INTERNATIONAL JOURNAL OF COMMUNICATIONS
Issue 2, Volume 5, 2011

Mutual Coupling of Near Collocated Monopoles

Motti Haridim, Boris Levin, Michael Bank, Yoav Trabelsi, S. Tapuchi

Abstract- The electrical characteristics of two monopoles with different lengths located in the near region of each other are analyzed. The self and mutual impedances of both radiators are calculated, the mutual coupling between two monopoles is considered. It is shown that as in the case of two monopoles with equal lengths the structure of two monopoles with different lengths can be modeled as a combination of two-wire transmission line and monopole with stepped change of equivalent radius. The current distribution along each conductor is found. Also the method is applied to the multiple-wire radiator. Calculations are based on the folded dipoles theory, on the theory of electrically coupled lines located under ground, and on the superposition principle.

Keywords - Folded dipoles, Monopole antennas, Mutual coupling, Near fields, Transmission lines.

I. INTRODUCTION

The requirement for creating a weak field area in the transmitting antenna near region stems from the necessity to protect vulnerable devices or phone users from RF irradiation. In accordance with the compensation method proposed by M. Bank [1], such problem can be efficiently solved by employing two radiators, the fields of which mutually suppress each other in a certain desired area. For this purpose, between the main radiator 1 and the user's head an auxiliary radiator 2 is placed in the vicinity of the main radiator, as depicted in Fig.1.

Development of the compensation method theory required calculation of fields produced by two linear electric radiators of finite lengths located in their near regions [2]-[7]. This calculation is based on the folded dipoles theory and on the superposition principle. The two radiators system is divided into two circuits: an open-ended long line and a two-wire linear radiator (for example, monopole) with an equivalent radius. If the wires have equal lengths, the line length and the monopole height equal the wire length. But, if the wires have different lengths, it is necessary to determine the input impedance of each circuit and the current distribution along each wire.

Analogous problems occur with multiple-wire radiators. For example, a radiator may consist of a long central rod with load and a system of identical shorter wires located around this rod, in parallel to it. Another example of such problem is the analysis of a mast influence on the characteristics of a vertical wire antenna suspended in parallel to the mast [8].

We shall consider these problems by the example of a weak field area creation. In accordance with the compensation method, in a point A inside the head two radiators create the fields, the vertical components of which have equal magnitudes and opposite signs. That point is called the compensation point. Around this point a weak-field area is produced.

This paper is organized as follows. In Section 2 the procedure, which allows us to analyze the antenna system as a superposition of two sub-systems with in-phase currents (even mode) and anti-phased currents (odd mode), is considered. In Section 3 it is shown that the input impedance of a line with wires of unequal lengths is equal to the input impedance of a line with the short wires, loaded by a small capacitance. From the results of Section 4 one can see that the currents along both sections of the monopole are distributed in accordance with sinusoidal law. In Section 5 the results of the mutual impedances calculation for the radiators of unequal lengths in the near region are given. In Section 6 the method of multiple-wire radiator calculation is considered.

II. SUBDIVIDING INTO TWO SYSTEMS

Fig.2 shows the equivalent circuit of the two radiators structure for the case when an emf e_1 is connected to the input of the first radiator (hereafter, the active antenna), and the second radiator is not driven. In Fig.2, R_1 is the output impedance of the first generator; R_2 is the input impedance of the second generator (it may be measured at the input of the cable leading to this generator), and usually $R_1 = R_2 = R$.

In accordance with the theory of folded monopoles, we consider an equivalent structure, in which two generators of equal emf $(e_1/2)$ are connected to the terminal of the second

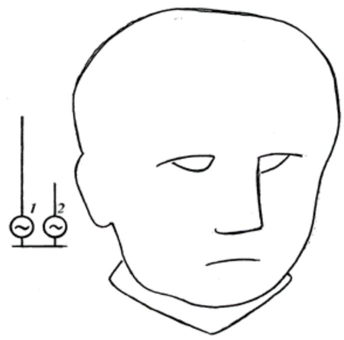

Fig.1. The compensation method

Manuscript received March 22, 2011.
Motti Haridim is with the Holon Institute of Technology, Holon, Israel (e-mail mharidim@hit.ac.il).
Boris Levin is with the Holon Institute of Technology, Holon, Israel (corresponding author, phone 972-3-5026694, e-mail levinpaker@gmail.com).
Michael Bank is with the Holon Institute of Technology, Holon, Israel (e-mail bankmichael1@gmail.com).
Yoav Trabelsi is with the Holon Institute of Technology, Holon, Israel (e-mail YTrabelsi@iai.co.il).
S. Tapuchi is with the Sami Shamoon College of Engineering, Israel

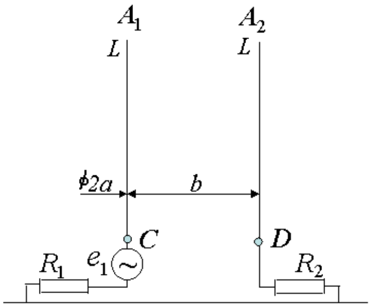

Fig.2. The two-radiator system

radiator in opposite directions, and the emf e_1 of the active radiator is split into two generators of equal emf $e_1/2$ and direction, as depicted in Fig. 3. This procedure allows us to analyze the antenna system as a superposition of two sub-systems with in-phase currents (even mode) and anti-phased currents (odd mode). The odd mode sub-system represents an open-ended transmission line, and the even mode sub-system represents a monopole.

