(s)}{P(s)} \tag{7}$$

in which $R(s)$, $Q(s)$ and $P(s)$ are from R_{MS} and

$$M(s) = A(s)P(s) + B(s)[R(s) + Q(s)] \tag{8}$$

corresponds to the characteristic (quasi)polynomial of the closed loop. Both external inputs, $W(s)$ and $D(s)$, are considered to be step functions, i.e.

$$W(s) = \frac{H_W(s)}{F_W(s)} = \frac{\frac{w_0}{m_w(s)}}{\frac{s}{m_w(s)}},\quad D(s) = \frac{H_D(s)}{F_D(s)} = \frac{\frac{d_0}{m_d(s)}}{\frac{s}{m_d(s)}} \tag{9}$$

where $m_w(s)$ and $m_d(s)$ are arbitrary stable polynomials of degree one and $H_W(s)$, $H_D(s)$, $F_W(s)$, $F_D(s) \in R_{MS}$.

IV. Direct Algebraic Controller Design in R_{MS} Ring

The algebraic controller design presented in this contribution supposes that all transfer functions and signals in the control system are in the form of ratios of elements in R_{MS}; thus, a field of fractions associated with the R_{MS} ring is introduced.

The control system scheme pictured in Fig. 3 can be grasped either as the whole system corresponding to transfer functions (6) or as an inner feedback loop with controller $G_Q(s)$ and outer loop with controller $G_R(s)$. Let us now describe the former, say "direct", approach.

Usual requirements on the control systems are these: closed-loop stability, asymptotical reference tracking and load disturbance attenuation.

A. Control System Internal Stabilization

It is natural to require that all signals in the control loop avoid impulse modes, which brings the notion of internal stability, see e.g. [12], [17]. Consider plant (3) where $A(s)$ and $B(s)$ are coprime elements in R_{MS}. If there exist functions $P_0(s)$, $T_0(s) \in R_{MS}$ where

$$T_0(s) = R_0(s) + Q_0(s) \tag{10}$$

satisfying the Diophantine equation

$$A(s)P_0(s) + B(s)T_0(s) = 1 \tag{11}$$

then the set of all controllers that internally stabilize the control loop is deduced from the parameterization of the particular solution

$$\begin{aligned} &P(s) = P_0(s) - B(s)Z(s) \\ &T(s) = T_0(s) + A(s)Z(s) \\ &Z(s) \in R_{MS},\ T_0(s) + A(s)Z(s) \neq 0 \end{aligned} \tag{12}$$

The proof of the previous statement can be done analogously as, e.g. in [13], [17], applied to control system shown in Fig. 3. A free parameter $Z(s)$ can be chosen properly to fulfill other control design requirements. Resultant controllers are given by (7) with respect to the distribution of the solution, (10).

Concretely, the particular solution of (11) for plant (3) arises from the solution of the following equation

$$\frac{s}{m_0(s)} P_0(s) + \frac{K\exp(-\tau s)}{m_0(s)} T_0(s) = 1 \tag{13}$$

Without loss of generality, let $T_0(s) = \alpha \in \Re$ and $P_0(s) = 1$, and the remaining task is to find a suitable stable quasipolynomial $m_0(s)$. Hence, (13) results in

$$\alpha = \frac{m_0(s) - s}{K\exp(-\tau s)} \tag{14}$$

The requirement is α to be real; therefore the simplest $m_0(s)$ has to be of the form

$$m_0(s) = \alpha K \exp(-\tau s) + s \tag{15}$$

The essential feature of retarded quasipolynomial $m_0(s)$ is its stability which can be studied e.g. using the Michajlov criterion, [18]-[19]. Via a computational procedure analogous to the one described for unstable systems in [16], one can derive the stability condition as

$$\alpha = \frac{1}{A_m} \frac{\pi}{2K\tau} \tag{16}$$

where $A_m > 1$ is the gain margin ($A_m = 1$ correspondences to the stability border).

B. Reference Tracking and Disturbance Rejection

As was mentioned above, the convenient option of $Z(s)$ in the parameterization (12) enables to find the solution of (11), so that requirements of reference tracking and disturbance rejection are accomplished. If both inputs are considered as step functions (9), it arises from transfer functions (6) that numerators of $P(s)$ and $Q(s)$ must have "derivative" pattern. In other words, unstable (zero) poles of $F_W(s)$ and $F_D(s)$ must be canceled by zero poles of $P(s)$ and $Q(s)$, i.e. their numerators must be either of the form

$$p(s) = s\tilde{p}(s), \quad q(s) = s\tilde{q}(s) \tag{17}$$

or, eventually for quasipolynomials

$$\begin{aligned} p(s) &= s\tilde{p}(s) + \tilde{p}_0[1 - \exp(-\tau s)] \\ q(s) &= s\tilde{q}(s) + \tilde{q}_0[1 - \exp(-\tau s)] \end{aligned} \tag{18}$$

Both numerators (17) and (18) of $P(s)$ and $Q(s)$ ensures at least one zero root. For the plant (3) and the particular solution (13)-(15), the choice $Z(s) = \alpha$ in parameterization (12) yields

$$\begin{aligned} P(s) &= \frac{s}{s + \alpha K \exp(-\tau s)} \\ T(s) &= \frac{\alpha[2s + \alpha K \exp(-\tau s)]}{s + \alpha K \exp(-\tau s)} \end{aligned} \tag{19}$$

Obviously, $P(s)$ and $T(s)$ are from R_{MS} and the form of $P(s)$ ensures reference tracking and disturbance rejection.

C. Parameterization of T(s)

The solution of the Bézout identity (11) with parameterization (12) gives $P(s)$ and $T(s)$; however, the controller transfer functions involve $Q(s)$ and $R(s)$ and these elements are obtained by parameterization of $T(s)$ according to (10). Hence, the last step in controller design is the "distribution" of $T(s)$ onto $P(s)$ and $R(s)$ with respect to demand on the form of $Q(s)$, see (17) and (18).

Thus, function $T(s)$ in (19), with respect to (10), (17) and (18) and taking a distribution parameter $0 \le \gamma \le 1$, can be formulated as

$$\begin{aligned} T(s) &= \frac{\alpha[2s + \alpha K \exp(-\tau s)]}{s + \alpha K \exp(-\tau s)} \\ &= \frac{\gamma 2\alpha s + \alpha^2 K \exp(-\tau s)}{s + \alpha K \exp(-\tau s)} + \frac{(1-\gamma)2\alpha s}{s + \alpha K \exp(-\tau s)} \\ &= R(s) + Q(s) \end{aligned} \tag{20}$$

which results in controllers

$$\begin{aligned} G_R(s) &= \frac{R(s)}{P(s)} = \frac{\gamma 2\alpha s + K\alpha^2 \exp(-\tau s)}{s} \\ G_Q(s) &= \frac{Q(s)}{P(s)} = (1-\gamma)2\alpha \end{aligned} \tag{21}$$

Hence, the proportional and generalized (delayed) proportional-integrative controllers are obtained.

