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Using software applications in teaching of slope-deflection method in subject Static Analysis of Constructions

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Abstract—Static analysis of building structures is an important engineering discipline. The presented paper deals with the teaching process of the slope-deflection method in the subject Static Analysis of Constructions at the Faculty of Civil Engineering of the Technical University in Košice. The slope-deflection method is the method for the analysis of statically indeterminate structures and serves to obtain their inner forces and deformations. For purpose of teaching this subject, teachers prepared programs in the Fortran code and with the help of the output of decisive values obtained from the programs. Teachers know if students have correct solutions of their tasks and can guide students to work to correct mistakes in their work.

Keywords—Static analysis of constructions, slope-deflection method, indeterminate structures, teaching.

I. INTRODUCTION

ONE from the basic parts of physics is the mechanics of solids. This scientific field deals with the study of mechanical motion of solids or their parts [1]. If the solid under investigation is a load-bearing building structure, the application of the mechanics represents a very large separate field called the structural mechanics. Static analysis of construction deals with the examination of load-bearing building structures from stationary loads and dynamic analysis of construction deals with the examination of load-bearing building structures from moving loads [2,3]. The aim of static analysis of structures is to master, create and improve computational methods and algorithms that provide the necessary data (internal forces and deformations) for load-bearing building structures to dimension and assess the load-bearing capacity and usability of the structure [4]. Computational methods and algorithms are constantly changing and improving, adapting to new construction technologies, the development of numerical mathematics and the ever-increasing efficiency and availability of computational technology [5-7].

The beginnings of the history of statics and static analysis of constructions as a scientific field date back to the 17th century [8]. In previous centuries, in which the load-bearing structures of buildings were designed based on empirical knowledge, the previous personalities such as Galileo and Newton began to deal with theoretical regularities [9]. Their most important successors until the 19th century were, for example, Varignon, Bernoulli, Navier, Maxwell, Cremona, Langrage, according to their names, the important basic definition of statics is still named until today [10].

The Technical University of Košice has currently nine faculties specialized in technology and even art: Faculty of Mining, Ecology, Process Control and Geotechnologies, Faculty of Materials, Metallurgy and Recycling, Faculty of Mechanical Engineering, Faculty of Electrical Engineering and Informatics, Faculty of Civil Engineering, Faculty of economics, Faculty of Manufacturing Technologies, Faculty of Arts, Faculty of Aeronautics [11].

The Faculty of Civil Engineering of the Technical University of Košice was founded in 1976, with effect from February 1977, as the fifth faculty of the Technical University of Košice. More than 40 years of experiences, more than 40 years of faculty have left more than 8,000 graduates in all three levels of study have left its gates, working in various positions, such as construction managers, managers, general managers, successful designers, builders, contractors, managers, researchers, and pedagogical staff [12].

The many different types of structures are found all around us [13]. Each structure has a specific purpose or function.

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Some structures are simple, while others are complex; however, there are two basic principles of composing structures [14]. They must support different parts of the external load that they are designed for without collapsing of structures [15].

They must support the various parts of the external load in the correct relative position, without big deformation of structures [16]. The structure refers to a system with connected parts used to support a load. Some examples related to civil engineering are buildings, bridges, towers and more another. However, these structures are very complex and difficult for analyse and design [17]. We will first consider simple examples of structures and parts of structures, such as beams, trusses, frames etc [18]. It is important that the civil engineer recognizes the different types of elements that make up a structure and to be able to classify them as to their shape and function [19].

II. The slope deflection method

A. Introduction

The *slope-deflection method* is an alternative way to analyse indeterminate structures.

In 1915, George A. Maney introduced the slope-deflection method as one of the classical methods of analysis of indeterminate structures [20]. The method considers flexural deformations but ignores axial and shear deformations. The unknowns are the rotations and the relative joint displacements in the slope-deflection method [21]. For the determination of the end moments of elements at the joint, this method requires the solution of simultaneous equations consisting of rotations of joints and elements, joint displacements, element stiffness, and lengths of elements [22].

