Integration of variance analysis and multi attribute methods of decision in application of optimal factor combination choice in one experiment

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Abstract – Beside one affirmation their hypothesis scientists make experiments to choose the optimal from available possibilities in one experiment. It is known that on the results of one experiment make influnce treatments and other uncontrolled factors called experimental error which must be smaller and for this reason scientists make different statistical plans. Mathematical apparatus for experiment organization are possible to search on the basis of total random distribution, random block distribution and special organized block distribution while they can most effectively represent complex most often multifactor even multivariate experiments. It is very dificult to make analysis of results and especially determine the optimal factor combination choice in these experiments with respondable apparatus of multiple regression analysis, or canonical analysis in multivarite case, and at any rate with help of variance analysis. Because of that for optimal factor combination choice in one experiment authors propose procedure based on integration of analysis of variance and multi attribute methods of decision. In the end of this paper are given three examples on which are demonstrated proposed procedure.

Keywords - Multivariate experiments, Analysis of variance, Multiple attribute methods

I. INTRODUCTION

Primary aim of one experiment is to provide necessary conditions to make smaller possible error of experiment and in this way enable to establish real differences between applied treatments. Therefore different plans of experiments have been developed. They was first in the form of total random distribution then as more precise random block distribution and in the end in the form special block distribution (see [1], [4]-[6] and [13]-[15]).

This plans are also applied in experiments with the group of treatments similar properties called multifactor experiments where practically each treatment consists from one combination of values of each factor. Multifactor experiments are often multivariate and just both they give possibility for greater precision and also considering of interaction in made experiment.

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The plans for multifactor experiment organization are same like for one factorial. For all of these experiments it is difficult to make analysis of results using known apparatus of classical statistics. Especially it is very difficult to solve a problem of the optimal factor combination choice usual like multiple regression analysis i.e. canonical analysis, in respectivelly univariate i.e. multivarite case, (see [4]-[6]).

Second task in one experiment can be the affirmation of given hypothesis or choice of the optimal available possibility in one experiment. The object of considering in this paper is the choice of optimal factor combination as independent variables in one multifactor and multivariate experiment towards the aim of this experiment i.e. minimum or maximum answer dependent variables in this experiment.

1. We can write for dependent variable y_i which is called the response surface $y_i=F(x_{1i},x_{2i},x_{3i},\ldots,x_{pi})+e_i$ where $i=1,2,\ldots,n$ represents the n observation in the multi factorial experiment and x_{pi} represents the level of p-th factor in the i-th observation and residual e_i measures the experimental error of the i-th observation. In the case when the mathematical form of function F isn't known, this function can be approximated satisfactorily, for example by a polynomial different degree in the independent variables x_{pi}. This considering is the basis of multiple regression analysis in the case of univariate, or canonical analysis in multivarite case besides obligatory using variance analysis and all three methods belong to the group of the methods of dependance. It is known that besides the group of the methods of dependance exist and the group of methods mutual dependence which are not the subject of considering in this paper(see [7]-[11]).

2. Theory of multiple criteria analysis gives possibility that we can make also the analysis of experiments results just in the case of the choice of optimal factor combination one multifactor and multivariate experiment. Authors jet have considered application one subgroup of this methods called multi attribute decision methods, which belong and ELECTRA, PROMETHEE and AHP methods. Application of multi attribute decision methods in the optimal factor combination choice of one experiments

is possible therefore exist necessary parameters for this application:

- More criteria – functions of aim for decision which are defined with defined explicit attributes
- More and that finite number of discreet alternatives
- One finite solution

II. AVALIABLE MATHEMATICAL APPARATUS

Mathematical apparatus for experiments results analysis towards the aim of optimal factor combination choice can be given in two basic groups([12]-[16]):

- *classic statistical analysis of variance and using*

- statistical analysis of multiple linear regression for univariate experiments i.e.
- canonical analysis for multivariate experiments

in addition it is important to notice that both subgroups belong to grop of so called methods of dependance in which group belong also and method analysis of variance and how we have noticed they are very dificult for application

- *multiple criteria analysis in which group belong*

- multi attribute decision methods
- data envelopment analysis (DEA) method

where the DEA method have not the heaviness coefficients for input and output criteria i.e attributes. In this paper author considers an integration of method analysis of variance in the multi attribute methods and demonstrates on one example that the application so obtained method gives the better resulat as standard application of mult attribute decision methods.

A. Analysis of variance

When the scientists consider the affirmation their hypothesis on the three or more different treatments in one basis set of experiment units they use the method analysis of variance. They can observe two elementary forms of experiments besides mathematical model analysis of variance for each form can be given with two different forms and that as additive (which we will use in this paper) and multiplicative:

- The first form of experiments is where we have only one criteria by classification units. By those experiments, called experiment with total random distribution, total variation is divided into two components: variation between and inside groups. First variation results from the use of different treatments and the other is a consequence of accidental swinging inside each sample.

