A Relationship between Marker and Inkdot for Four-Dimensional Automata

Yasuo Uchida, Takao Ito, Makoto Sakamoto, Ryoju Katamune, Kazuyuki Uchida, Hiroshi Furutani, Michio Kono, Satoshi Ikeda and Tsunehiro Yoshinaga

Abstract — A multi-marker automaton is a finite automaton which keeps marks as pebbles in the finite control, and cannot rewrite any input symbols but can make marks on its input with the restriction that only a bounded number of these marks can exist at any given time. An improvement of picture recognizability of the finite automaton is the reason why the multi-marker automaton was introduced. On the other hand, a multi-inkdot automaton is a conventional automaton capable of dropping an inkdot on a given input tape for a landmark, but unable to further pick it up. Due to the advances in many application areas such as moving image processing, computer animation, and so on, it has become increasingly apparent that the study of four-dimensional pattern processing has been of crucial importance. Thus, we think that the study of four-dimensional automata as a computational model of four-dimensional pattern processing has also been meaningful. This paper deals with marker versus inkdot over four-dimensional input tapes, and investigates some properties.

Keywords — finite automaton, four-dimension, inkdot, marker, recognizability

I. Introduction

A multi-marker automaton is a finite automaton which keeps marks as *pebbles* in the finite control, and cannot rewrite any input symbols but can make marks on its input with the restriction that only a bounded number of these marks can exist at any given time. An improvement of picture recognizability of the finite automaton is the reason why the marker automaton was introduced. That is, a two-dimensional multi-marker automaton can recognize connected pictures [3].

On the other hand, as is the well-known open problems in computational complexity, there is the historical open question whether or not the separation exists between deterministic and nondeterministic space (especially hard-level) complexity classes. Related to the historical open question, D. Ranjan et al. introduced a slightly modified Turing machine model, called a one-inkdot Turing machine [70]. An inkdot machine is a conventional Turing machine capable of dropping an inkdot on a given input tape for landmark, but unable to further pick it up. Against an earlier expectation, it was proved that nondeterministic inkdot Turing machines are more powerful than nondeterministic ordinary Turing machines for sublogarithmic space bounds. As is well-known result in the case of two-dimensional input tapes, there is a set of square tapes accepted by a nondeterministic finite automaton, but not by any deterministic Turing machine with sublogarithmic space bounds. Thus, it makes no sense to ask the same question whether the separation exists between deterministic and nondeterministic complexity classes for the two-dimensional Turing machines. However, there is an other important aspect in the inkdot mechanism : we can see a two-dimensional finite automaton with inkdot as a weak recognizer of the inherent properties of digital pictures. By this motivation, a two-dimensional multi-inkdot automaton was introduced [41,70]. (See [3,6,35,70,74,104] for another results concerning two-dimensional inkdot and marker machines.)

By the way, the question of whether processing three-dimensional digital patterns is much more difficult than two-dimensional ones is of great interest from the theoretical and practical standpoints. Thus, the research of three-dimensional automata as the computational model of three-dimensional pattern processing has been meaningful. From this viewpoint, we investigated a multi-marker automaton and a multi-inkdot automaton on three-dimensional input tapes [1,2,4,5,7-115]. Due to the advances in many application areas such as moving image processing, computer animation, and so on, it has become increasingly apparent that the study of four-dimensional pattern processing has been of crucial importance. Thus, we think that the study of four-dimensional automata as a computational model of four-dimensional pattern processing has also been meaningful[92,95-97,108,112].

This paper deals with a relationship between marker and inkdot for four-dimensional automata, and shows some properties (see Fig. 3,4.).

II. Definitions and Notations

Definition2.1. Let Σ be a finite set of symbols. A *four-dimensional tape* over Σ is a four-dimensional rectangular array of elements of Σ. The set of all four-dimensional tapes over Σ is denoted by $\Sigma^{(4)}$.

Given a tape $x \in \Sigma^{(4)}$, for each integer j ($1 \leq j \leq 4$), we let $l_j(x)$ be the length of x along the jth axis. The set of all $x \in \Sigma^{(4)}$ with $l_1(x) = n_1$, $l_2(x) = n_2$, $l_3(x) = n_3$, and $l_4(x) = n_4$ is denoted by $\Sigma(n_1, n_2, n_3, n_4)$. When $1 \leq i_j \leq l_j(x)$ for each j ($1 \leq j \leq 4$), let $x(i_1, i_2, i_3, i_4)$ denote the symbol in x with coordinates (i_1, i_2, i_3, i_4), as shown in Fig.1. Furthermore, we define $x[(i_1, i_2, i_3, i_4),(i'_1, i'_2, i'_3, i'_4)]$, when $1 \leq i_j \leq i'_j \leq l_j(x)$ for integer j ($1 \leq j \leq 4$), as the four-dimensional tape y satisfying the following conditions:

(i) for each j ($1 \leq j \leq 4$), $l_j(y) = i'_j - i_j + 1$;

(ii) for each r_1, r_2, r_3, r_4($1 \leq r_1 \leq l_1(y)$, $1 \leq r_2 \leq l_2(y)$, $1 \leq r_3 \leq l_3(y)$, $1 \leq r_4 \leq l_4(y)$), $y(r_1, r_2, r_3, r_4) = x(r_1+i_1-1, r_2+i_2-1, r_3+i_3-1, r_4+i_4-1)$. (We call $x[(i_1, i_2, i_3, i_4), (i'_1, i'_2, i'_3, i'_4)]$ the $[(i_1, i_2, i_3, i_4), (i'_1, i'_2, i'_3, i'_4)]$-*segment* of x.)

For each $x \in \sum^{(n_1,n_2,n_3,n_4)}$ and for each $1 \leq i_1 \leq n_1$, $1 \leq i_2 \leq n_2$, $1 \leq i_3 \leq n_3$, $1 \leq i_4 \leq n_4$, $x[(i_1, 1, 1, i_4),$

(i_1, n_2, n_3, i_4)], x[$(1, i_2, 1, i_4)$, (n_1, i_2, n_3, i_4)], x[$(1, 1, i_3, i_4)$, (n_1, n_2, i_3, i_4)], x[$(i_1, 1, i_3, i_4)$, (i_1, n_2, i_3, i_4)], and x[$(1, i_2, i_3, i_4)$, (n_1, i_2, i_3, i_4)] are called the i_1*th* (2-3) *plane of the* i_4*th three-dimensional rectangular array of x*, the i_2*th* (1-3) *plane of the* i_4*th three-dimensional rectangular array of x*, the i_3*th* (1-2) *plane of the* i_4*th three-dimensional rectangular array of x*, the i_1*th* row *on the* i_3*th* (1-2) *plane of the* i_4*th three-dimensional rectangular array of x*, and the i_2*th column on* the i_3*th* (1-2) *plane of the* i_4*th three-dimensional rectangular array of x*, and are denoted by $x(2\text{-}3)_{i_1 \cdot i_4}$, $x(1\text{-}3)_{i_2 \cdot i_4}$, $x(1\text{-}2)_{i_3 \cdot i_4}$, $x[i_1, *, i_3, i_4]$ and $x[*, i_2, i_3, i_4]$, respectively (See Fig.1.).

