Real-time Simulation of Stone Skipping

Jooyoung Do, Namkyung Lee*, Kwan Woo Ryu

***Abstract*—** The stone skipping has been a popular pastime for thousands of years. In this paper, we propose a method for simulating motion of stone skipping with physically based modeling. From a physical point of view, stone skipping is a collision response of objects with water. In order to handle the collision reaction, we compute the force acting on the stone due to the water and calculate deformation of the water. We also show that our method performs well in applications where interactive performance is preferred to realism. The techniques presented can easily be extended to simulate other interactive dynamics systems.

***Keywords*—** Physically-based modeling, Real-time simulation, Stone skipping.

I. INTRODUCTION

MOST people have an experience of throwing a stone with a flat surface across a lake or other body of water in such a way that it bounces off the surface of the water. The objective of this pastime is to see how many times a stone can be made to bounce before sinking. When a stone strikes water surface, it receives force from the water related to the speed with which it travels across it. From a physical point of view, stone skipping is a collision response of objects with water. There have been many results on this problem in the area of computer graphics[1]. However, at least to the best of our knowledge, no result of research focuses on motion of object that collides with water surface. Their main goal is to simulate motion of water after collision.

Researches on impacts and ricochets of solid objects against water surface have received a considerable amount of attention in physics area. In earlier attempts, the research was of importance in naval engineering concerning the impacts of canon balls on sea surface[2]. A projectile ricochets off the water surface if some conditions are satisfied. The conditions may rely on initial impact velocity, angle of impact with respect to water surface and mass densities of the object and the fluid. This research may not be as important as in a century ago. But recently it attracts renewed interest with the studies of locomotion of basilisk lizard[3] and stone-skip[4].

Bocquet investigated impacts of a circular disk (stone) on water surface and found the maximum number of bounces performed by the stone[4]. However, he used a simple and thin disk to model a stone, and hence this approach is not suitable for an irregular-shaped stone.

Nagahiro and Hayakawa studied the condition for bounce of a circular disk which obliquely impacted on fluid surface[5]. They applied an ODE model to the disk-water impact and obtained an analytical form for the required velocity and the maximum angle. They solved the Navier-Stokes equation by using the technique of Smoothed Particle Hydrodynamics (SPH) to perform a numerical simulation of the disk-water impact. Although this approach is suitable for scientific simulations requiring a high level of detail, it is computationally expensive, making it impractical to use for interactive computer animation.

In this paper, we propose a method for simulating motion of stone skipping with physically based modeling[6]. We also show that our method performs well in applications where interactive performance is preferred to realism.

Our method actually consists of three stages. First, we need a geometric model of the stone and the water. Then, we calculate the forces on the stone by using physically-based modeling approaches. Finally, we handle the motion of water surface. Since our goal is to provide a method suitable for computer animation, we compromise accuracy for reduced computation time. Experimental results show that visually plausible motion of stone skipping is achieved in real time.

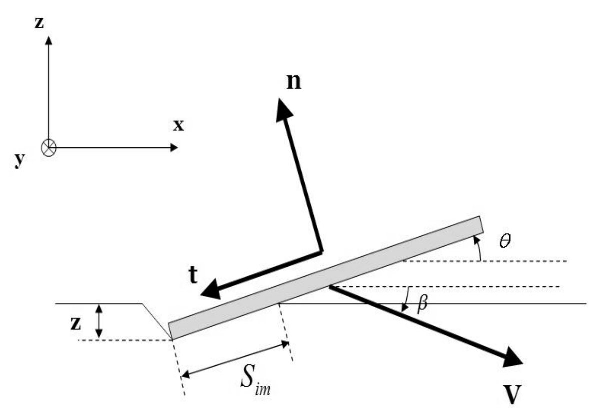

Fig. 1 An oblique entry of a circular disk on water surface with tilt angle θ and initial velocity V [4].

II. PHYSICAL ANALYSIS OF STONE SKIPPING

When one throws a stone on a water surface with small angle, the water surface exerts a reaction force on the stone, allowing it to rebound. We adopt the equation proposed by Bocquet which describes the motion of stone skipping by using the law of energy conservation[4]. The following is from his result in [4].