B. Degrees of Freedom

In structural analysis, we often conceptualize a real structure as a simplified stick model with elements interconnected at specific locations joint scalled *nodes*. Even though the elements have deformations between the nodes, using the methods of structural analysis we can characterize the behaviour and deformation of the structure based on the deformations in the nodes themselves [23].

The *Degree-Of-Freedom* (*DOF*) represents a one direction in which a node can move or rotate. Each node has three possible degrees-of-freedom: translation (movement) in one direction, translation in another direction perpendicular to the first one, and rotation [24]. The horizontal and vertical axes are usually considered to be two perpendicular translational degrees-of-freedom. Although three DOFs are possible for each node, individual directions may be limited, either by a support reaction or by one of the elements connected to the node [25].

As a results of assumption of the slope-deflection method - idealization of structures, we will usually assume that all frame elements are axially rigid for the purpose of determining the number of degrees of freedom in the system [26]. The assumption of the slope-deflection method is that the beams (elements) cannot elongation or compress. This is usually a good assumption for beam and frame analysis since the structural deformations are mostly caused by bending of frame elements, not axial elongation. Based on this assumption, we reduce the number of effective DOFs in the slope-deflection analysis, and therefore reduce the number of equations in the system of equations that we have to solve [27].

C. The Slope-Deflection Equations

The ground of the slope-deflection method is using and application of the *slope-deflection equations.* The *slope-deflection equations* are related the rotation of the element with the total moments at either end, see Fig.1:

- the rotation of the node *a* on the ends of the element,
- the rotation of the node *b* the ends of the element,
- solid rotation of the whole element [28].

The goal of the slope-deflection equations is to compile equations and acquisition values of end moments for all elements of the structure as a function of all of the DOFs associated with the structure elements [28]. We can apply equilibrium conditions at all of the joints of structure to solve for the unknown rotations. This is the system of equations that we will have to solve: the equations are the equilibrium equations for each node and the unknowns are the translations and rotations of the nodes [29].

The first step is that we currently need an expression for the moment at each end of an arbitrary element in an indeterminate structure in terms of the deflections: rotations and translations of the nodes at either end. If between the nodes along the length of the element are loads:

- distributed loads,
- point loads,
- point moments,

then we will need a way to consider their effect as well [20].

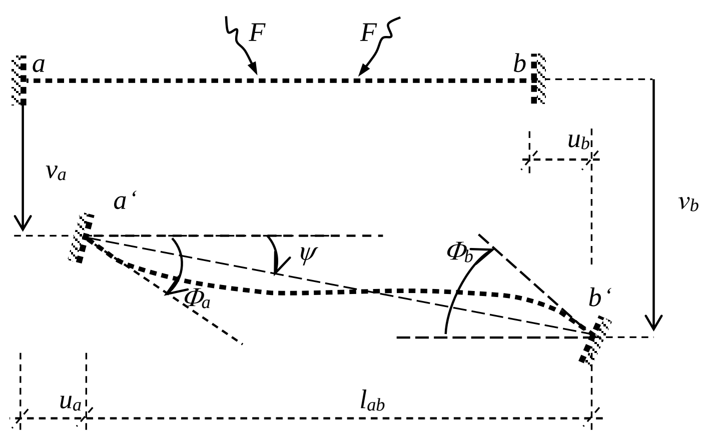

Fig. 1 Beam *a* - *b* and its deflections

To develop the general form of the slope-deflections, we will consider the beam *a-b* as the basic element - beam anchored at both ends with a fixit support) according to the principle of superposition is determined by folding the four loading states

a) the effect of external load

b) the effect of nodal rotation ϕ_a

c) the effect of nodal rotation ϕ_b

d) the effect of rotation of the beam ψ_{ab} due to the displacement of the nodes v_a, v_b, $\psi = (v_b - v_a)/l_{ab}$, resp. due to node displacement u_a, u_b, $\psi = (u_b - u_a)/l_{ab}$.