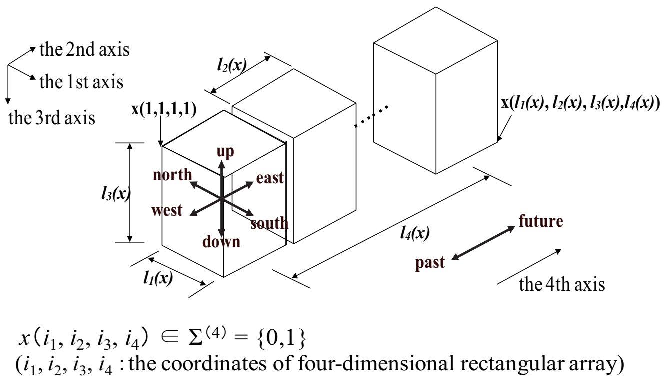

Fig. 1. Four-dimensional input tape.

Definition2.2. A *four-dimensional finite automaton* (4-FA), which can be considered as a natural extension of the three-dimensional finite automaton to four dimensions, consists of read-only four-dimensional input tape, a finite control, and an input head which can move east, west, south, north, up, down, in the past, or in the future [3]. A *four-dimensional alternating finite automaton* (4-AFA) is a sixtuple M=(Q,q_0,U,F,Σ,δ), where

(1) Q is a finite set of *states*,
(2) $q_0 \in Q$ is the *initial state*,
(3) $U \subseteq Q$ is the set of *universal states*,
(4) $F \subseteq Q$ is the set of the *accepting states*,
(5) Σ is a finite *input alphabet* ($\# \notin \Sigma$ is the *boundary symbol*), and
(6) $\delta \subseteq (Q \times (\Sigma \cup \{\#\})) \times (Q \times \{$*east, west, south, north, up, down, future, past, no more*$\})$ is the *next-move relation*.

A state q in $Q - U$ is said to be *existential*. As shown in Fig.2, the machine M has a read-only rectangular input tape with boundary symbols #'s, and finite control. A position is assigned to each cell of the input tape as shown in Fig.2.

At each moment, the machine M is in one of the states. A *step* of M consists of reading the symbol currently under the input head, changing its state, and moving the input head in specified direction (*east, west, south, north, up, down, future, past, or no more*) which is determined by the next-move relation δ. If the input head falls off the input tape, then M can move no further make.

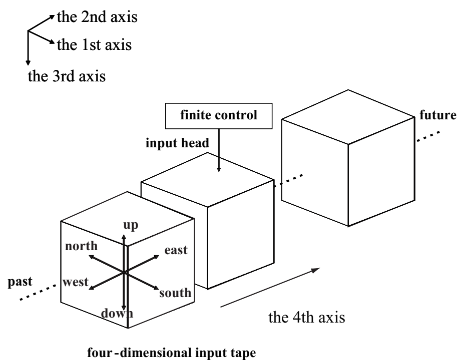

Fig. 2. Four-dimensional alternating finite automaton.

Definition2.3. A *configuration* of a 4-AFA M on an input $x \in \Sigma^{(4)}$is of the form ((i_1,i_2,i_3,i_4),q), where (i_1,i_2,i_3,i_4), $0 \leq i_1 \leq l_1(x)+1$, $0 \leq i_2 \leq l_2(x)+1$, $0 \leq i_3 \leq l_3(x)+1$, and $0 \leq i_4 \leq l_4(x)+1$, is a position of the input head, and q is a state of the finite control. If q is the state associated with configuration c, then c is said to be *universal* (*existential*, *accepting*) configuration if q is a universal (existential, accepting) state. The *initial configuration* of M on input x is $I_M(x)$=($(1, 1, 1, 1)$,q_0). For each input tape x, we write $c \vdash_{M,x} c$', and say that c' is an *immediate successor* of c (of M on x), if configuration c' is derived from configuration c in one step of M on x according to the next-move relation. A configuration with no immediate successor is called a *halting configuration*. Below, we assume that every accepting configuration is a halting configuration.

We can view the computation of M as a tree whose made are labelled by configurations. A *computation tree* of M on an input tape x is a tree whose nodes are labelled by configurations of M on x. The root of the tree is labelled by the initial configuration $I_M(x)$; the children of any node labelled by a universal configuration are all the immediate successors of that configuration on x; and any node labelled by an existential configuration has one child, which is labelled by one of the immediate successors of that configuration on x (provided there are any). An *accepting computation tree* of M on x is a computation tree of M on x whose leaves are all labelled by accepting configurations. We say that M *accepts* x if there is an accepting computation tree of M on input x. Define $T(M)$=$\{x \in \Sigma^{(4)} | M$ accepts $x\}$.

Definition2.4. A *four-dimensional alternating one-maker automaton* [6,21] is a 4-AFA with the capability of using one-marker which the finite control can use as a marker on the input tape, as shown in Fig.3. During the computation, the device can deposit (retrieve) a marker on (from) any cell of the input. The action of the machine depends on the current state of the finite control, the currently scanned input tape symbol, and on the presence of the marker on the current input tape cell. The action consists of moving the input head, changing the state of the finite control, and picking up or placing the marker on the currently scanned cell of the input tape.

A *configuration* of a four-dimensional alternating one-marker automaton M on an input tape x is of the form

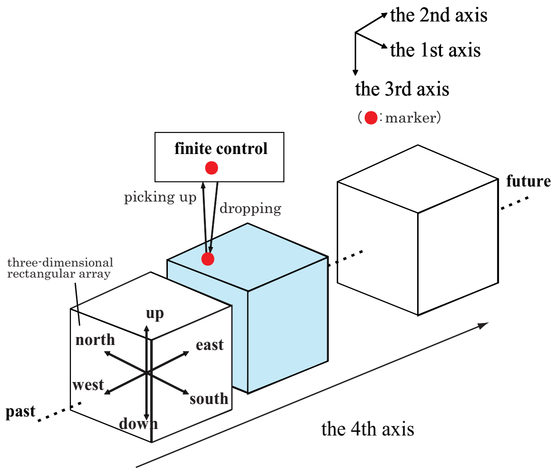

Fig. 3. Image of marker on four-dimensional input tape.

((i_1,i_2,i_3,i_4), *marker-position*,q), where (i_1,i_2,i_3,i_4) is the input head position, *marker-position* is the position of the marker on x (let *marker-position* be "*no*" if the marker is not placed on the input tape x), and component q represents a state of the finite control. The *initial configuration* of M on x is ((1, 1, 1, 1), *no*, q_0), where q_0 is the initial state of M. That is, the machine M starts with the marker in the finite control and with the input head on the upper-northwestmost corner of the first three-dimensional rectangular array of the input tape. An *accepting computation tree* of M on an input tape is defined as in the case of a four -dimensional alternating finite automaton. We say that M *accepts* an input tape x if there is an accepting computation tree of M on x. By $T(M)$, we denote the set of all the four-dimensional tapes accepted by M.

Definition2.5. A *four-dimensional alternating k-inkdot finite automaton* [41] is a 4-AFA capable of dropping inkdots on a given iput tape for a landmark, but unable to further pick it up. That is, a four-dimensional alternating k-inkdot finite automaton is a four-dimensional alternating multi-marker finite automaton which cannot pick up the pebbles again, once it has put down the marker on a given input tape, as shown in Fig.4. (See [41].)

