

Mutual Inductance and Magnetic Force Calculations Between Thick Bitter Circular Coil of Rectangular Cross Section With Inverse Radial Current and Thin Wall Superconducting Solenoid With Constant Azimuthal Current

S. Babic, and C. Akyel

Abstract— In this paper we calculate the mutual inductance and the magnetic force between the thick Bitter coil of rectangular cross section with the inverse radial current and the thin wall superconducting solenoid with the constant azimuthal current. The semi-analytical and the analytical expressions of these magnetic quantities are obtained over complete elliptic integrals of the first and second kind as well as Heuman's Lambda function. There is a simple integral which has to be solved numerically by some of numerical integrations. The results of this method are compared by those obtained by the modified filament method for the presented configuration. All results are in an excellent agreement.

Keywords— magnetic field, mutual inductance, magnetic force, Bitter coils, inverse radial current, azimuthal current.

I. INTRODUCTION

HIGH magnetic fields can be generated by a water-cooled magnet with Bitter coils or by hybrid magnet combined with an inner water-cooled magnet and an outset superconducting magnet. To have a strong and uniform magnetic field in the thin wall solenoid a high electric current is required but because of Joule effect and the resistance the coil will melt. This problem can be avoid where we can either cool the wire with water or use a superconducting wire. This is way the superconducting wall coil with the high uniform azimuthal current produces the strong uniform field. Otherwise, the high current density distribution of a Bitter coil in radial direction, however, is inversely proportional to the radius of Bitter coil, [1-5].

During a water-cooled magnet trip, the induced current in the superconducting coil changes as a function of decay time constant of the water-cooled magnet. The decay time constant

is determined by the self-inductance and the resistance of the water-cooled magnet. Thus, an accurate estimation of the inductance of Bitter coils is of primary importance in the hybrid magnet design. In addition, in such coils strong electromagnetic forces develop in an axial direction, causing mechanical stresses upon the supporting structures. For the proper design of such structures, the electromagnetic forces between coils must be calculated. Therefore, measurement of the mutual inductance and the magnetic force are of extremely important for the design viewpoint, [6-15]. To optimize the support structure of the Bitter coil, it is vital to have accurate evaluation of the magnetic force between Bitter coils. Y.Ren at al., [16-17] and J.T.Conway, [18] calculated the mutual inductance and the magnetic force for the following two coil arrangements, one between an ordinary coil and a Bitter coil, both with the rectangular cross sections, and the other between two Bitter coils with rectangular cross sections. These studies aim to help engineers and physicists calculate the mutual inductance and the magnetic force for when the thick Bitter coil of rectangular cross section structures are used. They obtained these magnetic quantities expressed by two integrations [16-17] and by the integration of Bessel functions [18]. Recently, S. Babic and C. Akyel calculated the mutual inductance and the magnetic force calculation between some Bitter coils [19-22] where they obtained these quantities in the analytical and the semi-analytical form. In this paper we calculate the mutual inductance and the magnetic force between the thick Bitter coil of rectangular cross section with the inverse radial current and the thin superconducting wall solenoid with the constant azimuthal current. In the approach presented in this paper, semi-analytical and analytical forms are expressed as functions of complete elliptic integrals of the first and second kind, the Heuman's Lambda function and a simple integral with continuous kernel function and very convenient for numerical integration. The results of this calculations will be compared to those obtained by the improved filament method for concerned configuration. The results obtained by these two methods are in an excellent agreement.

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II. NOMENCLATURE

I_1 (A): Current imposed in the thick coil of rectangular cross section
 I_2 (A): Current imposed in the wall solenoid
 N_1 : Number of turns of the thick coil of rectangular cross section
 N_2 : Number of turns of the wall solenoid
 R_1 (m) and R_2 (m): Inner and outer radius of the thick coil of rectangular cross section
 R (m): Radius of the thin wall solenoid
 z_1 (m): Axial position to the bottom of the thick coil of rectangular cross section
 z_2 (m): Axial position to the top of the thick coil of rectangular cross section
 z_3 (m): Axial position to the bottom of the thin wall solenoid
 z_4 (m): Axial position to the top of the thin wall solenoid
 M_B (H): Mutual inductance
 F_B (N): Magnetic force between coils
 J_1 and J_2 : Current densities at the thick coil of rectangular cross section and the thin wall solenoid respectively
 (r_I, z_I) and (r_{II}, z_{II}) : Arbitrary radial and axial positions along the thick coil of rectangular cross section and the thin wall solenoid respectively.