The resulting relations for the end bending moments a on the element *a-b* with constant *EI* consist of two parts according to the principle of superposition. We want to connect the beam's internal end moments M_{ab} and M_{ba} with its three degrees of freedom, namely, the linear displacement $\Delta = (v_b - v_a)$ and angular displacement ϕ_a and ϕ_b. As we develop a formula, moments and angular displacements will be considered positive when they act clockwise on the span, as shown in Fig. 1. Furthermore, the linear displacement $\Delta = (v_b - v_a)$ is considered positive as shown, because this displacement causes the cord of the span and the span's cord angle $\psi = (v_b - v_a)/l_{ab}$ to rotate clockwise [30].

We can develop the slope-deflection equations using the principle of superposition by considering separately the effects caused by each of the displacements and then the loads [30].

The internal moments M_{ab} and M_{ba} of beam *a-b* are in the near of the end:

$$M_{ab} = \bar{M}_{ab} + \Delta M_{ab} \tag{1}$$

$$M_{ba} = \bar{M}_{ba} + \Delta M_{ba} \tag{2}$$

this moment is positive clockwise, when acting on the end of beam.

$\bar{M}_{ab}$ are $\bar{M}_{ba}$ fixed-end moments, ΔM_{ab} a ΔM_{ba} are secondary moments.

The internal moments M_{ab} and M_{ba} of beam *a-b* are in the near of the span:

$$M_{ab} = \bar{M}_{ab} + \frac{2EI}{l}\left(2\phi_a + \phi_b + 3\frac{w_b - w_a}{l}\right) \tag{3}$$

$$M_{ba} = \bar{M}_{ba} + \frac{2EI}{l}\left(\phi_a + 2\phi_b + 3\frac{w_b - w_a}{l}\right) \tag{4}$$

resp.

$$M_{ab} = \bar{M}_{ab} + k_{ab}(2\phi_a + \phi_b - 3\psi_{ab}) \tag{5}$$

$$M_{ba} = \bar{M}_{ba} + k_{ba}(\phi_a + 2\phi_b - 3\psi_{ab}) \tag{6}$$

where

$\bar{M}_{ab}$ and $\bar{M}_{ba}$ are fixed-end moments at the near-end support; the moment is positive clockwise when acting on the span; refer to the table on the inside back cover for various loading conditions.

ϕ_a and ϕ_b are near--end and far-end slopes or angular displacements of the span at the supports; the angles are measured in radians and are positive clockwise [30].

ψ_{ab} is span rotation of its coefficient due to a linear displacement, that is, $\psi_{ab} = \Delta/l_{ab}$. On horizontal beam Δ is $\Delta = v_b - v_a$ and Δ is $\Delta = u_b - u_a$ on vertical beam, where *a* is left-end and *b* is right-end of beam *a-b*.

The beam stiffness k_{ab} is

$$k_{ab} = \frac{2EI_{ab}}{l_{ab}} \tag{7}$$

where *E* is Young modulus of elasticity of used material [30].

The slope-deflection equations for end shears V_{ab} and V_{ba} of beam a - b are in the near of the span

$$V_{ab} = V_{ab,0} - \frac{M_{ab} + M_{ba}}{l_{ab}} \tag{8}$$

$$V_{ba} = V_{ba,0} - \frac{M_{ab} + M_{ba}}{l_{ab}} \tag{9}$$

where $V_{ab,0}$ and $V_{ba,0}$ are internal shear of simply supported beam *a-b* from adequate loading [30].

Occasionally an end span of a beam with a hinge in end of beam *b*, than we have to consider

$$M_{ab} = \bar{M}_{ab} + k_{ab}(2\phi_a + \phi_b - 3\psi_{ab}) \tag{10}$$

$$M_{ba} = \bar{M}_{ba} + k_{ba}(\phi_a + 2\phi_b - 3\psi_{ab}) = 0. \tag{11}$$

We consider that $\phi_b = \phi_b^*$:

$$\phi_b^* = -\bar{M}_{ba}\frac{1}{2k_{ab}} - \frac{\phi_a}{2} + \frac{3}{2}3\psi_{ab} \tag{12}$$

so next

$$M_{ab} = \bar{M}_{ab} - \frac{1}{2}\bar{M}_{ba} + k_{ba}\left(\frac{3}{2}\phi_a - \frac{3}{2}\psi_{ab}\right) \tag{13}$$

and thus, for a EI constant and assumed positive end bending moment at the end of the beam *a-b* is then given by

$$M_{ab}^* = \bar{M}_{ab}^* + \frac{3}{2}k_{ab}(\phi_a - \psi_{ab}) \tag{14}$$

resp.