Mathematical model analysis of variance in this case of this form is given with following relation:

where: X_{ij} s random variable j-th unit and i-th treatment (i,j=1,2,…,n), α_i presents effect i-th treatment and ε_{ij} is random variation inside units. These experiments are applicable because they can include the big number of treatments without limitation of repetition and the statistics analysis of variance is very simple.

- The second form of experiments is with two or more criteria by classification units and we will consider in this paper those which have two criteria and that first criteria is treatment and second criteria is restriction of experiments error with the set apart from experiments error expected system variation which exists beside the treatments in experiment units. Total variation then have beside influence of treatments and variation inside units and third part and that is beside treatments influence and expected influence one other system variation and therefore exists random block-design which is the plan of experiment in which units beside treatments are grouped and by known controlled system variation.

For this form mathematical model analysis of variance is given as:

where X_{ij} is random variable j-th block and i-th treatment (i=1,2,…,t; j=1,2,…,b), α_i presents effect i-th treatment, β_j presents effect j-th block and ε_{ij} is random variation of basis set which has for middle 0 and σ^2.

This form of random block-design can be realized in two subforms:

1.) first subform of block-designs which have plane of complete random distribution so called balanced complete block design(BCBDs) where we first set apart homogenous groups i.e. blocks towards criteria of classification which don't result from treatments and afterwards is one treatment applied at each units from group (we have such a number of units how much treatments and the number of repetition of treatment is equal of the number of groups).

2.)second subform of block-designs which have plane of incomplete random distribution so called balanced incomplete block design(BIBDs) also and other special form of designs which are most effective like Latin square.

By multifactor experiments which have and some special planes like confounding and split-plot planes and so long, situation is more difficult. They have the number of considered units which, in the minimum case without repetition, present total number of combination equal t^f where t is the number of treatments and f is the number of factors.

Mathematical. model for multifactor experiment organization is same like for one factor and for two factor and n repetition is given with:

- for total random distribution

$$x_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ijk}$$

- and for block random distribution

$$x_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \gamma_k + \varepsilon_{ijk}$$

where a_i ,b_j ,are treatments each of two factors

$i=1,2,\ldots,a; j=1,2,\ldots,b; k=1,2,\ldots,n$ and γ_k effect from k repetition.
It is known that the best estimates of the unknown μ, α_i,β_j are obtained by the method of least squares. This method chooses estimates m,a_i,b_j, respectively from the mathematical model $x_{ijk}=m+a_i+b_j+\varepsilon_{ijk}$, which minimize the sum of squares of the residuals in general case notated as $S_1^2=\sum (x_{ijk}- m- a_i - b_j)^2$ taken over all observations. In order to find these minimizing values we differentiate the sum of squares with respect to each unknown in turn, and set the derivative equal to zero. All the procedures, like t and F tests use the residual sum of squares often called the error sum of squares. This quantity could be found by calculating for each observation x_{ijk} the value $(m- a_i + b_j)$ predicted by the least squares solution. This method is slow and the error sum of squares is much more quickly calculated by technique called analysis of variance. The analysis of variance provides much more than short-cut method of securing the error of squares. The sum of squares due to treatments is the quantity $S_2^2 - S_1^2$, where $S_2^2 = \sum(x_{ijk}-m'- a'- b_j')^2$ is residual sum of quadrate computed with restriction $a_1 = a_2 =\ldots= a_p$, which we need for the F-test of the hypothesis that no differences exist between the effects of the treatments. If we write $x_{ijk}=m+a_i+b_j+(x_{ijk}-m-a_i-b_j)$ then square both sides, and add over all observations we have the following analysis of sums of squares:
$\sum (x_{ijk})^2 = \sum m^2 +\sum (a_i)^2+ \sum(b_j)^2 +\sum(x_{ijk}-m-a_i-b_j)^2$.
This equations is the base of the analysis of variance of experiment results.
The situation in the case of multivariate experiments, which are not object of interesting in this paper, is based on identical basis.

