Demand-Side Management: Profit Function Formulation for Renewable Energy Power Market

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Abstract — A volume discounting model in the micro-grid power market discussed in this paper. The Generator Companies' (GenCo) discounting model and the special fares for Distribution Companies (DisCo) formulated. The concept of renewable energy intermittent electricity load transfers to the certain storage devices in the distribution level presented as a novel contribution to the field. According to demand-side needs, GenCo will periodically recharge DisCo energy storage devices by using the intermittent renewable energy resources as a set of micro grids. GenCo will sell certain volume of electricity to a subsidiary DisCo based on the bilaterally agreed list prices. In parallel, DisCo will sell the stored renewable electricity certain end-users. GenCo will set final unit price for each level of electricity consumption. Considering the final unit price that GenCo proposes to Disco for different power consumption levels, GenCo tries to maximize its profit margin by controlling DisCos' order behavior, in term of volumes per order, by using a discounting tool. Respectively DisCo tries to maximize its profit margin by considering the proposed special fares by GenCo. Finally, this paper presented cost functions for the power market players.

Keywords — Power market; demand-side management; discount policy; micro-grid; renewable energy resources

I. INTRODUCTION

The Quantity Discount Strategy is one of the usual methods used by sellers to reduce fixed ordering, shipment and material handling associated costs. Monahan [1] formulated the seller and buyer economical relationship ([2],[3]). As an expansion of this model, Parlar and Sarmah et al. [4] used Economic Order Quantity (EOQ) models. In inventory management, economic order quantity (EOQ) is the order amount which minimizes the holding and ordering costs. This model initially extended by Ford W. Harris in 1913 and further applied by R.H. Wilson [5]. An important application of the EOQ is in quantity discounting models. Two major quantity discounting models are "all-units discounting model" and "incremental or by-threshold discounting model" [6].

Considering recent studies, Jiamin Wang et al. [7] proposed a robust price-control model. Mehmet et al. [8] introduced and EOQ (Economic Order Quantity) respect to a group of perishable products. Maryam Akbari et el. [9] proposed a two calibrated meta-heuristic algorithm. Geoggrey A. Chuaa et al. [10] considered a short considered a short shelf-life and uncertain demand. In a similar work, T. Maiti et al. [11] presented a supply chain model for two periods of time horizons and applicable discounts as a Stackelberg play. The Stackelberg model came from the economics and management science [12]. This model initially applied in the industrial engineering practices and recently widely used in the power market literature. The Stackelberg models extended to the dynamic Stackelberg games [13]. A survey in the application of Stackelberg differential games in supply, marketing and sales channels held by He et al. [14]. In a review work, the possible market models studied by Mariano et al. [15]. This study reviewed most recent articles and technical publications respect to the electrical power market models. In a similar work, the examined approaches studied by Anke et al. [16]. Daniel J. Veit et al. [17] developed an agent based model to analyze the German electricity power market and Simona et al. [18] formulated a supply chain marginal cost function for the Italian electricity market. Considering the renewable energy, Allan et al. [19] provided an in depth review of the literature in the field. Olga [20] studied the electrical power market as a Stackelberg problem. Yu et al. [21] proposed a price-based demand response for a smart grid. Maciejowskaa et al. [22] used factoring model for the British power market. Guodong Liu et al. [23] proposed an optimal bidding strategy for the distributed generations (DGs) and Quang Duy et al. [24] discussed the interactions in the energy market. Similarly, Yang et al. [25] discussed a pricing scheme for consumers and distributors. Habib Allah Alami et al. [26] studied the impact of demand responses and the demand biddings and Wei et el. [27] proposed a Multiple Energies Trading problem (MET). Following the former studies which presented in the literature review, this paper presented cost functions for both GenCo and DisCo with a number of renewable-energy resources as a set of micro-grids. GenCo recharges DisCo storage devices or the power network against a price proposal which offered to the chain of subsidiary DisCos. GenCo tries to maximize its profit margin by controlling DisCo order volumes by a discounting tool. At the same time DisCo tries to maximize its profit

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margin by putting its orders in-line with the proposed discounting frame. This model expressed as a Stackelberg game between the micro-grid GenCo and DisCo considering the renewable-energy concept as an artwork to reach relevant profit functions for both parties. In the following, the model notation, formulation, and finally conclusion presented including the potential further steps following this work.

