Optical ZCZ Code Generators Using Sylvester-type Hadamard Matrix

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Abstract**—** In this paper, we propose the construction of two code generators for optical ZCZ codes of $Zcz = 4n-2$ with positive n and $Zcz = 1$ using the Sylvester-type Hadamard matrix, which are called ROM-type and non ROM-type code generators. The optical ZCZ code is a set of pairs of binary and bi-phase sequences with zero correlation zone. An optical code division multiple access (CDMA) system using optical ZCZ code can remove co-channel interference and influence of multi-path. This ROM-type code generator can be constructed by a ROM and an up-counter. Similarly, the non ROM-type code generator can be constructed by an up-counter and logic gates. The ROM-type and non ROM-type code generators are implemented on a field programmable gate array (FPGA) corresponding to $600,000$ logic gates, and the non ROM-type code generator can reduce logic elements and memory bits than the ROM-type code generator, and can operate faster than the ROM-type code generator.

Keywords**—** Optical communication, Optical ZCZ code, Optical CDMA system, Code generator, Field programmable gate array.

I. Introduction

The optical code division multiple access (CDMA) system can expect a high speed communication to be able to use a wide band [1], [2], [3]. An optical CDMA system using the optical ZCZ code, which is a set of pairs of binary and bi-phase sequences with zero correlation zone [4], [5], [6], [7] can remove co-channel interference and influence of multi-path.

We proposed the compact construction of a bank of matched filters [8], [9], [10], [11] for optical ZCZ codes in a receiver. But we have not proposed the construction of a code generator for optical ZCZ codes in a transmitter yet.

In this paper, we propose the construction of two code generators for optical ZCZ codes of zero correlation zone $Zcz = 4n-2$ with positive n and $Zcz = 1$ using the Sylvester-type Hadamard matrix, which are called ROM-type and non ROM-type code generators [12], [13], [14]. The ROM-type and non ROM-type code generators for optical ZCZ codes of $Zcz = 4n - 2$ with positive n and $Zcz = 1$ are implemented on a field programmable gate array (FPGA) corresponding to $600,000$ logic gates.

In Section **II**, we introduce optical ZCZ codes and its correlation properties, and explain the upper bound on zero correlation zone, and describe the construction of optical ZCZ codes with $Zcz = 4n - 2$ and $Zcz = 1$ using the Sylvester-type Hadamard matrix. In Section **III**, we describe the optical M-ary/DS-SS system using an optical ZCZ code , and the construction of two code generators, which are called a ROM-type and a non ROM-type code generators, respectively. In Section **IV**, we describe results of implementation of ROM-type and non ROM-type code generators on FPGA, and compare them.

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II. Optical ZCZ code

A. Definition of Optical ZCZ Code

Let a_N^j be a bi-phase sequence of length N whose elements take 1 or -1, written as

$$\begin{aligned} a_N^j &= (a_{N,0}^j, a_{N,1}^j, \cdots, a_{N,i}^j, \cdots, a_{N,N-1}^j), \\ & \quad a_{N,i}^j \in \{1,-1\}. \end{aligned} \tag{1}$$

Similarly, let $\hat{a}_N^{j,d}$ be a binary sequence of length N whose elements take 1 or 0, written as

$$\begin{aligned} \hat{a}_N^{j,d} &= (\hat{a}_{N,0}^{j,d}, \hat{a}_{N,1}^{j,d}, \cdots, \hat{a}_{N,i}^{j,d}, \cdots, \hat{a}_{N,N-1}^{j,d}), \\ & \quad \hat{a}_{N,i}^{j,d} \in \{1,0\}, \\ & \quad d \in \{1,0\}, \end{aligned} \tag{2}$$

where i denotes $i \bmod N$. Let A be a set of pairs of bi-phase sequences, a_N^j's, and binary sequences, $\hat{a}_N^{j,d}$'s , written as

$$\begin{aligned} A &= \{(a_N^1, \hat{a}_N^{1,d}), (a_N^2, \hat{a}_N^{2,d}), \cdots, (a_N^j, \hat{a}_N^{j,d}), \\ & \quad \cdots, (a_N^M, \hat{a}_N^{M,d})\}. \end{aligned} \tag{3}$$

A periodic correlation function between sequences a_N^j and $\hat{a}_N^{j',d}$ at shift i' is defined by

$$\rho_{a_N^j, \hat{a}_N^{j',d}, i'} = \sum_{i=0}^{N-1} a_{N,i}^j \hat{a}_{N,(i+i') \bmod N}^{j',d}. \tag{4}$$

In this paper, the above correlation function $\rho_{a_N^j, \hat{a}_N^{j',d}, i'}$ is called the auto-correlation function for $j = j'$ and the cross-correlation function for $j \neq j'$. The set is called an optical ZCZ code[4], [5], [6], [7], if the periodic auto- and cross-correlation functions satisfy

$$\rho_{a_N^j, \hat{a}_N^{j',d}, i'} = \begin{cases} w & ; i' = 0, j = j', d = 0 \\ -w & ; i' = 0, j = j', d = 1 \\ 0 & ; i' = 0, j \neq j', \\ 0 & ; 1 \leq |i'| \leq Zcz, \end{cases} \tag{5}$$

with $w = \sum_{i=0}^{N-1} \hat{a}_{N,i}^{j',d} < N$. The optical ZCZ codes are bounded by $M \leq N/(Zcz+1)$, where M is the number of sequences in a sequence family and is called family size.

B. Construction of Optical ZCZ Codes with $Zcz = 4n - 2$

Let b_{N_1} be the Legendre sequence [15] of length $N_1 = 4n_1 - 1$ with positive n_1 or an M-sequence of length $N_1 = 2^{n_1} - 1$ with $n_1 \geq 2$, whose elements take 1 or -1, written as

$$\begin{aligned} b_{N_1} &= (b_{N_1,0}, b_{N_1,1}, \cdots, b_{N_1,i}, \cdots, b_{N_1,N_1-1}), \quad (6) \\ & \quad b_{N_1,i} \in \{1, -1\} \end{aligned}$$

with $\sum_{i=0}^{N_1-1} b_{N_1,i} = 1$. A binary sequence $\hat{b}_{N_1,i} \in \{1, 0\}$ of length N_1 is given by

$$\hat{b}_{N_1,i} = \frac{1 + b_{N_1,i}}{2}.$$

The periodic correlation function between b_{N_1} and $\hat{b}_{N_1}$ is given by