B. Multiple linear regression
Method for examining the influence more different independent variables for example $x_{1i},x_{2i},x_{3i},\ldots,x_{pi}$ on one dependent variable for example y is called multiple regression and can be given in the form $y=a+ b_1x_1+ b_2x_2+ b_3x_3+\ldots+ b_px_p$.
where b_i $i=1,2,\ldots,p$ are partial coefficients of regression. In the case of fixed values independent variables x when we have and experimental error in each from fully n observation we can present multiple regression in the form
$y_i=\beta_0+ \beta_1x_{1i}+ \beta_2x_{2i}+ \beta_3x_{3i}+\ldots+ \beta_kx_{ki}+e_i$.
The calculation of parameters $a,b1,b2,b3,\ldots,b_p$
We can made with the method of smallest quadrates with minimization of expression

$$\sum_{i=1}^{n}(y_i - a - b_1x_{1i} - b_2x_{2i} - \cdots - b_px_{pi})^2.$$

Practical, algebraic algorithm for solving arising system of equation is rarely in use than known Gaussian method of multiplication all the more so this method are already used in calculation for regression valuation and therefore we consider this method.
With differentiation in in relation on $a,b1,b2,b3,\ldots,b_p$ and with exchange in notation $b_0=a$ we obtain next normal equation which must be solved to receive parameters:

$b_0(00)+b_1(01)+\ldots+b_p(0p)=(0y)$
$b_0(10)+b_1(11)+\ldots+b_p(1p)=(1y)$
.
.
.
$b_0(p0)+b_1(p1)+\ldots+b_p(pp)=(py)$

where
is the sum of products j-th and k-th variables x_j and y_k,

$$(jj)= \sum_{i=1}^{n}(x_{ji})^2$$

is the sum of squares j-th column variable xi,

$$(jy)= \sum_{i=1}^{n}x_{ji}yi$$

is the sum of products j-th column variable xj and variable y.
The matrix of independent variables x and vector y is the initial basis for calculation sum of squares and products of variables and can be given like:

x				y
x_{01}	x_{11}	∘∘∘	x_{p1}	y_1
x_{02}	x_{12}	∘∘∘	x_{p2}	y_2
x_{03}	x_{13}	∘∘∘	x_{p3}	y_3
∘	∘	∘∘∘	∘	∘
x_{0n}	x_{1n}	∘∘∘	x_{pn}	y_n

From this matrix and vector we form sums of square and product of variables x and products of x and y which form system of normal equation:

jk=x'x				jy=x'y
00	01	∘∘∘	op	0y
10	11	∘∘∘	1p	1y
20	21	∘∘∘	2p	2y
∘	∘	∘∘∘	∘	∘
p0	p1	∘∘∘	Pp	py

With inversion of matrix x'x we obtain Gauss' multipliers:

$C_{jk} = C_{kj} = x'x$			
C_{00}	C_{01}	∘∘∘	C_{0p}
C_{10}	C_{11}	∘∘∘	C_{1p}
C_{20}	C_{21}	∘∘∘	C_{2p}
∘	∘	∘∘∘	∘
C_{p0}	C_{p1}	∘∘∘	C_{pp}

Partial coefficients of regression are:

$b_i = \sum_{i=1}^{p} (C_{jk})(jy)$ i.e. the sum of products k-th column of C_{ij} with the column (jy). When the independent variables are mutually orthogonal normal equations are particularly easy to solve therefore in this case all sums of products (jk) vanish (j?k) and the normal equations for b_i reduces to:

$(jj)b_j=(jy)$

and the multiplier in inverse matrix become values $C_{jj}/(jj)$ and $C_{jk}=0$.

C. Canonical analysis

Considering of canonical analysis surpass the predicted level of presentation in this paper and because of that it will be explained briefly.

Canonical analysis belongs to the group of regression methods for data analysis. Regression analysis quantifies a relationship between a predictor variable and a criterion variable by the coefficient of correlation γ, coefficient of determination γ^2, and the standard regression coefficient β. Multiple regression analysis expresses a relationship between a set of predictor variables and a single criterion variable by the multiple correlationn R, multiple coefficient of determination R^2, and a set of standard partial regression weights β_1, β_2, etc. Canonical variate analysis captures a relationship between a set of predictor variables and a set of criterion variables by the canonical correlations ρ_1, ρ_2, ..., and by the sets of canonical weights C and D. Canonical analysis is a multivariate technique which is concerned with determining the relationships between groups of variables in a data set. The data set is split into two groups, lets call these groups A and B, based on some common characteristics. The purpose of Canonical analysis is then to find the relationship between A and B, IE can some form of A represent B. It works by finding the linear combination of A variables, IE A1, A2 etc and linear combination of B variables, IE B1, B2 etc which are most highly correlated. This combination is known as the "first canonical variates" which are usually denoted U1 and V1, with the pair of U1 and V1 being called a "canonical function". The next canonical function, U2 and V2 are then restricted so that they are uncorrelated with U1 and V1. Everything is scaled so that the variance equals 1.

D. Multi criteria decision methods

Multi criteria decision methods are grouped about two basis groups (see [2]):

- multi target methods
- multi attribute methods

and in each of this two basis groups we have a few methods.

