New Trends in Modeling and Simulation in Economic Sciences

Zuzana Chvátalová, Jiří Hřebíček

Abstract—The aim of this paper is to map selected tools offered the Maple system and user supports provided by the Canadian company Maplesoft Inc. for professional and modern implementation in economics, resp. finances. These are mainly fields of scientific computing, in the quantitative modeling, graphics visualizations and interactive simulations, both the direct using of built-in elements and the communication platform supported by Maplesoft. Maple is an efficient tool for the solving problems of various complexities in these areas. It uses the efficient algorithms and methods of mathematical disciplines and executes the numerical and also symbolic calculations. Interactive tools of Maple as Clickable Math, on-line and interactive statistical or numeric computations and visualizations and inspiration of worksheets and documents from the Application Maplesoft Centre are important for the application of quantitative methods in economics and finance. Selected methods are adjusted to article´s aims, i.e. to present the financial package and means in the example of mortgage loan. We used the last two current versions of Maple (Releases 16 and 17).

Keywords—Economics; finance; Maple system; loan; interactive modeling; simulation; visualization

I. INTRODUCTION

ECONOMIC and financial modeling and simulations represents an inevitable part of contemporary science. The magnitude of the problems in scientific computing [7], [8] in economics and finances is so high that their solving does not only need experts in their particular economic fields but also in computer science in order to perform high-performance computations on today's computing architectures like grids, clouds, and supercomputers [12].

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Scientists, teachers and all academicians must be ready to communicate and collaborate in heterogeneous teams to solve current problems in economics. It requires well-structured and highly-developed soft-skills which must be well founded in terms of psychological approaches and pedagogical methods, and supported by information and communication technology (ICT) tools. Such support can be a significant technical advantage as the worldwide trend in science, education and researches.Simon and Blume (1994) wrote: "*Within the last thirty years, mathematics has emerged as the language of economics. Today economists view mathematics as an invaluable tool at all levels of study, ranging from the statistical expression of real-world trends to the development of fully abstract economic system.*"[28]

The use of quantitative methods in economics (i.e. the methods based in particular on mathematic discipline outputs) is nowadays supported firmly by introduction of numerous mathematical software e.g. the Maple of the company Maplesoft Inc. (Canada), MathCAD of the company PTC Corporate Headquarters (USA), Mathematica of Wolfram Research Inc. (USA), MuPAD of SciFace Software GmbH & Co. KG (Germany), etc. The latest scientific computing in economics are connected with solution of the real problems by applying these ICT [13], [27], where we choose Maple because of its long term use at our Brno University Technology (BUT) since 1994.

The base for economic and financial modeling and analyses at decision making is focused on mathematical modeling of economic and financial phenomena. Economic or financial modeling is able to have a different character (effected the different access, the option of methods, the availability of means etc.). Mathematical, numerical and statistical methods in economic modeling lead to quantitative modeling and simulation based on so-called deep or shallow way. However primarily at formulation of a real problem from economics we have to provide a qualitative analysis based on intuition, estimation, experiences and common-sense reasoning with the use of efficient ICT tools.

Current economic models used in scientific computing [1], [7]and economic modeling[10], [15], [17], [18],[28], [32]are becoming large and the number of computations needed for their numerical simulation is increasing. Simultaneously, the requirements on economic model

performance and accuracy are growing. When simulation economic models are designed we encounter several problems that must be solved in order to achieve these goals. The mathematical software Maple of the Canadian company Maplesoft Inc. is an appropriate ICT tool and computer environment enabling to solve such problems.

Maplesoft's tools provide "whitebox" modeling analysis and optimization tools that give you full control and insight into economic models. With the powerful programming and analysis environment, economists can investigate models in ways not possible with other tools,[17], [30]:

- Extensive collection of powerful analytical tools, including multibody analysis, optimization, sensitivity analysis, and more.

- Sophisticated programming language for customizing existing analysis tools and creating new ones to meet requirements for solving economic problems.

- Easy access to complicated operations, such as symbolic differentiation, symbolic integration, order reduction, variable isolation, and analytical solving of sets of equations, at the click of a button or a single statement.

- Optimizations run with the fast auto-generated code, providing scalable solutions for enterprise (microeconomics), regional, national and global-level (macroeconomics) requirements.

- Unlimited analysis potential with access to the underlying system equations and the powerful Maple mathematical engine.

A Maple web site (Fig. 1) offers all of Maplesoft's key user resources in one central location.

There is also link to Financial modeling web page [22], which we will introduce later.

Financial and quantitative analysts can use Maplesoft's mathematical environment to analyze data and create forecasts, measure risks, and develop econometric and quantitative algorithms to support their market-making strategies. They are able [2] to use:

- Advanced mathematical and statistical modeling tools.

- Powerful symbolic and numeric solvers.

- Mathematics-aware programming language for fast prototyping and solution development.

- Code generation tools for further deployment.