$$\begin{aligned} \rho_{b_{N_1},\hat{b}_{N_1},i'} &= \sum_{i=0}^{N_1-1} b_{N_1,i} \hat{b}_{N_1,(i+i') \bmod N_1} \\ &= \frac{1}{2}\Big(1 + \rho_{b_{N_1},b_{N_1},i'}\Big) \\ &= \begin{cases} w_b & ; i' = 0 \\ 0 & ; otherwise \end{cases} \end{aligned} \quad (7)$$

with

$$\begin{aligned} \rho_{b_{N_1},b_{N_1},i'} &= \sum_{i=0}^{N_1-1} b_{N_1,i} b_{N_1,(i+i') \bmod N_1} \\ &= \begin{cases} N_1 & ; i' = 0 \\ -1 & ; otherwise \end{cases} \end{aligned} \quad (8)$$

and

$$w_b = \frac{N_1 + 1}{2}. \quad (9)$$

Let $\mathbf{H}_{N_2}$ be the Sylvester-type Hadamard matrix of order $N_2 = 2^{n_2}$ with $n_2 \geq 1$, written as

$$\begin{aligned} \mathbf{H}_{N_2} &= [h^0_{N_2}, h^1_{N_2}, \cdots, h^j_{N_2}, \cdots, h^{N_2-1}_{N_2}]^T, \quad (10) \\ h^j_{N_2} &= (h^j_{N_2,0}, h^j_{N_2,1}, \cdots, h^j_{N_2,i}, \cdots, h^j_{N_2,N_2-1}), \quad (11) \\ & \quad h^j_{N_2,i} \in \{1, -1\}, \end{aligned}$$

where the symbol T denotes the matrix transposition, which is defined by

$$\begin{aligned} \mathbf{H}_{N_2} &= \mathbf{H}_{\frac{N_2}{2}} \otimes \mathbf{H}_2, \quad (12) \\ \mathbf{H}_2 &= \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \quad (13) \end{aligned}$$

where the operation $\otimes$ denotes the Kronecker product. $h^j_{N_2}$ is called a Sylvester-type Hadamard sequence. On the other hand, a binary sequence $\hat{h}^{j,d}_{N_2,i} \in \{1, 0\}$ of length N_2 is given by

$$\hat{h}^{j,d}_{N_2,i} = \frac{1 + (-1)^d h^j_{N_2,i}}{2}. \quad (14)$$

The periodic correlation function at $i' = 0$ between $h^j_{N_2}$ and $\hat{h}^{j',d}_{N_2}$ except $j = j' = 0$ is given by

$$\begin{aligned} \rho_{h^j_{N_2},\hat{h}^{j',d}_{N_2},0} &= \frac{1}{2} \sum_{i=0}^{N_2-1} \Big\{ h^j_{N_2,i} + (-1)^d h^j_{N_2,i} h^{j'}_{N_2,i} \Big\} \\ &= (-1)^d \frac{1}{2} \rho_{h^j_{N_2},h^{j'}_{N_2},0} \\ &= = \begin{cases} w_h & ; j = j', d = 0 \\ -w_h & ; j = j', d = 1 \\ 0 & ; j \neq j' \end{cases} \end{aligned} \quad (15)$$

with

$$\begin{aligned} \sum_{i=0}^{N_2-1} h^j_{N_2,i} &= 0, \quad (16) \\ \rho_{h^j_{N_2},h^{j'}_{N_2},0} &= \sum_{i=0}^{N_2-1} h^j_{N_2,i} h^{j'}_{N_2,i} \\ &= \begin{cases} N_2 & ; j = j' \\ 0 & ; j \neq j' \end{cases} \quad (17) \end{aligned}$$

and

$$w_h = \frac{N_2}{2}. \quad (18)$$

If N_1 and N_2 are relatively prime, i. e., $gcd(N_1, N_2) = 1$, a bi-phase sequence $a^j_N = (a^j_{N,0}, a^j_{N,1}, \cdots, a^j_{N,i}, \cdots, a^j_{N,N-1})$ of length $N = N_1 N_2$ is produced by

$$a^j_{N,i} = b_{N_1, i \bmod N_1} \cdot h^j_{N_2, i \bmod N_2}. \quad (19)$$

Similarly, a binary sequence $\hat{a}^{j,d}_N = (\hat{a}^{j,d}_{N,0}, \hat{a}^{j,d}_{N,1}, \cdots, \hat{a}^{j,d}_{N,i}, \cdots, \hat{a}^{j,d}_{N,N-1})$ of length $N = N_1 N_2$ is produced by

$$\hat{a}^{j,d}_{N,i} = \hat{b}_{N_1, i \bmod N_1} \cdot \hat{h}^{j,d}_{N_2, i \bmod N_2}. \quad (20)$$

The periodic correlation function between a^j_N and $\hat{a}^{j',d}_N$ except $j = j' = 0$ is given by

$$\begin{aligned} \rho_{a^j_N,\hat{a}^{j',d}_N,i'} &= \sum_{i=0}^{N-1} a^j_{N,i} \hat{a}^{j',d}_{N,(i+i') \bmod N} \\ &= \sum_{i=0}^{N-1} \Big\{ b_{N_1, i \bmod N_1} \cdot \hat{b}_{N_1,(i+i') \bmod N_1} \Big\} \\ & \quad \cdot \Big\{ h^j_{N_2, i \bmod N_2} \hat{h}^{j',d}_{N_2,(i+i') \bmod N_2} \Big\} \\ &= \Big\{ \sum_{i=0}^{N_1-1} b_{N_1, i \bmod N_1} \cdot \hat{b}_{N_1,(i+i') \bmod N_1} \Big\} \\ & \quad \cdot \Big\{ \sum_{i=0}^{N_2-1} h^j_{N_2, i \bmod N_2} \hat{h}^{j',d}_{N_2,(i+i') \bmod N_2} \Big\} \\ &= \rho_{b_{N_1},\hat{b}_{N_1},i'} \rho_{h^j_{N_2},\hat{h}^{j',d}_{N_2},i'} \\ &= \begin{cases} w & ; i' = 0, j = j', d = 0, \\ -w & ; i' = 0, j = j', d = 1, \\ 0 & ; i' = 0, j \neq j', \\ 0 & ; 1 \leq |i'| \leq N_1 - 1, \end{cases} \end{aligned} \quad (21)$$

with

$$w = w_b \cdot w_h = \frac{(N_1 + 1) N_2}{4}. \quad (22)$$

Therefore a set of M pairs of a bi-phase sequence a_N^j and a binary sequence $\hat{a}_N^{j,d}$ is an optical ZCZ code with $Zcz = N_1 - 1 = 4n_1 - 2$ and $M = N_2 - 1 = N/(Zcz + 1) - 1$.