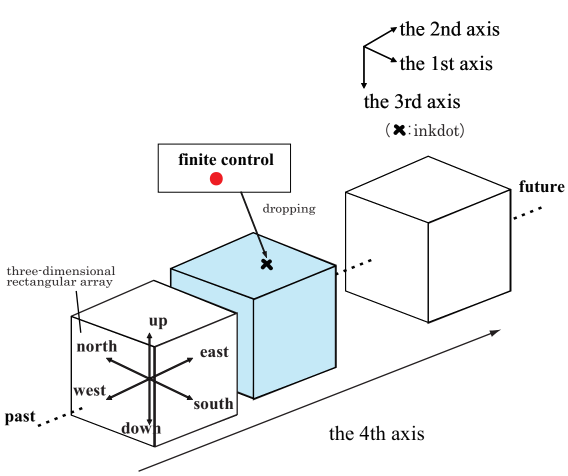

Fig. 4. Image of inkdot on four-dimensional input tape.

By 4-AFA (resp., 4-NFA, 4-DFA, 4-AM_1, 4-NM_1, 4-DM_1, 4-AI_k, 4-NI_k, 4-DI_k), we denote alternating (resp., nondeterministic, deterministic, alternating 1-marker, nondeterministic 1-marker, deterministic 1-marker, alternating k-inkdot, nondeterministic k-inkdot, deterministic k-inkdot) 4-FA. Furthermore, by 4-UFA (resp., 4-UM_1, 4-UI_k), we denote alternating (resp., alternating 1-marker, alternating k-inkdot) 4-FA with only universal states.

For each $X \in \{D, N, U, A\}$, we denote by $\mathcal{L}$[4-XFA] the class of sets of all four-dimensional tapes accepted by 4-XFA's. That is,

$\mathcal{L}$[4-XFA]=$\{T \mid T$=$T(M)$ *for some* 4-XFA $M\}$,

where $T(M)$ is the set of all four-dimensional tapes accepted by M. $\mathcal{L}$[4-XM_1] and $\mathcal{L}$[4-XI_k] are defined similarly.

Let M be a 4-AM_1 (or 4-AI_k), and x be an input tape. A sequence of configurations $c_1c_2\cdots c_m$ (m≥1) is called a *computation path* of M on x if $c_1 \vdash_{M,x} c_2 \vdash_{M,x} \cdots \vdash_{M,x} c_m$. For simplicity, we below call a computation path a *computation*. For any set S, $|S|$ denotes the cardinality of S.

III. Known Results and Related Results

This section surveys known results and related results in [50,83,84,86,89] concerning k-inkdot and 1-marker 4-FA's. The following result shows a relationship among the accepting powers of 4-FA's k-inkdot 4-FA's, and 1-marker 4-FA's.

Theorem 3.1[50,83,84,86,89]**.**
(1) $\mathcal{L}$[4-DFA] = $\mathcal{L}$[4-DI_k] $\subsetneq$ $\mathcal{L}$[4-DM_1],
(2) $\mathcal{L}$[4-NFA] $\subsetneq$ $\mathcal{L}$[4-NI_k] $\subsetneq$ $\mathcal{L}$[4-NM_1],
(3) $\mathcal{L}$[4-UFA] $\subsetneq$ $\mathcal{L}$[4-UI_k] $\subsetneq$ $\mathcal{L}$[4-UM_1], *and*
(4) $\mathcal{L}$[4-AFA] $\subsetneq$ $\mathcal{L}$[4-AI_k] $\subseteq$ $\mathcal{L}$[4-AM_1].

It is unknown whether $\mathcal{L}$[4-AI_k] $\subsetneq$ $\mathcal{L}$[4-AM_1]. What are the relationships between $\mathcal{L}$[4-NI_k] and $\mathcal{L}$[4-DM_1], between $\mathcal{L}$[4-UI_k] and $\mathcal{L}$[4-DM_1], and between $\mathcal{L}$[4-AI_k] and $\mathcal{L}$[4-NM_1]? The following theorem answer this question.

Theorem 3.2.
(1) $\mathcal{L}$[4-NI_k] *is incomparable with* $\mathcal{L}$[4-DM_1],
(2) $\mathcal{L}$[4-UI_k] *is incomparable with* $\mathcal{L}$[4-DM_1], *and*
(3) $\mathcal{L}$[4-NM_1] $\subsetneq$ $\mathcal{L}$[4-AI_k].

Proof: Let T_1=$\{x \in \{0,1\}^{(4)} \mid \exists n$≥1 [$l_1(x)$=$l_2(x)$=$l_3(x)$=$l_4(x)$=2$n$ ∧ (the top half of x is the same as the bottom half of x)]$\}$, and T_2=$\{x \in \{0,1\}^{(4)} \mid \exists n$≥2 [$l_1(x)$=$l_2(x)$=$l_3(x)$=$l_4(x)$=n ∧ ∃ i(2≤i≤n) { the first three-dimensional rectangular array of x is the same as the ith three-dimensional rectangular array of x }$\}\}$.

By using the same technique as in the proof of Lemma 5.1, Corollary 5.1, Theorem 6.4 in [41], we can show that the complement of T_1 is in $\mathcal{L}$[4-NI_k], T_1∈$\mathcal{L}$[4-UI_k], and T_2∉$\mathcal{L}$[4-NI_k]∪$\mathcal{L}$[4-UI_k]. Furthermore, we can easily prove that T_2∈$\mathcal{L}$[4-DM_1] [21,23].

By using the same technique as in the proof of Theorem 4.1 in [21], we can show that the complement of T_1 is not in $\mathcal{L}$[4-DM_1]. From these observation, (1) and (2) of the theorem follow. (3) of the theorem can be proved by using the same idea of Ref. [22,23]. □

The following result in [50,83,84,86,89] shows a relationship among the accepting powers of determinism, nondeterminism, alternation with only universal states, and alternation for k-inkdot 4-FA's.

Theorem 3.3[50,83,84,86,89]**.** (1) $\mathcal{L}$[4-DI_k] $\subsetneq$ $\mathcal{L}$[4-NI_k] $\subsetneq$ $\mathcal{L}$[4-AI_k], *and* (2) $\mathcal{L}$[4-DI_k] $\subsetneq$ $\mathcal{L}$[4-UI_k] $\subsetneq$ $\mathcal{L}$[4-AI_k].

A relationship between $\mathcal{L}$[4-NI_k] and $\mathcal{L}$[4-UI_k] is shown in the following theorem.

Theorem 3.4. $\mathcal{L}$[4-NI_k] *is incomparable with* $\mathcal{L}$[4-UI_k].

Proof: Let T_1 and T_2 be sets described in the proof of Theorem 3.2. We can easily prove that $T_1 \in \mathcal{L}$[4-UI_k]$-\mathcal{L}$[4-NI_k] [7], and the complement of T_2 is in $\mathcal{L}$[4-NI_k], but not in $\mathcal{L}$[4-UI_k]. From this fact, the theorem follows. □

For 1-marker 4-FA's, we can easily get the following result [21,23]. That is, alternation is better than nondeterminism, which is better than determinism.

Theorem 3.5. $\mathcal{L}$[4-DM_1] $\subsetneq$ $\mathcal{L}$[4-NM_1] $\subsetneq$ $\mathcal{L}$[4-AM_1].

IV. Main Results

This section investigates an open problem. That is, a relationship between $\mathcal{L}$[4-UMI_1] and $\mathcal{L}$[4-AI_k].

Here is some preliminaries. Let $c_1c_2......c_m(m\geq1)$ be a computation of M on an input tape x. Then, this computation is called:

· a *halting computation* of M on x if c_m is a halting configuration other than any accepting configuration,

· a *double-looping computation* of M on x if there exist some $i(1\leq i\leq m-2)$ and some (possibly empty) sequence of configurations s such that (i) $c_j\neq c_k$ for each $1\leq j\leq k\leq i$, (ii) $c_1c_2......c_m$ = $c_1c_2......c_{i-1}c_isc_isc_i$, and (iii) each configuration in c_is is different from each other, and different from each $c_r(1\leq r\leq i)$, and

· a *rejecting computation* of M on x if the sequence $c_1c_2......c_m$ is a halting, or double-looping computation.