- Full technical document environment which combines live calculations and interactive computations with explanations, images, and more.
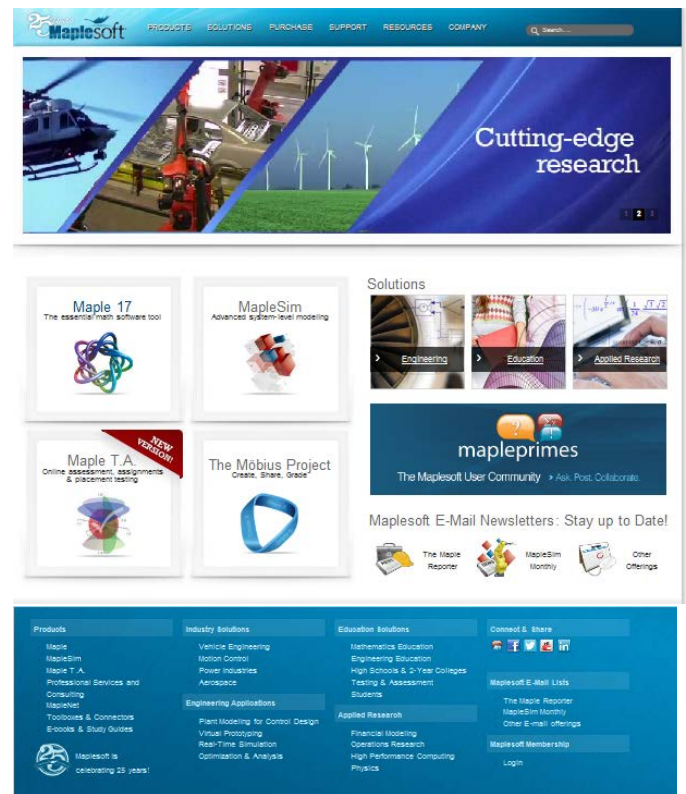

Fig. 1. Maplesoft web site. Source: [23]

Maple gives students an edge in their mathematic, economic and engineering courses. They can finish their assignments faster, improve their understanding of even the most difficult subjects, and explore on their own. Students get in Maple environment:

- They can start solving problems instantly with a click of the mouse on built-in toolbox and menu using *Clickable Mathematics*. Clickable Math techniques can be applied to problems from all your math, engineering, and science courses. Maple 17 increases the quality of the Clickable Math for Smart Popups and Drag-to-Solve.

- Maple brings the expert problem-solving power of expert mathematicians and economists to student computer.

- Maple provides interactive tutors, mathematical applications (Math Apps), and other learning tools that help students to understand the concept, not just give them the answer of solved tasks. Some tutors, including those for integration and differentiation, even display all the required steps of the solution.

- Maple covers thousands of built-in functions virtually every area of mathematics, including calculus, algebra, differential equations, linear systems, statistics, linear algebra, geometry, and transforms, further economics, including one hundred fifty-three matching applications in this category and finance engineering, including twenty-six matching applications in this category, [17], [23].

The Student Help Center of Maplesoft[24] offers a Maple student forum, on-line mathematic Oracles, training videos

and a mathematics homework resource guide. Maple 17 improves also products for students: Student Edition, The Mathematics Survival, Calculus Study Guide, Precalculus Study Guide, Advanced Engineering Mathematics for Maple, Maplets for Calculus, [23].

The *Teacher Resource Center* of Maplesoft[25] brings teachers the most of Maple users teaching experience with Maple. It provides sample applications, course material, training videos, white papers, e-books, podcasts, and tips. In Maple 17 (last version) the Möbius Project is Maplesoft's latest innovation for technical education, [23].

Maple's user community is now over two million people in the world. Together with this community the company Maplesoft has built large collections of Maple worksheets and Maple programs, many of them are freely available on at the *Application Centre* on the Maplesoft web [20] for Maple users to reuse or learn from them, [17].

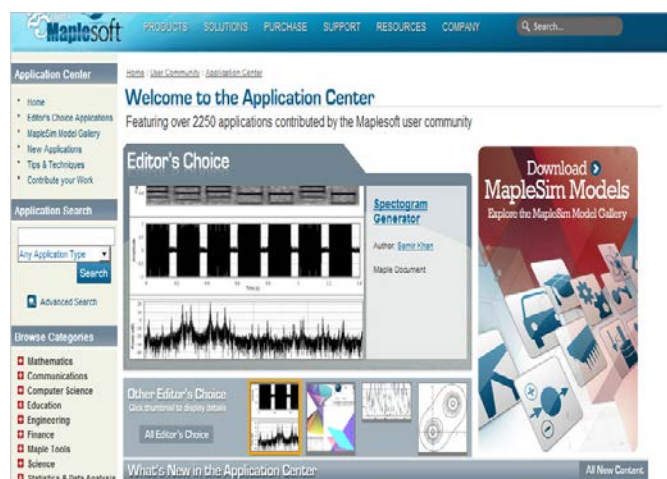Fig. 2 presents a Maple web site offers Application Center, [20].

Fig. 2. Maplesoft web site – Application Center. Source: [20]

Maple 17 presents interesting updates particularly in areas of: *Advanced Mathematics* (there are offered numerous advancements in a variety of branches of mathematics that push the frontiers of mathematical knowledge and Maple's capabilities), *Parallelism* (there are introduced new programming constructs to make it easier to write parallel code, as well as a parallel memory manager that significantly reduces running times), *Ease of Use Enhancements* (there is included a variety of enhancements to help ensure that new users are immediately productive while experienced users have the tools and flexibility they need to work efficiently.), further *Advanced Code Editor*, *Embedded Video*, *Performance*, *Signal Processing*, *Group Theory*, *Math Apps*, *One-Step App Creation*, *Language and Programming*, *Statistics*, *Differential Geometry*, *Physics*, *Control Design*, *Visualization*, *Multivariate Calculus for Students*, *Essay Tools*, *Maple Player*, *Customer Wish List*, etc., [23].