As an example, we generate an optical ZCZ code of $N = N_1 N_2 = 3 \times 4 = 12$, $Zcz = N_1 - 1 = 2$ and $M = N_2 - 1 = 3$. Let

$$b_3 = (+, +, -)$$

and

$$\mathbf{H}_4 = \begin{bmatrix} h_4^0 \\ h_4^1 \\ h_4^2 \\ h_4^3 \end{bmatrix} = \begin{bmatrix} + & + & + & + \\ + & - & + & - \\ + & + & - & - \\ + & - & - & + \end{bmatrix}, \quad (23)$$

where $+$ and $-$ denote $+1$ and -1, respectively. From Equation (19), we can generate bi-phase sequences, a_{12}^j's, as follows, respectively.

$$\begin{aligned} a_{12}^1 &= (+,-,-,-,+,+,+,-,-,-,+,+), \\ a_{12}^2 &= (+,+,+,-,+,-,-,-,-,+,-,+), \\ a_{12}^3 &= (+,-,+,+,+,+,-,+,-,-,-,-). \end{aligned}$$

Similarly, let

$$\hat{b}_3 = (+, +, 0)$$

and

$$\begin{aligned} \hat{h}_4^{1,0} &= (+,0,+,0), \\ \hat{h}_4^{2,0} &= (+,+,0,0), \\ \hat{h}_4^{3,0} &= (+,0,0,+), \\ \hat{h}_4^{1,1} &= (0,+,0,+), \\ \hat{h}_4^{2,1} &= (0,0,+,+), \\ \hat{h}_4^{3,1} &= (0,+,+,0), \end{aligned}$$

where $\hat{b}_{3,i} = (1 + b_{3,i})/2$, $\hat{h}_{4,i}^{j,d} = \{1 + (-1)^d h_{4,i}^j\}/2$ and $+$ denotes 1. From Equation (20), we can generate binary sequences, $\hat{a}_{12}^{j,d}$'s, as follows, respectively.

$$\begin{aligned} \hat{a}_{12}^{1,0} &= (+,0,0,0,+,0,+,0,0,0,+,0), \\ \hat{a}_{12}^{2,0} &= (+,+,0,0,+,0,0,0,0,+,0,0), \\ \hat{a}_{12}^{3,0} &= (+,0,0,+,+,0,0,+,0,0,0,0), \\ \hat{a}_{12}^{1,1} &= (0,+,0,+,0,0,0,+,0,+,0,0), \\ \hat{a}_{12}^{2,1} &= (0,0,0,+,0,0,+,+,0,0,+,0), \\ \hat{a}_{12}^{3,1} &= (0,+,0,0,0,0,+,0,0,+,+,0). \end{aligned}$$

A set of bi-phase sequences, a_{12}^j's, and binary sequences, $\hat{a}_{12}^{j,d}$'s, is an optical ZCZ code with $Zcz = 2$ and $M = 3$.

Its auto-correlation functions are given by

$$\begin{aligned} \rho_{a_{12}^1,\hat{a}_{12}^{1,0},i'} &= (4,0,0,-4,0,0,4,0,0,-4,0,0), \\ \rho_{a_{12}^1,\hat{a}_{12}^{1,1},i'} &= (-4,0,0,4,0,0,-4,0,0,4,0,0), \\ \rho_{a_{12}^2,\hat{a}_{12}^{2,0},i'} &= (4,0,0,0,0,0,-4,0,0,0,0,0), \\ \rho_{a_{12}^2,\hat{a}_{12}^{2,1},i'} &= (-4,0,0,0,0,0,4,0,0,0,0,0), \\ \rho_{a_{12}^3,\hat{a}_{12}^{3,0},i'} &= (4,0,0,0,0,0,-4,0,0,0,0,0), \\ \rho_{a_{12}^3,\hat{a}_{12}^{3,1},i'} &= (-4,0,0,0,0,0,4,0,0,0,0,0) \end{aligned}$$

and its cross-correlation functions

$$\begin{aligned} \rho_{a_{12}^1,\hat{a}_{12}^{2,0},i'} &= (0,0,0,0,0,0,0,0,0,0,0,0), \\ \rho_{a_{12}^1,\hat{a}_{12}^{2,1},i'} &= (0,0,0,0,0,0,0,0,0,0,0,0), \\ \rho_{a_{12}^2,\hat{a}_{12}^{3,0},i'} &= (0,0,0,4,0,0,0,0,0,-4,0,0), \\ \rho_{a_{12}^2,\hat{a}_{12}^{3,1},i'} &= (0,0,0,-4,0,0,0,0,0,4,0,0), \\ \rho_{a_{12}^3,\hat{a}_{12}^{1,0},i'} &= (0,0,0,0,0,0,0,0,0,0,0,0), \\ \rho_{a_{12}^3,\hat{a}_{12}^{1,1},i'} &= (0,0,0,0,0,0,0,0,0,0,0,0). \end{aligned}$$

C. Optical ZCZ Code with $Zcz = 1$

Let $\mathbf{H}_{N_2}$ be the Sylvester-type Hadamard matrix of order $N_2 = 2^{n_2}$ with $n_2 \geq 2$, written as Equations (10) and (11), which is defined by Equations (12) and (13).

A bi-phase sequence a_N^j with length $N = 2N_2$ is given by

$$\begin{aligned} a_{N,i}^j &= \alpha_{N,i} \cdot h_{N_2, i \bmod N_2}^j, \quad (24) \\ \alpha_{N,i} &= \begin{cases} h_{N_2,i \bmod N_2}^0 &= 1 \quad ; 0 \leq i < \frac{N}{2}, \\ -h_{N_2,i \bmod N_2}^1 &= (-1)^{i+1} \quad ; \frac{N}{2} \leq i < N, \end{cases} \end{aligned}$$

where i denotes $i \bmod N$. The mean value of a sequence a_N^j is given by

$$\sum_{i=0}^{N-1} a_{N,i}^j = \sum_{i=0}^{N_2-1} h_{N_2,i}^0 h_{N_2,i}^j - \sum_{i=0}^{N_2-1} h_{N_2,i}^1 h_{N_2,i}^j = 0, \quad (25)$$

where $j \neq 0, 1$. Therefore, a bi-phase sequence a_N^j is called a bi-phase balanced sequence. The periodic correlation function between a_N^j and $a_N^{j'}$ except $j = j' = 0, 1$ is given by