Theorem 4.1. $\mathcal{L}$[4-AI_k]$-\mathcal{L}$[4-UM_1] $\neq \phi$.

Proof: Let $V(m)$={$x_1c_1x_2c_2......x_mc_m$ | $\forall i(1\leq i\leq m)$ $\{x_i\in\{0,1\}^{(m,m,m,m)} \wedge c_i\in\{2\}^{(m,1,m,m)}\}$}, and T_3={xy | $\exists m\geq1\{x,y\in V(m)\}$ $\wedge$ $x\neq y$}, where for any two four-dimensional tapes x and y with $l_4(x)$=$l_4(y)$, we denote by xy the four-dimensional tape obtained by concatenating y to the future side of x. To prove the theorem, we below show that (1) T_3 $\in$ $\mathcal{L}$[4-AI_k], and (2) T_3 $\notin$ $\mathcal{L}$ [4-UM_1]. It is obvious that Part (1) of the theorem holds. Here we only prove (2). We suppose to the contrary that there is a 4-UM_1 M which accepts T_3. Let Q be the set of states of the finite control of M. We divide Q into two disjoint subsets Q^+ and Q^- which correspond to the sets of states when M holds and does not hold the marker in the finite control, respectively. M starts from the initial state in Q^+ with the input head on the upper-northwestmost symbol of first three-dimensional rectangular array of an input tape. We assume without loss of generality that M satisfies the following condition (A): ‘ M does not go out of the boundary symbols #'s. (Of course, M does not go into the input tape from the outside of the boundary symbols #'s.)’ For each $M\geq$1, let $W(m)$={xy | x,$y\in V(m)$}. Below we shall again consider the computations of M on tapes in $W(m)$ for large $m\geq$1. Let x be any tape in $V(m)$ that is supposed to be a future or past half on an input tape (in $W(m)$) to M, and let #'sx (resp., x#'s) be the tape obtained from x by attaching the boundary symbols #'s to the east, west, south, north, upper, lower, and in the future (resp., east, west, south, north, upper, lower, and in the past) sides. Note that, from the above condition (A), both the entrance points to #'sx (resp., x#'s) and the exit points from #'sx (resp., x#'s) are the future (resp., past) side of #'sx (resp., x#'s). Let $PT(m)$ be the set of these entrance (or exit) points. Clearly, $|PT(m)| = (m+2)^3$. Suppose that the marker of M is not placed on the #'sx (resp., x#'s). Then, we define a mapping M_x^w(resp., M_x^e), which depends on M and x, from $Q\times PT(m)$ to the power set of $(Q\times PT(m))\cup Q_{stop}\cup\{loop\}$ as follows (where Q_{stop} is the set of halting states other than accepting states, and *loop* is a new symbol):

· for any (s,p), (s',p')$\in Q^-\times PT(m)$, (s',p')$\in M_x^w$(s,p) (resp., M_x^e(s,p)) $\Leftrightarrow$ when M enters #'sx (resp., x#'s) in state s from entrance point p of the future (resp., past) edge of #'sx (resp., x#'s),there exists a computation of M in which M eventually exits #'sx (resp., x#'s) in state s' from exit point p' of the furture (resp., past) edge of #'sx (resp., x#'s),

· for any (s, p) $\in Q\times PT(m)$ and for any $q\in Q_{stop}$, $q\in M_x^w$(s, p) (resp., M_x^e(s, p)) $\Leftrightarrow$ when M enters #'sx (resp., x#'s) in state s from entrance point p of the future (resp., past) edge of #'sx (resp., x#'s), there exists a computation of M in which M eventually enters state q in #'sx (resp., x#'s), and halts, and

· for any (s, p)$\in Q\times PT(m)$, *loop*$\in M_x^w$(s, p) (resp., M_x^e(s, p)) $\Leftrightarrow$ when M enters #'sx (resp., x#'s) in state s from entrance point p of the future (resp., past) edge of #'sx (resp., x#'s), there exists a computation in which M enters a loop in #'s x (resp.,x #'s).

Let $x_1, x_2 \in V(m)$. We say that x_1 and x_2 are

· M*-equivalent* if two mappings $M_{x_1}^w$ and $M_{x_2}^w$ are equivalent, and two mappings $M_{x_1}^e$ and $M_{x_2}^e$ are equivalent, and

· M*-equivalent* if for any (s, p), (s', p')$\in Q^-\times PT(m)$, and for any $a\in$\{w, e\}, (s', p')$\in M_{x_1}^a$(s, p) if and only if (s', p')$\in M_{x_2}^a$(s, p).

(Note that if x_1 and x_2 are M-equivalent, then x_1 and x_2 are M-eqivalent.) Clearly, M-equivalence is an equivalence relation on $V(m)$. Clearly, there are at most $e(m) = (2^{|Q|(m+2)^3+d+1})^{|Q|(m+2)^3}$, where $d = |Q_{stop}|$, M-equivalence classes of $V(m)$. Let $P(m)$ be a largest M-equivalence classes of $V(m)$. Then, we have $|P(m)| \geq \frac{V(m)}{e(m)} = \frac{2^{m^4}}{e(m)}$.

Note that $|P(m)|\gg1$ for large m. By using the same technique as in the proof of Theorem 6 in [23] and the well-known counting argument, finally, we can prove that $T_3 \notin \mathcal{L}[4\text{-}UM_1]$. □

V. Conclusion

We investigated about marker versus inkdot on four-dimensional input tapes, and showed some accepting properties of various four-dimensional automata with markers or inkdots.

We conclude this paper by giving the following open problems : (1) $\mathcal{L}[4\text{-}AI_k] \subsetneq \mathcal{L}[4\text{-}AM_1]$? (2) What are the relationships between $\mathcal{L}[4\text{-}NI_k]$ and $\mathcal{L}[4\text{-}UM_1]$ and between $\mathcal{L}[4\text{-}UI_k]$ and $\mathcal{L}[4\text{-}NM_1]$? (3) Is $\mathcal{L}[4\text{-}NM_1]$ incomparable with $\mathcal{L}[4\text{-}UM_1]$? (4) $\mathcal{L}[4\text{-}UM_1] \subsetneq \mathcal{L}[4\text{-}AI_k]$?

References

[1] A. Aho, J. E. Hopcroft, and J. D. Ullman, *The design and analysis of computer algorithms*; Addison-Wesley, Reading, Mass., 1974.

[2] T. Beyer, Recognition of topological invariants by interative arrays, *Ph. D. Thesis, MIT*, 1970.

[3] M. Blum and C. Hewitt, Automata on a two-dimensional tape, *in IEEE Symposium on Switching and Automata Theory*, pp.155–160, 1967.

[4] M. Blum, and W. J. Sakoda, On the capability of finite automata in 2 and 3 dimensional space, *in Proceedings of the 18th Annual Symposium on Foundations of Computer Science*, pp.147-161, 1977.

[5] A. K. Chandra, D. C. Kozen, and L. J. Stockmeyer, Alternation; *Journal of the ACM*, Vol.28, No.1, pp.114-133, 1981.

[6] J. H. Chang, O. H. Ibarra, M. A. Palis, and B. Ravikumar, On pebble automaton, *Theoretical Computer Science*, Vol. 44, pp. 111-121, 1986.