D. Alternative Choice of Z(s)

The choice of a selectable element $Z(s) = \alpha$ as was proposed above, is not the only possibility how to find controllers satisfying simultaneous internal stability, reference tracking and disturbance rejection. To demonstrate this feature, let

$$Z(s) = \frac{s + \alpha K \exp(-\tau s)}{s + \lambda} \frac{\lambda}{K} \tag{22}$$

instead of $Z(s) = \alpha$. A selectable positive real parameter λ ensures that $Z(s) \in \mathrm{R}_{\mathrm{MS}}$ and brings an additional degree of freedom. Then

$$P(s) = \frac{s + \lambda[1 - \exp(-\tau s)]}{s + \lambda}$$
$$T(s) = \frac{(\alpha K + \lambda)s + \alpha K}{K(s + \lambda)} \tag{23}$$

The distribution of $T(s)$ onto $Q(s)$ and $R(s)$ is in the form

$$R(s) = \frac{\gamma_1\left(\alpha + \dfrac{\lambda}{K}\right)s + \alpha\lambda}{s + \lambda}$$
$$Q(s) = \frac{(1 - \gamma_1)\left(\alpha + \dfrac{\lambda}{K}\right)s}{s + \lambda} \tag{24}$$

Hence, the resulting set of controllers is the following

$$G_R(s) = \frac{R(s)}{P(s)} = \frac{\gamma\left(\alpha + \dfrac{\lambda}{K}\right)s + \alpha\lambda}{s + \lambda[1 - \exp(-\tau s)]}$$
$$G_Q(s) = \frac{Q(s)}{P(s)} = \frac{(1 - \gamma)\left(\alpha + \dfrac{\lambda}{K}\right)s}{s + \lambda[1 - \exp(-\tau s)]} \tag{25}$$

As can be seen, denominators of final controllers (25) are quasipolynomials, and this feature refers to so-called anisochronic form of the controllers. However, these types of controllers are as easy to implement either on PC or PLC, see [20], as the traditional PID controllers; which can be easily deducted from the Matlab-Simulink simulation block scheme of e.g. $G_R(s)$ displayed in Fig. 4.

V. Controller design With The Pre-stabilizing Inner Loop

The controller design procedure presented in Section 4 was based on the control system description in the form of (the whole) closed loop transfer functions (6). However, one can conceive the scheme in Fig. 3 as a control system with an inner pre-stabilization loop containing controller $G_Q(s)$ and outer loop with controller $G_R(s)$ which provides disturbance rejection and setpoint tracking.

To avoid the presence of input disturbance in the inner feedback for controllers design, let the control system scheme be rearranged as in Fig. 5. Obviously, all transfer functions (6) still hold; nevertheless, controllers design for the inner loop excludes the assumption of the input disturbance. The idea is that inner feedback pre-stabilizes the controlled process, i.e. zero pole is to be moved to the left, and the outer feedback controller ensures already mentioned requirements for pre-stabilized system $G_0(s)$.

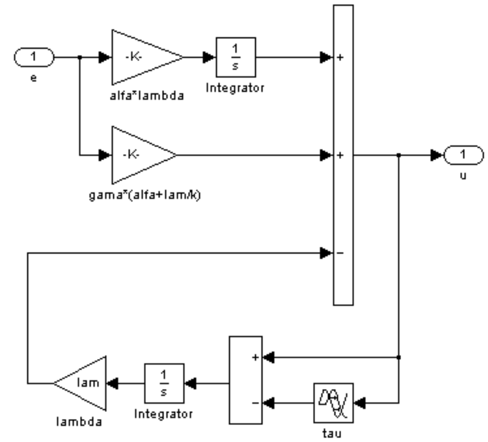

Fig. 4 The structure of anisochronic controller $G_R(s)$ in (25).

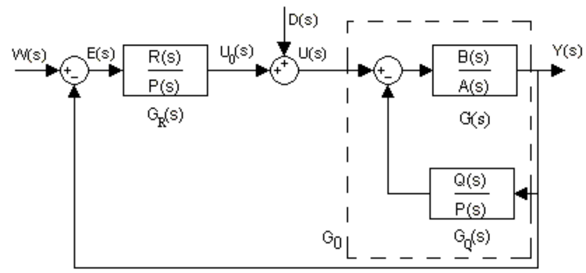

Fig. 5 Reconfigured control scheme.

A. Inner Loop Pre-stabilization

Let an integrating delayed plant (3) be pre-stabilized using a proportional controller $G_Q(s) = q_0$. The condition for closed-loop stability, see e.g. [17], is then given by the Diophantine equation

$$\frac{s}{m_0(s)} q_0 + \frac{K\exp(-\tau s)}{m_0(s)} = 1 \tag{26}$$

The natural task is to find a suitable stable retarded quasipolynomial $m_0(s)$ which provides a coprime factorization of the plant transfer function. The solution of (26) gives

$$q_0 = \frac{m_0(s) - s}{K\exp(-\tau s)} \tag{27}$$

The requirement is q_0 to be real; therefore, similarly as in (14) and (15), the simplest $m_0(s)$ has to be of the form

$$m_0(s) = q_0 K\exp(-\tau s) + s \tag{28}$$

where the stability of $m_0(s)$ is conditioned again by (16) in which q_0 is placed instead of α. Thus, the transfer function of the inner pre-stabilized feedback loop is

$$G_0(s)=\frac{K\exp(-\tau s)}{s+q_0K\exp(-\tau s)} \tag{29}$$

B. System Stabilization

Now the task is to control the pre-stabilized loop $G_0(s)$ using a simple feedback with controller $G_R(s)$. However, the numerator and denominator in (29) are not from R_{MS} and thus the transfer function must be factorized as

$$G_0(s)=\frac{\frac{K\exp(-\tau s)}{m_1(s)}}{\frac{s+q_0K\exp(-\tau s)}{m_1(s)}} \tag{30}$$

where $m_1(s)$ is a stable (quasi)polynomial. In order to have $G_0(s)$ as simple as possible, let

$$m_1(s)=s+\lambda \tag{31}$$

where $\lambda>0$ is a selectable real parameter which brings an additional degree of freedom. Naturally one can take another $m_1(s)$; an example demonstrating it is presented in Section 5D.

The outer closed-loop stability property is ensured by a solution of the Diophantine equation

$$\frac{s+q_0K\exp(-\tau s)}{s+\lambda}P(s)+\frac{K\exp(-\tau s)}{s+\lambda}R(s)=1 \tag{32}$$

whose a particular solution reads

$$R_0(s)=1,\ \ P_0(s)=\frac{s+\lambda-K\exp(-\tau s)}{s+q_0K\exp(-\tau s)} \tag{33}$$

This solution can be further parameterized according to (12) using the Youla-Kučera parameterization as

$$\begin{aligned}R(s)&=R_0(s)+A(s)Z(s)=1+\frac{s+q_0K\exp(-\tau s)}{s+\lambda}Z(s)\\P(s)&=P_0(s)-B(s)Z(s)\\&=\frac{s+\lambda-K\exp(-\tau s)}{s+q_0K\exp(-\tau s)}-\frac{K\exp(-\tau s)}{s+\lambda}Z(s);\quad Z(s)\in \mathrm{R}_{MS}\end{aligned} \tag{34}$$

to fulfill other control requirements, i.e. reference tracking and disturbance rejection, via an appropriate choice of the free element, $Z(s)$.