The Explore command (*One-Step App Creation*) has been updated in Maple 17 to provide new and enhanced functionality. The exploration parameters can be specified programmatically as well as through the Exploration Assistant. A single command gives a quick and easy way to create interactive applications, simulations and demonstrations. These applications let you use sliders to change the parameters of expressions and immediately see the results (Fig.3). The resulting application can be saved and shared with others, including through the Möbius Project.
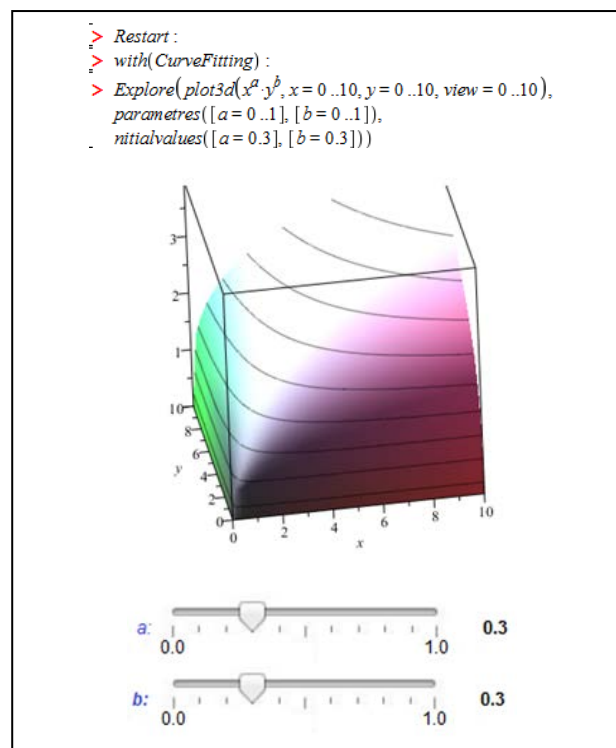

Fig. 3. Explore command - application. Source: Own work in Maple 17 according to [23]

Areas of the appropriate application are primarily *Parameters and initial values*, *Previously assigned data*, *Exploring a function call*, *Numerical ODE with parameters*, *Exploring an animation*, *Interactive plots and the Plot Builder*, etc., [23].

The paper introduces chosen features for scientific computing in economics and its visualization, including on-line economic modeling and its applications in scientific disciplines, especially in the area of economic/financial modeling. Students can learn mathematical modeling as an important interactive support for understanding and presenting solved real economic and financial problems, [31]. In this paper are presented chosen basic ideas on process of mathematical modeling of economic phenomena which are demonstrated in examples including Maple visualization tools, [2], [9].

Note: This paragraph was written also in correspondence with the information provided by the website providing Maplesoft Inc., [23].

II. Interactive Economic and Financial Modeling and Simulation with Maple

In this chapter we demonstrate chosen previously implemented cross-sectional examples of a utilization and usability of Maple tools to interactive economic modeling and simulation, i.e. construction, analysis, computing and modification of nontrivial economic models of selected real phenomena from practices which need high performance computing,[5], [6],], [29].

With built-in database connectivity, Maple allows economists and scientists to quickly develop and deploy powerful applications that combine large enterprise and open government datasets with the state-of-the-art analysis and visualization tools of Maple. Maple users can easily query, create, and update your databases within the interactive, user-friendly Maple environment, without any detailed SQL knowledge,[9], 11], [18].

The majority of the mathematical algorithms in Maple are written in the very efficient and user friendly Maple Programming Language [2]. Maple users can write simple programs using the same basic tools which the Maple developers themselves use. Moreover Maple users can easily view most of the code in the Maple library and they can even extend the Maple statements, trying their programs in with existing functionality.

A. Introduction to Maple environment

With the Maple environment, the user can create powerful interactive documents. This environment also allows every user to start solving problems right away by entering mathematical expressions in "2-D Math" form and solving these expressions using point-and-click interfaces [2]. With Maple, the user can drag input, output, or curves in a plot region into a new input region. This is done by highlighting the input or selecting the curve and dragging it with Maple user mouse into a new input region. The Fig.4 presents the example of the Maple worksheet environment improved in Maple 17 for automatic coloring surfaces. It is often an illustrative tool for economists and managers -for example, modeling of consumer surplus and producer surplus in demand models and supply models.
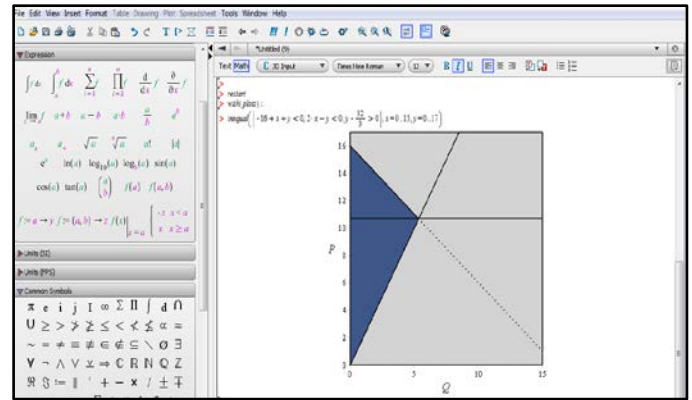

Fig. 4. Smart Maple environment. Source: Own work in Maple according to [23]

The basic Maple graphic user interface consists from worksheets [2]. They are files that document how to solve mathematical problems from the fields of mathematics, economic science, and engineering. Note that Maple provides users with two worksheet interfaces. Worksheets are both interactive and reusable. They can be used to replace calculators, spreadsheet application programs, and programs in languages such as FORTRAN, C, Java, Visual Basic, etc. Specifically, in worksheets Maple users can:

- perform computations;
- manipulate mathematical expressions;
- describe the problem-solving processes.