III. BASIC EXPRESSIONS

The thick Bitter circular coil of rectangular cross section with the inverse radial current and the thin superconducting wall solenoid with the constant azimuthal current are characterized by the following current densities [16-22] respectively:

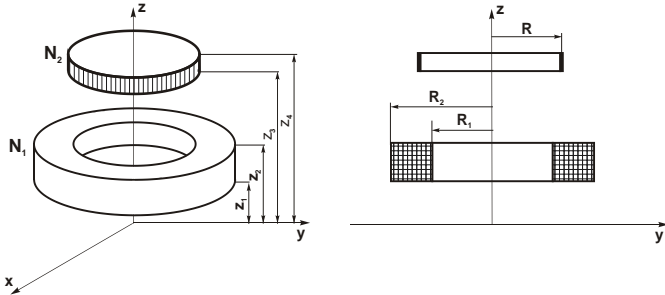


Fig.1. Bitter thick coil of rectangular cross section and thin wall solenoid

$$J_1 = \frac{N_1 I_1}{\ln \frac{R_2}{R_1}} \frac{1}{r_I} \quad (1)$$

and

$$J_2 = \frac{N_2 I_2}{(z_4 - z_3)} \quad (2)$$

The mutual inductance and the magnetic force between coils in question (See Fig. 1) can be calculated from the following expressions which can be easily obtained from

general expressions for these two important magnetic quantities [16-22]:

$$M_B = \frac{\mu_0 N_1 N_2 R \int_0^\pi \int_{R_1}^{R_2} \int_{z_1}^{z_2} \frac{\cos \theta dr_I dz_{II} d\theta}{r}}{(z_4 - z_3)(z_2 - z_1) \log \frac{R_2}{R_1}} \quad (3)$$

$$F_B = \frac{-\mu_0 N_1 N_2 I_1 I_2 R \int_0^\pi \int_{R_1}^{R_2} \int_{z_1}^{z_2} \frac{(z_{II} - z_I) \cos \theta dr_I dz_{II} d\theta}{r^3}}{(z_4 - z_3)(z_2 - z_1) \log \frac{R_2}{R_1}} \quad (4)$$

where

$$r = \sqrt{(z_{II} - z_I)^2 + r_I^2 + R^2 - 2r_I R \cos \theta}$$

IV. CALCULATION METHOD

Integrating in (3) and (4) over r_I, z_I, z_{II} and θ (substituting $\theta = \pi - \beta$) the mutual inductance and the magnetic force between previously mentioned coil combinations can be expressed respectively in an semi-analytical form as follow:

$$M_B = \frac{\mu_0 N_1 N_2}{(z_4 - z_3)(z_2 - z_1) \ln \frac{R_2}{R_1}} \sum_{n=1}^{n=8} (-1)^n T_n \quad (5)$$

$$F_B = \frac{\mu_0 N_1 N_2 I_1 I_2}{(z_4 - z_3)(z_2 - z_1) \ln \frac{R_2}{R_1}} \sum_{n=1}^{n=8} (-1)^{n-1} S_n \quad (6)$$