II. NOMENCLATURE

q_i	Volume of DisCo power order each time from a GenCo under scenario A and B
O_i	Volume of GenCo power order each time from a micro-grid under scenario A and B
T_i	Length of DisCo order cycle under strategy A and B
ts_c	GenCo storage associated cost per unit of time
ts_a	DisCo storage associated cost per unit of time
sc_c	GenCo order cost per power load request from the micro-grid
sc_a	DisCo order cost per power load request from the GenCo
p_c	GenCo unit acquisition price from the micro-grid
p_a	GenCo unit acquisition sales price to DisCo without discount, e.g. DisCo acquisition price
ξ_c	Discount rate; GenCo discounted fare (1- ξ_c) p_a while ($0 \leq \xi_c < 1$)
p_p	DisCo selling price
θ_c	Power deterioration rate at the GenCos storage devices
θ_a	Power deterioration rate at the DisCo storage devices
μ	Power market natural demand rate
$S^{(a)}(t)$	DisCo power storage devices' electricity recharge level
$S^{(c)}(t)$	GenCo power storage devices' electricity recharge level
$\alpha(T_1)$	Cumulative stored electricity in DisCo storage devises
$\beta(T_1)$	Cumulative stored electricity in GenCo storage devises

III. MODEL STRUCTURE

The Quantity Discount Strategy is one of the mostly used methods used by sellers to reduce fixed ordering, shipment and material handling associated costs. This paper presented a volume discounting model by using the profit functions of typical GenCo and DisCo, where the energy storage levels are continuously drained due to the regular market demand, the promo offers to the special group of customers and the technical loss of the electricity charges in the storage devices. Since the number of energy storage devices in the distribution layer is by far greater than its number in the generation layer, then the energy deterioration rate at the storage devices in DisCo is greater than the respected rate at GenCo. In this model, GenCo has a kind of purchase agreement with a number of renewable-energy resources as a set of micro-grids. To relax the total volume constraint at the GenCo layer, we assumed that GenCo has access to an unlimited volume of electricity with a contracted price through a number of limited intermittent energy storage recharges. GenCo recharges DisCo storage devices or the power network against a price proposal which offered to the chain of subsidiary DisCos. GenCo tries to maximize its profit margin by controlling DisCo order volumes by a discounted approach. At the same time DisCo tries to maximize its profit margin by putting its orders in-line with the proposed discounting frame which offered by GenCo. This model formulated a micro-grid GenCo and DisCo profit functions by considering the renewable-energy concept as an artwork. In this model we assumed that a dedicated GenCo exists for each market region. This GenCo uses certain energy storage devices (such as pomp storage) to reserve intermittent electricity loads from different renewable energy resources. Consequently, GenCo plays the role of a wholesaler which deals with several DisCos in that region. Depending on the electrical energy demand patterns, DisCo can either distribute the currency to its contracted end-users or can recharge its storage devices to be used in different time laps. Although we considered DisCo as part of the demand-side that is in contact with the end-users directly, nevertheless the relationship between DisCo and end-users and the possible sales tools and tactics ignored in this paper. The following two major scenarios presented in this paper,

- Scenario A – Demand-side is not controllable; DisCo orders based on EOQ
- Scenario B – Demand-side is controllable; DisCo orders based on GenCo offers

The following assumptions considered in this model formulation,

- GenCo and DisCo power reserves in the storage devices drains continuously
- Power recharge capacity from micro-grid to GenCo considered infinite
- Power volumes assumed to be continuous
- Power shortage (black-out) is not permitted
- Both GenCo and DisCo deciding purely based on a rational model
- GenCo order cycle is given by $N_i\,T_i\,; N_i$ is a positive number.