If the wires have equal lengths *L*, we can write for the two-wire line of Fig. 2

$$e_1 = J_l(Z_l + 2R), \qquad (1)$$

where J_l is the current at the line base, $Z_l = -jW_l \cot kL$ is the input impedance of a line with length L, $W_l = 120\ln(b/a)$ is the line's wave (characteristic) impedance, b is the distance between the wires, and 2 a is the diameter of each wire. The current at point *C* is equal to $J_{C1l} = e_1 Y_1$, and the current at point *D* is $J_{D1l} = -e_1 Y_1$, where

$$Y_1 = 1/(-jW_l \cot kL + 2R).$$

For the monopole we can write

$$e_1/2 = J_r(Z_r + R/2), \qquad (2)$$

where J_r is the current at the monopole base, and $Z_r = Z_m(L, a_e)$ is the input impedance of a monopole with length L and equivalent radius a_e, given by $\sqrt{ab}$. The currents at points *C* and *D* are the same

$$J_{C1r} = J_{D1r} = e_1/(4Z_r + 2R) = e_1 Y_2,$$

where $Y_2 = 1/[4Z_m(L, a_e) + 2R]$. So, if emf e_1 fed the

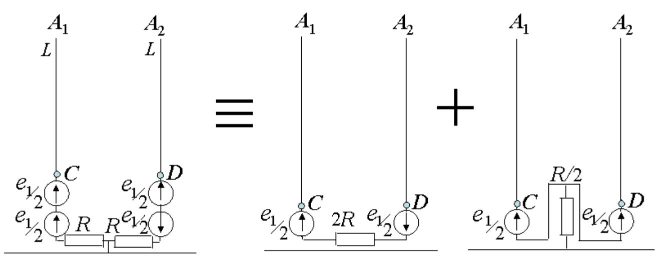

Fig.3. For calculation of the input impedance

first radiator, the currents at the first and the second radiator bases are

$$J_{11} = e_1(Y_1 + Y_2),\ J_{21} = e_1(-Y_1 + Y_2).$$

Similarly, if emf e_2 is connected to the second radiator input, the currents at the radiator's bases are

$$J_{12} = e_2(-Y_1 + Y_2), J_{22} = e_2(Y_1 + Y_2).$$

According to the superposition principle the currents at the radiators' terminals are

$$J_{A1} = (e_1 - e_2)Y_1 + (e_1 + e_2)Y_2,$$
$$J_{A2} = (e_2 - e_1)Y_1 + (e_1 + e_2)Y_2. \qquad (3)$$

And the input admittances of the radiators are

$$Y_{A1} = J_{A1}/e_1 = Y_1 + Y_2 + e_2(Y_2 - Y_1)/e_1,$$
$$Y_{A2} = J_A/e_2 = Y_1 + Y_2 + e_1(Y_2 - Y_1)/e_2. \qquad (4)$$

If the radiators have different lengths, the problem is complicated.

III. TWO-WIRE TRANSMISSION LINE

As shown in Fig.4*a*, a two-wire line consisting of parallel wires of unequal lengths has two sections: a lower section of length $L = l_2$ and an upper section of length $l = l_1 - l_2$, where l_1 is the length of the longer radiator, and l_2 is the length of the shorter one. The lower section consists of two parallel wires of circular cross section of the same lengths and radii. The capacity per unit length between such wires placed in a homogeneous medium of permittivity ε is given by

$$C_0 = \pi\varepsilon/\ln(b/a). \qquad (5)$$

Here *a* is the wire radius, and *b* is a distance between wires axes. The linear capacitance C_0 determines the wave impedance of the two-wire line lower section.

We shall take account of the upper section effect on the line input impedance by calculating the capacitance between the upper part of the longer wire (of length *l*) and the short radiator (Fig. 4*b*). This capacitance equals the difference of two capacitances:

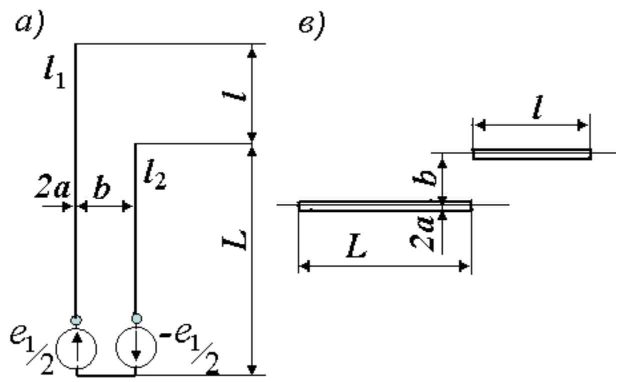

Fig. 4. The account of line's upper section

$$C = C_1 - C_0 L . \qquad (6)$$

Here C_1 is the total capacitance between the longer and the short wires, $C_0 L$ is the capacitance between the line wires of length L. At that C_1 is given by (see, for example [9])

$$C_1 = \left(\alpha_{11} + \alpha_{22} - 2\alpha_{12}\right)^{-1}, \qquad (7)$$

where

$$\alpha_{11} = \frac{1}{2\pi\varepsilon L}\left\{\ln\left[\frac{L}{a} + \sqrt{1+\left(\frac{L}{a}\right)^2}\right] + \frac{a}{L} - \sqrt{1+\left(\frac{a}{L}\right)^2}\right\},$$

$$\alpha_{22} = \frac{1}{2\pi\varepsilon(L+l)}\left\{\ln\left[\frac{L+l}{a} + \sqrt{\left(\frac{L+l}{a}\right)^2 + 1}\right] + \frac{a}{L+l} - \sqrt{\left(\frac{a}{L+l}\right)^2 + 1}\right\}$$

$$\alpha_{12} = \frac{1}{4\pi\varepsilon(L+l)}\left\{\ln\frac{L+\sqrt{L^2+b^2}}{b} - \frac{1}{L}\sqrt{L^2+b^2} + \right.$$

$$+\frac{L+l}{L}\ln\frac{L+l+\sqrt{(L+l)^2+b^2}}{b} + \frac{1}{L}\sqrt{l^2+b^2} -$$

$$\left. -\frac{l}{L}\ln\frac{l+\sqrt{l^2+b^2}}{b} + \frac{b}{L} - \frac{1}{L}\sqrt{(L+l)^2+b^2}\right\}.$$

At $L/a, l/a >> 1$, we obtain

$$\alpha_{11} = \frac{1}{2\pi\varepsilon L}\left(\ln\frac{2L}{a} - 1\right),$$

$$\alpha_{22} = \frac{1}{2\pi\varepsilon(L+l)}\left[\ln\frac{2(L+l)}{a} - 1\right],$$

