

Graham's anomalies in case of parallel computation electromagnetic phenomena

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Abstract—Parallel computing becomes common tool to accelerate long-lasting electromagnetic computations. In some cases their realization does not bring an expecting gain. In this paper the authors present the results of the simulations from application created in order to show selected anomalies in parallel computing. The way of parallel system implementation is presented on the basic of direct graph model with the set of precedence constraints. The Graham's anomalies are discussed during optimization tasks scheduling issue, towards makespan criterion. The research was focused on three most frequently applied cases during acceleration the computations i.e. shortened tasks times execution, add additional computational power and reduction of selected precedence constraints. All phenomena are discussed and presented with the usage of Gantt's charts.

Keywords—Graham's anomalies, parallel computing, task schedule, makespan criterion.

I. INTRODUCTION

DYNAMIC development of numerical algorithms and information technology enabled precisely analysis and implementation many physical phenomena using adequately mathematical models. One of these issue is analysis of dynamics of electromagnetic devices in application to automatics and robotics. The devices are modeled using 3D distributed parameters systems. After discretisation of partial or integral equations that describe electromagnetic phenomena, a large and sparse system of equations are obtained [10-13].

The computation performs execution many mathematical operations. In most cases it make possible to decrease the time of calculation. Then very often is made an effort to realise multi-processors system. Fortunately, some algorithms can be executed simultaneously on principle of parallel computation, what in most cases considerable allows to cut down the time of calculation.

This paper presents Graham's anomalies phenomenon as a menace that appears during fragmentation and parallel code of the application. There is discussed and presented a general problem in scheduling theory called the minimum makespan scheduling.

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II. MODEL OF AN ELECTROMAGNETIC SYSTEM

Using the magnetic vector potential \mathbf{A} and electric scalar potential V as electromagnetic field variables, the electric field intensity \mathbf{E} in conducting region (Ω_C) and magnetic flux density \mathbf{B} in conducting and non-conducting region ($\Omega_C \cup \Omega_N$) is defined as [11]:

$$\mathbf{B} = \nabla \times \mathbf{A} \text{ in } \Omega_C \cup \Omega_N \quad (1)$$

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} - \nabla V + \mathbf{v} \times (\nabla \times \mathbf{A}) \text{ in } \Omega_C \quad (2)$$

In this case, the boundary value problem in terms of potentials is expressed as follows:

$$\nabla \times \left(\frac{1}{\mu} \nabla \times \mathbf{A}(t) \right) = \mathbf{j}(t) - \sigma \left(\frac{\partial \mathbf{A}}{\partial t} + \nabla V \right) \text{ in } \Omega_C \cup \Omega_N \quad (3)$$

$$-\nabla \cdot \sigma \left(\frac{\partial \mathbf{A}}{\partial t} + \nabla V - \mathbf{v} \times (\nabla \times \mathbf{A}) \right) = 0 \text{ in } \Omega_C \quad (4)$$

where μ is a permeability, σ represents conductivity, \mathbf{v} represents velocity of movable armature and $\mathbf{j}(t)$ current density of the thin coil. If voltage excitation is given, the electric circuit system of equations expressed in term of magnetic vector potential must be considered as [13]:

$$\begin{aligned} \frac{d}{dt} \oint_{l_1} \mathbf{A}(t) d\mathbf{l} + R_1 i_1(t) &= u_1(t) \\ &\vdots \end{aligned} \quad (5)$$

$$\frac{d}{dt} \oint_{l_n} \mathbf{A}(t) d\mathbf{l} + R_n i_n(t) = u_n(t)$$

For n-phase voltage forced system above equations are expressed by matrix of dynamic impedances. Discrete form of (7) can be defined as bellow:

$$\begin{bmatrix} Z_{11} & Z_{12} & \dots & Z_{1n} \\ Z_{21} & Z_{22} & \dots & Z_{2n} \\ \dots & \dots & & \\ Z_{n1} & Z_{n2} & \dots & Z_{nn} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \dots \\ I_n \end{bmatrix} = \begin{bmatrix} U_1 \\ U_2 \\ \dots \\ U_n \end{bmatrix} \quad (6)$$

where:

$$Z_{ii} = R_i + \frac{j\omega}{I_i} \oint_{l_i} \mathbf{A} d\mathbf{l}_i, \quad Z_{ik} = \frac{j\omega}{I_i} \oint_{l_k} \mathbf{A} d\mathbf{l}_k \quad (7)$$