The subject of interesting in this paper are multi attribute decision methods. In this group we have two different subgroup of methods and that:

- subgroup without heaviness coefficients

which typical represent is data envelopment analysis (DEA) method and

- the methods with heaviness coefficients for considered units which well known represent this group are Elimination et choice translating reality (ELECTRE)method and Preference ranking organization method for enrichment evaluations (PROMETHEE) method in the subgroup of standard heaviness coefficients determining and Analytical hierarchical process(AHP)method in the subgroup for objective heaviness coefficients determining.

As we have noticed the multifactor also multivariate experiments, which are usually object of considering ones experiment therefore they give possibility for greater precision and also considering of interaction and where practically each treatment consists from one combination of values of each factor the application of multi attribute methods and that one concrete from enumerated method is possible so that it is easy to make the table of criteria which are in columns of this table and alternatives which are rows in this table with values from executed experiments take the values of factor combinations.

With the application of method of mathematical programming, which is in the basis of multi attribute methods, today we can find information support with suitable software package.

Multi attribute methods are given with mathematical model:

Max $\{f_1(x), f_2(x),\dots, f_n(x), n\geq 2\}$

by restriction

$x \in A=[a_1, a_2,\dots, a_m,]$

where are

n-number of criteria(attributes) $j=1,2,\dots,n$

m-number of alternatives(actions) $i=1,2,\dots,m$

f_j – criteria(attributes) $j=1,2,\dots,n$

a_i –alternatives(actions) $i=1,2,\dots,m$

A – set of all alternatives(actions).

Values f_{ij} of each considered criteria f_j which are received with each from possible alternatives a_i are known:

Usually the model of multi criteria methods are given with suitable matrix of attributes values for individual alternative :

	max f_1	*max f_2*	०००	*max f_n*
a_1	f_{11}	f_{12}	०००	f_{1n}
a_2	f_{21}	f_{22}	०००	f_{2n}
०००	०००	०००	०००	०००
a_m	f_{m1}	f_{m2}	०००	f_{mn}

Criteria type of minimization can be translated in criteria type of maximization for example with multiplication their values with -1.

For example one of the multi attribute methods method

ELECTRE is based on the next comparisons:
When is alternative a better then alternative b for majority criteria and additional don't exist criteria for which is alternative a strict worse then alternative b we can say without risk alternative a is better then b i.e. alternative a surpassed alternative b.
The basis of algorithm of decision for ELECTRE method form two conditions:

- condition of agreement defined trough desired level of agreement P and real index of agreement c(a,b)
- condition of disagreement defined trough desired level of disagreement Q and real index of disagreement d(a,b)

Indexes of agreement and disagreement express quantitative indexes of agreement or disagreement that the alternative a can be range before alternative b in the sense of all criteria simultaneous.
Index of agreement is the relation of the sum of relative importance each criteria which give that the alternative a is better or equals inn relation with alternative b and total sum relative importance w_j criteria K_j in the sense which we make range

$$c(a,b) = \frac{\sum_{j \in J_1} W_j}{\sum_{j=1}^{n} w_j} \cdot 100(\%)$$

Where J_1 is the set all criteria trough which is alternative a better then alternative b or equals.
Indexes of agreement (they are n(n-1)) take values from 0 to 1 end we notice they in matrix of agreement C_{nxn}.
Index of disagreement is the defined like maximum normalized interval of disagreement i.e. relation of the maximum of intervals for criteria where is alternative a worse then b and maximum interval of valuation for each criteria

$$d(a,b) = \begin{cases} 0, for I_2 = \varnothing \\ \dfrac{\max_j r(a,b)}{\max_j R_j}, contrary. \end{cases}$$

where is:
r(|a,b)-difference of valuations criteria values for alternatives a and alternatives b for individual criteria,
R_j – maximum span of valuations for each criteria (max a_j -min a_j)
I_2 – set of each criteria for which is alternative a worse then alternative b.
With the choice the biggest range of agreement(p=1) and the least range of disagreement(q=0) we separate only alternatives which are better for each criteria simultaneous.
The range is determined on the basis of relation index agreement and disagreement for even comparison i.e.