[7] C. R. Deyer and A. Rosenfeld, Parallel image processing by memory-augmented cellular automata; *IEEE Transactions on Pattern Analysis and Machine Intelligence*, Vol.PAMI-3, No.1, pp.29-41, 1981.

[8] P. Dietz, and S. R. Kosaraju, Recognition of topological equivalence of patterns by array automata, *Journal of Computer and System Sciences*, Vol.2, pp.111-116, 1980.

[9] E. M. Gurari, and O. H. Ibarra, (Semi-) alternating stack automata, *Mathematical Systems Theory*, Vol.15, pp.211-224, 1982.

[10] J. E. Hopcroft and J. D. Ullman, Some results on tape-bounded Turing machines, *Journal of the ACM*, Vol.16, pp.168-177, 1969.

[11] J. E. Hopcroft, and J. D. Ullman, Some results on tape-bounded Turing machines, *Journal of the ACM*, Vol.19, No.2, pp.283-295, 1972.

[12] J. E. Hopcroft, and J. D. Ullman, Introduction to Automata Theory, Languages, and Computation, *Addison-Wesley, Reading, Mass.*, 1979.

[13] J. Hromkovič, On the power of alternation in automata theory, *Journal of Computer and System Sciences*. Vol.31, No.1, pp.28-39, 1985.

[14] J. Hromkovič, How to organize the commmunication among parallel proceses in alternating computations, *manuscript*, January, 1986.

[15] J. Hromkovič, J. Karhumäki, B. Rován, and A. Slobodová, On the power of synchronization in parallel computations, *Tech. Report, comenius Univ., Bratislava, Czechoslovakia, Dept. of Theoretical Cybernetics and Institute of Computer Science*, 1989.

[16] J. D. Hromkovič, K. Inoue, B. Rován, and A. Slobodová, I. Takanami and K.W. Wagner, On the power of one-way synchronized alternating machines with small space, *International J. of Foundations of Computer Science*, Vol.3, No.1, pp.65-79, 1992.

[17] O. H. Ibarra and R. T. Melson, Some results concerning automata on two-dimensional tapes, *International Journal of Computer Mathematics*, Vol.4, No.A, pp.269-279, 1974.

[18] C.M. Ibara, and Trân N.Q., On space-bounded synchronizing alternating Turing machines, *Theoretical Computer Science 99*, pp.243-264, 1992.

[19] K.Iihoshi, M.Sakamoto, T.Ito, M.Saito, H.Furutani, M.Kono and, Katsushi Inoue, Leaf-size bounded computation for four-dimensional alternating Turing machines, Proceedings of the 11th International Symposium on Artificial Life and Robotics, Oita, Japan, 2006 (CD-ROM).

[20] K.Iihoshi, M.Sakamoto, T.Ito, N.Tomozoe, Y.Nagamizu, H.Furutani, M.Kono, S.Ikeda, T.Tamaki, and K.Inoue, Leaf-size hierarchy of four-dimensional alternating Turing machines, Proceedings of the 12th International Symposium on Artificial Life and Robotics, Oita, Japan, 2007 (CD-ROM).

[21] A. Inoue, K. Inoue, A. Ito, Y. Wang and T. Okazaki, A note on one-pebble two-dimensional Turing machines, *Information Sciences*, Vol.162, pp.295–314, 2004.

[22] A. Inoue, A. Ito, K. Hiraishi and K. Inoue, Alternating inkdot is better than nondeterministic pebble over two-dimensional languages, *to be announced in the 3rd International Conference on Information* (INFO'2004), 2004.

[23] A. Inoue, A. Ito and K. Inoue, Inkdot versus pebble over picture languages, *in 2004 Chugoku-Section Joint Connection Record of Institutes of Electrical and Information Engineerings, Japan*, pp.251–252, 2004.

[24] K.Inoue, and A. Nakamura, Some notes on n-dimensional on-line tessellation acceptors, *in Proceedings of the International Symposium on Uniformly Structured Automata and Logic*, pp.103-112, 1975.

[25] K. Inoue, and A. Nakamura, An n-dimensional on-line tessellation accepter (in Japanese); *The Transactions of the IECE*, Vol.J59-D, No.4, pp.229-236, 1976.

[26] K. Inoue, Investigations of two-dimensional on-line tessellation accptors (in Japanese), *Ph.D. Thesis, Nagoya University*, 1977.

[27] K. Inoue, and A. Nakamura, Some properties of two-dimensional nondeterministic finite automata and parallel sequential array acceptors (in Japanese), *The Transactions of the IECE*, Vol.J60-D, No.11, pp.990-997, 1977.

[28] K. Nakamura, and A. Nakamura, Some properties of two-dimensional on-line tessellation accepters, *Information Sciences*, Vol.13, pp.95-121, 1977.

[29] K. Inoue, and I. Takanami, Some properties of three-way two-dimensional multicounter automata over square tapes (in Japanese), *The Transactions of the IECE*, Vol.J62-D, No.10, pp.673-680, 1979.

[30] K. Inoue, I. Takanami, and H. Taniguchi, Three-way two-dimensional simple multihead finite automata -Closure properties-, *Trans. of IECE* (*Japan*), Vol.62-D, No.4, pp.273-280, 1979.

[31] K.Inoue, and I. Takanami, A note on bottom-up pyramid acceptors, *Information Processing Letters*, Vol.8, No.1, pp.34-37, 1979.

[32] K.Inoue, and I. Takanami, Three-way tape-bounded two-dimensional Turing machines, *Information Sciences*, Vol.17, pp.195-220, 1979.

[33] K. Inoue, I. Takanami, and H. Taniguchi, The accepting powers of two-dimensional automata over square tapes, *The Transactions of the IECE*, Vol.J63-D, No.2, pp.113-120, 1980.

[34] K. Inoue, and I. Takanami, A note on deterministic three-way tape-bounded two-dimensional Turing machines, *Information Sciences* 20, pp.41-55, 1980.

[35] K. Inoue, I. Takanami, and H. Taniguchi, Two-dimensional alternating Turing machines, *Theoretical Computer Science*, Vol. 27, pp. 61-83, 1983.

[36] K.Inoue, and I. Takanami, A survey of two-dimensional automata theory, *Information Sciences*, Vol.55, pp.99-121, 1991.

[37] A. Ito, K. Inoue, I. Takanami, and H. Taniguchi, Two-dimensional alternating Turing machines with only universal states, *Information and Control*, Vol.55, No.1-3, pp.193-221, 1982.

[38] A. Ito, K. Inoue, and I. Takanami, A note on three-way two-dimensional alternating Turing machines, *Information Sciences*, Vol.45, pp.1-22, 1988.

[39] A. Ito, K. Inoue, and I. Takanami, Complexity of acceptance problems for two-dimensional automata, *a perspective in Theoretical Computer Science, Series in Computer Science*, Vol.16, pp.70-94, 1989.

[40] A. Ito, K. Inoue, I. Takanami, and Y. Inagaki, Constant leaf-size hierarchy of two-dimensional alternating Turing machines, *International Journal of pattern Recognition and Artificial Intelligence*, Vol.8, No.2, pp.509-529, 1994.

[41] A. Ito, K. Inoue, I. Takanami and Y. Wang, The effect of inkdots for two-dimensional automata, *International Journal of Pattern Recognition and Artificial Intelligence*, Vol.9, No.5, pp.777-796, 1995.

