Study of Lyapunov function for different strategies of analogue circuit optimization

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Abstract—The problem of designing of analog network for a minimal computer time has been formulated as the functional minimization problem of the control theory. The design process in this case is formulated as the controllable dynamic system. The optimal sequence of the control vector switch points was determined as a principal characteristic of the minimal-time system design algorithm. The conception of the Lyapunov function was proposed to analyze the behavior of the process of designing. The special function that is a combination of the Lyapunov function and its time derivative was proposed to predict the design time of any strategy by means of the analysis of initial time interval of the process of network optimization. Thus, analyzing and comparing the behavior of function of Lyapunov for various strategies of optimization, it is possible to draw conclusions about the perspective strategies, from the point of view of speed. The parallel computing serves to compare the different strategies of optimization in real time and to select the best strategy that has the minimal computer time. This approach gives us the possibility to select the quasi optimal strategy of network optimization by analyzing the initial part of the total design process. Numerical results of optimization of various analog circuits confirm a possibility of the choice of the best, in sense of speed, strategy of the optimization allowing solving a problem of design in several orders of magnitude faster than traditional approach.

Keywords—Minimal-time system designing, control theory application, network optimization, Lyapunov function.

I. Introduction

THE problem of the reduction of computer time for a large system designing is one of the essential problems of the total quality design improvement. Besides the traditionally used ideas of sparse matrix techniques and decomposition techniques [1-5] some another ways were proposed to reduce the total computer design time [6-7]. The above described ideas of system designing can be named as the traditional approach or the traditional strategy because the method of analysis is based on the Kirchhoff laws.

The other formulation of the circuit optimization problem was developed on heuristic level some decades ago [8-9]. This idea was based on the Kirchhoff's laws ignoring for all the circuit or for the circuit part. The special cost function is minimized instead of the circuit equation solving. This idea was developed in practical aspect for the microwave circuit optimization [10] and for the synthesis of high-performance analog circuits [11] in extremely case, when the total system model was eliminated.

The generalized approach for the analog system design on the basis of control theory formulation was elaborated in some previous works [12-14]. This approach serves for the definition of minimal-time algorithm of designing. On the other hand this approach gives the possibility to analyze with a great clearness the design process while moving along the trajectory curve into the design space. The main conception of this theory is the introduction of the special control functions, which, on the one hand generalize the design process and, on the other hand, they give the possibility to control design process to achieve the optimum of the cost function of designing for the minimal computer time. This possibility appears because practically an infinite number of the different design strategies that exist within the bounds of the theory. The different design strategies have the different operation number and different executed computer time. On the bounds of this conception, the traditional design strategy is only a one representative of the enormous set of different strategies of designing. As shown in [13] the potential computer time gain that can be obtained by the new design problem formulation increases when the size and complexity of the system increase. However it can be realized for optimal or quasi optimal algorithm only.

We can define the formulation of the main properties of the quasi optimal design strategy as one of the first problems that needs to be solved for the optimal algorithm construction.

The second section includes the problem formulation on mathematic level. The Lyapunov function is introduced in section III. The numerical analysis and discussion of results are provided in section IV.

II. Problem Formulation

The designing process for any analog system design can be defined in discrete form [13] as the problem of the generalized cost function $F(X,U)$ minimization by means of the equation (1) with the constraints (2):

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$$X^{s+1} = X^s + t_s \cdot H^s \tag{1}$$

$$\left(1-u_j\right)g_j(X) = 0,\ j = 1,2,\dots,M \tag{2}$$

where $X \in R^N$, $X = \left(X', X''\right)$, $X' \in R^K$ is the vector of the independent variables and the vector $X'' \in R^M$ is the vector of dependent variables ($N = K + M$), $g_j\left(X\right)$ for all j presents the system model, s is the iterations number, t_s is the iteration parameter, $t_s \in R^1$, $H \equiv H(X,U)$ is the direction of the generalized cost function $F\left(X,U\right)$ decreasing, U is the vector of the special control functions $U = \left(u_1, u_2, \dots, u_m\right)$, where $u_j \in \Omega$; $\Omega = \{0;1\}$. The generalized cost function $F\left(X,U\right)$ is defined as:

$$F\left(X,U\right) = C\left(X\right) + \psi\left(X,U\right) \tag{3}$$

where $C\left(X\right)$ is the non negative cost function of the designing process, and $\psi\left(X,U\right)$ is the additional penalty function:

$$\psi\left(X,U\right) = \frac{1}{\varepsilon}\sum_{j=1}^{M} u_j \cdot g_j^2\left(X\right) \tag{4}$$

This formulation of the problem permits us to redistribute the computer time expense between the solution of problem (2) and the optimization procedure (1) for the function $F\left(X,U\right)$. The control vector U is the main tool for the redistribution process in this case. Practically an infinite number of the different design strategies are produced because the vector U depends on the optimization procedure current step. The problem of search of the optimal design strategy is formulated now as the typical problem for the functional minimization of the control theory. The functional that needs to minimize is the total CPU time T of the design process. This functional depends directly on the operations number and on the strategy of designing that has been realized. The main difficulty of this definition is unknown optimal dependencies of all control functions u_j.

The continuous form of the problem definition is more adequate for the control theory application. This form replaces Eq. (1) and can be defined by the next formula:

$$\frac{dx_i}{dt} = f_i\left(X,U\right),\quad i = 1,2,\dots,N \tag{5}$$

This system together with equations (2), (3) and (4) composes the continuous form of the design process. The structural basis of different design strategies that correspond to the fixed control vector U includes 2^M design strategies. The functions of the right hand part of the system (5) can be determined for example for the gradient method as:

$$f_i\left(X,U\right) = -\frac{\delta}{\delta x_i}F\left(X,U\right),\quad i = 1,2,\dots,K \tag{6}$$

$$f_i\left(X,U\right) = -u_{i-K}\frac{\delta}{\delta x_i}F\left(X,U\right) + \frac{\left(1-u_{i-K}\right)}{t_s}\left\{-x_i^s + \eta_i\left(X\right)\right\} \tag{6'}$$

$$i = K+1, K+2, \dots, N$$

where the operator $\frac{\delta}{\delta x_i}$ hear and below means $\frac{\delta}{\delta x_i}\varphi(X) = \frac{\partial\varphi(X)}{\partial x_i} + \sum_{p=K+1}^{K+M}\frac{\partial\varphi(X)}{\partial x_p}\frac{\partial x_p}{\partial x_i}$, x_i^s is equal to $x_i(t-dt)$; $\eta_i(X)$ is the implicit function ($x_i = \eta_i(X)$) that is determined by system (2).

The control variables u_j have the time dependency in general case. The equation number j is removed from (2) and the dependent variable x_{K+j} is transformed to the independent when u_j=1. This independent parameter is defined by the formulas (5), (6'). In this case there is no difference between formulas (6) and (6'). On the other hand, the Eq. (5) with the right part (6') is transformed to the identity $\frac{dx_i}{dt} = \frac{dx_i}{dt}$, when u_j=0, because $\eta_i\left(X\right) - x_i^s = x_i\left(t\right) - x_i\left(t-dt\right) = dx_i$. It means that at this time moment the parameter x_i is dependent one and the current value of this parameter can be obtained from the system (2) directly. This transformation of the vectors X' and X'' can be done at any time moment.

It is necessary to find the optimal behavior of the control functions u_j during the design process to minimize the total computer time of designing. The functions $f_i\left(X,U\right)$ are piecewise continued as the temporal functions and the optimal structure of these functions can be found by means of approximate methods of the control theory [15-16].

The idea of the system design problem formulation as the functional minimization problem of the control theory is not depend on the optimization method and can be embedded into any optimization procedures. In this paper the gradient method is used, nevertheless any optimization method can be used as shown in [13-14].