$$\begin{aligned} \rho_{a_N^j, a_N^{j'}, i'} &= \sum_{i=0}^{N-1} a_{N,i}^j a_{N,(i+i') \bmod N}^{j'} \\ &= \sum_{i=0}^{N-1} \left(\alpha_{N,i} \cdot h_{N_2, i \bmod N_2}^j \right) \\ &\quad \cdot \left(\alpha_{N, i+i' \bmod N} \cdot h_{N_2, i+i' \bmod N_2}^{j'} \right) \\ &= \sum_{i=0}^{N_2-1} \Big(\alpha_{N,i} \alpha_{N,i+i' \bmod N} \\ &\quad + \alpha_{N,i+N_2} \alpha_{N,i+N_2+i' \bmod N} \Big) \\ &\quad \cdot h_{N_2, i \bmod N_2}^j \cdot h_{N_2, i+i' \bmod N_2}^{j'} \\ &= \begin{cases} N = 2N_2 & ; i' = 0, j = j', \\ 0 & ; i' = 0, j \neq j', \\ 0 & ; i' = \pm 1. \end{cases} \quad (26) \end{aligned}$$

Therefore a set of bi-phase sequences, a_N^j's is a ZCZ code with $Zcz = 1$ and $M = N_2 - 2 = N/(Zcz + 1) - 2$.

A binary sequence $\hat{a}_{N,i}^{j,d} \in \{1, 0\}$ of length N is given by

$$\hat{a}_{N,i}^{j,d} = \frac{1 + (-1)^d a_{N,i}^j}{2}. \quad (27)$$

Let A be a set of M pairs of a bi-phase sequence a_N^j and a binary sequence $\hat{a}_N^{j,d}$ of $N = 2N_2$. The periodic correlation function between a_N^j and $\hat{a}_N^{j',d}$ except $j, j' \leq 1$ is given by

$$\begin{aligned}\rho_{a_N^j, \hat{a}_N^{j',d}, i'} &= \sum_{i=0}^{N-1} a_{N,i}^j \hat{a}_{N,(i+i') \bmod N}^{j',d} \\ &= \begin{cases} w & ; i' = 0, j = j', d = 0, \\ -w & ; i' = 0, j = j', d = 1, \\ 0 & ; i' = 0, j \neq j', \\ 0 & ; i' = \pm Zcz = \pm 1, \end{cases}\end{aligned} \quad (28)$$

with $w = \sum_{i=0}^{N-1} \hat{a}_{N,i}^{j,d} = \frac{N}{2}$ and Zcz is zero correlation zone. Therefore, the above set of M pairs of a bi-phase sequence a_N^j and a binary sequence $\hat{a}_N^{j,d}$ is called an optical ZCZ code with $Zcz = 1$ and $M = N/2 - 2 = N/(Zcz + 1) - 2$.

As an example, we generate an optical ZCZ code of $N = 2N_2 = 2 \times 4 = 8$, $Zcz = 1$ and $M = N/2 - 2 = 2$. From Equations (23) and (24), we can generate bi-phase sequences, a_8^j's, as follows, respectively.

$$\begin{aligned} a_8^2 &= (+,+,-,-,-,+,+,-), \\ a_8^3 &= (+,-,-,+,-,-,+,+). \end{aligned}$$

From Equation (27), we can generate binary sequences, $\hat{a}_8^{j,d}$'s, as follows, respectively.

$$\begin{aligned} \hat{a}_8^{2,0} &= (+,+,0,0,0,+,+,0), \\ \hat{a}_8^{2,1} &= (0,0,+,+,+,0,0,+), \\ \hat{a}_8^{3,0} &= (+,0,0,+,0,0,+,+), \\ \hat{a}_8^{3,1} &= (0,+,+,0,+,+,0,0). \end{aligned}$$

A set of pairs of a bi-phase sequence a_8^j and a binary sequence $\hat{a}_8^{j,d}$ is an optical ZCZ code with $Zcz = 1$ and $M = 2$.

Its auto-correlation functions are given by

$$\begin{aligned} \rho_{a_8^2, \hat{a}_8^{2,0}, i'} = \rho_{a_8^3, \hat{a}_8^{3,0}, i'} &= (4,0,-2,0,0,0,-2,0), \\ \rho_{a_8^2, \hat{a}_8^{2,1}, i'} = \rho_{a_8^3, \hat{a}_8^{3,1}, i'} &= (-4,0,2,0,0,0,2,0) \end{aligned}$$

and its cross-correlation functions are given by

$$\begin{aligned} \rho_{a_8^2, \hat{a}_8^{3,0}, i'} = \rho_{a_8^3, \hat{a}_8^{2,0}, i'} &= (0,0,2,0,-4,0,2,0), \\ \rho_{a_8^2, \hat{a}_8^{3,1}, i'} = \rho_{a_8^3, \hat{a}_8^{2,1}, i'} &= (0,0,-2,0,4,0,-2,0). \end{aligned}$$

III. CONSTRUCTION OF CODE GENERATOR

A. Optical M-ary/DS-SS system using an optical ZCZ code

Figure 1 shows an optical M-ary/DS-SS system using an optical ZCZ code. A transmitter selects a binary sequence $\hat{a}_N^{j,d}$ within an optical ZCZ code in according to input data, and send it as optical signal, which is converted by the electrical to optical (E/O) converter.

A receiver converts received optical signal to electrical signal by the optical to electrical (O/E) converter, and detects the selected sequence from the maximum value of correlations between the electrical signal and bi-phase sequences, a_N^j's in a bank of matched filters, and recover the data.

In optical M-ary/DS-SS system using optical ZCZ codes, binary sequences, $\hat{a}_N^{j,d}$'s and bi-phase sequences, a_N^j's are used in transmitter and receiver, respectively. Therefore a code generator for binary sequences, $\hat{a}_N^{j,d}$'s is necessary in transmitter.