C. Reference Tracking and Disturbance Rejection

Parameterization (34) enables to find the solution of (32), so that requirements of reference tracking and disturbance rejection will be accomplished. The requirement is that both denominators of Laplace forms of external inputs, $F_W(s)$ and $F_D(s)$, divide $P(s)$. If both inputs, $w(t)$ and $d(t)$, are considered as step functions (9), $P(s)$ must contain at least one zero pole. Let

$$Z(s)=\frac{\left(\frac{\lambda}{K}-1\right)(s+\lambda)}{s+q_0K\exp(-\tau s)} \tag{35}$$

then the outer feedback controller reads

$$\begin{aligned}G_R(s)&=\frac{R(s)}{P(s)}=\frac{\frac{\frac{\lambda}{K}[s+q_0K\exp(-\tau s)]}{s+q_0K\exp(-\tau s)}}{\frac{s+\lambda[1-\exp(-\tau s)]}{s+q_0K\exp(-\tau s)}}\\&=\frac{\lambda[s+q_0K\exp(-\tau s)]}{K[s+\lambda[1-\exp(-\tau s)]]}\end{aligned} \tag{36}$$

Obviously, this controller is of anisochronic structure as in (25) again and its structure is similar to the one pictured in Fig. 4.

Recall that the inner-feedback controller is proportional, $G_Q(s)=q_0$; however, in terms of algebraic philosophy it can be written also in factorization as

$$G_Q(s)=\frac{Q(s)}{P(s)}=\frac{\frac{q_0[s+\lambda[1-\exp(-\tau s)]]}{s+q_0K\exp(-\tau s)}}{\frac{s+\lambda[1-\exp(-\tau s)]}{s+q_0K\exp(-\tau s)}} \tag{37}$$

D. An Alternative Solution

As was mentioned in Section 5B, stable (quasi)polynomial $m_1(s)$ can be chosen unlike in (31). Another natural choice is

$$m_1(s)=m_0(s)=s+q_0K\exp(-\tau s) \tag{38}$$

which agrees with the denominator of the non-factorized inner-feedback transfer function. Thus the factorized one reads

$$G_0(s)=\frac{\frac{K\exp(-\tau s)}{s+q_0K\exp(-\tau s)}}{\frac{s+q_0K\exp(-\tau s)}{s+q_0K\exp(-\tau s)}}=\frac{\frac{K\exp(-\tau s)}{s+q_0K\exp(-\tau s)}}{1} \tag{39}$$

In this case, the stabilizing Diophantine equation

$$P(s)+\frac{K\exp(-\tau s)}{s+q_0K\exp(-\tau s)}R(s)=1 \tag{40}$$

has one of particular solutions

$$R_0(s)=1,\ P_0(s)=\frac{s+q_0K\exp(-\tau s)-K\exp(-\tau s)}{s+q_0K\exp(-\tau s)} \tag{41}$$

and by decision $Z(s)=q_0-1$ in the Youla-Kučera parameterization, the final outer-feedback controller structure ensuring reference tracking and disturbance rejection is then

$$G_R(s)=\frac{q_0\left[s+q_0K\exp(-\tau s)\right]}{s} \tag{42}$$

which is a delayed PI controller of the same structure as in (21); however, one can notice that a proportional and an integral coefficients cannot be simultaneously the same as those in (21). The inner-feedback controller is $G_Q(s) = q_0$ again.

VI. Tuning Of Controllers

The final sets of controllers, (21), (25), (36) and (42), still contain unknown parameters that have to be set properly. There are naturally plenty of approaches solving the problem of controller tuning.

In this contribution, the well applicable and relatively simple tuning method called direct pole placement, which was described e.g. in [14], is utilized. This method enables to prescribe the desired set of dominant poles of the closed loop, the maximum number of which is given by the number, k, of unknown parameters in the characteristic quasipolynomial. If the dominant poles are denoted as $\sigma_i,\ i=1...k$, the characteristic equation as $m(s)$, and a vector of r unknown parameters as $\mathbf{v}$, then the following system of k linear equations is obtained

$$m(\sigma_i,\mathbf{v})=0,\ i=1...k \tag{43}$$

For complex poles, one root from each complex conjugate pair is taken and (43) is divided into two equations of the form

$$\begin{aligned}\operatorname{Re}\{m(\sigma_i,\mathbf{v})\}=0\\ \operatorname{Im}\{m(\sigma_i,\mathbf{v})\}=0\end{aligned} \tag{44}$$

The significant feature is that sets (43) and (44) are linear with respect to unknown parameters, which makes the solution easy to find. If $r > k$, the equations are solved using Moore-Penrose pseudo-inversion of non-squared matrix [21]; on the other hand, if $r = k$, the set is full rank and can be solved using as a common set of algebraic linear equations, i.e. pseudo-inversion become an inversion.

In the particular case of delayed integrator, all sets of final controllers result in two disparate characteristic quasipolynomials

$$\begin{aligned}m(s)&=\left[s+q_0K\exp(-\tau s)\right]^2\\ m(s)&=\left[s+q_0K\exp(-\tau s)\right](s+\lambda)\end{aligned} \tag{45}$$

with unknown parameters q_0 (or α which has the same meaning) and λ.

Thus, let the first quasipolynomial in (45) be taken. Since there is a single parameter to be found, q_0, the only multiple real dominant root or a conjugate pair of complex roots can be prescribed. Moreover, stability condition (16) cannot be omitted. In many real applications, oscillatory modes in the output signal are undesirable; therefore the optimal choice of prescribed poles in the form of the leftmost dominant real roots is suggested. We will propose here two ways how to derive the condition for the prescription of these optimal poles.

The first deduction arises from the observation of the value of q_0 computed from (43) and (45) for a prescribed stable real pole. Starting the pole position in the left neighborhood of the stability border (i.e. $\sigma_1=0$) and continuing toward to negative infinity, the value of q_0 initially rises up until its maximum value is reached and consequently slopes down behind this point. A concrete example of this behavior can be seen in Fig. 6.

Thus, there exist two distinct values of the chosen pole for the same value of the controller parameter, except the maximum point. This means that whenever a pole to the left of the maximum point is chosen, there must exist another pole to the right which is dominant. Therefore, the task of choosing the leftmost dominant pole is converted to the searching the maximum of q_0, which indicates the position of the optimal real pole.

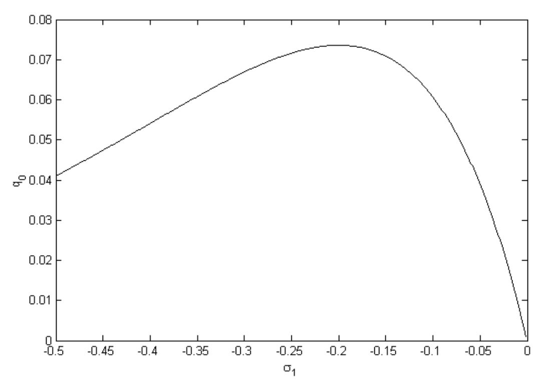

Fig. 6 Dependence of q_0 on a prescribed dominant pole σ_1; $K = 1$, $\tau = 5$.