Now the analog system design process is formulated as a dynamic controllable system. The time-optimal design process can be defined as the dynamic system with the minimal transition time in this case. So we need to find the special conditions to minimize the transition time for this dynamic system.

III. Lyapunov Function

On the basis of the analysis in previous section we can conclude that the minimal-time algorithm has one or some switch points in control vector where the switching is realize among different design strategies. As shown in [17] it is necessary to switch the control vector from like modified traditional design strategy to like traditional design strategy with an additional adjusting.

The main problem of the time-optimal algorithm construction is unknown optimal sequence of the switch points during the design process. We need to define a special criterion that permits to realize the optimal or quasi-optimal algorithm by means of the optimal switch points searching. On the other hand a Lyapunov function of dynamic system serves as a very informative object to any system analysis in the control theory. We suppose that the Lyapunov function can be used for the revelation of the optimal algorithm structure. First of all we can compare the behavior of the different design strategies by means of the Lyapunov function analysis.

There is a freedom of the Lyapunov function choice because of a non-unique form of this function. Let us define the Lyapunov function of the process of designing (2)-(6) by means of the following expression:

$$V(X) = \sum_i (x_i - a_i)^2 , \qquad (7)$$

where a_i is the stationary value of the coordinate x_i. In other words the set of all the coefficients a_i is the main objective of the process of designing. The function (7) satisfies all of the conditions of the standard Lyapunov function definition for the variables $y_i = x_i - a_i$. In fact the function $V(Y) = \sum_i y_i^2$ is the piecewise continue. Besides there are three characteristics of this function: i) V(Y)>0, ii) V(0)=0, and iii) $V(Y) \to \infty$ when $\|Y\| \to \infty$. Inconvenience of the formula (7) is an unknown point a=$(a_1, a_2, ..., a_N)$, because this point can be reached at the end of the design process only. We can use this form of the Lyapunov function if we already found the design solution someway. On the other hand, it is very important to control the stability of the design process during the optimization procedure. In this case we need to construct other form of the Lyapunov function that doesn't depend on the unknown stationary point. Let us define the Lyapunov function of the design process (2)-(6) by the following expression:

$$V(X,U) = [F(X,U)]^r \qquad (8)$$

$$V(X,U) = \sum_i \left(\frac{\partial F(X,U)}{\partial x_i} \right)^2 \qquad (9)$$

where *F(X,U)* is the generalized cost function of the design process. The formula (8) can be used when the general cost function is non-negative and has zero value at the stationary point *a*. Other formula (9) can be used always because all derivatives $\partial F / \partial x_i$ are equal to zero in the stationary point *a*.

We can define now the design process as a transition process for controllable dynamic system that can provide the stationary point (optimal point of the design procedure) during some time. The problem of the construction of the time-optimal design algorithm can be formulated now as the problem of the transition process searching with the minimal transition time. There is a well-known idea [18-19] to minimize the time of transition process by means of the special choice of the right hand part of the principal system of equations, in our case these are the functions $f_i(X,U)$. It is necessary to change the functions $f_i(X,U)$ by means of the control vector *U* selection to obtain the maximum speed of the Lyapunov function decreasing (the maximum absolute value of the time derivative of Lyapunov function $\dot{V} = dV/dt$). Normally the time derivative of Lyapunov function is non-positive for the stable processes. However we define more informative function as a relatively time derivative of the Lyapunov function: $W = \dot{V}/V$. In this case we can compare the different design strategies by means of the analysis of behavior of the function *W(t)*.

IV. Analysis of Different Strategies

All examples have been analyzed for the continuous form of the optimization procedure (5). Lyapunov function *V(t)* and some other functions that been produced from *V(t)* were the main objects of the analysis. The behavior of all these functions have been analyzed for all strategies that compose the structural basis of the general design methodology. We need to analyze some special functions for the definition of the rigorous correlation between the CPU time and the properties of Lyapunov function. The cost function $C(X)$ has been determined as the sum of the squared differences between beforehand-defined values and current values of the nodal voltages for some nodes. All results were obtained by parallel computing for different strategies of designing. This computing was emulated on PC.