Therefore, the input impedance of the line with wires of unequal lengths equal to the input impedance of the line with the short wires, loaded by a capacitance. Calculations show that this capacitance is small in comparison with C_0 of the line. In particular, for L=7.5, b=1.0, 2a=0.05 (all dimensions are in centimeters) we have C_0=7.5 pF, and C calculated in accordance with (6) for l from 1 to 4 cm changes from 0.05 to 0.1 pF, where it is assumed that the wires are located in air,

Table 1. Capacitive loads due to unequal wire lengths and elongations l_0 and l_{01} at 2a=0.05 cm

***l*, cm**	**l_0, cm**	**l_{01}, cm**	***C*, pF**
0.0	0	0	0.020
0.5	0.22	0.19	0.037
1.0	0.41	0.39	0.050
1.5	0.56	0.52	0.063
2.0	0.69	0.86	0.073
2.5	0.80	1.10	0.081
3.0	0.90	1.38	0.089
3.5	0.98	1.66	0.095
4.0	1.05	1.94	0.101
4.5	1.12	2.17	0.107

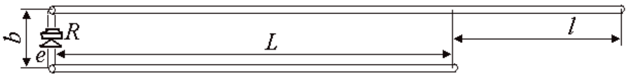

Fig.5. The simulation model for the two-wire transmission line

i.e. $\varepsilon = 10^{-9}/(36\pi)$.

The capacitive load at the end of the open line is actually equivalent to its elongation by l_0, beyond the line's length L. The elongation value is obtained from the expression

$$l_0 = \frac{1}{k}\,arc\cot\frac{1}{\omega C W_l} . \qquad (8)$$

The calculation results are given in Table 1. Table 1 presents the values of capacitance C, and also the distances l_0 for given wires dimensions at frequency 1GHz.

The obtained theoretical results were verified by the CST simulation. The system model used in the simulations is shown in Fig.5. In this Figure e is a discrete port (generator), and R is the output impedance of the generator set to 50 ohm. These calculation results are also presented in the Table 1 (the length l_{01}). At that the magnitudes l_0 and l_{01} are decreased by their values for l=0 cm. The calculation and simulation results are close to each other, if $l \le 0.1\lambda$, and show that the input impedance of a line with different wire lengths differs somewhat from the input impedance of a line with wires having the same lengths as the shorter wires.

The analogous results at 2a=0.2 are presented in Table 2.

IV. MONOPOLE OF PARALLEL WIRES

The not less important second problem is the input impedance calculation of a linear radiator (monopole) composed of two wires with different lengths (Fig. 6*a*). Fig. 6*b* shows an equivalent asymmetric line for this radiator. The current distribution along the monopole wires is calculated in accordance with the theory of electrically coupled lines located under ground, developed by A. Pistolkors [10].

In this case, since the line wires have different lengths, it is necessary to divide the equivalent line to two sections, as shown in Fig. 6*b*. The expressions for the current and potential of wire n at section m of the asymmetric line of N wires

Table 2. Capacitive loads due to unequal wire lengths and elongations l_0 and l_{01} at 2a=0.2

***l*, cm**	**l_0, cm**	**l_{01}, cm**	***C*, pF**
0.0	0	0	0.047
0.5	0.21	0.15	0.073
1.0	0.37	0.30	0.093
1.5	0.49	0.45	0.108
2.0	0.58	0.61	0.119
2.5	0.65	0.79	0.128
3.0	0.71	1.00	0.135
3.5	0.75	1.24	0.140
4.0	0.78	1.48	0.144
4.5	0.81	1.64	0.148

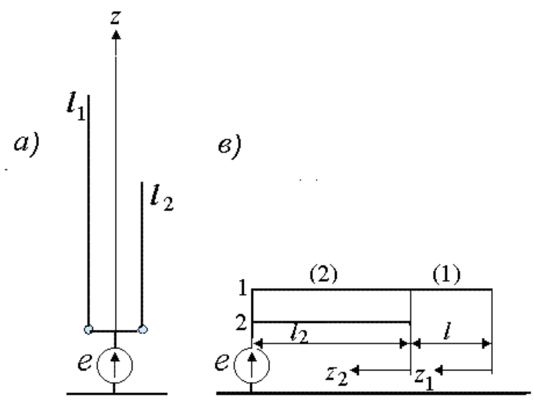

Fig.6. Monopole and equivalent asymmetric line

take the form

$$i_n^{(m)} = I_n^{(m)}\cos kz_m + j\left[\frac{2U_n^{(m)}}{W_{nn}^{(m)}} - \sum_{s=1}^{M}\frac{U_s^{(m)}}{W_{ns}^{(m)}}\right]\sin kz_m, \tag{9}$$

$$u_n^{(m)} = U_n^{(m)}\cos kz_m + j\sum_{s=1}^{M}\rho_{ns}^{(m)} I_s^{(m)}\sin kz_m,$$

where $I_n^{(m)}$ and $U_n^{(m)}$ are, respectively, the current and potential at the beginning of section m of wire n (at point $z_m = 0$), $n = 1,2$ $m = 1,2$, M is the quantity of wires at section m, and $W_{ns}^{(m)}$ and $\rho_{ns}^{(m)}$ are the electrostatic and electrodynamic wave impedances between wire n and wire s at section m. If the distance between the wires is small as compared with the wires lengths, one can assume

$$\rho_{nn}^{(m)} = const(n) = \rho_1^{(m)},\ \rho_{ns}^{(m)}\Big|_{n\neq s} = const(n) = \rho_2^{(m)},$$

$$W_{nn}^{(m)} = const(n) = W_1^{(m)},\qquad W_{ns}^{(m)}\Big|_{n\neq s} = W_2^{(m)}.$$

The zero currents at the ends of the wires and the continuity of the current and potential along each wire permit to write the boundary conditions

$$i_1^{(1)}\Big|_{z_1=0} = i_2^{(2)}\Big|_{z_2=0} = 0;\quad i_1^{(1)}\Big|_{z_1=l} = i_1^{(2)}\Big|_{z_2=0};$$

$$u_1^{(1)}\Big|_{z_1=l} = u_1^{(2)}\Big|_{z_2=0};\ u_1^{(2)}\Big|_{z_2=l_2} = u_2^{(2)}\Big|_{z_2=l_2} = e.$$