The pole placement condition combining (43) and (45) reads

$$m(\sigma_1,q_0)=\left[\sigma_1+q_0K\exp(-\tau\sigma_1)\right]^2=0 \tag{46}$$

The objective is to maximize the function $q_0(\sigma_1)$, $\sigma_1 < 0 \in \Re$, which is given by (46) as

$$q_0 = \frac{-\sigma_1}{K\exp(-\tau\sigma_1)} \tag{47}$$

The well-known procedure of analytic searching the maximum of continuous functions yields the optimal dominant pole choice

$$\sigma_{1,\mathrm{OPT}} = -\tau^{-1} \tag{48}$$

which gives the optimal controller parameter

$$q_{0,\mathrm{OPT}} = \frac{1}{K\tau \mathrm{e}} \tag{49}$$

The interesting feature of this result rests in the fact that the derived closed-loop pole is triplicate, which is evident from the following statement

$$\left.\frac{\mathrm{d}^2 m(\sigma_1, q_0)}{\mathrm{d}q_0^2}\right|_{\substack{\sigma_1=\sigma_{1,\mathrm{OPT}}\\ q_0=q_{0,\mathrm{OPT}}}} = 0 \tag{50}$$

The second possibility how to derive the optimal controller parameter choice to obtain the leftmost real dominant pole is based on the solution of combination of (44) and (45). If $\sigma_i = \alpha_i + \mathrm{j}\omega_i$, the following set of equations holds

$$\begin{aligned} &\alpha_i + q_0 K\exp(-\alpha_i\tau)\cos(\tau\omega_i) = 0 \\ &\omega_i - q_0 K\exp(-\alpha_i\tau)\sin(\tau\omega_i) = 0 \end{aligned} \tag{51}$$

Consider one conjugate pair of prescribed poles, i.e. i = 1, then the solution of (51) by cancellation of the factor $q_0 K$ reads

$$\alpha_1 = -\omega_1 \arctan(\omega_1\tau) \tag{52}$$

which describes the dependence of a real and an imaginary parts of the root of (45). Calculation of limit

$$\lim_{\omega_1\to 0}\left[-\omega_1\arctan(\omega_1\tau)\right] = -\tau^{-1} \tag{53}$$

leads to the same result as in (48).

The polynomial factor in the second characteristic quasipolynomial in (45) is linear, thus, an additional stable root, $-\lambda$, can be prescribed. Obviously, if $\lambda < \tau^{-1}$, then this pole becomes dominant and the influence of the quasipolynomial factor $s + q_0 K\exp(-\tau s)$ on the system dynamics is suppressed. In [10], the suggestion for the choice of λ is

$$\lambda = \frac{2}{\tau} \tag{54}$$

which, in comparison with (48), does not allow for the dominant pole.

As can be seen from above sentences, roots of the characteristic quasipolynomial (real or complex) must be chosen carefully because some attempts to place them excessively to the left in the complex plane can lead into the following situation. Due to the fact that the quasipolynomials (45) has the infinity number of roots, the chain of complex roots can move to the right near to the stability border (imaginary axis) and thus these roots can take over the role of dominant poles of the system.

Coefficient γ in (21) and (25) influences the feedback system behavior as well because it appears in the numerators of closed loop transfer functions; however, it does not impact the spectrum of the system. In the illustrative example below, various values of γ are set randomly.

There are naturally other possibilities how to set the unknown parameters of controllers, e.g. in more computational way via artificial intelligence approaches based on genetic algorithms, [21].

VII. LQ Polynomial Method

The methodology presented above in this contribution ought to be compared with another approach to demonstrate its usability. Take a method presented in [10] which is based on a rational approximation of exponentials in a plant transfer function followed by optimal pole-placement via minimization of a quadratic cost function.

Look at a brief yet more detailed description of the LQ method. The control system is considered as in Fig. 3 again. The plant transfer function (3) is approximated using the first order Padé approximation

$$\exp(-\tau s) \approx \frac{1 - 0.5\tau s}{1 + 0.5\tau s} \tag{55}$$

i.e. a rational fraction description is obtained. The ring of polynomials instead of R_{MS} is thus used. In this ring, stabilizing equation

$$A(s)P(s) + B(s)T(s) = M(s) \tag{56}$$

is solved where $M(s)$ is the characteristic polynomial and distribution (10) holds. Parameterization (12) is not used; on the other hand, $P(s)$ is set directly in the form ensuring the asymptotic tracking and load disturbance attenuation, i.e. as in (17), whereas the solution $T(s)$ is distributed according to

$$\begin{aligned} &T(s) = R(s) + s\tilde{Q}(s) \\ &T(s) = \sum_{i=0}^{n} t_i s^i,\ R(s) = \sum_{i=0}^{n} r_i s^i,\ \tilde{Q}(s) = \sum_{i=1}^{n} q_i s^{i-1} \\ &r_0 = t_0;\ r_i + q_i = \gamma_i t_i + t_i(1-\gamma_i)t_i \\ &\text{for } \gamma_i \in \langle 0,1\rangle,\ i = 1,\ldots n \end{aligned} \tag{57}$$

where coefficients γ_i divide a weight between $R(s)$ and $Q(s)$.

The polynomial $M(s)$ is considered as a product of two stable polynomials in the form

$$M(s) = M_1(s)M_2(s) \tag{58}$$

where $M_1(s)$ is a monic form of the polynomial $\tilde{M}_1(s)$ obtained by the spectral factorization

$$[sA(s)]^* \varphi [sA(s)] + B(s)^* B(s) = \tilde{M}_1^*(s)\tilde{M}_1(s) \tag{59}$$

where φ is the weighting coefficient and the asterisk denotes the spectral factor of an appropriate polynomial. Condition (59) holds for the minimization of the quadratic cost function

$$J = \int_0^\infty \left\{ e^2(t) + \varphi \dot{u}^2(t) \right\} dt \tag{60}$$

Finally, the second polynomial in (58) is suggested as

$$M_2(s) = s + \frac{2}{\tau} \tag{61}$$

which agrees with the choice (54).

As can be seen, there is one selectable real parameter, $\varphi > 0$, that influences the closed loop poles location. The final controllers for a plant (3) have the following forms of transfer functions

$$G_R(s) = \frac{r_2 s^2 + r_1 s + r_0}{s^2 + p_0 s}, \qquad G_Q(s) = \frac{q_2 s + q_1}{s + p_0} \tag{62}$$

VIII. Illustrative Example

This simulation example composed in the Matlab-Simulink environment demonstrates the usability of the proposed controller design method in the R_{MS} ring and it presents a benchmark test by comparison the simulation results with the polynomial LQ method. Two criteria as instrument for a quantitative comparison are implemented.

The first one follows criterion (60) and thus it is expected that the optimal LQ method should perform the best results. Let us call the criterion simply as ISE (Integrated Squared Error) criterion. A selection of a positive real parameter φ enables to determine the impact of control signal derivation, i.e. the higher value of φ results in a smoother course of $\dot{u}(t)$ function, and let $\varphi = 500$ is chosen.