Global impedance matrix includes own impedances Z_{ji} and mutual impedance $Z_{jk} = Z_{kj}$. For $I_i = 1$ A ($j=1,2, \dots, n$) and for $I_{k \neq i} = 0$ are performed calculations for matrix of impedance \mathbf{Z} , next the current vector \mathbf{I} in both coils are calculated from formulation:

$$\mathbf{I} = \mathbf{Z}^{-1} \mathbf{U} \quad (8)$$

when a voltage vector \mathbf{U} is known. Calculation of the force is performed using the Maxwell's stress tensor method. The force density is given by following formula:

$$\mathbf{f} = \nabla \cdot \mathbf{T} \quad (9)$$

where \mathbf{T} denotes modified Maxwell's stress tensor [9] proposed as follows (10):

$$\mathbf{T} = \begin{bmatrix} \frac{1}{2\mu_0} B_x^2 - \frac{1}{2} \mu_0 (H_y^2 + H_z^2) & H_y B_x & H_z B_x \\ H_x B_y & \frac{1}{2\mu_0} B_y^2 - \frac{1}{2} \mu_0 (H_x^2 + H_z^2) & H_z B_y \\ H_x B_z & H_y B_z & \frac{1}{2\mu_0} B_z^2 - \frac{1}{2} \mu_0 (H_x^2 + H_y^2) \end{bmatrix}$$

Then total force is defined

$$\mathbf{F} = \int_{\Omega} \mathbf{f} d\Omega \quad (11)$$

Motion problem is solved by sequentially coupled model with time step verification according to the fixed grid distance in motion direction [9]. Choosing the displacement and velocity as state of 1-DOF mechanical motion, the equation is solved by recurrence Euler's algorithm in state space form as a system of first order differential equations [13]:

$$\frac{d}{dt} \begin{bmatrix} \mathbf{x} \\ \mathbf{v} \end{bmatrix} = \begin{bmatrix} 0 & \mathbf{1} \\ -\frac{\mathbf{K}}{\mathbf{M}} & -\frac{\mathbf{B}}{\mathbf{M}} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{v} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{\mathbf{M}} \end{bmatrix} \mathbf{F} \quad (12)$$

where \mathbf{x} represents displacement and \mathbf{v} represents velocity of movable armature used in boundary value equations (3) - (4).

III. NUMERICAL REALISATION

For given voltage value an iterative procedure includes calculation of equation (6) in discrete form:

$$\begin{bmatrix} Z_{11} & Z_{12} & \dots & Z_{1n} \\ Z_{21} & Z_{22} & \dots & Z_{2n} \\ \dots & \dots & \dots & \dots \\ Z_{n1} & Z_{n2} & \dots & Z_{nn} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \dots \\ I_n \end{bmatrix} = \begin{bmatrix} U_1 \\ U_2 \\ \dots \\ U_n \end{bmatrix} \quad (13)$$

where own and mutual impedances are calculated as:

$$Z_{jj} = R_j + \frac{1}{\Delta t} \oint_{l_j} \mathbf{a}_{jj} d\mathbf{l} \quad Z_{ik} = \frac{j\omega}{I_i} \oint_{l_k} \mathbf{A} d\mathbf{l}_k, \text{ for } j \neq k \quad (14)$$

and right side of equation (13) performs:

$$b_j = u_j(t + \Delta t) - R_j i_j(t + \Delta t) + \frac{1}{\Delta t} \oint_{l_j} (\mathbf{a}_{jj}(t + \Delta t) - \mathbf{a}_{jj}^w(t)) d\mathbf{l} \quad (15)$$