where

$$\rho_1 = \rho_4 = \rho_5 = \rho_8 = R_2$$

$$\rho_2 = \rho_3 = \rho_6 = \rho_7 = R_1$$

$$t_1 = t_2 = z_4 - z_1; \quad t_3 = t_4 = z_4 - z_2;$$

$$t_5 = t_6 = z_3 - z_1; \quad t_7 = t_8 = z_3 - z_2$$

$$\begin{aligned}
T_n = & \frac{\pi}{8} |t_n| \text{sign}(\rho_n - R)(\rho_n^2 - 3R^2)[1 - \Lambda_0(\varepsilon_n, k_n)] - \frac{\pi}{12} |t_n| (t_n^2 - \\
& 3R^2)V_n + \frac{k_n t_n^2}{8\sqrt{R\rho_n}} \frac{(\rho_n - R)(\rho_n^2 - 3R^2)}{\rho_n + R} K(k_n) - \frac{k_n(t_n^2 - 3R^2)}{6\sqrt{R\rho_n}} [R^2 + \\
& t_n^2 - R\sqrt{R^2 + t_n^2}]K(k_n) + \frac{1}{18k_n^5\sqrt{R\rho_n}} [k_n^4(-58R^3\rho_n - 8R^4 - 8R^2t_n^2 - \\
& 36R^2\rho_n^2 - 3R\rho_n t_n^2 - 18R\rho_n^3) + k_n^2(64R^3\rho_n + 16R^4 + 16R^2t_n^2 + \\
& 72R^2\rho_n^2) - 64R^3\rho_n]E(k_n) + \frac{1}{72k_n^5\sqrt{R\rho_n}} [k_n^6(12t_n^4 - 36R^4 - 9\rho_n^2t_n^2 - \\
& 6R\rho_n t_n^2 - 51R^2t_n^2 - 108R^2\rho_n^2 - 108R^3\rho_n - 36R\rho_n^3) + k_n^4(344R^3\rho_n + \\
& 64R^4 + 64R^2t_n^2 + 12R\rho_n t_n^2 + 288R^2\rho_n^2 + 72R\rho_n^3) + k_n^2(-384R^3\rho_n - \\
& 64R^4 - 64R^2t_n^2 - 288R^2\rho_n^2) + 256R^3\rho_n]K(k_n) + R^2t_n I_{0n}
\end{aligned}$$

$$\begin{aligned}
S_n = & \frac{\pi}{8} \text{sign}(t_n)\text{sign}(\rho_n - R)(\rho_n^2 - 3R^2)[1 - \Lambda_0(\varepsilon_n, k_n)] - \\
& \frac{\pi}{4} \text{sign}(t_n)(t_n^2 - R^2)V_n + \frac{3k_n t_n}{8\sqrt{R\rho_n}} [(R + \rho_n)^2 + t_n^2]E(k_n) + \\
& \frac{k_n t_n}{8\sqrt{R\rho_n}} [t_n^2 - 2R^2 - 4\rho_n^2 + \frac{(\rho_n - R)(\rho_n^2 - 3R^2)}{\rho_n + R}] - \\
& \frac{4(t_n^2 - R^2)\sqrt{R^2 + t_n^2}}{\sqrt{R^2 + t_n^2} + R} K(k_n) + R^2 I_{0n}
\end{aligned}$$

$$k_n^2 = \frac{4R\rho_n}{(R + \rho_n)^2 + t_n^2}, \quad h_n = \frac{4R\rho_n}{(R + \rho_n)^2},$$

$$m_n = \frac{2R}{\sqrt{R + t_n^2} + R} \leq 1$$

$$\theta_{1n} = \arcsin \frac{|t_n|}{\sqrt{R^2 + t_n^2} + R}, \quad \theta_{2n} = \arcsin \sqrt{\frac{1 - m_n}{1 - k_n^2}}, \quad k_n^2 \leq m_n,$$

$$\varepsilon_n = \arcsin \sqrt{\frac{1 - h_n}{1 - k_n^2}}, \quad k_n^2 \leq h_n$$

$$V_n = 1 - \Lambda_0(\theta_{1n}, k_n) + \text{sgn}(\sqrt{R^2 + t_n^2} - \rho_n)[1 - \Lambda_0(\theta_{2n}, k_n)]$$

$$J_{0n} = \int_0^{\pi/2} \sinh^{-1} \frac{t_n}{\sqrt{R^2 + \rho_n^2 + 2R\rho_n \cos 2\beta}} d\beta$$