When a user uses Maple worksheet to perform computations or manipulate expressions, Maple displays the corresponding results which the user can use for subsequent processing. The request sending by the user to Maple worksheet is called *Maple input*, and the result is called *Maple output*. Maple input and output comprise together an *execution group* which is the fundamental element of the Maple worksheet.

Maple users enter, compute, and manipulate mathematical expressions of the worksheets by using the powerful Maple mathematical engine. During the problem-solving process, Maple can take the result of one calculation and use it as input for a second calculation, etc. In this way, Maple can solve very complicated mathematical problems in economics.

Maple is supported by an extensive range of academic resources, including curriculum content, e-books, and application add-on components,[9], [10], [11], [15], [17], [18].

We shortly introduce the most important interfaces in Maple, which enable on-line economic modeling.

B. Clickable Mathematics

Maplesoft has introduced the concepts of *Clickable Mathematics* with *Clickable Calculus, Clickable Algebra, Clickable Engineering*,[11], [12], [17],to make mathematics in economic modeling easy to do, easy to learn, and easy to teach through very visual, interactive point-and-click

methods. This approach allows scientists, teachers and students to focus on concepts and eliminates the need to memorize Maple statement names and syntax.

Maple worksheet interfaces allow for an unprecedented level of point-and-click economic problem solving and visualization. Scientists, teachers and students can combine text and mathematics in the same line of Maple environment (worksheet), add tables to organize the content of his work, or insert images, sketch regions, and spreadsheets. They can visualize and animate problems in two and three dimensions, format text for academic papers or books, powerpoint presentations and insert hyperlinks to other Maple files, web sites, or email addresses. They can also embed and program graphical user interface (GUI) components, as well as devise custom solutions using the Maple programming language,[2].

Clickable Math [21] refers to Maple's remarkable set of user-interface features that makes common mathematical operations as easy as pointing and clicking. For example users can use *Smart Popups* which are interactive pop up menus that are automatically displayed when users select an expression or individual terms in an expression (Fig. 5). They provide a choice of options for manipulating user selection, such as plotting, expanding, factoring, substituting trigonometric expressions, etc. *Smart Popups* work only on the output of a Maple expression or equation.
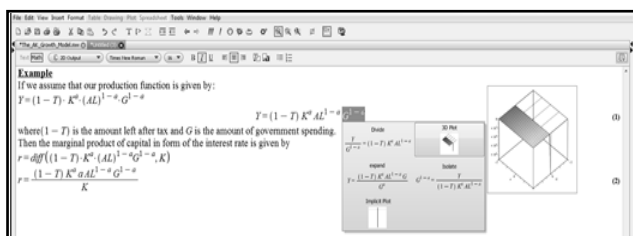

Fig. 5. Smart Popups menu in Maple 16. Source: Own work in Maple according to [23]

The concept of Clickable Maths for solving economic phenomena includes,[14], [21]:

- *Smart, context-sensitive right-click menus* for instant access to Maple solvers of economic phenomena and other free operations in mathematical modeling;

- *Extensive range of Maple palettes* for visual editing of mathematical expressions;

- *Interactive plotting and animation* controlled by the mouse and not by endless parameters and attributes in a Maple statement;

- *Drag and drop* and *Drag to Solve* operations on expressions, plots, text, and more;

- *Interactive assistants* from Maple menu that provide easy mechanisms to solve and explore advanced topics for economical modeling such as linear, nonlinear and differential equation-solving, optimization, and advanced visualization;

- A *Maple Portal for Students* from Maple help system [2], which acts as a guide for hundreds of common tasks from courses of mathematics, economics, econometrics, finance engineering, etc;

- Built-in selection of interactive *Tutors* from Maple Tools menu [2] that offer visual e-learning environments as for many important mathematical topics in precalculus, calculus, linear algebra, as for economics and economic modeling;

- *Handwriting recognition* of mathematical symbols and equations in economics;

- *WYSIWYG document* (worksheets) processing features that let Maple users create complex documents of economics courses more quickly and easily than in a word processor or LaTeX;

- An exploration *Assistant* from Maple Tools menu [2] that allows students and teachers to instantly create interactive mini economic applications and Math Apps examples from Maple Tools menu to explore the parameters of expressions and formulas.

We can summarize Maple provides both computational power and a new-generation user interface that allows scientists, teachers and students to become proficient with Maple without the burden of learning its commands (statements) and their related syntax. Overall, teaching, learning and solving problems in economics become more efficient and effective. With collection of Clickable Math tools, including palettes, interactive Assistants, context-sensitive menus, Tutors, and more, Maple has set the standard for making it easy to learn, teach, and do mathematics and solve mathematical problems in economics.

C. Interactive Statistical Computation

Statistical computations in Maple 16 combine the ease of working in a high-level, interactive environment with a very large and powerful set of statistical algorithms. Large data sets can be handled efficiently with thirty-five built-in statistical distributions, sampling, estimations, data smoothing, hypothesis testing, and visualization algorithms. In addition, integration with the Maple symbolic engine means that user can easily specify his distributions by combining existing distributions or simply by giving a formula for the probability or cumulative distribution function.

New in Maple are following possibilities,[2]:

- Discrete distributions, which are important in many areas from game theory to algorithm analysis, are significantly enhanced, with support for non-integer values as well as sampling of custom discrete distributions.

- Maximum likelihood estimation now allows for multiple parameters and is significantly faster.

- It is easier to split data into subsets based on particular criteria, enhancing your ability to analyse data and identify patterns.