Fig. 1 is a schematic of disk-water impact model proposed by him. A stone with small thickness and mass M is thrown over

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a flat water surface. The tilt angle between the stone surface and the water surface is θ. The velocity V is assumed to lie in a symmetry plane of the stone. Because the stone is only partially immersed in the water during the collision process, the force acting on the stone due to the water is proportional to the immersed surface. This force can be adequately decomposed into two components; One is a component along the direction of the stone (drag force) and the other is a component perpendicular to its direction (lift force). The force can be described by the following equation.

$$\mathbf{F} = \frac{1}{2} C_l \rho_w V^2 S_{im} \mathbf{n} + \frac{1}{2} C_f \rho_w V^2 S_{im} \mathbf{t}$$

where C_l and C_f are the lift and the friction coefficients, ρ_w is mass density of the water, S_{im} is area of the immersed surface, $\mathbf{n}$ is the unit normal vector to the stone and $\mathbf{t}$ is the unit tangent vector to it.

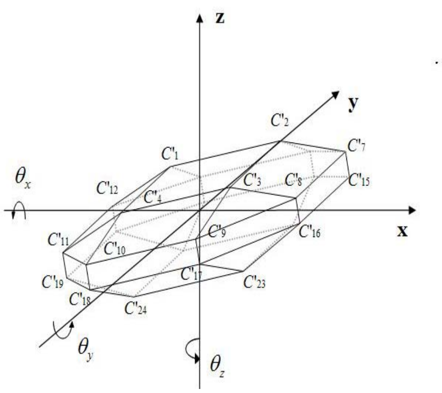

Fig. 2 The mesh structure for the stone.

III. Simulation of Stone Skipping

In this section, we present a method for collision reaction between the stone and the water surface. Since the collision generates hydrodynamics drag force, and since this drag force mainly affects the motion of the stone, the collision detection and reaction is an essential process in the simulation of stone skipping.

We construct our stone model as shown in Fig. 2, and our stone model is composed of 38 triangles. We use the general collision detection mechanism for detecting intersections between the triangles of the stone and the water surface[7]. When an intersection is found, we compute the force acting on the stone due to the water, and then handle the deformed water surface by the collision, too.

3.1 Calculation of the force acting on the stone

In order to handle the collision reaction, we first compute the force acting on the stone due to the water. As mentioned above, this force can be decomposed into lift force and drag force[8-11].

The magnitude of drag force is known as follows:

$$|F_D| = \frac{1}{2} C_f \rho_w |V|^2 S \sin\theta$$

The magnitude of the lift force $|F_L|$ can be expressed similarly as follows:

$$|F_L| = \frac{1}{2} C_l \rho_w |V|^2 S \cos\theta$$

When $\mathbf{n}_i$ denotes the unit normal of the *i*-th triangle of the stone model, the angle between $\mathbf{n}_i$ and V is $\pi/2-\theta$, and $|\mathbf{n}_i \cdot \hat{V}|$ is $\sin\theta$, where $\hat{V}$ is $V/|V|$. Hence, the drag force is proportional to $|\mathbf{n}_i \cdot \hat{V}|$. Since the direction of the drag force is opposite to the velocity, the drag force acting on the *i*-th triangle can be described as follows:

$$F_{i,D} = -\frac{1}{2} C_f \rho_w S_{i,im} |V|^2 |\mathbf{n}_i \cdot \hat{V}| \hat{V} \qquad (1)$$

Since the direction of the lift force is perpendicular to the direction of the velocity, the direction of the lift force $\mathbf{u}_i$ of the *i*-th triangle can be described as follows:

$$\mathbf{u}_i = \begin{cases} (\mathbf{n}_i \times \hat{V}) \times \hat{V}, & \text{if } \mathbf{n}_i \cdot \hat{V} > 0 \\ (-\mathbf{n}_i \times \hat{V}) \times \hat{V}, & \text{otherwise.} \end{cases}$$

Hence, the lift force $F_{i,L}$ acting on the *i*-th triangle can be described as follows:

$$F_{i,L} = (\frac{1}{2} C_l \rho_w S_{i,im} |\cos\theta| |V|^2) \mathbf{u}_i \qquad (2)$$

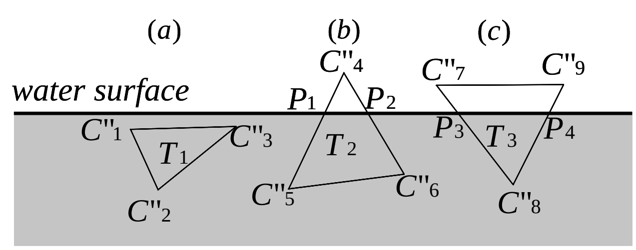

Fig. 3 Three types of interpenetration between the triangle and the water surface

Next, we need to calculate the immersed areas $S_{i,im}$ of the triangles into the water. There are three types of interpenetration between a triangle and the water surface as shown in Fig. 3.