The second ISTE (Integrated Squared Time Error) criterion is formulated as

$$J_{ISTE} = \int_0^\infty t \left\{ e^2(t) + \varphi \dot{u}^2(t) \right\} dt \tag{63}$$

The significant feature of this criterion is that it handicaps latter signal values, i.e. its higher value indicates slower control response settlement.

Let $K = 1$ and $\tau = 5$. The reference signal is $w(t) = 1$ for $0 \le t < 100$ and $w(t) = 2$ for $100 \le t < 300$. The step input disturbance $d(t) = -0.1$ enters at time $t = 200$; hence, the process of restoration of zero control error due to the input disturbance influences ISTE criterion significantly. Simulation results for every single final controller are discussed independently.

A. Results for Direct Controller Design

First assume controllers (21) which comprise two adjustable parameters: α, γ. The optimal value of α is given by (49) as $\alpha = 0.0736$ which ensures the triple leftmost real dominant system pole ($\sigma_{1,2,3} = -0.2$). For the sake of comparison, choose two complex conjugate dominant poles, according to (52), as $\sigma_{4,5} = -0.18 \pm 0.1\mathrm{j}$ and $\sigma_{6,7} = -0.13 \pm 0.2\mathrm{j}$ which gives $\alpha = 0.0835$ and $\alpha = 0.125$, respectively. Two different distribution parameters γ are evenly chosen as $\gamma = 0.25$ and $\gamma = 0.75$. The simulation results are in Fig. 7 - Fig. 10. A quantitative results collation in the form of values of ISE and ISTE criterions are in Table I.

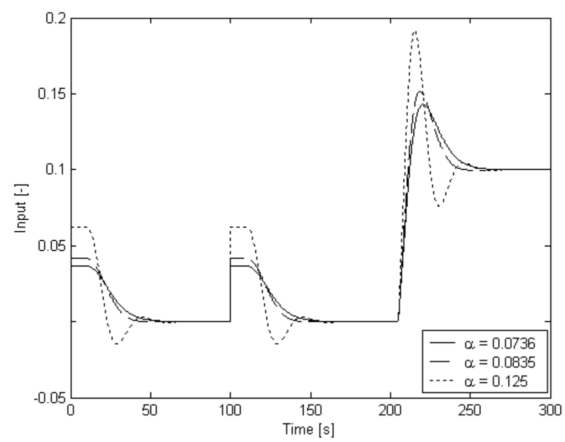

Fig. 7 Step setpoint and load disturbance responses of $u(t)$ using controllers (21); $K = 1$, $\tau = 5$, $\gamma = 0.25$, $d = -0.1$.

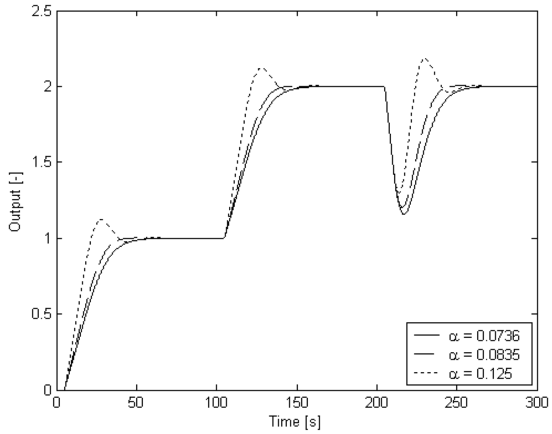

Fig. 8 Step setpoint and load disturbance responses of $y(t)$ using controllers (21); $K = 1$, $\tau = 5$, $\gamma = 0.25$, $d = -0.1$.

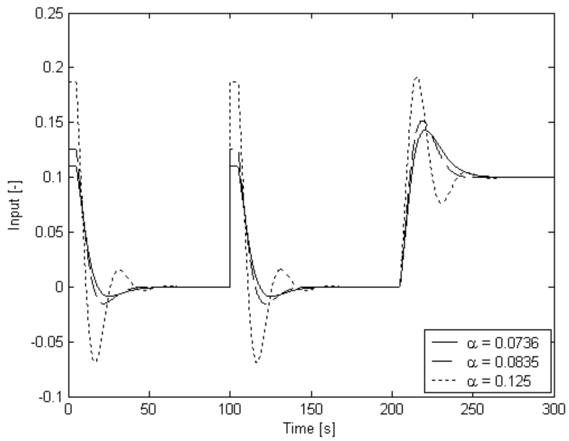

Fig. 9 Step setpoint and load disturbance responses of $u(t)$ using controllers (21); $K = 1$, $\tau = 5$, $\gamma = 0.75$, $d = -0.1$.

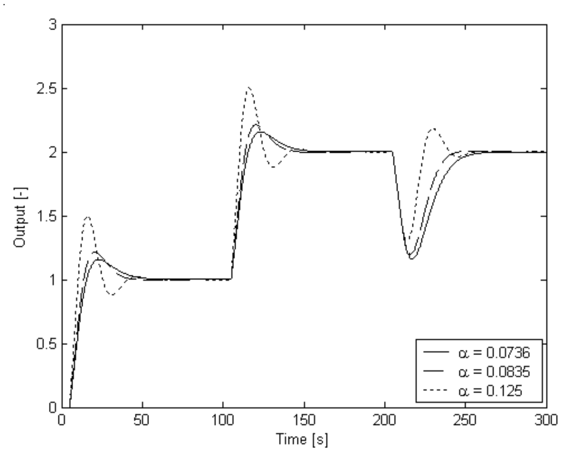

Fig. 10 Step setpoint and load disturbance responses of $y(t)$ using controllers (21); $K = 1$, $\tau = 5$, $\gamma = 0.75$, $d = -0.1$.

Table I: Values of ISE and ISTE criterions when using controllers (21); $K = 1$, $\tau = 5$

γ	α	ISE	ISTE
0.25	0.0736	26.174	1528.9
0.25	0.0835	22.645	1209.4
0.25	0.125	17.873	887.1
0.75	0.0736	20.671	1448.2
0.75	0.0835	18.42	1153.8
0.75	0.125	19.679	913.7

The results in Table I indicate that dominant poles with appropriately small (but not zero) ratio of their imaginary and real parts can improve both criterions, especially for lower γ values. However, as can be seen from Fig. 7 - Fig. 10, complex conjugate poles naturally cause the propensity to oscillations and overshoots, which are undesirable in many applications. Fig. 9 and Fig. 10 in contrast to Fig. 7 and Fig. 8 show also that increasing of the distribution parameter γ (i.e. parameters in the numerator of $G_R(s)$ rise) leads to faster changes of control signal $u(t)$ and more apparent overshoots; on the other hand, control responses are faster.

Consider now alternative controllers (25) comprising an additional parameter λ which is set by (54) as $\lambda = 0.4$. Control system responses are in Fig. 11 – Fig. 14 and the numerical comparison in Table II.

In contrast to controllers (21), Table II together with the figures above, evidently show that the alternative anisochronic controllers (25) are much convenient to control the plant (3). Both ISE and ISTE criterions are notably less, responses are faster and also undershoots caused by the input disturbance are reduced. Higher values of γ result in faster changes of control signal $u(t)$ and more apparent overshoots again, which spoils the quality criterions; on the contrary, it reduces undershoots a little.