From these boundary conditions we get:

$$I_1^{(1)} = I_2^{(2)} = 0;\ I_1^{(2)} = j\frac{U_1^{(1)}}{W_1^{(1)}}\sin kl;\ U_1^{(2)} = U_1^{(1)}\cos kl;$$

$$U_2^{(2)} = U_1^{(1)}\left[\cos kl - \frac{\rho_1^{(2)}-\rho_2^{(2)}}{W_1^{(1)}}\sin kl\cdot\tan kL\right];$$

$$U_1^{(1)} = \frac{e}{\cos kl\cos kL}\left[1 - \frac{\rho_1^{(2)}}{W_1^{(1)}}\tan kl\tan kL\right]^{-1}.$$

The current along the first section of the longer wire as a function of z-coordinate is given by

$$i_1^{(1)} = j\frac{U_1^{(1)}}{W_1^{(1)}}\sin k(l_1 - z), \tag{10}$$

The current along the second section is

$$i_1^{(2)} = jU_1^{(1)}\left\{\frac{\sin kl\cos kz_2}{W_1^{(1)}} + \cos kl\left[\frac{1}{W_1^{(2)}} - \frac{1}{W_2^{(2)}}\cdot\right.\right.$$

$$\left.\left.\cdot\left\langle 1 - \frac{\rho_1^{(2)}-\rho_2^{(2)}}{W_1^{(1)}}\right\rangle\tan kl\tan kL\ \right]\sin kz_2\ \right\}.$$

The current along the shorter wire is

$$i_2^{(2)} = jU_1^{(1)}\cos kl\left\{\frac{1}{W_1^{(2)}} - \frac{1}{W_2^{(2)}}\cdot\right.$$

$$\left.\cdot\left[1 - \frac{\rho_1^{(2)}-\rho_2^{(2)}}{W_1^{(1)}}\right]\tan kl\tan kL\ \right\}\sin kz_2.$$

The total current along the second section is

$$i_1^{(2)} + i_2^{(2)} = jU_1^{(1)}\left\{\frac{\sin kl\cos k(L-z)}{W_1^{(1)}} + 2\cos kl\ \cdot\right.$$

$$\cdot\left[\frac{1}{W_1^{(2)}} - \frac{1}{W_2^{(2)}}\left\langle 1 - \frac{\rho_1^{(2)}-\rho_2^{(2)}}{W_1^{(1)}}\right\rangle\tan kl\tan kL\right]\cdot$$

$$\left.\cdot\sin k(l_2 - z)\ \right\} \tag{11}$$

One can see from the obtained expressions that the current along both sections of the monopole is distributed in accordance with sinusoidal law as in the known case of a monopole consisting of two segments each with different wave impedances (for example, with different wire diameters).

Let us write the expression for the total current along the monopole in the form

$$J_m(z) = A_m\cos(kl_m - z) + jB_m\sin(kl_m - z),\ l_{m+1}\le z\le l_m$$

In accordance with presented earlier formulas

$$A_1 = 0,\ B_1 = \frac{U_1^{(1)}}{W_1^{(1)}},\ A_2 = j\frac{U_1^{(1)}}{W_1^{(1)}}\sin kl,$$

$$B_2 = 2U_1^{(1)}\cos kl\ *$$

$$*\left[\frac{1}{W_1^{(2)}} - \frac{1}{W_2^{(2)}}\left\langle 1 - \frac{\rho_1^{(2)}-\rho_2^{(2)}}{W_1^{(1)}}\right\rangle\tan kl\tan kL\right].$$

The input reactance of the monopole is equal to the input impedance of the equivalent long line:

$$Z_A = \frac{e}{J(0)} = -j\frac{\left[1 - \frac{\rho_1^{(2)}}{W_1^{(1)}}\tan kl\tan kL\right]\cos^2 kL}{\frac{\tan kl\cos^2 kL}{W_1^{(1)}} + D\sin 2kL}, \tag{12}$$

where $D=\frac{1}{W_1^{(2)}}-\left[1-\frac{\rho_1^{(2)}-\rho_2^{(2)}}{W_1^{(1)}}\right]\frac{\tan kl\tan kL}{W_2^{(2)}}$.

The radiation resistance of the monopole is

$$R_\Sigma = 40k^2h_e^2, \qquad (13)$$

where h_e is the effective height

$$h_e=\frac{1}{kJ(0)}\sum_{m=1}^{2}\{A_m\sin k(l_m-l_{m+1})+jB_m[1-\cos k(l_m-l_{m+1})]\}==\frac{j}{kJ(0)}$$
$$\frac{U_1^{(1)}}{W_1^{(1)}}\{1+\sin kl\sin kL+\cos kl[2DW_1^{(1)}(1-\cos kL)-1]\} \qquad (14)$$

In the presented equations the wave impedances $W_{ns}^{(m)}$ and $\rho_{ns}^{(m)}$ are used. The magnitudes of these impedances are determined by potential coefficients:

$$\rho_{nn}^{(m)}=\rho_1^{(m)}=\frac{p_{nn}^{(m)}}{2\pi\varepsilon c}=60\,p_{nn}^{(m)},$$
$$\rho_{ns}^{(m)}\big|_{s\neq n}=\rho_2^{(m)}=\frac{p_{ns}^{(m)}}{2\pi\varepsilon c}=60\,p_{ns}^{(m)}$$

At the section with one wire

$$W_1^{(1)}=\rho_1^{(1)}=\frac{p_1^{(1)}}{2\pi\varepsilon c}=60\,p_1^{(1)},$$

at the section with two wires

$$\frac{1}{W_1^{(2)}}=\frac{\rho_1}{\rho_1^2-\rho_2^2},\ \frac{1}{W_2^{(2)}}=\frac{\rho_2}{\rho_1^2-\rho_2^2}.$$