- a is better then alternative b if c(a,b) ≥p and d(a,b) ≤ q
- b is better then alternative a if c(b,a) ≥p and d(b,a) ≤q
- in other cases alternatives a and b are uncomparable

III. MAIN RESULTS

Therefore the solving a problem of the optimal factor configuration choice in one multifactor experiment with repetition understood application a very complex apparatus of multiple regression analysis or in multivariate experiments with repetition more complex canonical analysis the author of this paper proposes an integration of method analysis of variance in the multi attribute methods. An application of ELECTRA multi attribute method is presented in next several lines also on three examples in the end of this section.
Namely, like result one application of one multifactor experiment with repetition we have results organized in one table with rows which are factor combination and columns which are repetition of this factor combination.
In the ELECTRA method we make the beginning matrix which is given like table of criteria which are in columns of this table and alternatives, i.e combination of factors, which are rows in this table with values from obtained results from executed experiments which take the middle value of one factor combination and for all that last row take values of heaviness coefficients of this criteria. Sum of values this heaviness coefficients is normalized on value 1.
It is known that exist a methods for exact determining the heaviness coefficients of applied criteria, which are unfortunately also very dificult.
Therefore , without generalization we understood that the heaviness coefficients for applied criteria are equal for a group of output and a group of input criteria.
For the group of input criteria author proposes using of F parameters computed using the method of analysis of variance, which are obviously already used to consider the results of one experiment in the sense of affirmation supposed hypothesis, in the way that the values of heaviness coefficients of criteria can take whichever values, which sum is obviously identical one, if this F parameters have not a significant values for each input criteria.
In this way with connection the methods analysis of variance and multi attribute decision method we obtain the new procedure which evident enables an easier and efficacious way for considering a results of one experiment.
Example 1. Consider the results one two factor experiment .
Each factor has three degree of values and that the application of different quantity of nitro fertilizer from 66, 101, 136 kg/ha and density sowing of wheat from 450, 600, 750 sowed kernel of wheat on m2.
Experiment is organized with complete random distribution (BCBDs) in three repetition.The result of

application of treatments given as gain given in g/m2 is showed in Table 1. Let us to solve beside affirmation of our hypothesis and optimal factor configuration choice for maximum of wheat gain.
The results of application of method of analysis of variance is given Table 2.
From the values of F parameter we can see that the variance no one criteria is not significant

Table 1. The result in example 1.

Density Kernel/ m2	Fertilizer kg/ha	repetition 1	2	3	Sum
450	66	542	488	556	1586
	101	530	514	504	1548
	136	526	492	516	1534
600	66	529	438	556	1523
	101	508	508	522	1538
	136	540	454	530	1524
750	66	538	506	472	1516
	101	506	504	468	1478
	136	540	566	554	1660
Total		4759	4470	4678	13907

Table 2. The results of application of method of analysis of variance in example 1.

Variation source	Range of right	Sum of square	Middle of square	F-parameter
Blocks	2	4938,74	2469,37	
Fertiliz. F	2	1336,51	668,25	0,75
Densit. D	2	438,74	219,37	0,25
Intera.FxD	4	5337,93	1334,48	1,5
Error	16	14225,93	889,12	
Total	26	26277,85		

If all heaviness coefficients have equal values like in the application of method ELEKTRA given in Table3.

Table 3. ELEKTRA method applicated in example 1.

	x1(F)	x2(D)	y(G)
A1	66	450	528.66
A2	101	450	514
A3	136	450	511.33
A4	66	600	507.66
A5	101	600	512.66
A6	136	600	508
A7	66	750	505.33
A8	101	750	492.66
A9	136	750	553.33
Hea. Co.	0.3333	0.3333	0.3334

we obtain results which is given in Table 4.

Table 4. Results of ELEKTRA method in example 1.
a1 dominant over: a2 a3 a4 a5 a6 a7 a8 a9
a2 dominant over: a3 a5 a6 a8 a9
a3 dominant over: a6 a9
a4 dominant over: a5 a6 a7 a8 a9
a5 dominant over: a6 a8 a9
a6 dominant over: a9
a7 dominant over: a8 a9
a8 dominant over: a9
a9 non dominant
The best action is a1.

Next four tables show that when we take all one heaviness coefficients with extremely small value for one of two input criteria we obtain identical result i.e that the best action is a1. For example in the case when the criteria of fertilizer like in the application of method ELEKTRA given in Table 5. we obtain results which is given in Table 6. i.e in the case when the criteria of density like in the application of method ELEKTRA given in Table 7. we obtain results which is given in Table 8.