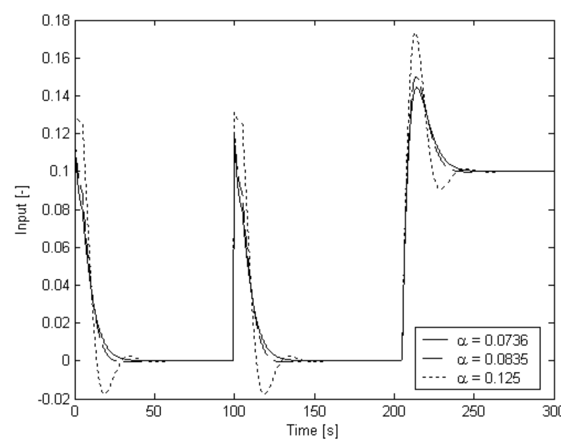

Fig. 11 Step setpoint and load disturbance responses of $u(t)$ using controllers (25); $K = 1$, $\tau = 5$, $\lambda = 0.4$, $\gamma = 0.25$, $d = -0.1$.

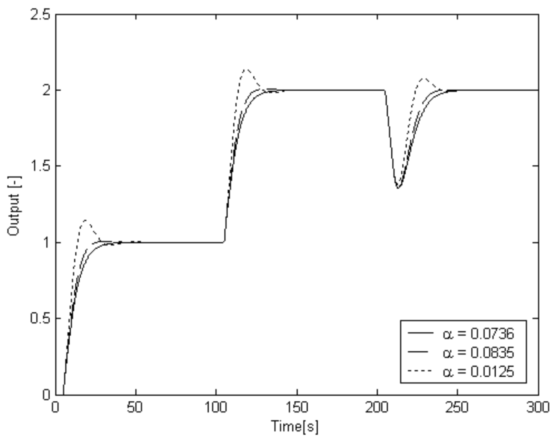

Fig. 12 Step setpoint and load disturbance responses of $y(t)$ using controllers (25); $K = 1$, $\tau = 5$, $\lambda = 0.4$, $\gamma = 0.25$, $d = -0.1$.

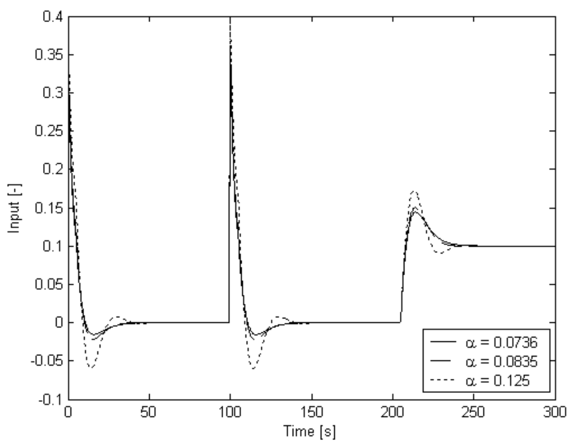

Fig. 13 Step setpoint and load disturbance responses of $u(t)$ using controllers (25); $K = 1$, $\tau = 5$, $\lambda = 0.4$, $\gamma = 0.75$, $d = -0.1$.

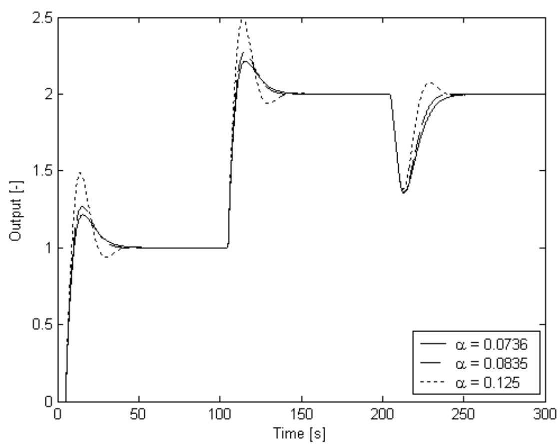

Fig. 14 Step setpoint and load disturbance responses of $y(t)$ using controllers (25); $K = 1$, $\tau = 5$, $\lambda = 0.4$, $\gamma = 0.75$, $d = -0.1$.

Table II: Values of ISE and ISTE criterions when using controllers (25); $K = 1$, $\tau = 5$, $\lambda = 0.4$

γ	α	ISE	ISTE
0.25	0.0736	15.62	766.254
0.25	0.0835	15.003	727.551
0.25	0.125	14.727	705.407
0.75	0.0736	22.21	765.6
0.75	0.0835	22.307	736.061
0.75	0.125	24.933	746.489

B. Results for Controller Design with the Inner Loop

Controller design utilizing the pre-stabilizing inner feedback loop and the outer loop gives e.g. controllers (36) and (37), and (42). Consider the former set of controllers first, which contains two selectable parameters: q_0 and λ. The double dominant pole is given by q_0 as for α in the previous section, see Fig. 15 - Fig. 18. The influence of a change of λ is demonstrated in Fig. 19 and Fig. 20. The ISE and ISTE criterions valuation presents Table III.

Fig. 15 – Fig. 18 clearly show that any change in q_0 does not affect the setpoint response. Appropriately chosen parameter q_0 corresponding to conjugate dominant poles improve particularly ISTE criterion, which reveals from Table III; however, there is a tendency to overshoots after the disturbance enters. Fig. 19 and Fig. 20 disclose that higher values of λ cause faster changes of the control signal yielding a deterioration of the ISE criterion.

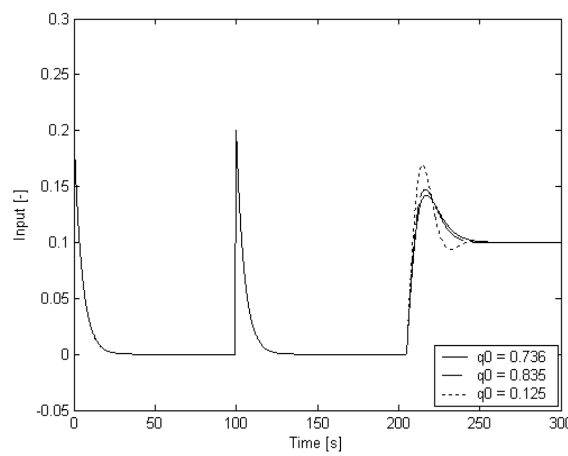

Fig. 15 Step setpoint and load disturbance responses of $u(t)$ using controllers (36) and (37); $K = 1$, $\tau = 5$, $\lambda = 0.2$, $d = -0.1$.

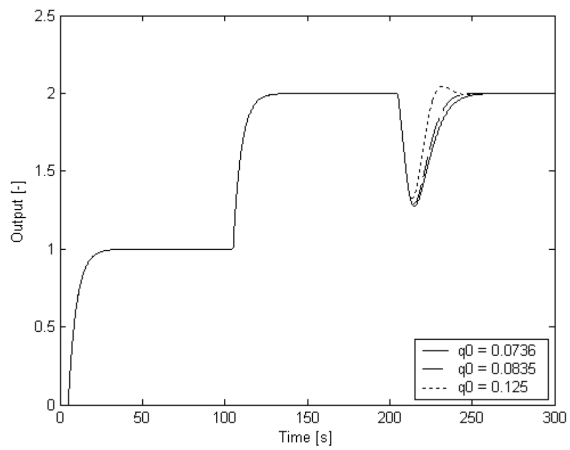

Fig. 16 Step setpoint and load disturbance responses of $y(t)$ using controllers (36) and (37); $K = 1$, $\tau = 5$, $\lambda = 0.2$, $d = -0.1$.