The potential coefficients $\rho_n^{(m)}$ are calculated by the method of mean potentials in accordance with a real location of antenna wires. The simplest variant of this method is the method of Howe. It is easy to show that the mutual potential coefficient of two parallel wires with the same length, the dimensions and location of which are given in Fig. 7, equals

$$p(L,l,b)=\frac{1}{4\pi\varepsilon L}\Big[(L+l)arsh\frac{L+l}{b}-2L\,arsh\frac{l}{b}+$$
$$+(L-l)arsh\frac{L-l}{b}-\sqrt{(L+l)^2+b^2}-$$
$$-\sqrt{(L-l)^2+b^2}+2\sqrt{l^2+b^2}\Big].$$

Then the self potential coefficient of n-wire at m-section taking into account mirror image equals

$$p_{nn}^{(m)}=p\left[l_m-l_{m+1},0,a_n^{(m)}\right]-p\left[l_m-l_{m+1},l_m+l_{m+1},a_n^{(m)}\right],$$

where l_m and l_{m+1} are the coordinates of m-section tips, l_m+l_{m+1} is the distance between lower tips of

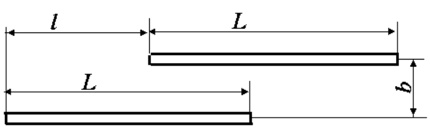

Fig. 7. The mutual location of wires

m-section and its mirror image, $a_n^{(m)}$ is the n-wire radius at the m-section. The mutual potential coefficient between the m-sections of n-wire and s-wire equals $p_{ns}^{(m)}=p\left[l_m-l_{m+1},0,b_{ns}^{(m)}\right]-p\left[l_m-l_{m+1},l_m+l_{m+1},b_{ns}^{(m)}\right]$. Here $b_{ns}^{(m)}$ is the distance between axes of n-wire and s-wire at the m-section.

V. MUTUAL INFLUENCE OF RADIATORS

As is well known, a current and an input impedance of a radiator depend on neighboring radiators currents and mutual impedances. One can write for a system of two radiators

$$e_1=J_{A1}Z_{11}+J_{A2}Z_{12},\ e_2=J_{A1}Z_{21}+J_{A2}Z_{22} \qquad (15)$$

Here e_1 and e_2 are the electromotive forces (emf) connected in the bases of the first and second monopoles, Z_{11} and Z_{22} are the self-impedances of the radiators, Z_{12} and Z_{21} are their mutual impedances.

Either of the two expressions (15) is Kirchhoff equation for a circuit. A set of Kirchhoff equations is valid at an arbitrary relative position of radiators. From (15) in particular it follows that

$$J_{A1}=\frac{e_1Z_{22}-e_2Z_{12}}{Z_{11}Z_{22}-Z_{12}Z_{21}},\ J_{A2}=\frac{e_2Z_{11}-e_1Z_{21}}{Z_{11}Z_{22}-Z_{12}Z_{21}},\qquad(16)$$

that is the current in each radiator is the sum of the currents produced by the self generator, as well as the generator of the neighboring radiator (because of the mutual coupling between radiators). The relation of these currents depends on the mutual coupling size that is depends on the radiators dimensions and location.

In order to determine the self and the mutual radiators impedances (with equal wire radii), we compare expressions (16) with expressions (3). Considering that

$$J_{11}=\frac{e_1Z_{22}}{Z_{11}Z_{22}-Z_{12}^2}=e_1(Y_1+Y_2),\ J_{12}=\frac{-e_2Z_{12}}{Z_{11}Z_{22}-Z_{12}^2}=e_2(Y_2-Y_1),$$
$$J_{22}=\frac{e_2Z_{11}}{Z_{11}Z_{22}-Z_{12}^2}=e_2(Y_1+Y_2),\ J_{21}=\frac{-e_1Z_{12}}{Z_{11}Z_{22}-Z_{12}^2}=e_1(Y_2-Y_1),$$

we obtain:

$$Z_{11}=Z_{22},Y_1+Y_2=\frac{Z_{11}}{Z_{11}^2-Z_{12}^2},Y_1-Y_2=\frac{Z_{12}}{Z_{11}^2-Z_{12}^2}.$$

Adding and subtracting left and right parts of last two expressions, we find:

$$2Y_1=\frac{1}{Z_{11}-Z_{12}},2Y_2=\frac{1}{Z_{11}+Z_{12}},$$

that is $Z_{11}+Z_{12}=\frac{1}{2Y_2},\ Z_{11}-Z_{12}=\frac{1}{2Y_1},$

and consequently

$$Z_{11}=Z_{22}=\frac{1}{4}\left(\frac{1}{Y_1}+\frac{1}{Y_2}\right),Z_{12}=\frac{1}{4}\left(\frac{1}{Y_2}-\frac{1}{Y_1}\right),$$

from which

$$Z_{11} = Z_{22} = Z_m(L+l,a_e) - j\frac{W_l}{4}\cot k(L+l_0) + R,$$
$$Z_{12} = Z_m(L+l,a_e) + j\frac{W_l}{4}\cot k(L+l_0) \quad .(17)$$

The calculation method for the transmission line is considered in Section 3. In accordance with it the wave impedance of the line is

$$W_l = 120\ln b/a\,,$$

and the elongation value is obtained from the expression (8). The input impedance of the monopole equals

$$Z_m = R_\Sigma + jX_A\,, \quad (18)$$

where the input resistance is $R_\Sigma = 40k^2h_e^2$, and the input reactance is $X_A = -W_2 \cot k(L+l_e)$.

At that the effective height h_e is equal to $h_e = h_1 + h_2$, where

$$h_1 = \frac{\sin kl_e}{k\sin k(L+l_e)}\tan\frac{kl}{2},$$
$$h_2 = \frac{\cos kl_e - \cos k(L+l_e)}{k\sin k(L+l_e)},$$
$$W_2 = 60\left[\ln\frac{2(L+l)}{\sqrt{ab}} - 1\right],\; W_1 = 60\left[\ln\frac{2(L+l)}{a} - 1\right],$$
$$l_e = \frac{1}{k}arc\cot\frac{W_1\cot kl}{W_l}.$$

The results of the self and mutual impedances calculation of the radiators with unequal lengths in the near region are given in Fig. 8. They are accomplished in accordance with the described method for variant with L=7.5, b=1.0, $2a$=0.05 depending from l (all dimensions are in centimeters).