SINGULAR CASES

For $t_n = 0$ and $k_n^2 \neq 1$

$$\begin{aligned}
T_{n0} = & \frac{1}{18k_{n0}^5\sqrt{R\rho_n}} [k_{n0}^4(-58R^3\rho_n - 8R^4 - 36R^2\rho_n^2 - 18R\rho_n^3) + \\
& k_{n0}^2(64R^3\rho_n + 16R^4 + 72R^2\rho_n^2) - 64R^3\rho_n]E(k_{n0}) + \\
& \frac{1}{72k_{n0}^5\sqrt{R\rho_n}} [k_{n0}^6(36R^4 - 108R^2\rho_n^2 - 108R^3\rho_n - 36R\rho_n^3) + \\
& k_{n0}^4(344R^3\rho_n + 64R^4 + 288R^2\rho_n^2 + 72R\rho_n^3) + k_{n0}^2(-384R^3\rho_n - \\
& 64R^4 - 288R^2\rho_n^2) + 256R^3\rho_n]K(k_{n0}) \\
k_{n0}^2 = & \frac{4R\rho_n}{(R + \rho_n)^2} = h_n
\end{aligned} \tag{7}$$

$$S_{n0} = 0 \tag{8}$$

For $t_n = 0$ and $k_n^2 = 1$

$$T_{n01} = -\frac{16}{9}R^3 \text{ or } -\frac{16}{9}\rho_n^3 \tag{9}$$

$$S_{n01} = 0 \tag{10}$$

$E(k)$ and $K(k)$ are complete integrals of the first and second kind and $\Lambda_0(\varepsilon, k)$ is Heuman's lambda function, [23-24].

V. MODIFIED FILAMENT METHOD

In [16-17] the mutual inductance is calculated between the Bitter coil of rectangular cross section and the superconducting coil with uniform current density by using the filament method. Here we give the modified formulas for the mutual inductance and the magnetic force for the thick Bitter coil of rectangular cross section with inverse radial current and the thin wall solenoid with the constant azimuthal current (See Fig. 2) using the filament method. Applying some modification in the mutual inductance calculation given in [16-17] we deduced the mutual inductance and the magnetic force between presented coils as follows:

$$M_{BF} = \frac{N_1 N_2 (R_4 - R_3) \sum_{g=-K}^{g=K} \sum_{p=-m}^{p=m} \sum_{l=-n}^{l=n} \frac{M_F(g, p, l)}{r_{II}(l)}}{(2K+1)(2m+1)(2n+1) \ln \frac{R_4}{R_3}} \tag{11}$$

$$F_{BF} = \frac{N_1 N_2 I_1 I_2 (R_4 - R_3) \sum_{g=-K}^{g=K} \sum_{p=-m}^{p=m} \sum_{l=-n}^{l=n} \frac{F_F(g, p, l)}{r_{II}(l)}}{(2K+1)(2m+1)(2n+1) \ln \frac{R_4}{R_3}} \quad (12)$$

where

$$M_F(g, p, l) = \frac{\mu_0 \sqrt{R r_{II}(l)}}{k(g, p, l)} [(2 - k^2(g, p, l)) K(k(g, p, l)) - 2E(k(g, p, l))]$$

$$F_F(g, p, l) = \frac{\mu_0 I_1 I_2 z(p) k}{4\sqrt{R r_{II}(l)}} \left[\frac{2 - k^2(g, p, l)}{1 - k^2(g, p, l)} E(k(g, p, l)) - 2K(k(g, p, l)) \right]$$

$$r_{II}(l) = R_{II} + \frac{h_{II}}{(2n+1)} l, \quad l = -n, \dots, 0, \dots, n$$

$$R_{II} = \frac{R_3 + R_4}{2}, \quad h_{II} = R_4 - R_3$$

$$z(g, p) = c - \frac{a}{(2K+1)} g + \frac{b}{(2m+1)} p$$

$$g = -K, \dots, 0, \dots, K; \quad p = -m, \dots, 0, \dots, m$$

$$k^2(g, p, l) = \frac{4R r_{II}(l)}{(R + r_{II}(l))^2 + z^2(g, p)}$$

Expressions (11) and (12) will be used to confirm the validity of formulas (5) and (6).