A. Example 1

The two-node network is shown in Fig. 1.

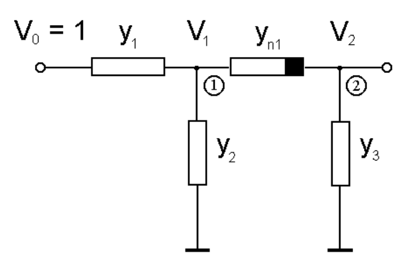

Fig. 1 Two-node nonlinear passive network

The nonlinear element has the following dependency: $y_{n1} = y_0 + b(V_1 - V_2)^2$. The vector X includes five components: $x_1^2 = y_1$, $x_2^2 = y_2$, $x_3^2 = y_3$, $x_4 = V_1$, $x_5 = V_2$. The model of this network (2) includes two equations (M=2) and the optimization procedure (5) includes five equations. The cost function $C(X)$ has been determined by the formula $C(X) = (x_5 - m_1)^2$, where m_1 is a beforehand-defined output voltage of the divider. This network is characterized by two dependent parameters (two nodal voltages) and the control vector includes two control functions: $U=(u_1, u_2)$. The structural basis of the design strategies includes four design strategies with the control vectors: (00), (01), (10), and (11). The Lyapunov function was calculated by formula (8) for r=0.5.

The results of the analysis of complete structural basis of different strategies of designing for network in Fig. 1 and initial point $x_{i0} = 1$, i = 1,2,…,5 are shown in Table I.

Table I. Data of complete structural basis of designing strategies

N	Control vector	Iterations number	Total design time (sec)
1	(0 0)	406308	8.52
2	(0 1)	455191	3.96
3	(1 0)	226909	3.31
4	(1 1)	451090	2.81

The behavior of the functions *V(t)* and *W(t)* for the network in Fig. 1 is shown in Fig. 2.

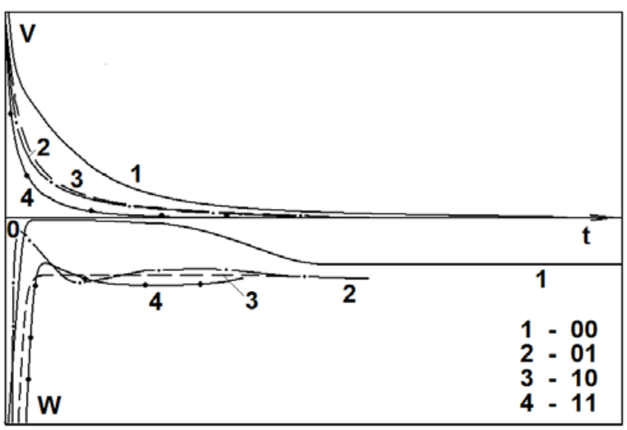

Fig. 2 Behavior of the functions *V(t)* and *W(t)* for four design strategies during the design process for network in Fig.1

As we can see from Fig. 2 the functions *V(t)* and *W(t)* can give an exhaustive explanation for the design process characteristics. A greater absolute value of the function *W(t)* corresponds to a more rapid decreasing of the function *V(t)*. We can state that the greater absolute value of the function *W(t)* on initial part of the design process provoke the lesser computer time. On the other hand the function *W(t)* is a normalized derivative and for this reason it is very sensitive. The behavior of this function for various strategies is non monotonic, and there are some intersections between the functions belonging to different strategies as we can see in Fig. 2. This complicates the identification of the best and the worst strategies. One of the strategies can be identified as the best for one time interval, and another strategy is the best for other time interval. We can assume that the area under the curve - *W(t)* may be the best way to predict the CPU time, as important to the behavior of this function at a certain time range, rather than a specific point. In this case, it makes sense to introduce a new function defined by the integral of the function *W(t)*, which will serve as a criterion for analyzing of dynamic properties for a Lyapunov function.

$$S(t) = -\int_0^t W(t)dt = -\int_0^t \frac{dV}{dt}\cdot\frac{1}{V}dt = -\int_{V(0)}^{V(t)} \frac{dV}{V} = -\ln\left|\frac{V(t)}{V(0)}\right| \quad (10)$$

The behavior of the function *S(t)* for all strategies of the Table 1 is presented in Fig. 3.