B. ROM-type code generator

The code generator for optical ZCZ codes using Sylvester-type Hadamard matrix has two constructs, which uses ROM and does not use ROM. The former and latter are called ROM-type and non ROM-type code generators. The ROM-type code generator is constructed by a ROM and an up-counter. Figure 2 shows a ROM-type code generator for optical ZCZ codes, where A_i, D are ROM address and ROM data, respectively. The sequence number j and the order variable i are expressed in a binary notation as follows:

$$\begin{aligned} j &= (j_{n_2-1}, j_{n_2-2}, \cdots, j_0)_2, \\ i &= (i_{n-1}, i_{n-2}, \cdots, i_0)_2, \end{aligned}$$

where $n_2 = \log_2 N_2$ and $n = \lceil \log_2 N \rceil$. As an example, Tables

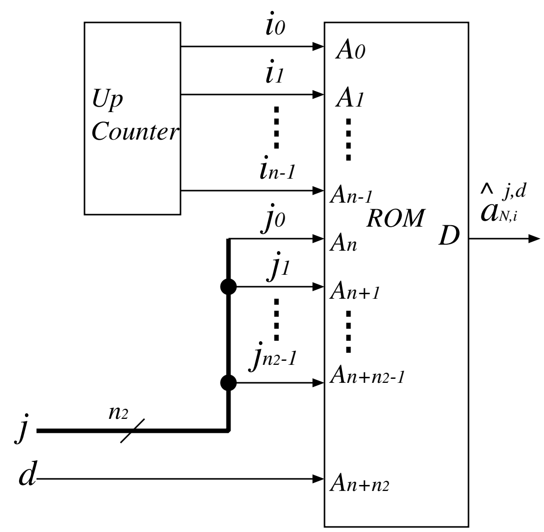

Fig. 2. ROM-type code generator for optical ZCZ codes of zero correlation zone $Zcz = 4n - 2$ with positive n and $Zcz = 1$ using the Sylvester-type Hadamard matrix.

I and II show memory map of ROM in a ROM-type code generator for optical ZCZ codes with $N = 12$ and $Zcz = 2$, and $N = 8$ and $Zcz = 1$, respectively.

C. Non ROM-type code generator for an optical ZCZ code with $Zcz = 4n - 2$

From Equation (20), a binary sequence $\hat{a}_N^{j,d} = (\hat{a}_{N,0}^{j,d}, \hat{a}_{N,1}^{j,d}, \cdots, \hat{a}_{N,i}^{j,d}, \cdots, \hat{a}_{N,N-1}^{j,d})$ of length $N = N_1 N_2$ is written as following Boolean expression.

$$\begin{aligned} \hat{a}_{N,i}^{j,d} &= \hat{b}_{N_1, i \bmod N_1} \cdot \hat{h}_{N_2, i \bmod N_2}^{j,d} \\ &= \hat{b}_{N_1, i \bmod N_1} \cdot (d \oplus \hat{h}_{N_2, i \bmod N_2}^{j,0}), \end{aligned} \quad (29)$$

where the operation $\cdot$ and $\oplus$ denote the logic operation AND and exclusive-OR (XOR), respectively. This means it is possible to construct by the generator for M-sequence and Sylvester-type Hadamard sequences. M-sequence generator can be easily constructed by a linear feedback shift register[15]. Sylvester-type Hadamard sequence $\hat{h}_{N_2,i}^{j,0}$ of

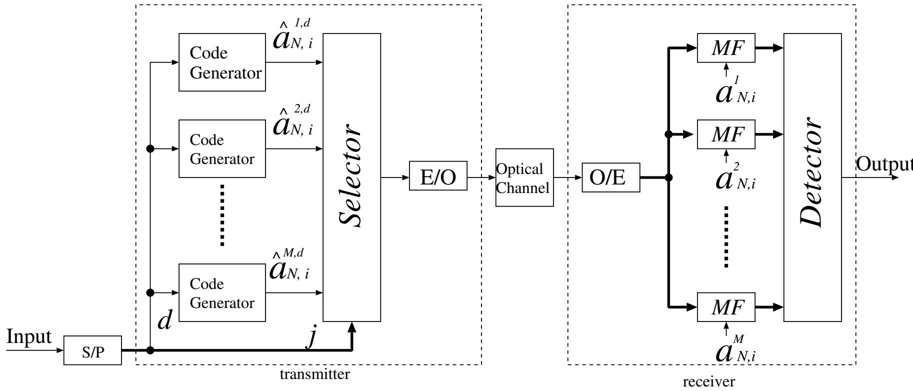

Fig. 1. Optical M-ary/DS-SS system using an optical ZCZ code of length N and family size M .

TABLE I
MEMORY MAP OF ROM IN A ROM-TYPE CODE GENERATOR FOR AN OPTICAL ZCZ CODE OF LENGTH 12 AND ZERO CORRELATION ZONE $Zcz = 2$.

Address							Data
A_6	A_5	A_4	A_3	A_2	A_1	A_0	D
d	j_1	j_0	i_3	i_2	i_1	i_0	$\hat{a}^{j,d}_{12,i}$
0	0	0	0	0	0	0	$\hat{a}^{0,0}_{12,0}$
0	0	0	0	0	0	1	$\hat{a}^{0,0}_{12,1}$
⋮	⋮			⋮			⋮
0	0	0	1	0	1	1	$\hat{a}^{0,0}_{12,11}$
⋮	⋮			⋮			⋮
0	1	1	0	0	0	0	$\hat{a}^{3,0}_{12,0}$
0	1	1	0	0	0	1	$\hat{a}^{3,0}_{12,1}$
⋮	⋮			⋮			⋮
0	1	1	1	0	1	1	$\hat{a}^{3,0}_{12,11}$
1	0	0	0	0	0	0	$\hat{a}^{0,1}_{12,0}$
1	0	0	0	0	0	1	$\hat{a}^{0,1}_{12,1}$
⋮	⋮			⋮			⋮
1	0	0	1	0	1	1	$\hat{a}^{0,1}_{12,11}$
⋮	⋮			⋮			⋮
1	1	1	0	0	0	0	$\hat{a}^{3,1}_{12,0}$
1	1	1	0	0	0	1	$\hat{a}^{3,1}_{12,1}$
⋮	⋮			⋮			⋮
1	1	1	1	0	1	1	$\hat{a}^{3,1}_{12,11}$

TABLE II
MEMORY MAP OF ROM IN A ROM-TYPE CODE GENERATOR FOR AN OPTICAL ZCZ CODE OF LENGTH $N = 8$ AND ZERO CORRELATION ZONE $Zcz = 1$.