VI. MULTIRADIATOR ANTENNA

One can apply the calculation method based on the theory of electrically coupled lines located under ground, which is used at the input impedance calculation of a linear radiator (monopole) composed of two wires with different lengths, for analysis of multiple-wire radiators. One of possible multi-radiators antenna variants is presented in Fig. 9*a*. An equivalent asymmetrical line is given in Fig. 9*b*. Antenna consists of the central radiator 1 with complex loading impedance Z_0 and side radiators 2 located around the central one along cylinder generatrixes and connected to the base of the radiator 1. The sake of simplicity let us consider that geometric dimensions of the side radiators are the same, though one can solve the problem in the general case. Then one may reduce the asymmetrical line to two-wire one and to obtain the solution for the current in an explicit form. At that first wire of an equivalent asymmetrical line is the central radiator, and the second wire is a system of *N-1* side radiators (*N* is the total quantity of radiators).

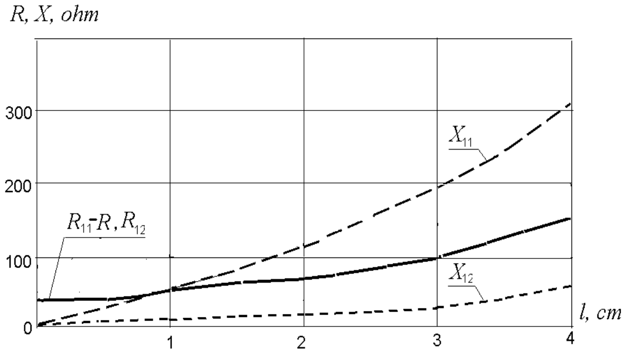

Fig. 8. The self and mutual impedances of the radiators with unequal lengths

If the wires of the line have different lengths, at that loading impedance is connected to one wire, it is necessary to divide the line into three sections. The expressions for the current and potential of *n*-wire in *m*-section look like (9). The boundary conditions for the two-wire asymmetrical line shown in Fig. 9*b* look in the following way

$$i_1^{(1)}\big|_{z_1=0} = i_2^{(3)}\big|_{z_3=0} = 0;\; i_1^{(1)}\big|_{z_1=l_1-l_2} = i_1^{(2)}\big|_{z_2=0};$$
$$i_1^{(2)}\big|_{z_2=l_2-l_3} = i_1^{(3)}\big|_{z_3=0};\; u_1^{(1)}\big|_{z_1=l_1-l_2} = u_1^{(2)} - i_1^{(2)}Z_0\big|_{z_2=0}; \quad (19)$$
$$u_1^{(2)}\big|_{z_2=l_2-l_3} = u_1^{(3)}\big|_{z_3=0};\; u_1^{(3)}\big|_{z_3=l_3} = u_2^{(3)}\big|_{z_3=l_3} = e.$$

These conditions mean the absence of the currents at the free ends of the wires and continuity of the current and the potential along each wire with the exception of the point where the load Z_0 is placed and the potential step occurs.

Substituting (9) into (19) and solving the equations system, we find all coefficients $I_n^{(m)}, U_n^{(m)}$ and afterwards the total current along the antenna as function of the coordinate $\varsigma = l_m - z_m$:

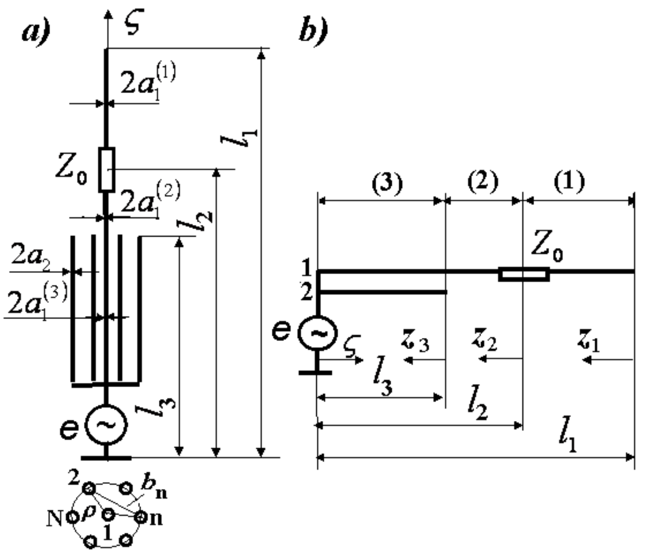

Fig. 9. Multi-radiators antenna with complex loading impedance (*a*) and an equivalent asymmetrical line (*b*)

$$J(\varsigma)=\begin{cases} j\dfrac{U_1}{\rho_1}\sin k(l_1-\varsigma), l_2\le\varsigma\le l_1, \\ jD_1\dfrac{U_1\sin k(l_{1e}-\varsigma)}{\rho_1\sin k(l_{1e}-l_3)}, l_3\le\varsigma\le l_2, \\ j\dfrac{U_1}{\rho_1}\left[D_1\cos k(l_3-\varsigma)+D_2\sin k(l_3-\varsigma)\right], 0\le\varsigma\le l_3. \end{cases} \quad (20)$$

where

$$D_1=\frac{\sin k(l_1-l_2)\sin k(l_{1e}-l_3)}{\sin k(l_{1e}-l_2)},$$

$$D_2=\rho_2\frac{\sin k(l_1-l_2)\cos k(l_{1e}-l_3)}{\sin k(l_1-l_2)}\left\{\frac{1}{W_{11}^{(3)}}-\frac{1}{W_{12}^{(3)}}+\right.$$

$$\left.+\left[\frac{1}{W_{22}^{(3)}}-\frac{1}{W_{12}^{(3)}}\right]\left[1-\frac{\rho_{11}^{(3)}-\rho_{12}^{(3)}}{\rho_{11}^{(3)}}\tan k(l_{1e}-l_3)\tan kl_3\right]\right\}$$