In equations (14)-(15) exists vector potential \mathbf{a} for one-ampere test. In the last equation (14) is given potential \mathbf{a}^w which takes eddy current into consideration. In second step the boundary equation (3) - (4) in integral form are solved by a finite element method with linear shape functions of the potentials. The global matrix system iterative procedure of the system becomes [13]:

$$\begin{bmatrix} \left(\mathbf{C} + \frac{\mathbf{D}}{\Delta t_i} \right) & \mathbf{E} \\ \mathbf{F} + \mathbf{H} & \mathbf{G} \Delta t_i \end{bmatrix} \times \begin{bmatrix} \mathbf{z}_{i+1} \\ \mathbf{y}_{i+1} \end{bmatrix} = \begin{bmatrix} \mathbf{r}_{i+1} + \frac{\mathbf{D}}{\Delta t_i} \mathbf{z}_i \\ \mathbf{F} \mathbf{z}_i \end{bmatrix} \quad (16)$$

In above equations \mathbf{z} represents vector of unknown of magnetic vector potential \mathbf{A} , \mathbf{y} represents vector of unknown electric scalar potential \mathbf{V} and \mathbf{r} is a vector of current density $\mathbf{j}(t)$. Matrices \mathbf{C} , \mathbf{D} , \mathbf{E} , \mathbf{F} , \mathbf{G} and \mathbf{H} are obtained as result of discretisation of the following formulations:

$$\int_{\Omega} \nabla \times \left(\frac{1}{\mu} \nabla \times \mathbf{A} \right) d\Omega - \int_{\Omega} \sigma (\mathbf{v} \times (\nabla \times \mathbf{A})) d\Omega, \int_{\Omega} \sigma \frac{\partial \mathbf{A}}{\partial t} d\Omega,$$

$$\int_{\Omega} \sigma \nabla V d\Omega, \oint_S \sigma \frac{\partial \mathbf{A}}{\partial t} dS, \oint_S \sigma \nabla V dS, - \oint_S \sigma (\mathbf{v} \times (\nabla \times \mathbf{A})) dS,$$

Next \mathbf{r} as result of discretisation $\int_V \mathbf{j} dV$. Obtained system of

equation is non-symmetric. Matrix \mathbf{C} is non-symmetric and diagonally dominant, \mathbf{D} is a diagonal matrix. Matrix \mathbf{G} is symmetric. Coefficients of \mathbf{C} , \mathbf{D} , \mathbf{E} , \mathbf{F} , \mathbf{G} , \mathbf{H} depends on materials description and type of discretisation. Included in formulations (5) and (6) terms $\int_{\Omega} \sigma (\mathbf{v} \times (\nabla \times \mathbf{A})) d\Omega$, and

$-\oint_S \sigma (\mathbf{v} \times (\nabla \times \mathbf{A})) dS$ produces non-symmetric coefficients

in global matrix structure. The magnetic vector potential \mathbf{A} and electric scalar potential \mathbf{V} are calculated by step by step computation process of equation system (16). For this type of computation the conjugate gradient method with preconditioner called BiCG is worked up [13].

Total force influencing on movable armature is obtained from (11)-(13) as a combination of calculated potentials \mathbf{A} , \mathbf{V} . The method is widely described in [9]. Then is solving the discrete state space equation of mechanical system to obtain velocity and displacement. The differential equation of a motion in discrete form

$$\begin{bmatrix} \mathbf{x}_{i+1} \\ \mathbf{v}_{i+1} \end{bmatrix} = \begin{bmatrix} 1 & \Delta t_i \\ -\frac{\mathbf{K}}{\mathbf{M}} \Delta t_i & 1 - \frac{\mathbf{B}}{\mathbf{M}} \Delta t_i \end{bmatrix} \begin{bmatrix} \mathbf{x}_i \\ \mathbf{v}_i \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{\Delta t_i}{\mathbf{M}} \end{bmatrix} \mathbf{F}_i \quad (17)$$

is successively solved in each iterative step Δt_i to get the armature displacement \mathbf{x} .