Address						Data
A_5	A_4	A_3	A_2	A_1	A_0	D
d	j_1	j_0	i_2	i_1	i_0	$\hat{a}^{j,d}_{8,i}$
0	0	0	0	0	0	$\hat{a}^{0,0}_{8,0}$
0	0	0	0	0	1	$\hat{a}^{0,0}_{8,1}$
⋮	⋮			⋮		⋮
0	0	0	1	1	1	$\hat{a}^{0,0}_{8,7}$
⋮	⋮			⋮		⋮
0	1	1	0	0	0	$\hat{a}^{3,0}_{8,0}$
0	1	1	0	0	1	$\hat{a}^{3,0}_{8,1}$
⋮	⋮			⋮		⋮
0	1	1	1	1	1	$\hat{a}^{3,0}_{8,7}$
1	0	0	0	0	0	$\hat{a}^{0,1}_{8,0}$
1	0	0	0	0	1	$\hat{a}^{0,1}_{8,1}$
⋮	⋮			⋮		⋮
1	0	0	1	1	1	$\hat{a}^{0,1}_{8,7}$
⋮	⋮			⋮		⋮
1	1	1	0	0	0	$\hat{a}^{3,1}_{8,0}$
1	1	1	0	0	1	$\hat{a}^{3,1}_{8,1}$
⋮	⋮			⋮		⋮
1	1	1	1	1	1	$\hat{a}^{3,1}_{8,7}$

length $N_2 = 2^{n_2}$ and $d = 0$ is written as following Boolean expression.

$$\begin{aligned} \hat{h}^{j,0}_{N_2,i \bmod N_2} &= \overline{\hat{h}^{j,1}_{N_2,i \bmod N_2}} \\ &= \overline{(j_0 \cdot i_0) \oplus (j_1 \cdot i_1) \oplus \cdots \oplus (j_{n_2-1} \cdot i_{n_2-1})}, \end{aligned} \tag{30}$$

where the operation $\cdot$, $\oplus$ and $\overline{(\ \cdot\ )}$ denote the logic operation AND, XOR and NOT, respectively, and the order variable i is expressed in a binary notation as follows:

$$i \bmod N_2 = (i_{n_2-1}, i_{n_2-2}, \cdots, i_0)_2.$$

Therefore, from Equations (29) and (30), a binary sequence $\hat{a}^{j,d}_N$ of length $N = N_1 N_2$ is written as following Boolean expression.

$$\begin{aligned} \hat{a}^{j,d}_{N,i} = \hat{b}_{N_1,i \bmod N_1} \cdot \Big\{ d \oplus & \overline{(j_0 \cdot i_0) \oplus (j_1 \cdot i_1)} \\ & \overline{\oplus \cdots \oplus (j_{n_2-1} \cdot i_{n_2-1})} \Big\}. \end{aligned} \tag{31}$$

Figure 3 shows a non ROM-type code generator for an optical ZCZ code of length $N = N_1 N_2$ and zero correlation zone $Zcz = 4n-2$, which can be constructed by a up-counter, flip-flops and logic gates.

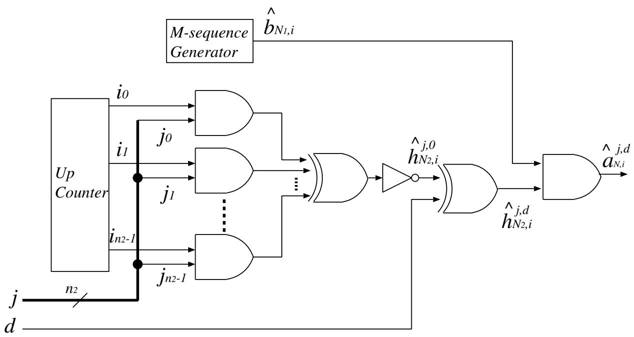

Fig. 3. Non ROM-type code generator for an optical ZCZ code of length $N = N_1 N_2$ and zero correlation zone $4n - 2$.

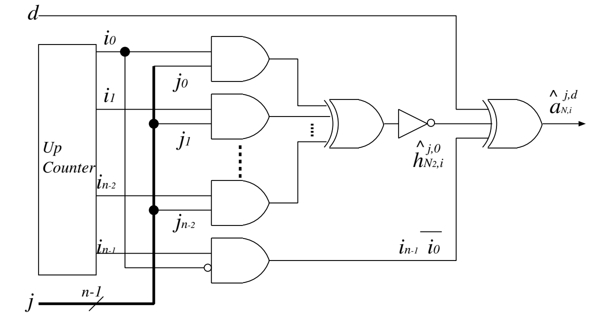

Fig. 4. Non ROM-type code generator for an optical ZCZ code of length $N = 2N_2$ and zero correlation zone $Zcz = 1$.

D. Non ROM-type code generator for an optical ZCZ code with $Zcz = 1$

From Equations (14), (24) and (27), a binary sequence $\hat{a}^{j,d}_N = (\hat{a}^{j,d}_{N,0}, \hat{a}^{j,d}_{N,1}, \cdots, \hat{a}^{j,d}_{N,i}, \cdots, \hat{a}^{j,d}_{N,N-1})$ of length $N = 2N_1$ is written as following Boolean expression.

$$\begin{aligned}
\hat{a}^{j,d}_{N,i} &= \begin{cases} \overline{\hat{h}^{0,0}_{N_2,i \bmod N_2}} \oplus \hat{h}^{j,d}_{N_2,i \bmod N_2} & ;0 \le i < N/2 \\ \hat{h}^{1,0}_{N_2,i \bmod N_2} \oplus \hat{h}^{j,d}_{N_2,i \bmod N_2} & ;N/2 \le i < N \end{cases} \\
&= \begin{cases} d \oplus \hat{h}^{j,0}_{N_2,i \bmod N_2} & ;0 \le i < N/2 \\ d \oplus (\overline{i_0} \oplus \hat{h}^{j,0}_{N_2,i \bmod N_2}) & ;N/2 \le i < N \end{cases} \\
&= d \oplus \left\{ \left(i_{n-1} \cdot \overline{i_0}\right) \oplus \hat{h}^{j,0}_{N_2,i \bmod N_2} \right\}, \qquad (32)
\end{aligned}$$

where i denotes $i \bmod N$, and the operation $\cdot$, $\oplus$ and $\overline{(\ \cdot\ )}$ denote the logic operation AND, exclusive-OR (XOR) and NOT, respectively. This means it is possible to construct by up-counter and the generator for Sylvester-type Hadamard sequence $\hat{h}^{j,0}_{N_2}$. The Sylvester-type Hadamard sequence $\hat{h}^{j,0}_{N_2}$ of length $N_2 = 2^{n_2}$ and $d = 0$ is written as Equation (30). Therefore, from Equations (30) and (32), a binary sequence $\hat{a}^{j,d}_N$ of length $N = 2N_2$ is written as following Boolean expression.

$$\begin{aligned}
\hat{a}^{j,d}_{N,i} &= d \oplus \left\{ \left(i_{n-1} \cdot \overline{i_0}\right) \oplus \overline{(j_0 \cdot i_0) \oplus (j_1 \cdot i_1)} \right. \\
&\qquad \left. \overline{\oplus \cdots \oplus (j_{n-2} \cdot i_{n-2})} \right\}, \qquad (33)
\end{aligned}$$

where $n = n_2 + 1$.