Table 5. ELEKTRA method applicated in example 1.

	x1(F)	x2(D)	y(G)
A1	66	450	528.66
A2	101	450	514
A3	136	450	511.33
A4	66	600	507.66
A5	101	600	512.66
A6	136	600	508
A7	66	750	505.33
A8	101	750	492.66
A9	136	750	553.33
Hea. Co.	0.0001	0.6665	0.3334

Table 6. Results of ELEKTRA method in example 1.
a1 dominant over: a2 a3 a4 a5 a6 a7 a8 a9
a2 dominant over: a3 a4 a5 a6 a7 a8 a9
a3 dominant over: a4 a5 a6 a7 a8 a9
a4 dominant over: a7 a8 a9
a5 dominant over: a4 a6 a7 a8 a9
a6 dominant over: a4 a7 a8 a9
a7 dominant over: a8
a8 non dominant
a9 dominant over: a7 a8

The best action is a1

Table 7. ELEKTRA method applicated in example 1.

	x1(F)	X2(D)	y(G)
A1	66	450	528.66
A2	101	450	514
A3	136	450	511.33
A4	66	600	507.66
A5	101	600	512.66
A6	136	600	508
A7	66	750	505.33
A8	101	750	492.66
A9	136	750	553.33
Hea. Co.	0.6665	0.0001	0.3334

Table 8. Results of ELEKTRA method in example 1.
a1 dominant over: a2 a3 a4 a5 a6 a7 a8 a9
a2 dominant over: a3 a5 a6 a8 a9
a3 dominant over: a6
a4 dominant over: a2 a3 a5 a6 a7 a8 a9
a5 dominant over: a3 a6 a8 a9
a6 non dominant
a7 dominant over: a2 a3 a5 a6 a8 a9
a8 dominant over: a3 a6 a9
a9 dominant over: a3 a6
The best action is a1
The obtained results shows that same alternative notated with a_6 is dominant in all presented cases.

Example 2. Compare effect in three factor experiment for corn . First factor is number of plants in hectare and that 70000, 105800 and 128600 , second factor is density of nitrogen fertilizers in kg/ha and that 50,100 and 150 and the third factor is the time of harvest and that in two ripeness milky and wax which are quantified with respectively with values 0.75 and 1. This 3x3 factorial experiment is performed so that the factors are applied in total random distribution plane of experiment plane with 4 repetition. Gain of dried matter is given in kg/7m^2.
Results are given in the table 9.

Table 9. Results of experiment gave in example 2

Number of plants N	Density of fert. D	Harvest H	Repetition (Gain of dried matter) G 1	2	3	4	Sum
70000	50	0.75	5.28	6.66	7.78	5.78	25.50
		1	8.49	8.20	8.39	8.38	33.46
	100	0.75	9.34	8.28	8.55	8.43	34.60
		1	10.34	8.86	9.81	8.96	37.97
	150	0.75	9.60	10.35	9.08	9.07	38.10
		1	10.10	10.46	11.51	13.80	45.87
105800	50	0.75	7.10	6.33	6.76	7.34	27.53
		1	8.86	9.07	9.11	9.23	36.27
	100	0.75	8.19	7.52	8.66	9.45	33.82
		1	10.17	9.73	10.97	10.27	41.14
	150	0.75	9.94	9.78	9.49	8.81	38.02
		1	11.52	9.94	12.14	12.08	45.68
128600	50	0.75	7.08	6.98	6.67	6.71	27.44
		1	6.77	7.06	8.01	8.12	29.96
	100	0.75	8.17	7.80	8.99	7.94	32.90
		1	9.70	8.37	11.26	10.14	39.47
	150	0.75	11.89	9.03	10.85	8.94	40.71
		1	11.54	11.00	11.93	11.84	46.31
Total			164.08	155.42	169.96	165.29	654.75

Table 10. Analysis of variance of experiment results given in example 2.

Regression Statistics

Multiple R	0.973683
R Square	0.948059
Adjusted R Square	0.936928
Standard Er.	0.039782
Observatios	18

ANOVA

	df	*SS*	*MS*	*F*	*Significane F*
Regression	3	0.404402	0.134801	85.17809	3.13E-09
Residual	14	0.022156	0.001583		
Total	17	0.426558			

	Coefficients	*Standard Er.*	*t Stat*	*P-value*	*Lower 95%*
Intercept	0.023922	0.080496	0.297185	0.770689	-0.14872
X Variable 1	0.015558	0.038877	0.400185	0.69506	-0.06782
X Variable 2	0.310542	0.022968	13.52067	1.99E-09	0.26128

X Variable 3	0.639	0.075013	8.518536	6.54E-07	0.478113

From the relation of values for F distribution we see that only the variance of density of fertilizer and

harvest are significant.
Statistical analysis of multiple linear regression, using Excel Data analysis option, we obtain results which are given in also in Table 10.
Output multiple linear regression give us relation between output parameter and factors of experiment in example 2 in the form

G = 0,023922 + 0,015558N + 0,310542D + 0,639H

from which we can calculate optimal factor combination like sixth combination
N=105800 , D= 150 and H=wax ripeness in for example a_6 notation..
Let us to solve example 2 with procedure proposed in this paper with ELECTRE method of multi attribute decision and :

1.) with heaviness coefficients for criteria i.e factors given in Table 11. which are equal between input factors i.e. with values for number of plants N, density of fertilizer D and harvest H equal 0,1666 or equal 0,5 for sum all three input criteria i.e. factors and like authors suppose value 0,5 for gain of dried matter like only one output criteria i.e. factor.