- Statistical visualization is easier than ever before. In addition to the new Live Data Plots, enhancements like variable bin-width histograms and new options for pie charts provide Maple user with extra control over how data is presented.

D. Live Data Plots

Live Data Plots in Maple help with insight, understanding, and publication of Maple user data, all at the click of a button. These plots make it even easier for user to present his data in a form that is visually appealing and conveys meaning. Using the new Live Data Plots Maple user can quickly generate and modify: *Area Charts, Bubble plots, Pie charts, Bar charts, Histograms, Scatter plots, Box plots* and *Line charts*.

Clicking on any of the items (e.g. Scatter plot) in the *Live Data Plots palettes* (Fig. 6) Maple user inserts a task template that lets him to create a customized statistical plot. Maple user simply replaces the placeholder with his own dataset. Both the plot and the plotting statement are displayed, and either can be copied into other parts of Maple user worksheet, [2].
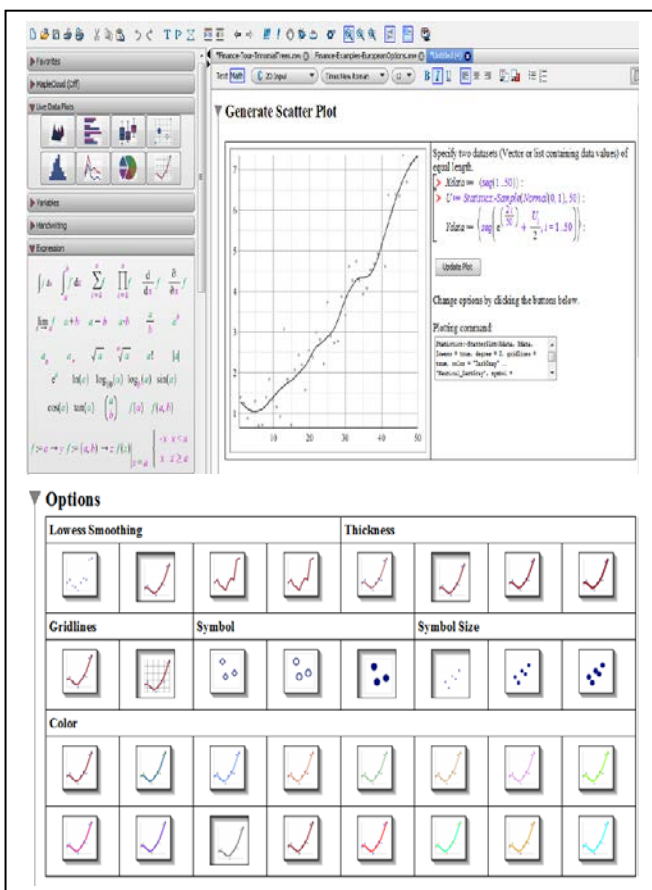

Fig. 6. Example of Live Data Plots palettes for Scatter plot in Maple.
Source: Own work in Maple according to [23]

III. INTERACTIVE MODELING AND SIMULATION OF THE MORTGAGE LOAN WITH MAPLE

The Financial Modeling package is a collection of tools for mathematical finance. This package builds on the functionality available in other packages of Maple such as LinearAlgebra, Statistics, Optimization, and CurveFitting. This package supports a wide range of common tasks such as date arithmetic, cash flow analysis, option pricing, term structure analysis, and simulation,[23]. The Maple system provides co-called *Quick Help* for Finance package in the *Finance Tour*. (Take a Tour: *Building Trinomial Trees*; *Term Structures of Interest Rates*; *Simulating SVJJ Processes and Sample Documen*ts: *Calendars and Day Count Conventions*; *Local Volatility and Implied Volatility*; *Pricing Arithmetic Asian Options*; *Pricing European Options*.), [23].

The Finance package obtains following topics: *Date Arithmetic*; *Cash Flow Analysis*; *Financial Instruments*; *Interest Rates; Lattice Methods*; *Short Rate Models; Stochastic Processes*; *Personal Finance*; *Notes*. Furthermore Maple system refers to other applications in the subsection named *See Also*, [23].

The Financial Modeling package gives to companies the important advantage of the technological and informatics support, which can provide in a short period of time a clear position in which the business is located. This simple, effective, and flexibly modifiable system is able to evaluate the current situation, respectively, possible to simulate the situation of the company. It can also provide the support for decision-making and management of the top management of any company.

E. Mortgage Loan

The performance of a company is determined by a number of indicators of economic, environmental, social and corporate governance character. The credit policy of the country offers various types of loans. A low debt ratio may be advantageous for the company.

A mortgage is the basic method for individuals and businesses how they can purchase real property without the need to pay its value immediately using their own resources, saving. A mortgage is a loan secured by real estate.

We present the example using the Maple which provides important information for the modeling of mortgage. We will use both the interactive elements of the Maple environment (clickable calculus, sliders, visualization) and easily modifiable with built-in support in the area of finance *(Help>Manuals*, *Resources>Math Apps>Finance and Economics*) and its statistical package (Statistics).

Note: Based on the theoretical bases in [3], [4], [19], [26]we lead our considerations bellow.

The mortgage loan (mortgage) is a loan intended for natural and legal persons. Several parameters determine the

basic characteristics, the structure and the total cost of a mortgage.

Let us consider, [22], [23], [24]:

- *DU* (we use the term *DU*, because the term *D* is a fixed predefined the Maple term) – the designation of the principal (equity);
- *i*– annual nominal interest rate (a decimal number corresponding to an interest rate in percentage);
- *m* – frequency of payments (the number of times the interest is compounded per year);
- *n* – number of years (the amortization period, total time for repayment);
- *a* – annuity (the mortgage payment);
- *v* – discount factor;
- *A* – amortization;
- *I* – interest.