IV. DISCO TOTAL PROFIT MARGIN FORMULATION

In this model GenCo assumed to use a volume discount strategy to manage demand-side consumption behavior. GenCo proposes electrical energy volumes together with a possible discounting proposal to encourage DisCo to align its power consumption behavior in terms of order volumes in-line with the GenCo preferences. Below are two major scenarios that studied in this paper. In this section DisCos total profit margin under two different scenarios formulated.

- **Scenario A – Demand-side is not controllable; DisCo orders based on EOQ**

Under this scenario, for what so ever reason, DisCo cannot make its orders in-line with the proposed discounting frame. DisCo mainly is not capable to steer business based on GenCos' proposal and therefore prefers to order electricity against market price and according to a volume which is defined by the EOQ model. Since DisCo preference is to have adhoc orders against market price, then it cannot theoretically reach any kind of service level agreement and promotional offers from GenCo to push an special power volume in the market for sales.

- **Scenario B – Demand-side is controllable; DisCo orders based on GenCos' offer**

In this scenario DisCo uses the proposed discount offer which offered by GenCo. Respectively DisCo has certain service level agreement with a nominated GenCo which enables him to sell special prices to the power market by using a discount grid based on GenCo discount proposal.

A. DisCo Profit Margin under Scenario A

If DisCo selects scenario A, its power load order volume each time and respected acquisition cost are given by $q_1 = q\ (T_1)$ and p_a, where p_a is per load acquisition cost without existence of GenCo discount. In this situation DisCo determines the optimal volume $q_1 = q_1^*$ which maximizes its profit margin per unit of time by using an EOQ model. The power storage electricity level $S^{(a)}(t)$ at the time t in $[0, T_1\ )$ is equal to the following equation,

$$dS^{(a)}(t)/dt = -\theta_a S^{(a)}(t) - \mu \tag{1}$$

Equation (1) respect to $S^{(a)}(T_1) = 0$ gives power storage level at time t as below,

$$S^{(a)}(t) = \lambda[e^{\theta_a(T_1 - t)} - 1] \tag{2}$$

Where $\lambda = \mu / \theta_a$

Therefore, $S^{(a)}(0)(= q_1 = q(T_1))$ in order cycle is

$$q(T_1) = \lambda(e^{\theta_a T_1} - 1) \tag{3}$$

And the cumulative stored electricity power in DisCo storage devises, $\alpha(T_1)$, for period $[0, T_1)$ is

$$\alpha(T_1) = \int_0^{T_1} S^{(a)}(t)dt = \chi\ [\frac{(e^{\theta_a T_1} - 1)}{\theta_a} - 1] \tag{4}$$

DisCo profit margin per unit of time under scenario A is

$$R_1(T_1) = \frac{p_p \int_0^{T_1} \mu dt - p_a q(T_1) - ts_a \alpha(T_1) - sc_a}{T_1}$$

$$= \lambda(p_p\ \theta_a + ts_a) - \frac{(p_a + \frac{ts_a}{\theta_a})q(T_1) + sc_a}{T_1} \tag{5}$$

According to Nash equilibriums [8] [9], a unique $T_1 = T_1^*$ (>0, Finite) exists that maximizes $R_1(T_1)$ in equation (5) therefore optimal order quantity for DisCo is

$$q_1^* = \lambda(e^{\theta_a T_1^*} - 1) \tag{6}$$

And respectively, the total profit margin per unit of time is

$$R_1(T_1^*) = \lambda[(p_p \theta_a + ts_a) - \theta_a(p_p + \frac{ts_a}{\theta_a})e^{\theta_a T_1^*}] \tag{7}$$

B. DisCo Profit Margin under Scenario B

If DisCo selects scenario B, the power recharge volume per order and the discounted electricity price are $q_2 = q_2(T_2) = \lambda(e^{\theta_a T_2} - 1)$ and $(1-\xi)p_a$ respectively. Hence the profit function will be as the following,

$$R_2(T_2, \xi) = \lambda(p_p \theta_a + ts_a) - \frac{[(1-\xi)p_a + \frac{ts_a}{\theta_a}]q_2(T_2) + sc_a}{T_2} \tag{8}$$

V. GENCO TOTAL PROFIT MARGIN FORMULATION

In this section GenCo total profit margin per unit of time formulated. The presented formulation is based on DisCo decision respect to the introduced scenarios A and B.