IV. MODEL DEFINITION, NOTATION AND PRELIMINARIES

The calculating cluster of four PC computers with Linux operating system is used to solve presented problem. Linux includes BSD (Berkeley Software Distribution) socket interface, which is the standard network communication protocol. TCP/IP connections as well as communication between processors in Unix domain are used. Hardware cluster consists of: one Intel Pentium IV 1.8 GHz with 1.5 GB RAM system memory running as client application and two Intel Pentium IV 1.8 GHz 256 MB as server applications.

Let's describe matrix $\begin{bmatrix} \left(C + \frac{D}{\Delta t_i} \right) & E \\ F + H & G\Delta t_i \end{bmatrix}$ from (16) as M

and right side of (16) as vector b . The idea of parallel calculation is to cut matrix A and vector b for three fragments simultaneously keeping non zero extortion in all new part of vector b .

For the sake of individuality of each solving problem and to optimise load balancing between cluster nodes the expert's knowledge was used by giving cut dimensions. The total space Ω is split into for three calculating subspace ($\Omega \in \Omega_1 \cup \Omega_2 \cup \Omega_3$) taking consideration boundary conditions on division area into account. Algorithm of solving field issue is divided on parallel field calculation in each from three domains.

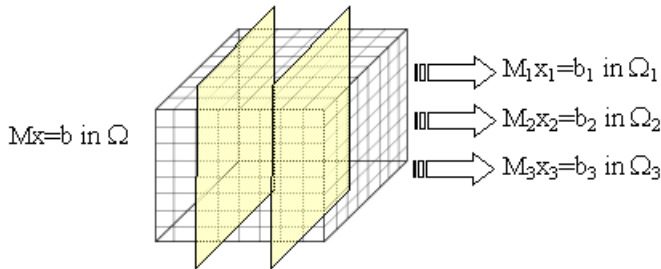


Fig.1 The model space Ω split into three parts

In this case we get three systems of equations type $A_N X_N = b_N$ solving by PCG algorithm on N node. Continuity of field functions at cutting area has to be kept and internal boundary condition at data divide and integrate should be retained. Then the potentials of next cutted walls in calculate subspace have to accept the same values.

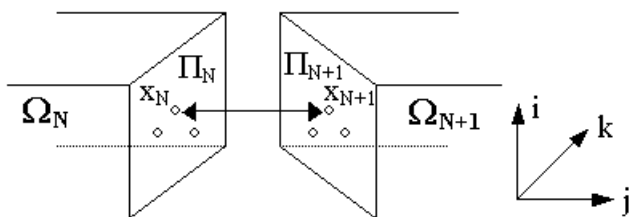


Fig.2 The way of two subspace marge

On $\Pi_N \subset \Omega_N$ and $\Pi_{N+1} \subset \Omega_{N+1}$ areas must occur following relation:

$$\forall_{M} X_{N,M} = X_{N+1,M} \text{ where } X_N \in \Pi_N \text{ and } X_{N+1} \in \Pi_{N+1} \quad (18)$$

Message passing quite often has application in matrix calculus, where individual processors execute computation on selected part of main matrix. In the discussed approach N nodes solve Ω_N part of space Ω . If stopping criteria in the iterative algorithm are achieved on each servers then the results are sent to a client application. Next they are integrated. The whole algorithm is recurred till the maximal number of iteration or stopping criteria is reached.

To realise the communication in the cluster architecture the authors used sockets service. It is implemented at kernel in systems type Unix and execution the operations is enable using system's functions similarly to files. Stream sockets (SOCK_STREAM) use TCP protocol, which main virtues are: sequencing, error control and connection-orient.

V. PROBLEM STATEMENT

The authors created an application, which allows to detect potential anomalies on the basis of structure of tasks' schedule graph (Fig. 1). As an example specific model is presented. The Graham's anomalies towards minimum makespan scheduling are studied. Each parallel system could be independently applied for resources [6].