Then in Maple we can determine the designation *DU*:

$$DU := a\frac{1}{1+i} + a\left(\frac{1}{1+i}\right)^2 + \ldots + a\left(\frac{1}{1+i}\right)^n \quad (1)$$

Let us denote $v:=(1+i)^{-1}$ discount factor, then from the convenient formula for a geometric series ($v \neq 1$)) follows:

$$DU := \frac{a\,v\,(v^n - 1)}{v - 1}. \quad (2)$$

We use Maple commands `subs` and `solve` to receive the corresponding formula for annuity:

$$a := -\frac{DU\,i}{\left(\frac{1}{1+i}\right)^n - 1} = \frac{i \cdot DU}{1 - (v)^n} \quad (3)$$

and we obtain for frequency of payments *m* times per year formula:

$$DU := a\frac{1 - \left(\frac{1}{1+\frac{i}{m}}\right)^{mn}}{\frac{i}{m}}. \quad (4)$$

Top managers of a company often have to deal with the evaluation of different situations to make decisions about mortgage.

F. Example

Let us visualize the development of the schedule for repayment of principal. Following parameters are needed to determine the total cost of a mortgage and the schedule for repayment of principal: principal *DU* = 400 000 CZK, the annual nominal interest rate (briefly interest) *i* = 0.05 (corresponding to an interest rate in percentage 5 %), the frequency of payments *m* =1 (the number of times the interest is compounded per year), thenumber of years *n* =

10 (the amortization period, total time for repayment, briefly total period).

In this case the annuity *a* (the mortgage payment) is constant. To do this, we need to know the values of the payment schedule.

We can use Maple commands `seq`which is used to construct a sequence of these values[26]. The most typical calling command in Maple is `seq(f(i), i = 1..n)`, which generates the sequence $f(1), f(2),\ldots,f(n)$.

More generally, the command `seq(f(i), i = m..n)`generates the sequence $f(m), f(m + 1),\ldots,f(n)$, [23].

After setting and calculating inputs:

$$i := 0.05 : n := 10 : DU := 400000 :$$

$$v := \frac{1}{1+i}; \quad v := 0.9523809524$$

$$a := \left(\frac{DU \cdot i}{\left(1 - (v(i)^n)\right)}\right); \quad a := 51801.83001$$

and using command according to[26]:

$$matrix\left(10, 5, \left[seq\left(\left[j, a, a \cdot \left(1 - (v(i))^{(n-j+1)}\right), a\right.\right.\right.$$
$$\left.\left.\left.\cdot (v(i))^{(n-j+1)}, a \cdot \frac{\left(1 - (v(i))^{(n-j)}\right)}{i}\right], j = 1..n\right)\right]\right)$$

we receive the values of the amortization and interest payments (Tab. 1).

TABLE I.

THE SCHEDULE FOR REPAYMENT OF PRINCIPAL SOURCE:OWEN WORK IN MAPLE ACCORDING TO[26]

Year n	Annuity a	Interest	Amortization	Rest
0				$4.0000000\,10^5$
1	51801.83001	20000.00000	31801.83001	$3.681981700\,10^5$
2	51801.83001	18409.90850	33391.92151	$3.348062486\,10^5$
3	51801.83001	16740.31243	35061.51758	$2.997447310\,10^5$
4	51801.83001	14987.23655	6814.59346	$2.629301376\,10^5$
5	51801.83001	13146.50688	38655.32313	$2.242748144\,10^5$
6	51801.83001	11213.74072	40588.08929	$1.836867251\,10^5$
7	51801.83001	9184.336256	42617.49375	$1.410692314\,10^5$
8	51801.83001	7053.461571	44748.36844	96320.86302
9	51801.83001	4816.043151	46985.78686	49335.07618
10	51801.83001	2466.753809	49335.07620	0
Σ	$5.180183001\,10^5$ $n \cdot a$	$1.180182999\,10^5$ $n \cdot a - DU$	$4.0000000\,10^5$ DU	--

Note: Most mortgage lenders offer on their web pages the interactive mortgage payment calculator to determine at least the approximate plan for principal repayment.

We continue using the Maple System.

Firstly, we can apply *the statistical* package (Statistics) in Maple (especially, subsections *regression*, *plots*) and create

the graphic model for the visualization of evaluation development proportions or simulation of development repayments over time (as the composition of the value of amortization paid and interest paid). Both models are exponential functions (Fig. 7).

Note: The graphs only approximate given values it can be seen especially near the intersections of curves with axes. But for our orientation, it is enough. The Maple enables to use the weight function *W = weights* to strengthen the chosen values of the vector, they are chosen according to economic interpretation.
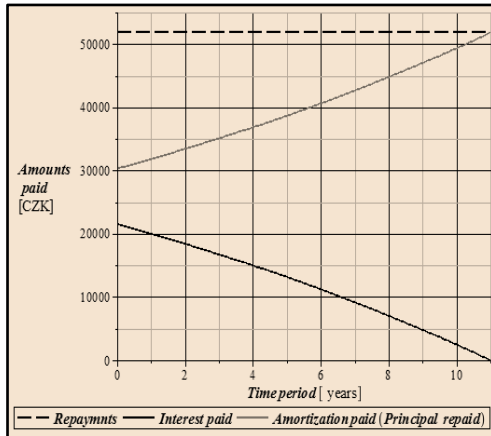

Fig. 7. Regression models – development: Interest paid and Amortization paid using the Maple statistical package (Statistics). Source: Own work in Maple

Secondly, we can apply the modification of *built-in tools: Math Apps -Finance and Economics* in Maple according to [22], [23].