A. GenCo Profit Margin under Scenario A

If GenCo uses strategy A, its storage load volume per time and the unit acquisition cost are respectively q_1 and p_a. The length of GenCo order cycle from the respected micro-grid can be divided to N_i recharge cycles (N_1=1,2,…) as earlier explained in the assumption number 6. Based on this assumption, Ni is a decision variable for GenCo. The GenCo storage volume is drained due to deterioration effect during $[(j-1)T_1, jT_1)$ in the j^{th} electricity recharge cycle (j =1,2,…, N_1). Therefore GenCo power storage level at time t is

$$dS^{(c)}(t)/dt = -\theta_c S^{(c)}(t) \tag{9}$$

Where $S^{(c)}(jT_1) = \varepsilon_j(T_1)$ and $\varepsilon_j(T_1)$ shows the remaining electricity volume in GenCo storage devices at the end of the j^{th} recharge cycle. Based on equation 9, GenCo storage level at time t in the j^{th} recharge is

$$S_j^{(c)}(t) = \varepsilon_j(T_1)e^{\theta_c(jT_1-t)} \tag{10}$$

The currency load level at the end of the $(N_1 -1)^{th}$ recharge cycle is equal to q_1, i.e. $\varepsilon_{N_1-1}(T_1) = q_1$, where

$$\varepsilon_j(T_1) = \frac{q(T_1)[e^{\theta_c(N_1-j)T_1} - 1]}{[e^{\theta_c T_1} - 1]} \tag{11}$$

GenCo order volume from the available micro-grid, $O_1 = O(N_1, T_1)$ $(= \varepsilon_0(T_1))$, per order is equal to

$$O(N_1, T_1) = \frac{q(T_1)[e^{\theta_c N_1 T_1} - 1]}{[e^{\theta_c T_1} - 1]} \tag{12}$$

And the cumulative stored electricity power in GenCo storage devises which is held during the j^{th} power recharge cycle, $\beta_j(T_1)$, is

$$\beta_j(T_1) = \int_{(j-1)T_1}^{jT_1} S^{(c)}(t)dt = \frac{q(T_1)}{\theta_c}[e^{\theta_c(Ni-j)T_1} - 1] \tag{13}$$

Therefore GenCo cumulative power storage on [0, N_1T_1) is

$$\beta(N_1, T_1) = \sum_{j=1}^{N_1-1} \beta_j(T_1)$$

$$= \frac{q_1(T_1)}{\theta_c}\left(\frac{e^{\theta_c N_1 T_1} - 1}{e^{\theta_c T_1} - 1} - N_1\right) \tag{14}$$

And for a specific N_1, the GenCo total profit margin per unit of time under scenario A is

$$\bar{R}_1(N_1, T^*{}_1) = \frac{1}{N_1 T^*{}_1}.[p_a N_1 q(T^*{}_1)$$

$$- p_c O(N_1, T^*{}_1) - ts_c \beta(N_1, T^*{}_1) - sc_c] \tag{15}$$

B. GenCo Profit Margin under Scenario B

When DisCo uses scenario B, it will purchase $q_2 = q(T_2)$ volume of electricity power against the discounted price $(1-\xi)p_a$ from GenCo. The GenCo electrical power volume per order under scenario B is $O_2 = O(N_2, T_2)$ and respectively GenCo total profit margin per unit of time under scenario B is

$$\bar{R}_2(N_2, T_2, \xi) = \frac{1}{N_2 T_2}.[(1-\xi)p_a N_2 q(T_2)$$

$$- p_c O(N_2, T_2) - ts_c \beta(N_2, T_2) - sc_c] \tag{16}$$

Where $q(T_2) = \lambda(e^{\theta_a T_2} - 1)$,

$$O(N_2, T_2) = \frac{q(T_2)[e^{\theta_c N_2 T_2} - 1]}{[e^{\theta_c T_2} - 1]} \tag{17}$$