$$M_{ab}^* = \bar{M}_{ab}^* + \frac{3}{4}k_{ab}(2\phi_a - 2\psi_{ab}) \tag{15}$$

and

$$M_{ba}^* = 0 \tag{16}$$

$$M_{ab} = \bar{M}_{ab} - \frac{1}{2}\bar{M}_{ba} + k_{ba}\left(\frac{3}{2}\phi_a - \frac{3}{2}\psi_{ab}\right) \tag{17}$$

$$M_{ab}^* = \bar{M}_{ab}^* + \frac{3}{2}k_{ab}(\phi_a - \psi_{ab}) \tag{18}$$

or

$$M_{ab}^* = \bar{M}_{ab}^* + \frac{3}{4}k_{ab}(2\phi_a - 2\psi_{ab}) \tag{19}$$

and

$$M_{ba}^* = 0 \tag{20}$$

where $\bar{M}_{ab}^*$ are fixed-end moments of a beam *a-b* is supported by a pin or roller on end span *b*.

$$\bar{M}_{ab}^* = \bar{M}_{ab} - \frac{1}{2}\bar{M}_{ba} \tag{21}$$

k_{ab}^* is span stiffness of beam *a-b* is supported by a pin or roller on end beam.

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$$k_{ab}^{*} = k_{ba}^{*} = \frac{3}{4} k_{ab} \tag{22}$$

Then

$$M_{ab}^{*} = \bar{M}_{ab}^{*} + k_{ab}^{*}(2\phi_a - 2\psi_{ab}) \tag{23}$$

The slope-deflection equations for end shears V_{ab}^{*} and V_{ba}^{*} of beam *a-b* are in the near of the span

$$V_{ab}^{*} = V_{ab,0} - \frac{M_{ab}^{*}}{l_{ab}} \tag{24}$$

$$V_{ba}^{*} = V_{ba,0} - \frac{M_{ab}^{*}}{l_{ab}} \tag{25}$$

Occasionally an end of a beam or frame is supported by *a* hinge at its far end, then we have to consider

$$M_{ab} = \bar{M}_{ab} + k_{ab}(2\phi_a + \phi_b - 3\psi_{ab}) = 0 \tag{26}$$

$\phi_a = \phi_a^{*}$

$$\phi_a^{*} = -\bar{M}_{ab}\frac{1}{2k_{ab}} - \frac{\phi_b}{2} + \frac{3}{2}3\psi_{ab} \tag{27}$$

$$M_{ba} = \bar{M}_{ba} - \frac{1}{2}\bar{M}_{ab} + k_{ba}\left(\frac{3}{2}\phi_b - \frac{3}{2}\psi_{ab}\right) \tag{28}$$

$$M_{ba}^{*} = \bar{M}_{ba}^{*} + \frac{3}{4}k_{ab}(2\phi_b - 2\psi_{ab}) \tag{29}$$

or

$$M_{ba}^{*} = \bar{M}_{ba}^{*} + \frac{3}{2}k_{ab}(\phi_b - \psi_{ab}) \tag{30}$$

and

$$M_{ab}^{*} = 0, \tag{31}$$

where $\bar{M}_{ba}^{*}$ are fixed-end moments of a beam *a-b* is supported by a pin or roller on end span *a*.