In case (a) of Fig. 3, the immersed area is easily computed by using Heron's formula[12]. In case (b), we find points P_1 and

P_2 whose z coordinates are zero in lines $\overline{C''_4C''_5}$ and $\overline{C''_4C''_6}$, respectively, and then calculate the immersed area by subtracting $\Delta C''_4P_1P_2$ from $\Delta C''_4C''_5C''_6$. Similarly, in case (c), we can calculate the immersed area $\Delta C''_8P_3P_4$ by finding points P_3 and P_4 .We now can compute the force acting on the stone due to the water by summing up all the drag forces and all the lift forces on all the triangles. Note that the drag force and the lift force of a triangle out of contact with the water are zero.

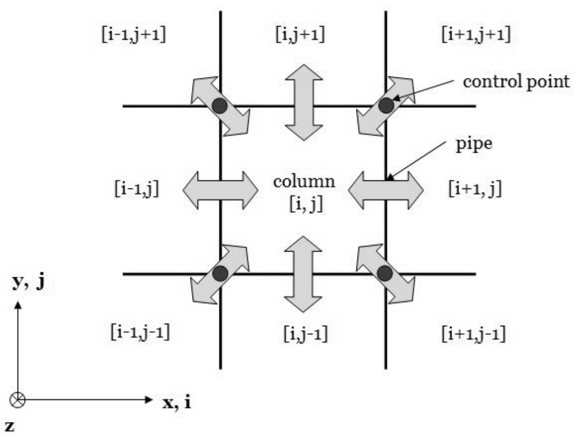

Fig. 4 Volume model of [13]. Each column is connected to other columns in its eight neighborhood through virtual pipes

3.2 Simplified fluid model using height field

When a stone gives an impact on a water surface, the impact creates ripples on the water. To handle this phenomenon, we use a simplified fluid model based on the approach by O'Brien and Hodgins[13].

They used a height field coupled with a volume model to simulate behavior of fluid. The height field plays a role in representing the shape of fluid and the volume model takes care of interaction between the stone and the fluid. To construct the volume model, they divided the body of water into a rectilinear grid of connected columns as shown in Fig. 4.

When the stone that collides on the water surface exerts forces on the height field, these forces are propagated as external pressure to the volume model. Water at column $[i, j]$ can flow into any one of its eight neighbors by difference of pressure through the pipe between the two columns, and the volume at column $[i, j]$ can change. The vertical positions of the height field are determined by the volume model.

Since a negative volume does not make sense in each column, we need the following inequality to ensure that the volume in each column remains positive:

$$C_{ij}^{t+\Delta t} \geq 0 \Leftrightarrow C_{ij}^{t} \geq -\Delta C_{ij}^{t} \qquad (3)$$

where C_{ij}^t and ΔC_{ij}^t are the volume and the volume change at the column $[i, j]$ at time t, respectively. The volume of each column is tested at the end of each iteration with (3). If a column has negative volume, then all pipes that moved fluid from that column are scaled back. This test procedure is repeated until each column in the volume model has positive volume.

Since this procedure consumes a lot of computation time, we need to improve this stabilization step for achieving real time simulation. Actually a column with negative volume occurs when the water surface deforms significantly. However, in case of stone skipping, water surface does not involve in a large amount of motion. Hence, we can approximate (3) with the following equation for reducing computational cost.

$$\text{if}\,(C_{ij}^t + \Delta C_{ij}^t < 0) \ \text{then} \ \mathrm{C}_{ij}^{t+\Delta t} = 0 \qquad (4)$$

Equation (4) ensures that the volume in each column remains positive and hence makes the system stable at each time.