VI. CONCLUSION

The Quantity Discount Strategy is one of the mostly used methods by the sellers to reduce fixed ordering, shipment and material handling associated costs. In inventory management, the Economic Order Quantity (EOQ) is the order amount which minimizes the holding and ordering costs. From the other side, the Stackelberg model brought from the economics and management science to the power market literature and in the resent years, it is widely used in electricity market studies. In this paper a volume discounting model in the micro-grid power market presented. The Generator Company (GenCo) discounting model and the special fares for Distribution Companies (DisCo) formulated respectively. The concept of intermittent electricity load transmissions generated by a set of renewable energy resources, to the certain storage devices in distribution level presented as a novel contribution to the field. In this model we assumed that a GenCo uses certain energy storage devices (such as pomp storage) to reserve intermittent electricity loads from different renewable energy resources. Consequently, GenCo plays the role of a wholesaler and deals with several DisCos in a region. Depending on the electrical energy demand patterns, DisCo can either distribute the electricity to the contracted end-users or can recharge the storage devices to use the energy in different time laps. The discounting proposal structured by considering the flow restrictions of the intermittent renewable-energy resources round the clock. This model structured by considering the expected behavior of the renewable-energy resources as an artwork to reach an optimal discounting and pricing policy which maximizes GenCo and DisCo profit margins per unit of time and under two different scenarios. Developing more heuristic models to optimize power market reliability, and considering network failure, are potential next steps which identified by the authors.

References

[1] J. P. Monahan, “A quantity discount pricing model to increase vendor’s profit,” Management Sci., Vol. 30, No. 6, pp. 720–726, 1984.

[2] M. Data and K. N. Srikanth, “A generalized quantity discount pricing model to increase vendor’s profit,” Management Sci., Vol. 33, No. 10, pp. 1247–1252, 1987.

[3] M. J. Rosenblatt and H. L. Lee, “Improving pricing profitability with quantity discounts under fixed demand,” IIE Transactions, Vol. 17, No. 4, pp. 338–395, 1985.

[4] S. P. Sarmah, D. Acharya, and S. K. Goyal, “Buyer vendor coordination models in supply chain management,” Eu. J. of Ops. R., Vol. 175, No. 1, pp. 1–15, 2006.

[5] Harris, Ford W., “How Many Parts to Make at Once”, Operations Research. Vol. 38 (6), pp. 947, 1990.

[6] Nahmias, Steven, “Production and operations analysis”, McGraw Hill Higher Education, 2005

[7] Jiamin Wang , Baichun Xiao, “A minmax regret price control model for managing perishable products with uncertain parameters”, European Journal of Operational Research, Volume 258, Issue 2, pp. 652–663, 2017.

[8] Mehmet Önala, Arda Yenipazarlib, O. Erhun Kundakcioglu<u>c</u>, “A mathematical model for perishable products with price- and displayed-stock-dependent demand”, Computers & Industrial Engineering, Volume 102, pp. 246–258, 2016.

[9] Maryam Akbari Kaasgaria, Din Mohammad Imanib, Mehdi Mahmoodjanlooa, “Optimizing a vendor managed inventory (VMI) supply chain for perishable products by considering discount: Two calibrated meta-heuristic algorithms”, Computers & Industrial Engineering, Volume 103, pp. 227–241, 2017.

[10] Geoffrey A. Chuaa, Reza Mokhlesia, Arvind Sainathana, “Optimal Discounting and Replenishment Policies for Perishable Products”, International Journal of Production Economics, Vol. 186, , pp. 8–20, 2017.

[11] T. Maiti, B.C. Giri, “Two-period pricing and decision strategies in a two-echelon supply chain under price-dependent demand”, Applied Mathematical Modelling, Vol 42, pp. 655–674, 2017.