Figure 4 shows a non ROM-type code generator for an optical ZCZ code of length $N = 2N_2$ and zero correlation zone $Zcz = 1$, which can be constructed by an up-counter and logic gates.

IV. Code Generator Implementation on FPGA

Code generators for optical ZCZ codes of length $N = 12$, 24, 48, 96, 192, 384 and 768, with $Zcz = 2$ and ones of length $N = 8$, 16, 32, 64, 128, 256 and 512 with $Zcz = 1$ have been implemented on a field programmable gate array (FPGA) with $600,000$ logic gates. This FPGA has 488 pins which the user can freely use and $24,320$ logic elements (LEs) which the basic building blocks of an FPGA, containing a 4-input look up table (LUT), a register, and additional logic. The word-length of the output is 1bit, respectively. Table III shows the resultant specification of code generators. The Synplify Pro which is logic synthesis tool, is used to synthesize the design file of code generators. Similarly, the Quartus II which is place-and-route tool, is used to place and route the design file of code generators, and the ModelSim which is a simulation tool is used to simulate and debug the design file.

TABLE III
Specifications of code generators.

Spreading sequence	Optical ZCZ code
Zero cor. zone Zcz	1,2
Sequence length N	12, 24, 48, 96, 192, 384, 768 (Zcz=2) 8, 16, 32, 64, 128, 256, 512 (Zcz=1)
Family size M	3, 7, 15, 31, 63, 127, 255 (Zcz=2) 2, 6, 14, 30, 62, 126, 254 (Zcz=1)
Output word-length	1bit
Samples per chip	1
FPGA	Altera, EP20K600EBC652-1x
Max. logic gates	600,000
Max. LEs	24,320
Max. memory bits	311,296
Max. pins	488
Logic synthesis tool	Synopsys, Synplify Pro D-2010.03
Place and route tool	Altera, Quartus II v.8.1
Simulation tool	Mentor, ModelSim SE 6.1d

Figure 5, 6 and 7 show the number of logic elements (LEs), the memory bits and the maximum clock frequency of code generators for ZCZ codes of length 12, 24, 48, 96, 192, 384 and 768, and family size 3, 7, 15, 31, 63, 127 and 255, respectively. Similarly, Figure 8, 9 and 10 show the number of logic elements (LEs), the memory bits and the maximum clock frequency of code generators for ZCZ codes of length 8, 16, 32, 64, 128, 256 and 512, and family size 2, 6, 14, 30, 62, 126 and 254, respectively.

As a result of implement on FPGA, from Figures 5 and 8, the non ROM-type code generators can reduce logic elements than the ROM-type code generators. The non ROM-type code generator don't use the ROM. On the other hand, from Figures 6 and 9, memory bits of ROM-type code generator increase exponentially with sequence length N. From Figures 7 and 10, the maximum clock frequency of non ROM-type code generators can operate faster than the ROM-type code generators. Therefore, non ROM-type code generator is more suitable for optical ZCZ-CDMA system than ROM-type code generator.

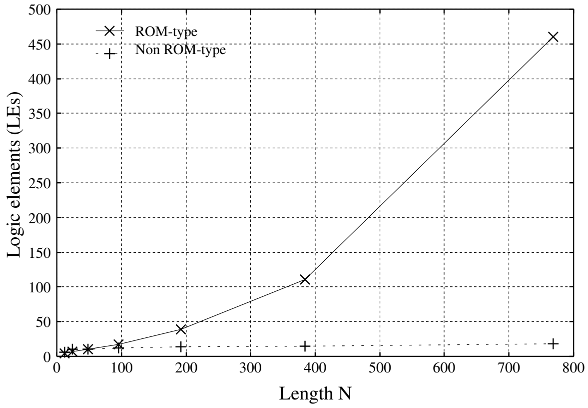

Fig. 5. Logic elements (LEs) of code generators for optical ZCZ codes of $Zcz = 2$.

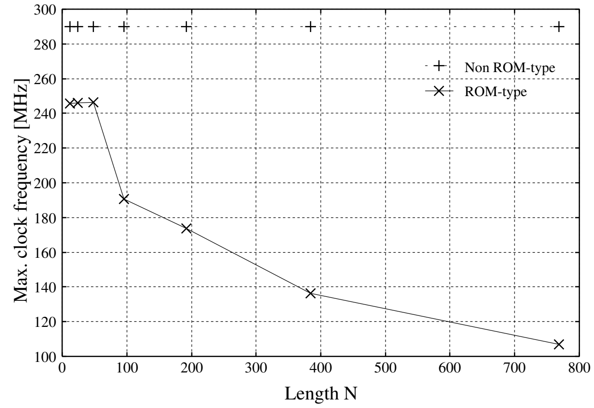

Fig. 7. Maximum clock frequency of code generators for optical ZCZ codes of $Zcz = 2$.

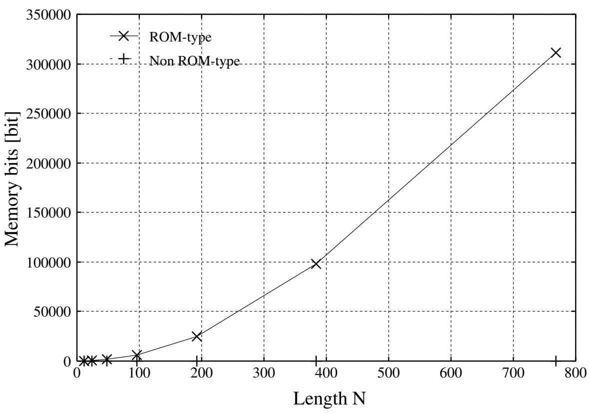

Fig. 6. Memory bits of code generators for optical ZCZ codes of $Zcz = 2$.

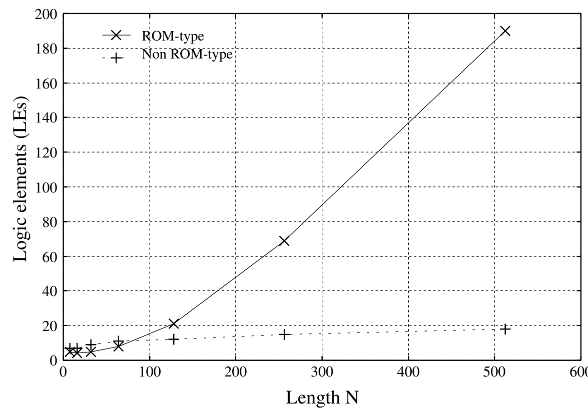

Fig. 8. Logic elements (LEs) of code generators for optical ZCZ codes of $Zcz = 1$.