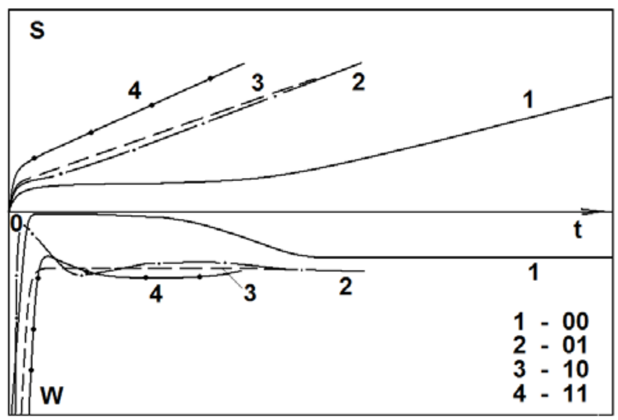

Fig. 3 Behavior of the functions *W(t)* and *S(t)* for all strategies of structural basis during the design process for network in Fig. 1

The curves of the functions *W(t)* are presented also in this figure for comparing both types of dependencies. The curves *W(t)* have intersections but the curves *S(t)* do not have intersections. We can see that all curves corresponding to the function *S(t)* are very well regulated as in design time and in absolute value of this function. There is a correlation between the function *S(t)* and a computer time. The strategy that has a lesser computer time of designing, at the same time it has a greater value of the function *S(t)* at any time moment.

Hypothesis 1. There is a strong correlation between the behavior of the Lyapunov function of the process of designing and a full CPU time of designing.

B. *Example 2*

Another passive nonlinear network with three nodes (Fig. 4) was analyzed below.

The nonlinear elements have been defined by following dependencies: $y_{n1} = a_{n1} + b_{n1} \cdot (V_1 - V_2)^2$, $y_{n2} = a_{n2} + b_{n2} \cdot (V_2 - V_3)^2$. The vector X includes seven components: $x_1^2 = y_1$, $x_2^2 = y_2$, $x_3^2 = y_3$, $x_4^2 = y_4$, $x_5 = V_1$, $x_6 = V_2$, $x_7 = V_3$.

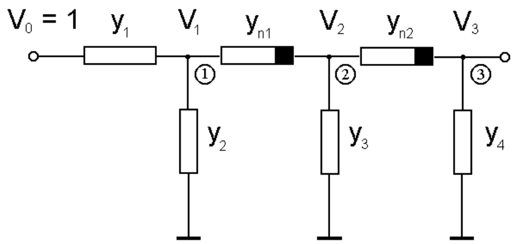

Fig. 4 Three-node nonlinear passive network

The model of this network (2) includes three equations (M=3) and the optimization procedure (1) includes seven equations. This network is characterized by three dependent parameters and the control vector includes three control functions: $U=(u_1, u_2, u_3)$. The cost function $C(X)$ has been determined as: $C(X) = (x_7 - m_1)^2 + (x_6 - m_2)^2$, where m_1 and m_2 are the beforehand-defined voltages of the circuit.

The results of the analysis for a complete structural basis of the design strategies and for initial point $x_{i0} = 1$, i = 1,2,…,7 are shown in Table II.