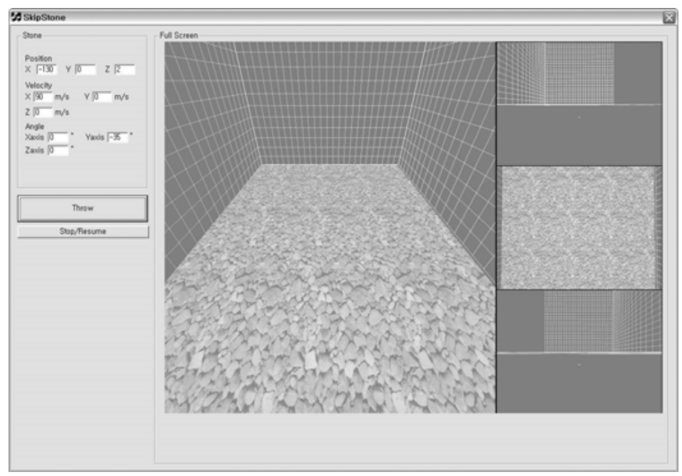

Fig. 5 Simulator for stone skipping

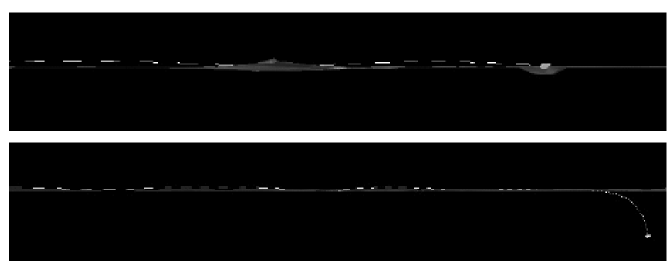

Fig. 6 Simulation of stone skipping in side view. The top image shows a part of the trajectory of a moving stone and the bottem one shows its whole trajectory.

IV. RESULTS

A prototype system is implemented using Visual C++ and OpenGL libraries on the Windows XP operating system. We used an Intel Pentium IV 3.0 GHz-based PC with 2G byte memory and GeForce 7600 GPU with 256M byte video RAM.

Fig. 5 shows our simulator for stone skipping. Our simulator displays the motion of stone, given an initial tilt angle and an initial velocity. Snapshots from the simulation are shown in Fig. 6 and 7. The stone bounces off the water surface a few times, and gradually slows down due to the drag force. Our simulation shows more than 60 frames per second, hence shows that our system has sufficient processing speed for real-time control.

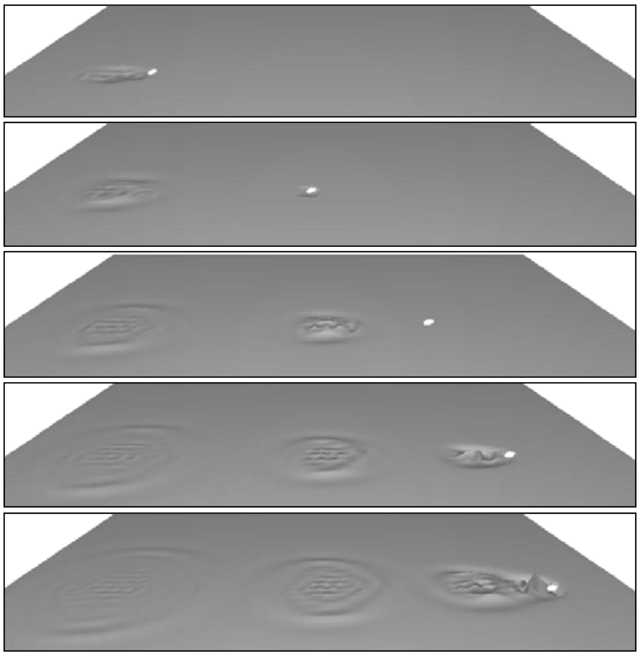

Fig. 7 The stone skips on the water.

V. CONCLUSION

In this paper, we represent a dynamics model of the stone skipping which has been a popular pastime for thousands of years. Our approach to simulate the motion of the stone skipping is based on the hydrodynamics and implemented as a real-time system.

User can generate various results with handling parameters such as the tilt angle, the weight of the stone, the shape of the stone, and an initial velocity.

Since the stone and the water skier receive a force from the water related to the speed with which they travel across it, our method can be used to simulate a water skier in virtual reality and computer games. For more realistic scenes, we need to extend our parameters to rotational velocity.

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