$$\bar{M}_{ba}^{*} = \bar{M}_{ba} - \frac{1}{2}\bar{M}_{ab}. \tag{32}$$

k_{ab}^{*} is span stiffness of beam *a-b* is supported by a pin or roller on end span

$$k_{ab}^{*} = k_{ba}^{*} = \frac{3}{4}k_{ab} \tag{33}$$

then

$$M_{ba}^{*} = \bar{M}_{ba}^{*} + k_{ab}^{*}(2\phi_b - 2\psi_{ab}) \tag{34}$$

The slope-deflection equations for end shears V_{ab}^{*} and V_{ba}^{*} of beam *a-b* are in the near of the span

$$V_{ab}^{*} = V_{ab,0} - \frac{M_{ab}^{*}}{l_{ab}} \tag{35}$$

$$V_{ba}^{*} = V_{ba,0} - \frac{M_{ab}^{*}}{l_{ab}}. \tag{36}$$

D. *Moment equilibrium equations*

The slope deflection method features an a priori compatibility of displacements. In order to grasp the basic idea, image that rotations are imposed to each element. This is associated with elastic deformation of the connecting elements. Compatibility is conserved in this process [30].

The rotated node *m* with four neighbours is shown in Fig. 2. Owing to elastic deformations, internal forces and moments are induced in elements. The end moments are assigned an extended subscript to distinguish the element. This notation is not standard and is utilized here just temporality for the sake of mathematical rigor [30].

Moment equilibrium of each joint requires that the sum of all end moments at the connecting elements has to be equal zero:

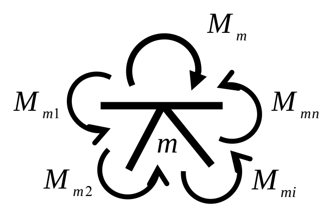


Fig. 2 Node *m*

$$M_m - \sum_{i=1}^{n} M_{mi} = 0 \tag{37}$$

$$M_{m1} + M_{m2} + M_{mi} + M_{mn} - M_m = 0 \tag{38}$$

where M_{mi} are given

$$M_{mi} = \bar{M}_{mi} + k_{mi}(2\phi_m + \phi_i - 3\psi_{mi}) \tag{39}$$

E. *Storey equations*

When solving constructions with sliding nodes, where displacements in the horizontal direction can occur (most of the storey frames belong here), in addition to unknown rotations of the nodes φ, also unknown horizontal displacements u. Due to the simplistic assumptions of neglecting the compression of the centre of the beams of the structure, the horizontal displacements of the nodes on the same floor of the frame will be the same. The equations for detecting the horizontal displacements of the individual floors of the floor frame are called floor equations. The floor equation expresses the conditions of equilibrium of horizontal forces above the horizontal section ρ of any floor with shear forces acting in the section below the individual floor, which replace the effect of the removed lower part of the system intersected by the cut on the upper part of the floor frame (Fig. 3).

In the case of constructions with sliding nodes, where horizontal displacements can occur, it is necessary to compile both moment and storey equations too. One storey equation must be compiled for each horizontal floor displacement [30].

The equation of equilibrium is

$$\sum_{\rho} V_{m,m-1} - \sum_{\rho} F = 0 \tag{40}$$

applies to any end shear force on bilaterally embedded beam

$$V_{m,m-1} = V_{m,m-1,0} - \frac{M_m + M_{m-1}}{l_{m,m-1}} \tag{41}$$

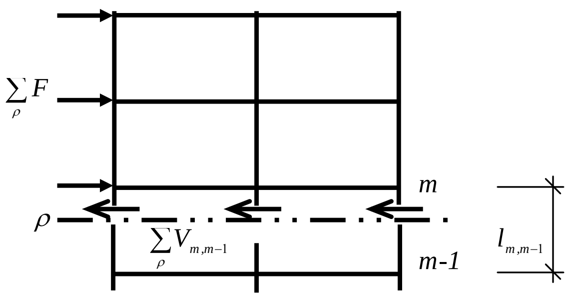
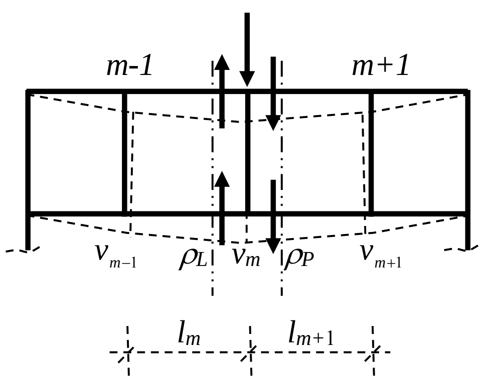