The great advantage is that the Maple user (without having to create a Maple program) can easily adapt into an existing command structure of the Maple program own commands according to his needs and take the advantage of the built-in worksheet which Maple offers.

The Maple programming commands (statements) are very transparent and time-efficient. The modification can be provided using icon `Startup Code For` in the menu bar in worksheet (Fig. 8).
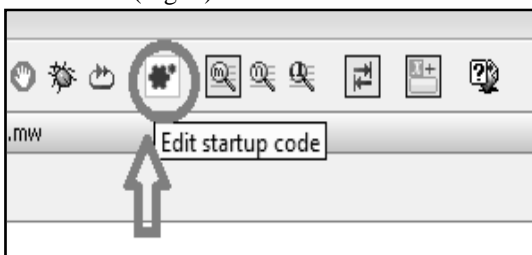

Fig. 8. Startup Code in application in the Menu bar. Source: Worksheet in Maple

Initially, we apply the same situation from above. We set the inputs (parameters for modeling of the mortgage structure) using *sliders*. *Startup code* is structured and managed so that after setting the input parameters there are

respected *DU, i, m, n* and are computed *Total Cost*. Graphs for *Interest* and *Amortization paid* are curves of exponential functions. They are designed as the combination of commands `pointplot` and `seq`. Images are supplemented by placing actual values and calculations using the Maple statement `textplot`(in our case for *Total Cost*). There are procedures used cycles and other statements for formatting, legends, descriptions, etc. Therefore, any modification of such transparently maintained code is easy. This Maple support is very convenient for its users.

We can determine the schedule for repayment of principal with the combination of Maple commands `matrix` and `seq` (as we mentioned above (Tab. 1)). Fig. 9 shows the development of this situation.

In this case (let us remind) parameters *n, m, DU, i* are fixed. We have to determine the annuity *a,* which is constant in this case.
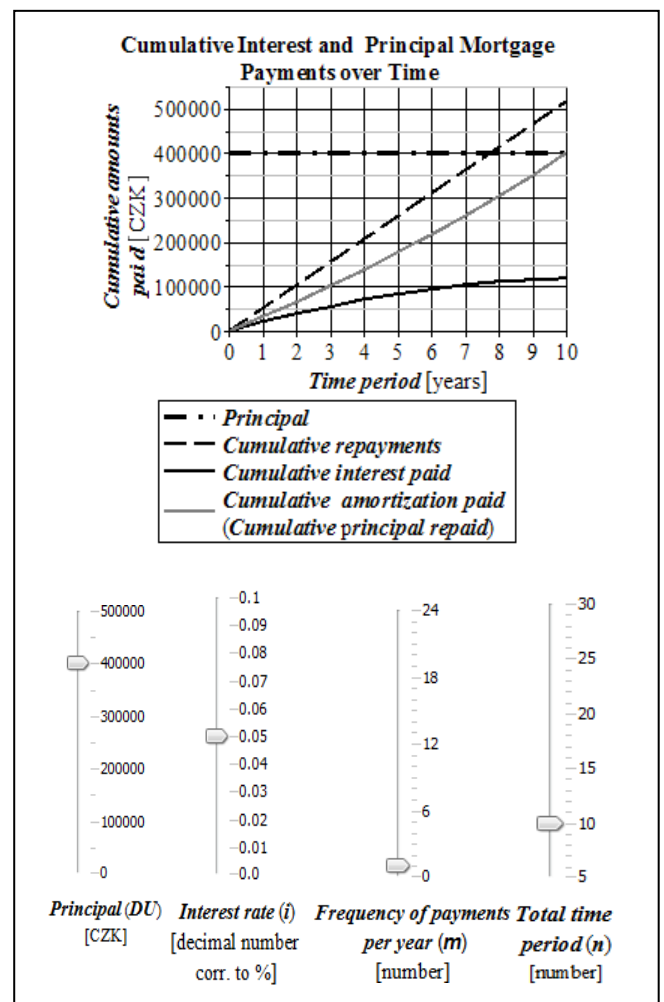

Fig. 9. Development: Interest paid and Amortization paid / Cumulative Interest paid and Amortization paid and Input settings with interactive elements *(sliders)* in Maple with using bulit-in support *(Finance and Economics).* Source: Own work in worksheet *Mortgage* in Maple [23]

Results: If mortgage parameters are $DU = 400\ 000$ CZK, $n = 10$, $m = 1$, $i = 0.05$ then the annuity a is equal to 51 801,83001 CZK and the mortgage total cost is equal to 518 018,30 CZK thereof the interest is equal to 118 018,30 CZK.

Now, we want to compare how the situation will change if we pay every month (during 10 years), i.e. now $m = 12$. Therefore using the slider just set the appropriate value of m. After setting inputs $DU := 400\ 000$ and $mn := 120$ calculating $i := 0.00416666667$, $v := 0.9958506221$ and $a := 4242.620368$ in Maple, we can determine in (Tab. 2) the matrix of corresponding values analogy with (Tab. 1). (Formally, we assign mn and i/m as n and i for reasons of simplification of designation encoded in the source Maple programs, i.e. $n := mn = 120$, $i := i/m = 0.05/12$).

Fig. 10 shows the development of this situation.

TABLE II.