[42] T. Ito, M. Sakamoto, M. Saito, K. Iihoshi, H. Furutani, M. Kono, and K. Inoue, Three-dimensional synchronized alternating Turing machines, *Proceedings of the 11th International Symposium on Artificial Life and Robotics, Oita, Japan*, OS 9-4, 2006.

[43] T. Ito, M. Sakamoto, N. Tomozoe, K. Iihoshi, H. Furutani, M. Kono, T. Tamaki, and K. Inoue, Some properties of three-dimensional synchronized alternating Turing machines, *Proceedings of the 10th WSEAS International Conference on COMPUTERS, Athens, Greece*, pp.665-670, 2006.

[44] T. Ito, M. Sakamoto, N. Tomozoe, K. Iihoshi, H. Furutani, M. Kono, T. Tamaki, and K. Inoue, Two topics concerning three-dimensional synchronized alternating Turing machines, *WSEAS Transactions on Computers*, Issue 10, Vol.5, pp.2149-2155, 2006.

[45] T. Ito, M. Sakamoto, Y. Nagamizu, K. Iihoshi, N. Tomozoe, H. Furutani, M. Kono, S. Ikeda, T. Tamaki, and K. Inoue, Three-dimensional parallel Turing machines, *Proceedings of the 12th International Symposium on Artificial Life and Robotics, Oita, Japan*, pp.131-134, 2007.

[46] T.Ito, M.Sakamoto, A.Taniue, T.Matsukawa, Y.Uchida, H.Furutani, and M.Kono, Parallel Turing machines on four-dimensional input tapes, Proceedings of the 15th International Symposium on Artificial Life and Robotics, Oita, Japan, OS5-5,pp.918-921, 2010 (CD-ROM).

[47] K. N. King, Measures of parallelism in alternating computation trees, *Proceedings of the 13th ACM Symposium on Theory of Computing*, pp.189-201, 1981.

[48] K. N.King, Alternating multihead finite automata, *Theoretical Computer Science*, Vol.61, pp.149-174, 1985.

[49] R. E. Ladner, R. J. Lipton, and L. J. Stockmeyer, Alternating pushdown automata, *Proceedings of the 19th IEEE Symposium on Foundations of Computer Science*, pp.92-106, 1978.

[50] T. Makino, M. Sakamoto, K. Inoue, H. Furutani and S. Katayama, Some note on three-dimensional multi-inkdot automata, *in Record of* 2004 *Joint Conference of Electrical and Electronics Engineerings in Kyushu, Japan*, No.12-1A-11, 2004(CD-ROM).

[51] T. Makino, H. Okabe, S. Taniguchi, M. Sakamoto, and K. Inoue, Three-dimensional multi-inkdot automata, *International Journal of Artificial Life and Robotics*, Vol.9, No.2, pp.99-101, 2005.

[52] H. Matsuno, Multihead automata and alternation, *Ph.D. Thesis, Kyusyu University* (*Japan*), 1994.

[53] P.Michel, A survey of space complexity, *Theoretical Computer Science*, Vol.101, pp.99-132, 1992.

[54] B.Monien, Two-way multihead automata over a one-leter alphabet, *RAIRO Information Theory* 14, pp.67-82, 1980.

[55] D. G. Morgenthaler, Three-dimensional digital topology : the genus, *Technical Report TR-980*, Computer Science Center, University of Maryland, 1980.

[56] K.Morita, H. Umeo, and K. Sugata, Computational complexity of L (m, n) tape-bounded two-dimensional tape Turing machines (in Japanese), *The Transactions of the IEICE*, Vol.J60-D, No.11, pp.982-989, 1977.

[57] K.Morita, Computational complexity in one- and two-dimensional tape automata, *Ph.D Thesis Osaka University*, 1978.

[58] K. Morita, H. Umeo, K. Sugata, Language recognition abilities of several two-dimensional tape automata and their relation to tape complexities (in Japanese), *The Transactions of the IEICE*, Vol.J60-D, No.12, pp.1077-1084, 1977.

[59] K. Morita, H. Umeo, and K. Sugata, Accepting ability of offside-free two-dimensional marker automata−The simulation of four-way automata by three-way tape-bounded Turing machines, *Technical Report of the IEICE*, No.AL79-2, 1979.

[60] Y.Nagamizu, M. Sakamoto, T. Ito, K. Iihoshi, N. Tomozoe, H. Furutani, M. Kono, S. Ikeda, I. Takanami, and K. Inoue, Remarks on recognizability of topological components by three-dimensional automata, *Proceedings of The 12th International Symposium on Artificial Life and Robotics*, $Oita$, $Japan$, pp.135-139, 2007.

[61] Y.Nakama, M.Sakamoto, M.Saito, S.Taniguchi, T.Ito, K.Inoue, H.Furutani, and S.Katayama, Accepting powers of four-dimensional alternating Turing machines with only universal states, Proceedings of the 10th International Symposium on Artificial Life and Robotics, Oita, Japan, 2005.

[62] A.Nakamura, and K. Aizawa, On the recognition of properties of three-dimensional pictures, *IEEE Transactions on Pattern Analysis and Machine Intelligence*, Vol.PAMI-7, pp.708-713, 1985.

[63] A. Nakamura, and K. Aizawa, Detection of interlocking components in three-dimensional digital pictures, *Information Sciences*, Vol.40, pp.143-153, 1986.

[64] A. Nakamura, and A. Rosenfeld, Three-dimensional connected pictures are not recognizable by finite-state acceptors, *Technical Report TR-566, Computer Science Center, University of Maryland*, 1991.

[65] Okabe H., Sakamoto M., and Inoue K., Computational complexity for the simulation of four-dimensional marker automata by seven-way four-dimensional Turing machines, *Proceedings of the 8th international symposium on artificial life and robotics*, $Oita$, $Japan$, Vol.1, pp.114-117, 2003.

[66] H. Okabe, S.Taniguchi,T. Makino, S.Nogami, Y. Nakama, M.Sakamoto, and K. Inoue, Computation for motion image processing by 4-dimensional alternating Turing machines, Proceedings of the 7th World Multiconference on Systemics, Cybernetics and Informatics, Orlando, USA, pp.241-246, 2003.

[67] K. Okinaka, K. Inoue, A. Ito, and Y. Wang, A note on hardware-bounded parallel Turing machines, *Proc. of the 2nd International Conference on Information*, $Beijing$, $China$, pp. 90-100, 2002.

[68] W. J. Paul, E. J. Prauss, and R. Reischuk, On alternation; *Acta Informatica*, Vol.14, pp.243-255, 1980.

[69] H. Petersen, Some results concerning two-dimensional Turing machines and finite automata, *Lecture Notes on Computer Science* 965, *FCT*'95, pp.394-382, 1995.

[70] D. Ranjan, R. Chang and J. Hartmanis, Space bounded computations : review and new separation results, *Theoretical Computer Science*, Vol.80, pp.289-302, 1991.

[71] A. L. Rosenberg, On multihead finite automata, *IBM J. Res. and Dev.*, Vol. 10, pp. 388-394, 1996.

[72] A. Rosenfeld, and D. L. Milgram, Parallel / sequential array automata, *Information Processing Letters*, Vol.2, pp.43-46, 1973.

[73] A.Rosenfeld, and A. C. Kak, *Digital Picture Processing*, Academic Press, New York, 1976.

[74] A. Rosenfeld, *Picture Languages−Foumal Models for Picture Recognition*, Academic Press, 1979.