V. Conclusion

In this paper, we propose the construction of two code generators for optical ZCZ codes with $Zcz = 4n - 2$ with positive n and $Zcz = 1$ using the Sylvester-type Hadamard matrix, which are called ROM-type and non ROM- type code generators. This ROM-type code generator can be constructed by a ROM and an up-counter. Similarly, this non ROM-type code generator can be constructed by an up-counter and logic gates. The ROM-type and non ROM-type code generators are implemented on FPGA, and the non ROM-type code generator can reduce logic elements and memory bits than the ROM-type code generator, and can operate faster than the ROM-type code generator. Therefore, non ROM-type code generator is more suitable for optical ZCZ-CDMA system than ROM-type code generator.

Acknowledgment

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References

[1] J. G. Zhang and G. Picchi, Tunable prime-code encoder/decoder for all-optical CDMA applications, *IEE Electronics Letters*, vol. 29, no. 13, 1993, pp.1211–1212.

[2] J. A. Salehi, A. M. Weiner and J. P. Heritage, Coherent ultrashort light pulse code-division multiple access communication systems, *IEEE Journal of Lightwave Technology*, vol. 8, no. 3, 1990, pp. 478–491.

[3] H. Fathallah, L. A. Rusch and S. LaRochelle, Passive optical fast frequency-hop CDMA communications system, *IEEE Journal of Lightwave Technology*, vol. 17, no. 3, 1999, pp. 397–405.

[4] T. Takahashi, S. Matsufuji, T. Matsumoto and Y. Tanada, An Optical CDMA System Using Extended ZCZ Sequences, *IEICE Technical Report*, WBS2003-83, 2003, pp.19–23(in Japanese).

[5] T. Takahashi, S. Matsufuji, T. Matsumoto and Y. Tanada, Study on an Optical ZCZ Code and its Applications to Optical CDMA Communication, *IEICE Technical Report*, RCS2004-308, 2005, pp.107–112 (in Japanese).

[6] S. Matsufuji, T. Matsumoto, Y. Tanada and N. Kuroyanagi, Extended ZCZ Codes for ASK-CDMA system, *Proceedings of The Second International Workshop on Sequence Design and Its Applications in Communications*, 2005, pp. 90–93.

[7] S. Matsufuji, T. Matsumoto, Y. Tanada and N. Kuroyanagi, ZCZ Codes for ASK-CDMA System, *IEICE Trans. Fundamentals.*, vo. E89-A, no.9, 2006, pp. 2268–2274.

[8] T. Matsumoto, S. Tsukiashi, S. Matsufuji and Y. Tanada, A Trial of a Digital Matched Filter for an M-ary/DS-SS System Using an Optical ZCZ Code, *IEICE Technical Report, RCS2004-309*, 2005, pp.113–118(in Japanese).

[9] T. Matsumoto, S. Tsukiashi, S. Matsufuji and Y. Tanada, A Design of a Digital Matched Filter Bank for an Optical ZCZ Code Using a Sylvester Type Hadamard Matrix, *Proceedings of The Second International Work-*

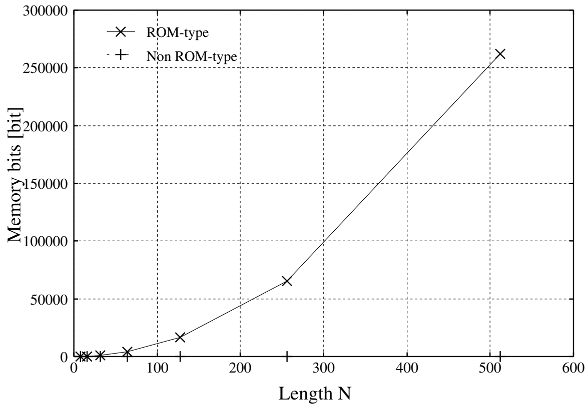

Fig. 9. Memory bits of code generators for optical ZCZ codes of $Zcz = 1$.

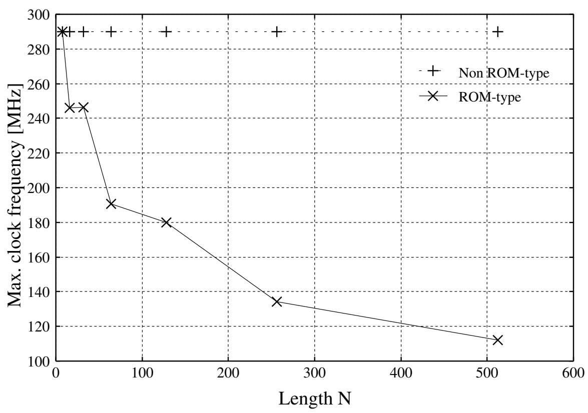

Fig. 10. Maximum clock frequency of code generators for optical ZCZ codes of $Zcz = 1$.

shop on Sequence Design and Its Applications in Communications, 2005, pp. 184–188.

[10] T. Matsumoto, S. Tsukiashi, S. Matsufuji and Y. Tanada, The Bank of Matched Filters for an Optical ZCZ Code Using a Sylvester Type Hadamard Matrix, *IEICE Trans. Fundamentals.* , vol.E89-A, no.9, 2006, pp. 2292–2298.

[11] T. Matsumoto and S. Matsufuji, Compact Bank of Matched Filters for Optical ZCZ Codes Using Fast Algorithm for M-sequence Type Hadamard Matrix, *Proceedings of the 2010 RISP International Workshop on Nonlinear Circuits, Communications and Signal Processing*, 2010, pp. 628–631.

[12] T. Matsumoto, S. Tsukiashi, S. Matsufuji and Y. Tanada, Design of an Optical ZCZ Code Generator, *IEICE Technical Report*, WBS2005-41, 2005, pp.13–18(in Japanese).

[13] T. Matsumoto and S. Matsufuji, Code Generator Implementation on FPGA for an Optical ZCZ Code Using a Sylvester Type Hadamard Matrix, *Proceedings of The Third International Workshop on Signal Design and Its Applications in Communications*, 2007, pp. 228–232.

[14] T. Matsumoto and S. Matsufuji, Construction of Optical ZCZ Code Generators With the Smallest Zero Correlation Zone, *New Aspects of Applied Informatics, Biomedical Electronics & Informatics and Communications*, 2010, pp. 216–220.

[15] P. Z. Fan and M. Darnell, Sequence Design for Communications Applications, *Research Studies Press*, 1996.

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