THE SCHEDULE FOR REPAYMENT OF PRINCIPAL ($m = 12$). SOURCE: OWEN WORK ACCORDING TO[26]

Year n	Annuity	Interest	Amortization	Rest
0				$4.0000000\ 10^5$
1	4242.620368	1666.666667	2575.953701	$3.974240462\ 10^5$
⋮ ⋮	⋮ ⋮	⋮ ⋮	⋮ ⋮	⋮ ⋮
120	4242.620368	17.60423519	4225.016133	0
$\sum$	$5.091144442\ 10^5$ $n \cdot a$	$1.091144442\ 10^5$ $n \cdot a - DU$	$4.0000000\ 10^5$ DU	--

Results: If mortgage parametres are $DU = 400\ 000$ CZK, $n = 10$, $m = 12$, $i = 0.05$ then annuity a is equal to 4242,620368 CZK and mortgage total cost is equal to 509 114.44420 CZK thereof the interest is equal to 109 114.44420 CZK.
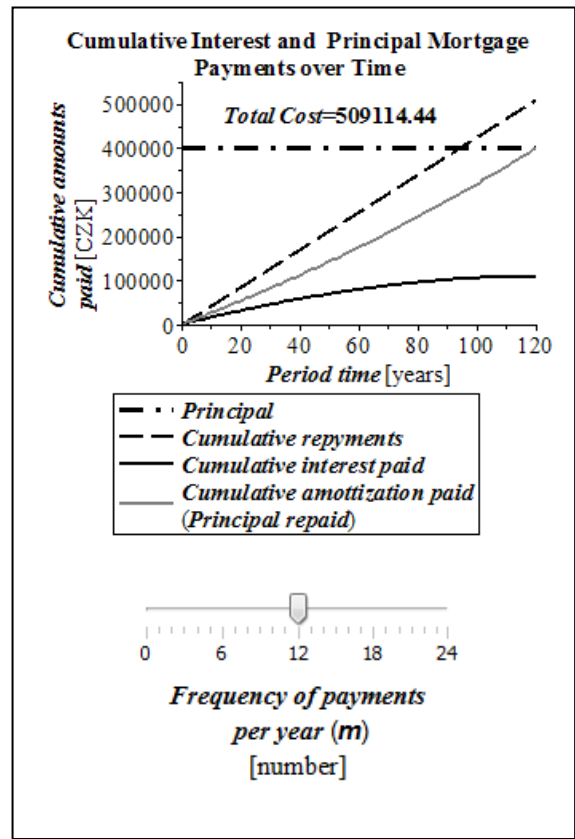

Fig. 10. Development: Interest paid and Amortization paid / Cumulative Interest paid and Amortization paid and Input settings with interactive elements *(sliders)* in Maple with using bulit-in support (*Finance and Economics*). Source: Own work in worksheet *Mortgage* in Maple, [23]

Finally, we remark that, similarly, we can proceed (using a modification of the presented supports in Maple) when the annuity a is a fixed input pre-chosen value and we must calculate the number of years n and a residual value for the last installment using derived implementations of mathematical formulas in the source program:

$$n - 1 := \left[\frac{\ln\left(1 - \frac{i \cdot DU}{a}\right)}{\ln(v)} \right], (5)$$

where for every $x \in \mathbf{R}$: floor(x) in Maple is the greatest integer less than or equal to x (defined by the set equations $[x] \leq x < [x] + 1$). Then the residual value for the last installment is equal to:

$$b := \frac{DU - a\frac{1 - v^{n-1}}{i}}{v^n}. \quad (6)$$

(In our case if we choose the annuity for example $a = 125\ 000$ CZK, $n = 4$ years ($n - 1 := \text{floor}(3.573535571) = 3$ and $b = 72436.87508$).

Further, for the fixed amortization value (a_m) in our case for $DU := 400\ 000$, $n := 10$, $m := 1$ a_m is equal to 40 000.

G. Discussion

Our experience in scientific computing and visualization in economics and finances with Maple has followed from teaching, practice and current research in the field of using mathematical modeling of economic phenomena discovered and confirmed a number of important aspects and trends. New competencies of professionals and managers (such as teamwork, communication, other soft-skills, interactivity, intensive knowledge sharing and transfer) are supported by modern tools, as well as by the progress in information and communication technologies represented by grids, clouds, and virtualization in general.

The philosophy of the Maple developers is not only in dealing with mathematics, but they take into account the different needs of the users. They continually improve and make a comfortable the working environment. They support the user groups with the specific topics of expertise. They have developed the targeted packages and applications in the documents for other disciplines, courses and competencies of the users. Also, the area of economics and finance is strongly supported. Maple system supports a wide range of professional activities and communications in their specialized and maintained centers.

The basic principle of use of Maple system is to create:

- models of economic or financial reality and analyze them; Thus economic phenomena can have both quantitative and qualitativecharacter;
- the most oftheexperimentscannotberepeated;wecanobtain often economic data withdifficulty];
- theycanbe very complex;
- data setscanbealso very large;

Problems tend to have aninterdisciplinarycharacter,[12], [14], [15], [16]. [17]. [18]. The professionals and managers behavior is often based on expert decisions and analysis. Therefore we need a fast and reliable calculations, visualizations, simulations and animations.

IV. CONCLUSION

We have demonstrated through implemented cross-sectional examples of a utilization and usability of Maple tools in scientific computing and visualization of chosen economic and financial problems and phenomena for interactive economic modeling and simulation, i.e. construction, analysis, computing and modification of nontrivial economic or financial models of selected real phenomena from practices. There are also economic problems which need high performance computing. We have also mentioned these properties of Maple system for cloud and grid computing.

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