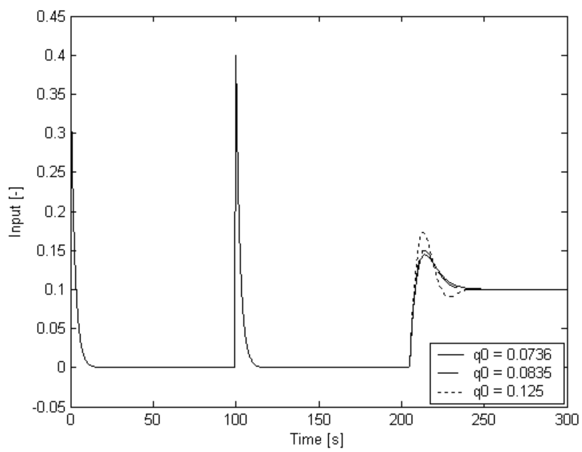

Fig. 17 Step setpoint and load disturbance responses of $u(t)$ using controllers (36) and (37); $K = 1$, $\tau = 5$, $\lambda = 0.4$, $d = -0.1$.

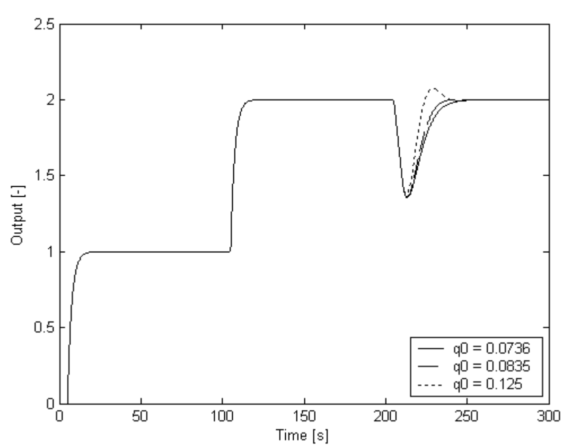

Fig. 18 Step setpoint and load disturbance responses of $y(t)$ using controllers (36) and (37); $K = 1$, $\tau = 5$, $\lambda = 0.4$, $d = -0.1$.

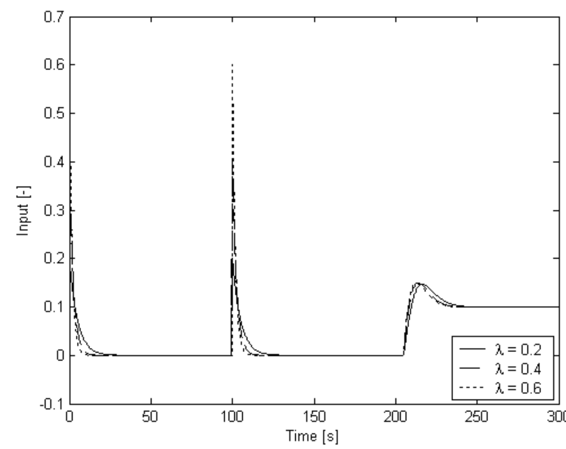

Fig. 19 Step setpoint and load disturbance responses of $u(t)$ using controllers (36) and (37); $K = 1$, $\tau = 5$, $q_0 = 0.835$, $d = -0.1$.

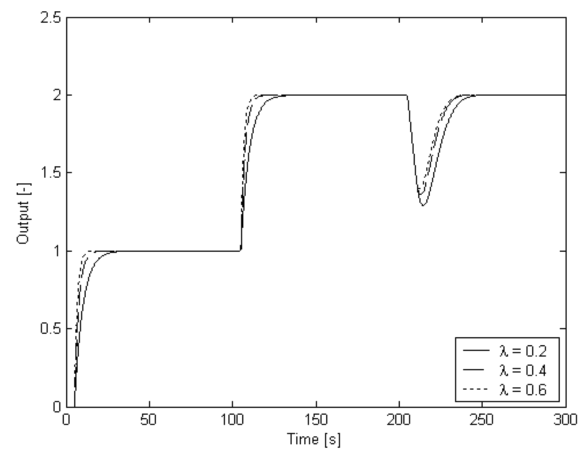

Fig. 20 Step setpoint and load disturbance responses of $y(t)$ using controllers (36) and (37); $K = 1$, $\tau = 5$, $q_0 = 0.835$, $d = -0.1$.

Table III: Values of ISE and ISTE criterions when using controllers (36) and (37); $K = 1$, $\tau = 5$.

λ	q_0	ISE	ISTE
0.2	0.0736	17.566	971.198
0.2	0.0835	16.931	890.314
0.2	0.125	15.823	748.452
0.4	0.0736	28.617	959.431
0.4	0.0835	28.353	725.04
0.4	0.125	28.229	702.907
0.6	0.0835	65.814	724.13

Look at the alternative solution (42) for which the parameter q_0 is set as for controllers (36) and (37). The corresponding results are presented in Fig. 21, Fig. 22 and Table IV.

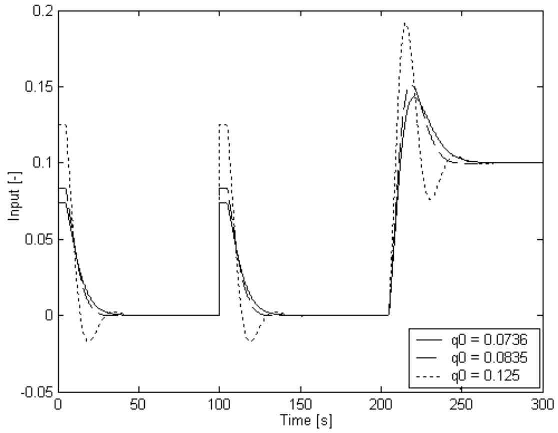

Fig. 21 Step setpoint and load disturbance responses of $u(t)$ using controller (42); $K = 1$, $\tau = 5$, $d = -0.1$.

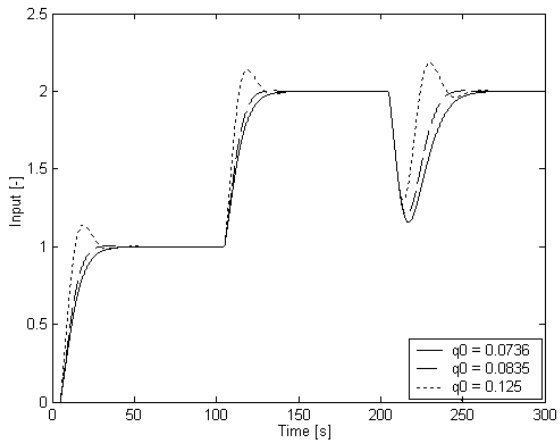

Fig. 22 Step setpoint and load disturbance responses of $y(t)$ using controller (42); $K = 1$, $\tau = 5$, $d = -0.1$.