Table 11. Beginning matrix for ELECTRA method for example 2. for given heaviness coefficients in 1)

	x1(N)	x2(D)	x3(H)	y(G)
a1	0.7	0.5	0.75	0.6375
a2	0.7	0.5	1	0.8365
a3	0.7	1	0.75	0.865
a4	0.7	1	1	0.94925
a5	0.7	1.5	0.75	0.9525
a6	0.7	1.5	1	1.14675
a7	1.058	0.5	0.75	0.68825
a8	1.058	0.5	1	0.90675
a9	1.058	1	0.75	0.8455
a10	1.058	1	1	1.0285
a11	1.058	1.5	0.75	0.9505
a12	1.058	1.5	1	1.142
a13	1.286	0.5	0.75	0.686
a14	1.286	0.5	1	0.749
a15	1.286	1	0.75	0.8225
a16	1.286	1	1	0.98675
a17	1.286	1.5	0.75	1.01775
a18	1.286	1.5	1	1.15775
	0.16665	0.16665	0.1667	0.5

Obtained results with ELECTRE method and such values for criteria are given in Table 12.

Table 12. Application of ELECTRA method for example 2.
a1 non dominant
a2 dominant over: a1 a7 a13 a14 a15
a3 dominant over: a1 a7 a9 a13 a15
a4 dominant over: a1 a7 a13 a14 a15
a5 dominant over: a11
a6 dominant over: a1 a3 a4 a5 a7 a9 a11 a12 a13 a14 a15 a17
a7 dominant over: a13
a8 dominant over: a1 a3 a7 a9 a13 a14 a15
a9 dominant over: a1 a15
a10 dominant over: a2 a3 a5 a7 a9 a11 a13 a14 a15 a16 a17
a11 non dominant
a12 dominant over: a4 a5 a7 a9 a11 a13 a14 a15 a17
a13 non dominant
a14 non dominant
a15 non dominant
a16 dominant over: a9 a11 a13 a14 a15
a17 dominant over: a1 a15
a18 dominant over: a11 a13 a14 a15 a17
The obtained results shows that same alternative notated with a6 is dominant.

2.) with heaviness coefficients for criteria i.e factors given in Table 13 which are for input factors i.e. criteria with values for number of plants N, density of fertilizer D and harvest H proportional to values corresponding F parameters respectively 0,00274, 0,27737 and 0.21989 and equal 0,5 for sum all three input criteria i.e. factors and like authors suppose value 0,5 for gain of dried matter like only one output criteria i.e. factor.

Table 13. Beginning matrix for ELECTRA method for example 2. for given heaviness coefficients in 2)

	x1(N)	x2(D)	x3(H)	y(G)
a1	0.7	0.5	0.75	0.6375
a2	0.7	0.5	1	0.8365
a3	0.7	1	0.75	0.865
a4	0.7	1	1	0.94925
a5	0.7	1.5	0.75	0.9525
a6	0.7	1.5	1	1.14675
a7	1.058	0.5	0.75	0.68825
a8	1.058	0.5	1	0.90675
a9	1.058	1	0.75	0.8455
a10	1.058	1	1	1.0285
a11	1.058	1.5	0.75	0.9505
a12	1.058	1.5	1	1.142
a13	1.286	0.5	0.75	0.686
a14	1.286	0.5	1	0.749
a15	1.286	1	0.75	0.8225
a16	1.286	1	1	0.98675
a17	1.286	1.5	0.75	1.01775
a18	1.286	1.5	1	1.15775
	0.00274	0.27737	0.21989	0.5

Obtained results with ELECTRE method for such values for criteria are given in Table 14