Table II. Data of complete structural basis of strategies of designing for network in Fig. 4

N	Control vector	Iterations number	Total design time (sec)
1	(0 0 0)	104961	5.72
2	(0 0 1)	270001	5.71
3	(0 1 0)	74428	1.65
4	(0 1 1)	80317	0.93
5	(1 0 0)	102500	2.53
6	(1 0 1)	253473	4.34
7	(1 1 0)	157583	2.63
8	(1 1 1)	246776	1.92

The behavior of the functions *W(t)* and *S(t)* during the design process is shown in Fig. 5.

There is a strong correspondence between the time of the designing shown in Table II and the dependencies *S(t)* in Fig. 5. The less time is one or another strategy, the higher is its graph. One can note an important difference between the behavior of the function *S(t)* and the function *W(t)*. The relative derivative *W(t)*, corresponding to the different strategies have intersections, which prevents the unequivocal determination of the best strategy. Dependence of *S(t)* such intersections do not have.

Consider the second option of designing the same circuit,

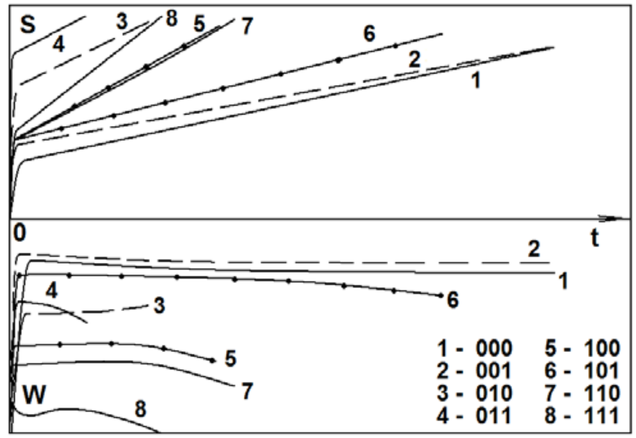

Fig. 5 Behavior of the functions *W(t)* and *S(t)* for all strategies of structural basis during the design process for network in Fig. 4

but with another initial approximation: $x_{10} = 1$ and $x_{i0} = 2$, for i = 2, 3,..., 7. The results of the designing process for a complete structural basis of design strategies are presented in Table III.

Table III. Data of complete structural basis of strategies of designing for network in Fig. 4

N	Control vector	Iterations number	Total design time (sec)
1	(0 0 0)	198989	10.61
2	(0 0 1)	586750	10.71
3	(0 1 0)	272611	5.87
4	(0 1 1)	541099	6.11
5	(1 0 0)	118901	2.64
6	(1 0 1)	278663	4.72
7	(1 1 0)	198162	3.35
8	(1 1 1)	274751	2.14

The behavior of the functions *W(t)* and *S(t)* during the design process is shown in Fig. 6.

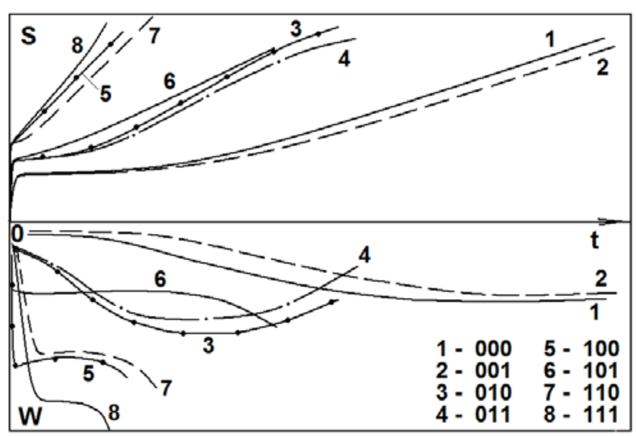

Fig. 6 Behavior of the functions *W(t)* and *S(t)* for all strategies of structural basis during the design process for network in Fig. 4

It can be stated that the change in the initial approximation leads to a redistribution strategies required CPU time. On the role of the best strategies other strategy pretends now, than in the previous case. Nevertheless we can see a complete correspondence between the CPU time and the behavior of the function *S(t)*.