Table IV: Values of ISE and ISTE criterions when using controller (42); $K = 1$, $\tau = 5$.

q_0	ISE	ISTE
0.0736	521.828	51453
0.0835	518.887	51149
0.125	515.967	50857

Obviously, these results give the worst ISE and ISTE criterions, which stems from very slow control responses. On the other hand, only one controller parameter, q_0, is to be set and the changes of control signal $u(t)$ are slow, which contributes to a long working life of actuators.

C. Results for LQ Controllers

The methodology proposed in this paper is further compared with the polynomial LQ approach which serves as a benchmark for the method utilizing the ring R_{MS}.

There are three selectable parameters, γ, λ, φ. To obtain consistent results with those presented in Section 8A and Section 8B, let $\lambda = 0.4$ in all cases, $\gamma_1 = \gamma_2 = 0.25$ and $\gamma_1 = \gamma_2 = 0.75$ for the comparison. The weighting factor, φ, has three various values, $\varphi = 200, 500, 900$, to study its influence again. Graphic results are displayed in Fig. 23 – Fig. 26, and ISE and ISTE criterions are evaluated in Table V.

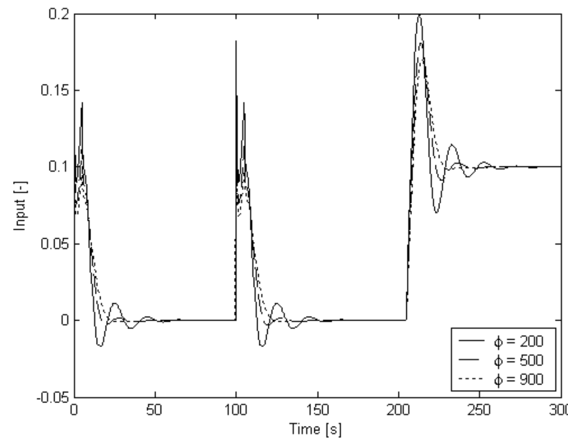

Fig. 23 Step setpoint and load disturbance responses of $u(t)$ using controllers (62); $K = 1$, $\tau = 5$, $d = -0.1$, $\gamma_1 = \gamma_2 = 0.25$

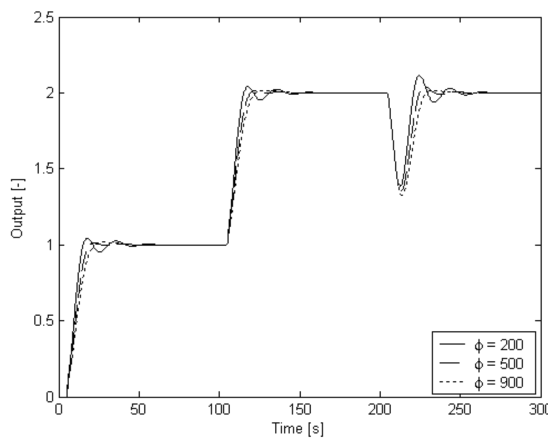

Fig. 24 Step setpoint and load disturbance responses of $y(t)$ using controllers (62); $K = 1$, $\tau = 5$, $d = -0.1$, $\gamma_1 = \gamma_2 = 0.25$.

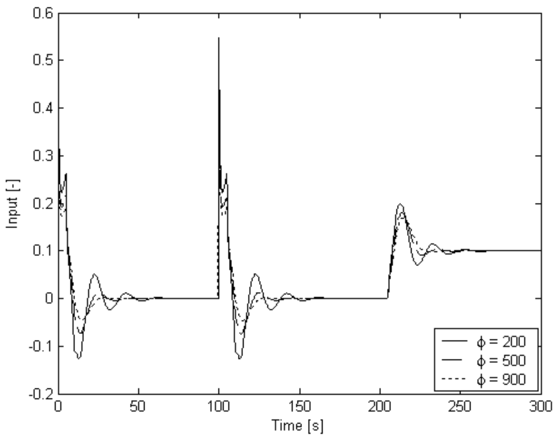

Fig. 25 Step setpoint and load disturbance responses of $u(t)$ using controllers (62); $K = 1$, $\tau = 5$, $d = -0.1$, $\gamma_1 = \gamma_2 = 0.75$.

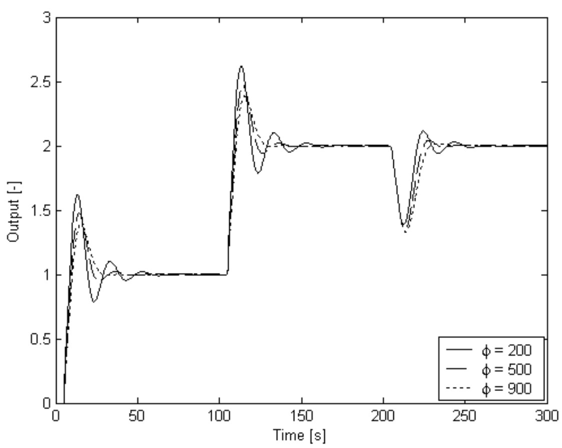

Fig. 26 Step setpoint and load disturbance responses of $y(t)$ using controllers (62); $K = 1$, $\tau = 5$, $d = -0.1$, $\gamma_1 = \gamma_2 = 0.75$.

Table V: Values of ISE and ISTE criterions when using controllers (62); $K = 1$, $\tau = 5$.

γ	φ	ISE	ISTE
0.25	200	22.831	863.89
0.25	500	17.514	749.402
0.25	900	16.887	764.729
0.75	200	88.582	1062.2
0.75	500	38.879	812.107
0.75	900	26.652	789.973

Since the ISE criterion (60) is calculated with $\varphi = 500$, one would expect that the LQ method using this option gives the best result. However, this does not hold as it arises from Table V. This is because of this method uses the linear approximation and thus the optimization (60) is made for an approximated plant transfer function instead of an original one.

Looking at Table V, it can be affirmed that the algebraic method utilizing the ring R_{MS} gives results reconcilable with the optimal polynomial LQ method for both, the direct solution and also for successive design of the inner and the outer controller.

IX. Conclusion

This paper developed the problem of algebraic control design in the ring of stable and proper RQ meromorphic functions for integrating time delay processes. The proposed method does not involve the delay approximation as it is customary; however, it utilizes transfer function parameterization without any loss of information. The controller structure was derived through the solution of the Bézout equation together with the Youla-Kučera parameterization. The methodology enables to find various controllers that satisfy requirements on the closed loop stability, step reference tracking and step load disturbance attenuation. The novel combination of this algebraic methodology and the control system structure combining conventional 1DOF and 2DOF schemes was proposed. The control structure is conceived in double meaning: either as a whole (one) system or a inner (pre-stabilizing) feedback loop plus the outer one. The final controllers were tuned using the dominant pole assignment method where the optimal setting yielding the leftmost dominant real poles was derived. The efficiency and usability of the proposed methodology was verified on a simulation example and compared with the polynomial LQ method.

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