Here $U_1=U_1^{(1)}, W_{11}^{(1)}=\rho_{11}^{(1)}, \rho_2=W_{11}^{(2)}=\rho_{11}^{(2)}$; l_{1e} is a complex magnitude, which is obtained from the expression

$$Z_0-j\rho_{11}^{(1)}\cot k(l_1-l_2)=-j\rho_{11}^{(2)}\cot k(l_{1e}-l_2).$$

The input impedance of the asymmetrical line is

$$Z_l=e/J(0). \quad (21)$$

This expression with allowance for (20) permits to calculate approximately the reactance of the multi-radiators antenna, similarly to the fact that the expression for impedance of the equivalent long line permits to calculate approximately the reactance of the linear radiator. One can find the antenna impedance more exact, if to consider that antenna is the linear radiator, the current along which equals the total current of the multi-radiators antenna.

In accordance with the second statement of emf method the impedance of the linear radiator with the concentrated load is defined by the expression

$$Z_A=-\frac{1}{J^2(0)}\left[\int_0^{l_1}E_\varsigma J(\varsigma)d\varsigma-Z_0J^2(l_2)\right], \quad (22)$$

where E_ς is the tangent component of electric field, created at the radiator space by current $J(\varsigma)$ along its axis, and the current $J(\varsigma)$ is found from (20). The free term in square brackets of expression (22) is the power, which is dissipated by the complex load Z_0. The field E_ς is calculated in accordance with common expression. The function $J(\varsigma)$ is continuous in the all interval $0\le\varsigma\le l_1$ and has sinusoidal character in each antenna section. However the function $\frac{dJ}{d\varsigma}$ has a break at the section boundaries. Therefore

$$E_\varsigma=-j\frac{30}{k}\left\{\frac{2e^{-jkR_0}}{R_0}\frac{dJ(0)}{d\varsigma}-\left(\frac{e^{-ikR_1}}{R_1}+\frac{e^{-jkR_2}}{R_2}\right)\frac{dJ(l_1)}{d\varsigma}+\right.$$
$$\left.+\sum_{m=2}^{3}\left(\frac{e^{-jkR_{m1}}}{R_{m1}}+\frac{e^{-jkR_{m2}}}{R_{m2}}\right)\left[\frac{dJ(l_m+0)}{d\varsigma}-\frac{dJ(l_m-0)}{d\varsigma}\right]\right\}, \quad (23)$$

where $R_0=\sqrt{a^2+\varsigma^2}$, $R_{m1}=\sqrt{a^2+(l_m-\varsigma)^2}$, $R_{m2}=\sqrt{a^2+(l_m+\varsigma)^2}$, $\frac{dJ(l_m+0)}{d\varsigma}$ and $\frac{dJ(l_m-0)}{d\varsigma}$ are the values of derivatives on the right and on the left from the point $z=l_m$, a is the equivalent radius of antenna in the point ς. Substitution (20) into (23) gives

$$E_\varsigma=\frac{30U_1^{(1)}}{\rho_{11}^{(1)}}\left[\sum_{m=1}^{3}D_{1m}\left(\frac{e^{-jkR_{m1}}}{R_{m1}}+\frac{e^{-jkR_{m2}}}{R_{m2}}\right)+D_{14}\frac{e^{-jkR_0}}{R_0}\right],$$

where

$$D_{11}=1, D_{12}=D_1\frac{\cos k(l_{1e}-l_2)}{\sin k(l_{1e}-l_3)}-\cos k(l_1-l_2),$$

$$D_{13}=D_2-D_1ctgk(l_{1e}-l_3), D_{14}=2(D_1\sin kl_3-D_2\cos kl_3)$$

An electrical field in the far region at the distance r is

$$E_\theta=j30kJ(0)\frac{e^{-jkr}}{r}H(\theta), \quad (24)$$

where $H(\theta)$ is a generalized effective height, equal to

$$H(\theta)=\frac{\sin\theta}{J(0)}=\int_0^{l_1}J(\varsigma)e^{jk\varsigma\cos\theta}d\varsigma,$$

from which an effective height of asymmetric multi-radiators antenna is

$$h_e=\frac{1}{k(D_1\cos kl_3+D_2\sin kl_3)}\left\{1-\cos k(l_1-l_2)+\right.$$
$$\left.+D_1\left[\sin kl_3+\frac{\cos k(l_{1e}-l_2)-\cos k(l_{1e}-l_3)}{\sin k(l_{1e}-l_3)}\right]+D_2(1-\cos kl_3)\right\} \quad (25)$$

Radiation resistance of antenna is

$$R_\Sigma=R_A-R_{ls}, \quad (26)$$

where R_A is active component of an input impedance calculated with help of (22)), and R_{ls} is loss resistance in the load Z_0 referred to an antenna input:

$$R_{ls}=\mathrm{Re}\frac{J^2(l_2)Z_0}{J^2(0)}. \quad (27)$$

In Fig. 10 the characteristics of the multi-radiator antenna with 6 the same side radiators are given (N=7). The geometric dimensions (in meters) are $l_1=10, l_2=7, l_3=6.5, a_1^{(1)}=0.007,$ $a_1^{(2)}=a_1^{(3)}=0.02, a_2=0.01, \rho=0.15$. The load Z_0 is the parallel connection of resistor with active impedance $R=200$ ohm and of the coil with inductance £=14·10[-6] H.