Table 14. Obtained results with ELECTRE method for

values for criteria are given in Table 13.
a1 non dominant
a2 dominant over: a1 a7 a13 a14 a15
a3 dominant over: a9 a15
a4 non dominant
a5 dominant over: a11
a6 dominant over: a5 a11 a12
a7 dominant over: a1 a13
a8 dominant over: a1 a2 a3 a7 a9 a13 a14 a15
a9 dominant over: a15
a10 dominant over: a3 a4 a5 a9 a11 a15 a16 a17
a11 non dominant
a12 dominant over: a5 a11
a13 dominant over: a1
a14 non dominant
a15 non dominant
a16 dominant over: a4 a5 a11 a15
a17 dominant over: a5 a11
a18 dominant over: a5 a6 a11 a12
The obtained results shows that alternatives notated with a_8 and $a_{10}$are dominant.
Example 3. Compare effect of three factor experiment in cow feeding organized in four groups each with five cows. First factor is grouped in two sort of fodder - noodles of sugar beet and cornstalks which are quantified with values respectively 1 and 0.5, second factor is grouped in two races of cows - Frisian and domestic variegated which are quantified with values respectively 1 and 0.5 and the third factor is the period of time - first like 28 days and second next 28 days which are quantified with values respectively 1 and 0.5. Gain of quantity of milk for 28 days is given in liter. Results are given in the table15. and results with statistical analysis of multiple linear regression, using Excel Data analysis option, are given in Table 16. Output multiple linear regression gives us relation between output parameter and factors in example 3
G = 160,49 + 90,33S + 119,75D + 147,03P
and we can calculate optimal factor combination like first combination and that first factor like noodles of sugar beet , second factor like Frisian race and third factor first period time of 28 days.
Let us solve example 3 with procedure proposed in this paper with ELECTRE method of multi attribute decision and with heaviness coefficients for criteria i.e factors which are equal between each factor in group and between groups of input and output factors.

Table 15. Results of experiment gave in example 3.

Sort of fodder S	Race of cows D	Period of time P	Repetition (Gain of of milk) G					Sum
			1	2	3	4	5	
1	1	1	496.7	438.5	586.6	453.1	518.9	2493.8
		0.5	392.3	284.2	678.9	309.4	576.2	2241.0
	0.5	1	444.9	434.2	485.9	555.5	298.9	2219.4
		0.5	496.9	409.2	411.3	307.9	438.6	2063.9
0.5	1	1	411.0	348.6	781.3	356.1	523.7	2420.7
		0.5	311.2	368.2	514.9	362.6	452.8	2009.7
	0.5	1	553.1	366.2	456.6	323.2	468.6	2167.7
		0.5	286.2	365.3	279.1	382.1	204.0	1516.7
Total			3392.3	3014.4	4194.6	3049.9	3481.7	17132.9

Table 16. Results of application multiple linear regression,using Excel Data analysis, for example3.
SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.933755
R Square	0.871898
Adjusted R Square	0.775821
Standard Error	28.46464
Observations	8

ANOVA

	df	*SS*	*MS*	*F*	*SignificancF*
Regression	3	22058.7	7352.899	9.075013	0.029422
Residual	4	3240.943	810.2357		
Total	7	25299.64			

	Coef-ficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	160.49	53.25246	3.013757	0.039404	12.63746	308.3425	12.63746	308.3425

X Variable1	90.33	40.25508	2.243941	0.088236	-21.436	202.096	-21.436	202.096
X Variable2	119.75	40.25508	2.97478	0.04095	7.983986	231.516	7.983986	231.516
X Variable3	147.03	40.25508	3.652458	0.021724	35.26399	258.796	35.26399	258.796

Table 17. Beginning matrix for ELECTRA method for example 3.

	x1(S)	x2(D)	X3(P)	y(G)
A1	1	1	1	498.76
A2	1	1	0.5	448.2
A3	1	0.5	1	443.88
A4	1	0.5	0.5	412.78
A5	0.5	1	1	484.14
A6	0.5	1	0.5	401.94
A7	0.5	0.5	1	433.54
A8	0.5	0.5	0.5	303.34

Table 18. Result of application ELECTRA method for example 3.
a1 dominant over: a2 a3 a4 a5 a6 a7 a8
a2 dominant over: a4 a6 a8
a3 dominant over: a4 a7 a8
a4 dominant over: a8
a5 dominant over: a6 a7 a8
a6 dominant over: a8
a7 dominant over: a8
a8 non dominant

The obtained results for data in Table 17. showed in Table 18. demonstrate that the a_1 i.e. same alternative like with using multiple linear regression method is dominant.
It is necessary to notice that this procedure connection methods analysis of variance and some from multi attribute methods is also applicable in the case multivariate experiments but for this case we have not considered in this paper.
The multivariate experiments are special difficult for considering of optimal factor combination choice in one experiment with respondable apparatus of canonical analysis and because of that the application from author proposed method have a especial importance and must be the thema of particular study.

IV. CONCLUSION

The application classical statistic mathematical apparatus for considering results one experiment is very complex and especially in the case of solving a problem of optimal factor configuration choice in one multifactor or multivariate experiment with repetition. Because of that the authors have proposed in this paper one application mathematical apparatus of multi attribute analysis for analysis of experiment results. Proposed application multi attribute decision methods is based on one connection between analysis of variance and selected multi attribute decision method so that their very important obviously present heaviness coefficients for input criteria i.e. factors are whichever value if the values of corresponding F parameters obtained in analysis of variance are not significant.

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