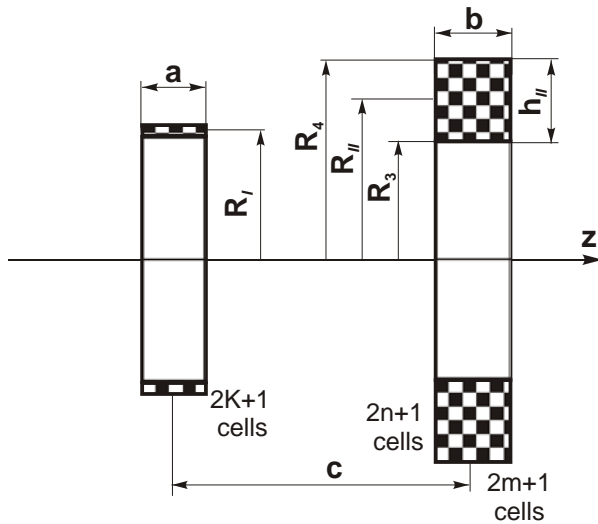


Fig. 2. Configuration of mesh matrix: Thin wall solenoid -Thick Bitter coil with rectangular cross section

VI. NUMERICAL VALIDATION

6.1. Example 1.

To prove the new approach we give many examples which cover either the regular or the singular cases. In these examples all coils are with the unit currents. Also, the coil dimensions and the number of turns are given. For the comparative method (improved filament method) the number of subdivisions for each coil is given also. Obviously, the precision of the filament method depends on the number of subdivisions of coils. Increasing the number of the subdivisions of coils will increase the computational time. Our goal is to verify the results of the presented semi-analytical method so that we will fix the number of subdivisions of coils ($N = n = m = 50$) in the following examples without taking into consideration the computational time in the calculations.

First coil: $R_1 = 0.3$ m, $R_2 = 0.4$ m, $z_1 = -0.4$ m, $z_2 = 0.4$ m, $N_1 = 100$.

Second coil: $R = 0.5$ m, $z_3 = -0.2$ m, $z_4 = 0.2$ m, $N_2 = 100$.

Applying the presented method (5) and (6) we obtain:

$$M_B = 3.9026415 \text{ mH}$$

$$F_B = 0 \text{ N}$$

By using the modified filament method (11) and (12) we obtain:

$$M_{BF} = 3.9026558 \text{ mH}$$

$$F_{BF} = 0 \text{ N}$$

6.2. Example 2.

First coil: $R_1 = 0.3$ m, $R_2 = 0.4$ m, $z_1 = -0.2$ m, $z_2 = 0.2$ m, $N_1 = 100$.

Second coil: $R = 0.5$ m, $z_3 = 0.4$ m, $z_4 = 0.8$ m, $N_2 = 100$.

Applying the presented method (5) and (6) we obtain:

$$M_B = 1.2320891 \text{ mH}$$

$$F_B = -3.7145291 \text{ mN}$$

By using the modified filament method (11) and (12) we obtain:

$$M_{BF} = 1.2320724 \text{ mH}$$

$$F_{BF} = -3.71446853 \text{ mN}$$

6.3. Example 3.

First coil: $R_1 = 0.3$ m, $R_2 = 0.4$ m, $z_1 = -0.2$ m, $z_2 = 0.2$ m, $N_1 = 100$.

Second coil: $R = 0.3$ m, $z_3 = 0.4$ m, $z_4 = 0.8$ m, $N_2 = 100$.

Applying the presented method (5) and (6) we obtain:

$$M_B = 653.81529 \mu\text{H}$$

$$F_B = -2.4739581 \text{ mN}$$

By using the modified filament method (11) and (12) we obtain:

$$M_{BF} = 653.80025 \mu\text{H}$$

$$F_{BF} = -2.4738708 \text{ mN}$$

6.4. Example 4.

First coil: $R_1 = 0.3 \text{ m}$, $R_2 = 0.4 \text{ m}$, $z_1 = -0.2 \text{ m}$, $z_2 = 0.2 \text{ m}$, $N_1 = 100$.

Second coil: $R = 0.3 \text{ m}$, $z_3 = 0.2 \text{ m}$, $z_4 = 0.8 \text{ m}$, $N_2 = 100$.

This is the singular case.

Applying the presented method (5), (6), (7) (8), (9) and (10) we obtain:

$$M_B = 1.1581969 \text{ mH}$$

$$F_B = -5.3490210 \text{ mN}$$

By using the modified filament method (11) and (12) we obtain:

$$M_{BF} = 1.1581287 \text{ mH}$$

$$F_{BF} = -5.3484753 \text{ mN}$$

6.5. Example 5.

First coil: $R_1 = 0.3 \text{ m}$, $R_2 = 0.4 \text{ m}$, $z_1 = -0.2 \text{ m}$, $z_2 = 0.2 \text{ m}$, $N_1 = 100$.