C. Example 3

Other examples correspond to the designing of transistors' networks. The cost function $C(X)$ in these cases has been determined as the sum of the squared differences between beforehand-defined values of voltages on transistors' junctions and the current values of these voltages.

The first example corresponds to the designing of a single-stage transistor amplifier shown in Fig. 7.

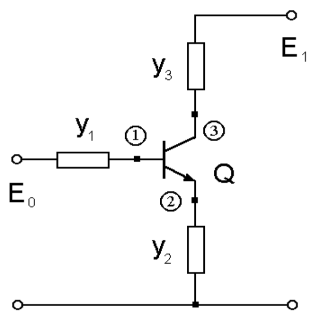

Fig. 7 One-stage transistor amplifier

The vector X includes six components: $x_1^2 = y_1$, $x_2^2 = y_2$, $x_3^2 = y_3$, $x_4 = V_1$, $x_5 = V_2$, $x_6 = V_3$. The model of this network (2) includes three equations (*M*=3).

The optimization procedure (5) includes six equations. The total structural basis contains eight different design strategies. The control vector includes three control functions: $U=(u_1, u_2, u_3)$. The Ebers-Moll static model of the transistor has been used [20].

The results of the process of designing for all strategies of the complete structural basis are given in Table IV.

Table IV. Data of complete structural basis of strategies of designing for one-stage transistor amplifier

N	Control vector	Iterations number	Total design time (sec)
1	(0 0 0)	418791	26.970
2	(0 0 1)	95396	5.051
3	(0 1 0)	615254	39.722
4	(0 1 1)	53218	2.581
5	(1 0 0)	393730	22.310
6	(1 0 1)	56821	2.913
7	(1 1 0)	292356	14.834
8	(1 1 1)	7234	0.111

The corresponding dependences of the function *S(t)* during the design process are presented in Fig. 8.

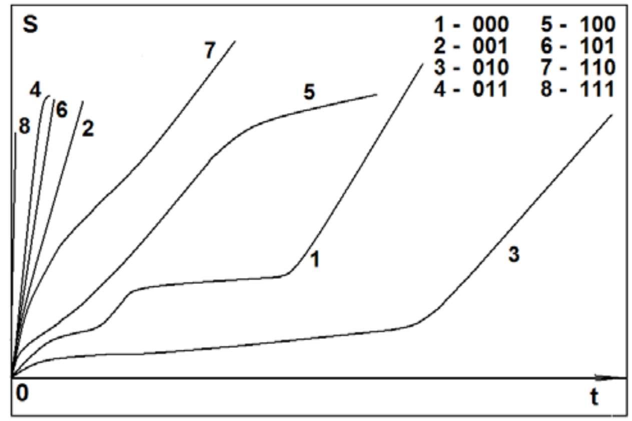

Fig. 8 Behavior of the functions *S(t)* for different design strategies of structural basis during the design process for one-stage transistor amplifier

Comparing the behavior of *S(t)* during the design process in this figure and the behavior of *W(t)* for the same example in paper [21] shows the advantages of using the function *S(t)*. The graphs of W(t) have many intersections, and the ranking of strategies for the best and the worst may be done from the data [21] only in average sense, but the analysis of Fig. 8 gives us the reasonably argued what of the strategies is the best or the worst directly.

D. Example 4

This example corresponds to the design of a two-stage transistor amplifier showed in Fig. 9.

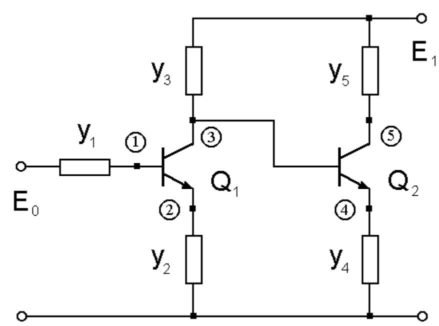

Fig. 9 Two-stage transistor amplifier

The vector X includes ten components: $x_1^2 = y_1$, $x_2^2 = y_2$, $x_3^2 = y_3$, $x_4^2 = y_4$, $x_5^2 = y_5$, $x_6 = V_1$, $x_7 = V_2$, $x_8 = V_3$, $x_9 = V_4$, $x_{10} = V_5$. The model of this network (2) includes five equations (*M*=5) and the optimization procedure (5) includes ten equations. The total structural basis contains 32 different design strategies. The control vector includes five control functions: $U=(u_1, u_2, u_3, u_4, u_5)$.