[75] A. Rosenfeld, Three-dimensional digital topology, *Information and Control*, Vol.50, pp.119-127, 1981.

[76] W. L. Ruzzo, Tree-size bounded alternation, *Journal of Computer and System Sciences*, Vol.21, pp.218-235, 1980.

[77] M.Saito, M.Sakamoto, Y.Nakama, T.Ito, K.Inoue, H.Furutani, and S.Katayama, Some properties of four-dimensional multicounter automata, Proceedings of the 10th International Symposium on Artificial Life and Robotics, Oita, Japan, 2005.

[78] M.Saito, M.Sakamoto, K.Iihoshi, T.Ito, H.Furutani, M.Kono, and K.Inoue, Hierarchies based on the number of counters or the space allowed by four-dimensional multicounter automata, Proceedings of the 11th International Symposium on Artificial Life and Robotics, Oita, Japan, 2006 (CD-ROM).

[79] M.Saito, M.Sakamoto, Y.Nakama, T.Ito, K.Inoue, H.Furutani, and S.Katayama, Some properties of four-dimensional multicounter automata, International Journal of Artificial Life And Robotics, Springer, Vol.l0,No.2, pp.166-170, 2006.

[80] M. Sakamoto, I. Sakuramoto, K. Inoue, and I. Takanami, A space lower-bound technique for two-dimensional alternating Turing machines, *IEICE Trans.*, D-I, J74-DI(4), pp.311-314, 1991.

[81] M. Sakamoto, K. Inoue, and I. Takanami, A note on three-dimesional alternating Turing machines with space smaller than $log(m)$, *Information Sciences*, Vol.72, pp.225-249, 1993.

[82] M. Sakamoto, K. Inoue, and I. Takanami, Three-dimensionally fully space constructible functions, *IEICE Transactions on Information and Systems*, Vol.E77-D, No.6, pp.723-725, 1994.

[83] M. Sakamoto, A. Ito and I.Takanami, Simulation of three-dimensional one-marker automata by five-way Turing machines, *Information Sciences*, Vol.77, pp.77–99, 1994.

[84] M. Sakamoto and K. Inoue, Recognizability of connected pictures by three-dimensional multi-inkdot finite automata, *in 1996 Chugoku-Section Joint Convention Record of Institutites of Electrical and Information Engineerings, Japan*, p.289, 1996.

[85] M. Sakamoto, and K. Inoue, Three-dimensional alternating Turing machines with only universal states, *Information Sciences−Information and Computer Science*, Vol.95, pp.155-190, 1996.

[86] M. Sakamoto, K. Inoue and A.Ito, Basic properties of three-dimensional multi-inkdot automata, *in 1997 IEICE General Conference*, p.5, 1997.

[87] M. Sakamoto, T. Okazaki, and K. Inoue, Three-dimensional multicounter automata, *in Proceedings of the 6th International Workshop on Parallel Image Processing and Analysis−Theory and Applications*, $Madras$, $India$, pp.267-280, 1999.

[88] M. Sakamoto, Three-dimensional alternating Turing machines, *Ph.D Thesis Yamaguchi University*, 1999

[89] M. Sakamoto, K. Inoue and A.Ito, Some properties of three-dimensional multi-inkdot automata, *in 1999 Chugoku-Section Joint Convention Record of Institutites of Electrical and Information Engineerings, Japan*, pp.219–220, 1999.

[90] M. Sakamoto, H. Okabe, and K. Inoue, Some properties of four-dimensional finite automata, *in 2002 Chugoku-Section Joint Convention Record of Institutes of Electrical and Information Engineerings, Japan*, pp.351, 2002.

[91] M. Sakamoto, S. Nogami, K. Inoue, and M. Kono, A relationship between the accepting powers of three-dimensional finite automata and time-bounded bottom-up pyramid cellular acceptors with three-dimensional layers, *Trans. of SCI* (*Japan*), Vol.17, No.10, pp.451-458, 2004.

[92] M.Sakamoto, H.Okabe, S.Nogami, S.Taniguchi, T.Makino, Y.Nakama, M.Saito, M.Kono, and K.Inoue, A note on four-dimensional finite automata, WSEAS Transactions on Computers, Issue 5,Vol.3, pp.1651-1656, 2004.

[93] M. Sakamoto, T. Ito, N. Tomozoe, and H. Furutani, A survey of three-dimensional automata, *The papers of Technical Meeting on Information Systems, IEE, Japan*, IS-07-12, pp.11-17, 2007.

[94] M.Sakamoto, T. Ito, H. Furutani, and M. Kono, Some accepting powers of three-dimensional parallel Turing machines, *in 1st European Workshop on Artificial Life and Robotics, Vienna, Austria*, pp.73-79, 2007.

[95] M.Sakamoto, S.Okatani, M.Fukuda, T.Ito, and H.Furutani, and M.Kono, A relationship between Turing machines and finite automata on four-dimensional input tapes, International Journal of Artificial Life And Robotics, Springer, Vol.l3, No.1, pp.58-60, 2008.

[96] M.Sakamoto, T.Matsukawa, R.Katamune, H.Furutani, M.Kono, S.Ikeda, T.Ito, Y.Uchida, and T.Yoshinaga, Four-dimensional synchronized alternating Turing machines, Proceedings of the 10th American Conference on Applied Mathematics, Harvard University, Cambridge, USA, pp.195-200, 2010 (CD-ROM).

[97] M.Sakamoto, R.Katamune, T.Matsukawa, H.Furutani, M.Kono, S.Ikeda, T.Ito, Y.Uchida, and T.Yoshinaga, Some results about hierarchy and recognizability of four-dimensional synchronized alternating Turing machines, Proceedings of the 10th American Conference on Applied Mathematics, Harvard University, Cambridge, USA, pp.201-205, 2010 (CD-ROM).

[98] S. Sakurayama, K. Inoue, I. Takanami, H. Taniguchi, and H. Matsuno, A note on multihead on-line Turing machines, *Technical Report of the Yamaguchi University*, Vol.3, No.2, pp.183-192, 1983.

[99] A. N. Shah, Pebble automata on arrays, *Computer Graphics Image Processing*, Vol.3, pp.236-246, 1974.

[100] A. N. Shah, Pushdown automataon arrays, *Information Sciences*, Vol.25, pp.175-193, 1981.

[101] R. E. Stearns, J. Hartmanis, and P. M. Lewis, Hierarchies of memory limited computations, *Proceedings of the IEEE Conference on Switching Circuit Theory and Logical Design*, pp.179-190, 1965.

[102] I. H. Sudborough, Efficient algorithms for paths system programs and applications to alternating and time-space complexity classes, *Proceedings of the 21st IEEE Symposium on Foundations of Computer Science*, pp.62-73, 1980.

[103] A.Szepietowski, On three-way two-dimensional multicounter automata, *Information Sciences*, Vol.55, pp.35-47, 1991.

[104] A. Szepietowski, Turing machines with sublogarithmic space, *Lecture Notes in Computer Science* 843, 1994.

[105] H. Taniguchi, K. Inoue, and I. Takanami, A note on three-dimensional finite automata, *Information Sciences*, Vol.26, pp.65-85, 1982.

[106] H. Taniguchi, K. Inoue, and I. Takanami, k-neigh-borhood template $\mathcal{A}$-type 2-dimensional bounded cellular acceptor (in Japanese), *The Transactions of the IEICE*, Vol.J69-D, No.3, pp.291-301, 1986.