Second coil: $R = 0.3 \text{ m}$, $z_3 = 0 \text{ m}$, $z_4 = 0.8 \text{ m}$, $N_2 = 100$.

This is the singular case.

Applying the presented method (5), (6), (7) (8), (9) and (10) we obtain:

$$M_B = 2.0423685 \text{ mH}$$

$$F_B = -6.1491443 \text{ mN}$$

By using the modified filament method (11) and (12) we obtain:

$$M_{BF} = 2.042447774531412 \text{ mH}$$

$$F_{BF} = -6.148900903658425 \text{ mN}$$

6.6. Example 7.

First coil: $R_1 = 1 \text{ m}$, $R_2 = 3 \text{ m}$, $z_1 = 1 \text{ m}$, $z_2 = 2 \text{ m}$, $N_1 = 100$.

Second coil: $R = 3 \text{ m}$, $z_3 = 2 \text{ m}$, $z_4 = 3 \text{ m}$, $N_2 = 100$.

This is the singular case.

Applying the presented method (5), (6), (7) (8), (9) and (10) we obtain:

$$M_B = 21.492989 \text{ mH}$$

$$F_B = -11.951579 \text{ mN}$$

By using the modified filament method (11) and (12) we

obtain:

$$M_{BF} = 21.493041 \text{ mH}$$

$$F_{BF} = -11.951458 \text{ mN}$$

6.7. Example 8.

First coil: $R_1 = 1 \text{ m}$, $R_2 = 3 \text{ m}$, $z_1 = 1 \text{ m}$, $z_2 = 2 \text{ m}$, $N_1 = 100$.

Second coil: $R = 1 \text{ m}$, $z_3 = 0 \text{ m}$, $z_4 = 1 \text{ m}$, $N_2 = 100$.

This is the singular case.

Applying the presented method (5), (6), (7) (8), (9) and (10) we obtain:

$$M_B = 6.9589585 \text{ mH}$$

$$F_B = 6.3730223 \text{ mN}$$

By using the modified filament method (10) and (11) we obtain:

$$M_{BF} = 6.9589276 \text{ mH}$$

$$F_{BF} = 6.3727538 \text{ mN}$$

6.8. Example 9.

First coil: $R_1 = 2 \text{ m}$, $R_2 = 4 \text{ m}$, $z_1 = 0 \text{ m}$, $z_2 = 2 \text{ m}$, $N_1 = 100$.

Second coil: $R = 4 \text{ m}$, $z_3 = 1 \text{ m}$, $z_4 = 3 \text{ m}$, $N_2 = 100$.

This is the singular case.

Applying the presented method (5), (6), (7) (8), (9) and (10) we obtain:

$$M_B = 43.886289 \text{ mH}$$

$$F_B = -13.012766 \text{ mN}$$

By using the modified filament method (10) and (11) we obtain:

$$M_{BF} = 43.886310 \text{ mH}$$

$$F_{BF} = -13.013218 \text{ mN}$$

As we can see all results obtained by two different approaches are in an excellent agreement. We give more examples which represent the singular cases so that all cases either regular or singular were covered. We kept the eight significant figures and the bold digits illustrate the significant digits with the same accuracy in both calculation.

VII. CONCLUSION

The new accurate mutual inductance and magnetic force formulas for the system of the thick Bitter coaxial coil of rectangular cross section with the inverse radial current and the thin superconducting wall solenoid with the constant azimuthal current in air are derived and presented in this paper. All expressions either for the mutual inductance or the magnetic force is obtained in the semi-analytical form expressed over complete elliptic integrals of the first and second kind and the Heuman's Lambda function. There is a simple integral which has not the analytical solution, and it is solved numerically by the Gaussian numerical integration. All singular case are obtained in the close form. Also we gave in this paper the improved formulas for the mutual inductance and the magnetic

force between treated coils by using the modified filament method. All results obtained by two presented methods are in an excellent agreement.

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