It is shown parallel system in MPI (ang. Message Passing Interface) technology using three nodes cluster with memory system concentrated. The system was made up of tasks set $F = \{F_1, F_2, \dots, F_n\}$. Each task could be executed on one at the most machine [8]. The simulated cluster system is made of three parallel machines $M = \{M_1, M_2, M_3\}$ that each could calculated simultaneously any but only one task from set F [1].

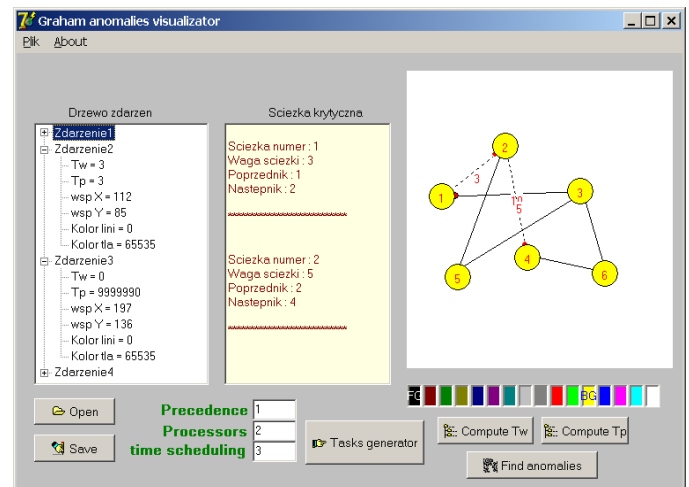


Fig.3 Graham anomalies simulator

The authors focused one's attention on set nonsplitable tasks, because these algorithms are used frequently in electromagnetic field calculation. In general, the MPI system is a set that depends on tasks determined precedence constraints. Between F_1 and F_2 tasks is precedence constraint (notation $F_1 \prec F_2$) when F_1 has to be done before beginning

task F2 calculation. It is necessary to take into consideration that binary relation \prec partially order set F [4].

In the application the relation \prec is presented in the form of direct acyclic graph $G(V,E)$ with node convention. In this model the nodes (V) answer a task and the edges (E) describes precedence constraints [2][3]. The authors' application can models tasks schedule such as portray three Graham's anomalies on base modification the original system. User has to only precise how many machines he wants to use in cluster, parallel algorithms execution times and precedence is constraints.

The graph was implemented as neighborhood matrix $V \times V$ dimension and the memory complexity this algorithm is $O(V^2)$ [7]. In the aim to simulate Graham's anomalies it was necessary to appropriately match tasks execution time vector $[\tau_{ij}]$, where τ_{ij} is time of execution task F_i on machine M_j . The parallel computation is made using set three identical machines:

$$\forall_i \tau_{ij} = \tau_i, j=1,2,3 \quad i=1,2,\dots,9. \quad (19)$$

The problem is to find optimal tasks schedule with assurance all tasks execution which meets all imposed constraints in schedule length (makespan) criterion [5]:

$$C_{\max} = \max_i \{C_i\} \quad (20)$$

where : C_i is completion time task F_i , $F_i \in F$

As the results the optimal schedule is presented in graphical way using Gantt chart. Next chart illustrated individual Graham's anomalies on the basis of research modifications.

VI. THE TASK CLUSTERING SCHEDULES

Individual tasks should be identify with parallel opetations in electromagnetic calculation, where:

F_1 - socket combination and opening, F_2 - data transfer to server1, F_3 - data transfer to server2, F_4 - server1 computation, F_5 - server2 computation, F_6 - client computation, F_7 - test of data transfer correctness, F_8 - marge the results, F_9 - verification of cluster cohesion.

Knowing executions times all tasks and precedence constraints the issue is solved and optimal schedule is found. In the basic model time C_{\max} amount 48 second. Solution is presented at Gantt chart (Figure 4).