[107] H. Taniguchi,K. Inoue, and I. Takanami, Hierarchical properties of the k-neighborhood template $\mathcal{A}$-type 2-dimensional bounded cellular acceptor (in Japanese), *The Transactions of the IEICE*, Vol.J69-D, No.7, pp.1025-1034, 1986.

[108] N.Tomozoe, M.Sakamoto, K.Iihoshi, Y.Nagamizu, T.Ito, H.Furutani, M.Kono, S.Ikeda, T.Tamaki, and K.Inoue, A relationship between the accepting powers of alternating finite automata and nondeterministic on-line tessellation acceptors on four-dimensional input tapes, Proceedings of the 12th International Symposium on Artificial Life and Robotics, Oita, Japan, 2007 (CD-ROM).

[109] A. M. Turing, On computable number, with an application to the Entscheidungsproblem, *Proceedings of the London Math. Soc.*, Vol.2, No.42, pp.230-265, 1936.

[110] Y.Uchida, T.Ito, H.Okabe, M.Sakamoto, H.Furutani, and Michio Kono, Recognizability of multi-inkdot finite automata on four-dimensional input tapes, Proceedings of the 12th WSEAS International Conference on Computers, pp.408-413, Heraklion, Greece, 2008.

[111] Y.Uchida, T.Ito, H.Okabe, M.Sakamoto, H.Furutani, and M.Kono, Four-dimensional multi-inkdot finite automata, WSEAS Transactions on Computers, Issue 9, Vol. 7, pp.1437-1446, 2008.

[112] Y.Uchida, M.Sakamoto, A.Taniue, R.Katamune, T.Ito, H.Furutani, and M.Kono, Some properties of four-dimensional parallel Turing machines, Proceedings of the 15th International Symposium on Artificial Life and Robotics, Oita, Japan, OS5-6, pp.922-925, 2010 (CD-ROM).

[113] J. Wiedermann, Parallel Turing machines, *Technical Report RUU-CS-84-11*, Dep. of Computer Science, University of Utrecht, the Netherlands, 1984.

[114] Y. Yamamoto, K. Morita, and K. Sugata, Space complexity for recognizing connected patterns in a three-dimensional tape (in Japanese), *Technical Report of the IEICE*, No.AL79-104, pp.91-96, 1979.

[115] A. C. Yao , and R. L. Rivest , k+1 heads are better than k, *J. ACM*, Vol.25, No.2, pp.337-340, 1978.

Yasuo Uchida is a professor in the Department of Business Administration at Ube National College of Technology, Japan. His current research interests are in the areas of administration simulation game and e-Learning system. He has published several research papers and books in that area. His articles have appeared in journals such as *WSEAS TRANSACTIONS* on *COMPUTERS*, and proceedings of major conferences such as *ISAGA* and *AROB*.

Takao Ito is a professor of the Department of Business Administration at Ube National College of Technology, Japan. He is serving concurrently as professor of Harbin Institute of Technology (*HIT*) in Weihai, China. He received his Doctor degree of Economics from Kyoto University and Ph.D. degree of Engineering from Miyazaki University, Japan. His current research interests include automaton theory, quantitative analysis of supply chain relationships using graph theory, theoretical analysis of the relationship between IT and corporate strategy, and engineering analysis of organizational structures using complex system theory. He was one of the winners of the Best Paper Award in the International Symposium on Artificial Life and Robotics (*AROB*) in January 2006. He has published in several refereed journals and proceedings, particularly in the area of corporate management, IT strategy, and computer science. Dr. Ito is a member of the international advisory board of Scientific Journals International and the editorial review board of International Journal of Business Data Communications and Networking, International Journal of Data Mining, Modeling and Management, and International Journal of Information and Communication Technologies for the Advanced Enterprise. He sits in the International Program Committee of Artificial Life and Robotics (*AROB*). And he also is a member of the Board of Directors of Asian Symbiosis Association, Japan.

Makoto Sakamoto received the Ph.D. degree in computer science and systems engineering from Yamaguchi University in 1999. He is presently an associate professor in the Faculty of Engineering, University of Miyazaki. His first interests lay in hydrodynamics and time series analysis, especially the directional wave spectrum. His current main research interests are automata theory, languages,and computation. He is also interested in digital geometry, digital image processing, computer vision, computer graphics, complex systems. and so forth. He has published many research papers in that area. His articles have appeared in journals such as Information Sciences, Pattern Recognition and Artificial Intelligence, *WSEAS*, *AROB*, *SJI*,and *IEICE* (Japan), and proceedings of conferences such as Parallel Image Processing and Analysis, *SCI*, *WSEAS*, and *AROB*.

Ryoju Katamune received B.E. degree in computer science from University of Miyazaki in 2009. He is presently a postgraduate student in University of Miyazaki. His main interests are automata theory and digital geometry. He is also interested in image processing, computer graphics, and so on. He has published some research papers in that area. His articles have appeared in journals such as *WSEAS TRANSACTIONS* on *COMPUTERS*, and proceedings of conferences such as *WSEAS* and *AROB*.

Kazuyuki Uchida received B.E. degree in computer science from University of Miyazaki in 2009. He is presently a postgraduate student in University of Miyazaki. His main interests are automata theory and computational complexity. He is also interested in moving image processing, computer animation, and so on. He has published some research papers in the area of theoretical computer science. His articles have appeared in proceedings such as *WSEAS* and *AROB*.

Hiroshi Furutani received the Ph.D. degree in science from Kyoto University in 1981. He is presently a professor in the Faculty of Engineering, University of Miyazaki. His intersts were problems in theoretical nuclear phisics. His current main research interests are automata theory, evolutionary computation and bio-informatics.

His articles have appeared in journals such as Complex systems, Foundations of Genetic Algorithms, and so on.

Michio Kono received the B. Eng., M. Eng. and D.Eng. Degrees in Aeronautical Engineering from Nagoya University, Japan in 1969, 1971 and 1977, respectively. He joined the Department of Control Engineering, Tokyo University of Mercantile Marine, in 1979, where he became a Professor in 1990. Since 1992, he has been a Professor at the Department of Computer Science and Systems Engineering, University of Miyazaki. His research interests include decoupling control, guaranteed cost control, periodic systems, time-delay systems and networked control systems. His articles have appeared in journals such as *WSEAS*, *AROB*, *SICE*(Japan), *ISCIE*(Japan), and proceedings of conferences such as *WSEAS* and *AROB*.

Satoshi Ikeda received the BSc degree in Mathematics from Waseda University, the MSc degree in Mathematics from Shizuoka University and the Ph.D. degree from Hiroshima University. Though a researcher of Japan Society for the Promotion of Science, he was a research assistant of Tokyo University of Agriculture and Technology from 1996 to 2002. He has been an associate professor in Miyazaki University from 2002. His research interest includes graph theory and measure theory. His articles have appeared in journals such as Theoretical Computer Science, Journal of human ergology, Topology and its Applications, and *IEEE* Transactions on Parallel and Distributed Systems, and proceedings of conferences such as *ICALP*, *ICM*, *VLSI*, and *AROB*.

Tsunehiro Yoshinaga received the Ph.D. degree in computer science and systems engineering from Yamaguchi University in 1996. He is presently a professor in the Department of Computer Science and Electronic Engineering at Tokuyama National College of Technology. His research interests are automata theory, computational complexity, and database. He has published research papers in that area. His articles have appeared in journals such as Thoeretical Computer Science, *WSEAS*, *JCST*, *IEICE*, and *IPSJ*, and in proceedings of conferences such as Information and *WSEAS*.