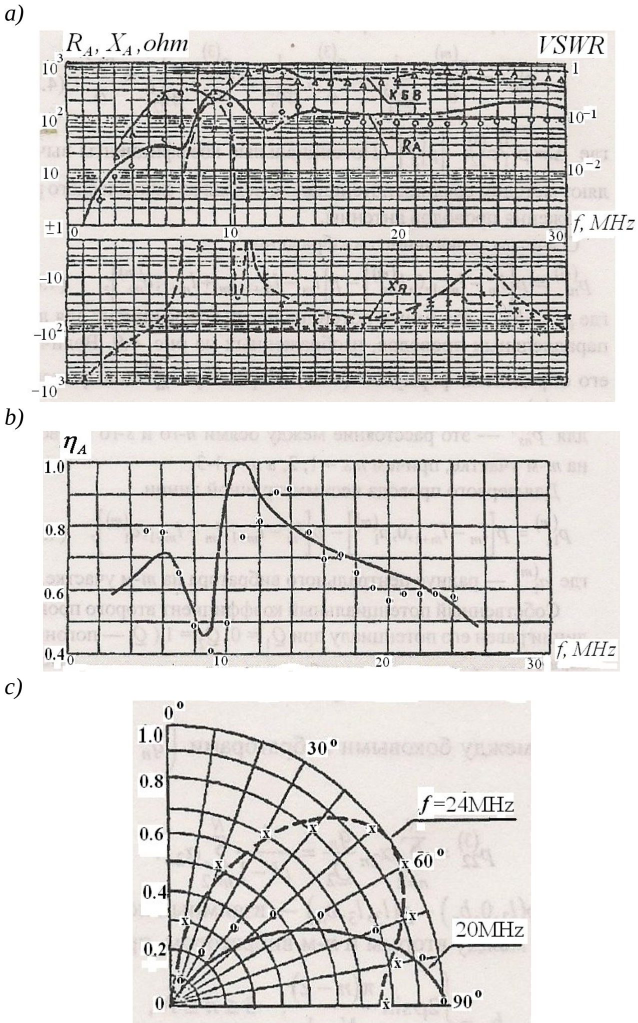

Fig. 10. The input characteristics (a), efficiency (b) and pattern (c) of multi-radiators antenna with complex loading impedance

The calculations are accomplished on the basis of described procedure. The experimental values are given together with calculated curves. The agreement is enough good.

VII. CONCLUSIONS

The obtained results show that using the folded dipoles theory and the superposition principle one can analyze the near region behavior of a system from two linear electric radiators with different lengths. For this purpose the system is divided into two circuits: a two-wire open-ended transmission line and a two-wire linear radiator (monopole). As a first order approximation, the length of the equivalent line's wires is quite close to that of the shorter wire, and the monopole length is equal to the length of the longer wire.

REFERENCES

[1] M. Bank and B. Levin, "The development of the cellular phone antenna with a small radiation of human organism tissues," *IEEE Antennas Prop. Magazine*, vol. 49, pp. 65-73, Aug. 2007.

[2] M. Bank, M. Haridim, and B. Levin, "Linear radiators in near zone," *in Proc. 8th Intern. Symposium on Electromagn. Compatibility and Electromagn. Ecology* (June 16-19, 2009), St. Petersburg (Russia), 2009, pp. 90-93.

[3] M. Haridim, B. Levin, M. Bank, and Z. Ibragimov. "Highly efficient SAR reduction using PIFA and MB antenna in mobile handsets," *in Proc. 3th Intern. Conference on Communic. and Inform. Technology* (December 29-31, 2009), Athens (Greece), 2009, pp. 165-167.

[4] M. Bank, M. Haridim, and B. Levin, "Impact of heterogeneous media on antennas' near field," ibid., pp. 183-185.

[5] M. Haridim, B. Levin, Z. Ibragimov, and M. Bank, "Analysis of SAR reduction of mobile phones in heterogeneous media," *International Journal of Communications*, vol. 3, no. 4, 2009, pp. 87-95.

[6] B. Levin, M. Bank, M. Haridim, and V. Tsingauz, "Dimensions of a dark spot produced by the compensation method," *in Proc. 9th Intern. Symposium on Electromagn. Compatibility joint with 20th Intern. Wroclaw Symposium on EMC* (September 13-17, 2010), Wroclaw (Poland), 2010, pp. 317-322.

[7] M. Haridim, B. Levin, M. Bank, S. Tapuchi, and Y. Trabelsi, "Line and monopole with wires of different length," *in Proc. 10th WSEAS Intern. Confer. on Electronics, Hardware, Wireless and Optical Communic.* (February 20-22, 2011), Cambridge (UK), 2011, pp. 314-317.

[8] B. M. Levin, *Monopole and Dipole Antennas for Ship Radio Communication.* Abris, 1998 (*in Russian).*

[9] U. J. Iossel, E. S. Kochanov, and M.G. Strunsky, *Calculation of Electrical Capacity.* Energoisdat, 1981, pp. 9 and 169 (*in Russian).*

[10] A. Pistolkors, *Antennas*. Sviyazizdat, 1947 (*in Russian).*

Motti Haridim received his M.Sc. degree in electrical engineering from the University of Washington in 1986 and the Ph.D. degree in electrical engineering from Technion of Israel in 1992. Since 1994 he has joined Holon Institute of Technology (HIT). During 2002-2008 Prof. Haridim was the head of the Dept. of Communication Engineering at HIT. His research activities focus mainly on the physical layer of communication systems, including optical systems, microwave photonics, RF communications, and antennas. He has published over 60 papers. Prof. Haridim acts as a consultant in RF communication systems and antennas to several large Israeli companies.

Boris Levin received his Ph.D. in radio physics from Central Research Institute of Automatic Devices in Leningrad in 1969 and Doctor of Sciences Degree in Physics and Mathematics (Russian equivalent of professor degree) from S.-Petersburg Polytechnic University in 1993. From 1963 to 1998 he worked for the Design Office "Svyazmorproyekt" of Russia Shipbuilding Department. From 2000 he works in Holon Institute of Technology (with interruption). He has authored 3 books about antennas and 77 papers in technical journals. His main research interests are in the fields of the electromagnetic theory, the theory of linear antennas and antenna optimization.

Michael Bank received the BA and M.Sc. degrees in communicational engineering from the Leningrad Institute of Communications in 1960, received the Ph.D. degree in 1969 in the field of FM signal detection. He received Doctor of Science degree (Russian equivalent of professor) in 1990. Since 1992 he is a consultant of Israel communicational company Bezeq and a lecturer in the Holon Academic Institute of Technology. His research interests include digital modulation, digital signal processing and RF receiving theory.