The results of the process of designing for some strategies of the complete structural basis are given in Table. V. The corresponding dependences of the function *S(t)* during the designing process are presented in Fig. 10 for all strategies of Table V.

Comparing the behavior of curves corresponding to the function *S(t)* in this figure with the data of CPU time from the Table V can be stated a very strong correlation of two these characteristics.

Table V. Data of some strategies of designing from total structural basis for two-stage transistor amplifier

N	Control vector	Iterations number	Total design time (sec)
1	(0 0 0 0 0)	165962	107.872
2	(0 0 0 0 1)	337487	263.481
3	(0 0 1 0 0)	44118	24.610
4	(0 0 1 0 1)	14941	6.540
5	(0 0 1 1 1)	21971	7.361
6	(0 1 1 0 1)	4544	1.543
7	(1 0 1 0 1)	2485	0.592
8	(1 0 1 1 1)	7106	1.212
9	(1 1 1 0 1)	2668	0.440
10	(1 1 1 1 1)	79330	3.411

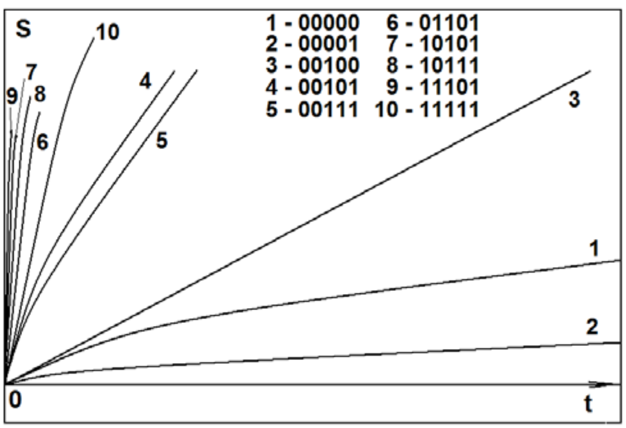

Fig. 10 Behavior of the functions *S(t)* for different design strategies of structural basis during the design process for two-stage transistor amplifier

This example, as well as all the previous shows an unambiguous correlation between the behavior of the function *S(t)* and total CPU time required to optimize the circuit. Parallel computing gives us a possibility to compare all different strategies in real time and select the best strategies.

Summarizing the results of the analysis can be argued that Hypothesis 1 is confirmed in full, that is, the behavior of the function, the derivative of Lyapunov function of the designing process and that is calculated as the logarithm of the Lyapunov function related to the total CPU time that is required to optimize the circuit. Knowledge of the behavior of this function at the initial stage of the optimization process serves to estimate the total CPU time of designing of the electronic system.

V. Conclusion

The problem of the construction of minimal-time algorithm of designing can be solved adequately on the basis of the control theory. The designing process in this case is formulated as the controllable dynamic system. The Lyapunov function and its time derivative include the sufficient information to select more perspective design strategies from infinite set of the different strategies of designing that exist into the general methodology of designing. The special functions *W(t)* and *S(t)* have been proposed to predict the better designing strategies with a minimal designing time. These functions can be used as the principal tool to the prediction of the optimal in time algorithm of designing. The successful solution of this problem permits us to select the best strategies for circuit optimization and to construct the algorithm with a minimal CPU time. The obtained results serve as a next step for the revelation of the properties of the best optimization strategy. We suppose that these results clarify the problem of circuit optimization and can be used for the future analysis to construct the optimal or quasi optimal design algorithm.

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