[12] M. Simaan and J.B. Cruz, Jr., “Additional Aspects of the Stackelberg Strategy in Nonzero-Sum Games”, Journal of Optimization Theory and Applications, Vol. 11, No. 6, pp. 613–626, 1973.

[13] Yoav Shoham, “Multiagent Systems, Algorithmic, Game-Theoretic, and Logical Foundations”, Kevin Leyton-Brown, University Press. pp. 60-88, 2009.

[14] He, X., Prasad, A., Sethi, S.P., and Gutierrez, G., “A Survey of Stackelberg Differential Game Models in Supply and Marketing Channels”, Journal of Systems Science and Systems Engineering (JSSSE), Vol. 16(4), pp. 385-413, 2007.

[15] Mariano Ventosa, Alberto Aguilera, Andrés Ramos, Michel Rivier, “Electricity market modeling trends”, Energy Policy, Vol. 33, Issue 7, pp. 897-913, 2005.

[16] Anke Weidlich, Daniel Veit, “A critical survey of agent-based wholesale electricity market models”, Energy Economics, Vol. 30, Issue 4, pp. 1728-1759, 2008.

[17] Daniel J. Veit, Anke Weidlich, Jacob A. Krafft, “An agent-based analysis of the German electricity market with transmission capacity constraints”, Energy Policy, Vol. 37, Issue 10, pp. 4132-4144, 2009.

[18] Simona Bigerna, Carlo Andrea Bollino, Paolo Polinori, “Marginal cost and congestion in the Italian electricity market: An indirect estimation approach”, Energy Policy, Vol. 85, pp. 445-454, 2015.

[19] Grant Allan, Igor Eromenko, Michelle Gilmartin, Ivana Kockar, Peter McGregor, “The economics of distributed energy generation: A literature review”, Renewable and Sustainable Energy Reviews, Vol. 42, pp. 543-556, 2015.

[20] Olga Spiridonova, “Transmission capacities and competition in Western European electricity market”, Energy Policy, Vol. 96, pp. 260-273, 2016.

[21] Mengmeng Yu, Seung Ho Hong, “Supply–demand balancing for power management in smart grid: A Stackelberg game approach”, Applied Energy, Vol. 164, pp. 702-710, 2016.

[22] Katarzyna Maciejowskaa, Jakub Nowotarskia, Rafał Werona, “Probabilistic forecasting of electricity spot prices using Factor Quantile Regression Averaging”, International Journal of Forecasting, Vol. 32, Issue 3, pp. 957–965, 2016.

[23] Guodong Liu, Yan Xu, Kevin Tomsovic, “Bidding Strategy for Microgrid in Day-Ahead Market Based on Hybrid Stochastic/Robust Optimization”, IEEE Transactions on Smart Grid, Vol.7, Issue: 1, pp. 227 - 237, 2016.

[24] Quang Duy La, Yiu Wing Edwin Chan, Boon-Hee Soong, “Power Management of Intelligent Buildings Facilitated by Smart Grid: A Market Approach”, IEEE Transactions on Smart Grid, Vol. 7, Issue: 3, pp. 1389-1400, 2016.

[25] Hongming Yang, Jun Zhang, Jing Qiu, Sanhua Zhang, Mingyong Lai, Zhao Dong, “A Practical Pricing Approach to Smart Grid Demand Response Based on Load Classification”, IEEE Transactions on Smart Grid, Vol. PP, Issue: 99, 2016.

[26] Habib Allah Aalami, Ahmadali Khatibzadeh, “Regulation of market clearing price based on nonlinear models of demand bidding and emergency demand response programs”, Electrical Engineering System, Vol. 26, Issue 11, pp. 2463–2478, 2016.

[27] F. Wei, Z.X. Jing, Peter Z. Wu, Q.H. Wu, “A Stackelberg game approach for multiple energies trading in integrated energy systems”, Applied Energy, Vol. 200, pp. 315–329, 2017.