τ_i [s]	2	8	8	30	30	30	6	9	10
F_i	1	2	3	4	5	6	7	8	9

Table 1 The times of the tasks execution

$$\prec : F_1 \prec F_9 ; F_2 \prec F_4 ; F_3 \prec F_5 ; F_3 \prec F_6 ; F_4 \prec F_8 ;$$

$$F_5 \prec F_8 ; F_6 \prec F_8 ; F_7 \prec F_4 ; F_7 \prec F_5$$

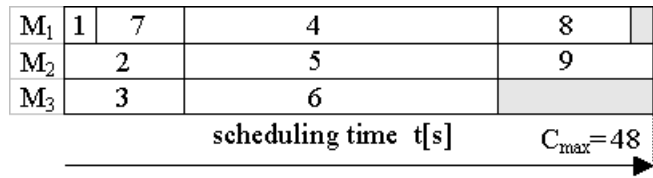


Fig.4 Gantt chart for optimization parallel system in basic mode

To such identified parallel system tasks execution times is modified. All execution time vector's $[\tau_i]$ parameters is decreased – the tasks has been shortened about 1[s]. Such changed system is optimised again keeping identical precedence constraints. Figure 3 revealed first Graham's anomaly, so shortened tasks execution times caused elongated minimum makespan scheduling.

τ_i [s]	1	7	7	29	29	29	5	8	9
F_i	1	2	3	4	5	6	7	8	9

Table 2 The modified times of the tasks execution

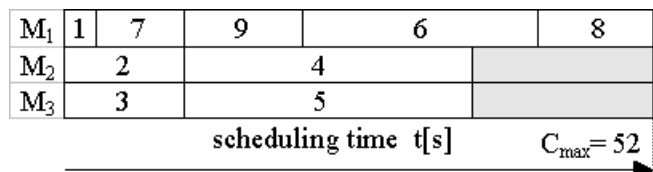


Fig.5 Gantt chart for optimization parallel system in mode with tasks' execution times shortened

Due to present the second Graham's anomaly fourth computing machine is added. The precedence constraints still remain identical to original version of the problem. Optimal scheduling presented at diagram number 4 proves, that even such intuitive operation like increasing parallel computation power of the cluster in special cases could cause worse solution towards makespan criterium.

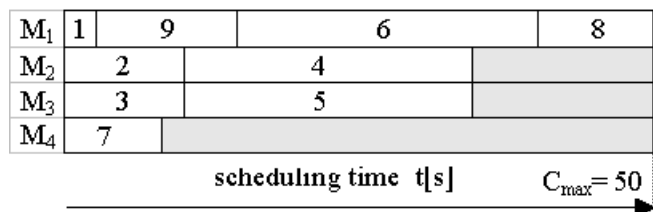


Fig.6 Gantt chart for optimization parallel system in mode with additional computation machine

Third Graham's anomaly concern elongated optimal time scheduling (C_{\max}) even though reduction selected precedence constraints. According to figure nr 5 presented below excepting two constraints from set \prec ($F_7 \prec F_4$ and $F_2 \prec F_4$) causes deterioration solution time by five second with relation to original problem. The constraints set is following:

$$\prec : F_1 \prec F_9 ; F_3 \prec F_5 ; F_3 \prec F_6 ; F_4 \prec F_8 ;$$

$$F_5 \prec F_8 ; F_6 \prec F_8 ; F_7 \prec F_5$$

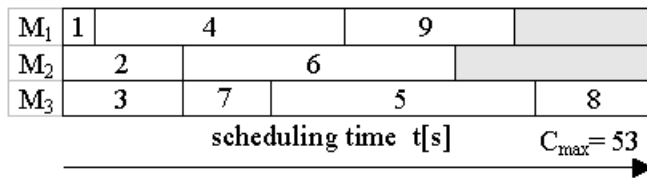


Fig.7 Gantt chart for optimization parallel system in

VII. CONCLUSION

The results of the simulations presented in this paper show interesting phenomena which is opposite to standard parallel-based computations. Sometimes the attempt at improvement tasks scheduling and gain increasing may causes significant elongated calculation's cycle. An identification problem's essence could be very difficult.

The measure of single tasks taking execution time into account allows to simulate and detect potential Graham's anomalies and consider it in parallel systems. The application created by the authors takes into considerations data transmission described by the user. Future research will concern the attempts at including the real image of network packages movement during TCP/IP transmission.

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