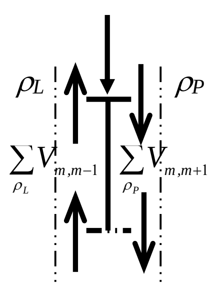
Fig. 3 Section ρ to create storey equation

F. Column equations

For structures with noncontinuous columns, the nodes can also be moved in the vertical direction, see Fig. 4.



a)



b)

Fig. 4 Section ρ to create column equation, a) on part of structure, b) detail of section

Due to the simplistic assumptions of neglecting the compression of the centre of the elements of the structure, the whole column has the same vertical displacements in the individual nodes. The equations that make it possible to determine the vertical displacements of individual columns are called column equations. It is necessary to compile as many column equations as the planar construction of continuous columns, one column equation must be constructed for each vertical displacement. Thus, other unknowns that occur in the conditions of the balance of vertical forces are also the vertical displacement, so the unknown vertical displacements in. The column equation expresses the condition of equilibrium of vertical forces acting on the cut part of the structure, i.e. to a continuous column of the structure with possible vertical displacements of the nodes of the system (Fig. 3).

The equation of equilibrium is

$$\sum_{\rho_L} V_{m,m-1} - \sum_{\rho_P} V_{m,m+1} - F_m = 0 \tag{42}$$

$$V_{m,m-1} = V_{m,m-1,0} - \frac{\bar{M}_m - \bar{M}_{m-1}}{l_{m,m-1}} - 3\frac{k_{m,m-1}}{l_m}(\Phi_m - \Phi_{m-1}) + 6\frac{k_{m,m-1}}{l_m^2}(\mathrm{v}_m - \mathrm{v}_{m-1}) \tag{43}$$

$$V_{m,m+1} = V_{m,m+1,0} - \frac{\bar{M}_m - \bar{M}_{m+1}}{l_{m,m+1}} - 3\frac{k_{m,m+1}}{l_{m+1}}(\Phi_m - \Phi_{m+1}) + 6\frac{k_{m,m+1}}{l_{m+1}^2}(\mathrm{v}_m - \mathrm{v}_{m+1}) \tag{44}$$

G. Solution using the slope deflection method

Procedure of solution by solving of the slope deflection method have next steps:

1. *Degree of deformation indeterminacy*

Determination of deformation indeterminacy and determination of nodal rotations and independent horizontal and vertical displacements [30].

2. *Basic deformation-specific system*

Designation of nodal rotations and horizontal and vertical displacements, obtaining the basic deformation-certain system to determining the basic elements of the slop-deflection method.

3. *Geometric equations*

Determination of nodal rotations and independent horizontal and vertical displacements and recast of the solved construction as a system. Obtaining the dependences between individual displacements, as well as determining the dependences of the rotation of elements due to independent horizontal and vertical displacements of nodes of a statically indeterminate construction, the expression of interdependencies between individual displacements.

4. *Compilation of conditional equations*

General compilation of conditional equations, the solution of which is to obtain numerical values of unknown components of displacements:

$$[K]\cdot\{\delta\} = \{R\}, \tag{45}$$

where $[K]$ is stiffness matrix,

$\{\delta\}$ is vector of unknown displacements,

$\{R\}$ is loading vector.

5. *Analysis of elements*

a) Stiffness of elements – to calculating the real stiffness of all elements of the construction,

b) End primary moments from a given load - calculation of values of primary moments from load on loaded beam (element) - embedded on both sides or with hinge in end one side of beam, respectively

c) End shear forces on a simple supported beam from a given load - determination of shear forces values on individual determining beams (elements).

6. *Assembly end-bending moments and end-shear forces on individual elements*

Preparation of relations for end-bending moments and end-shear forces of individual beams (elements) in general form by using of known numerical values from step 5.

7. *Substitution into the conditional equation, solution and calculation of the unknowns*

Compilation of conditional equations, assignment of relations for end-bending moments and end-shear forces from step 6 to conditional equations from step 4 for the purpose obtaining the system of conditional equations, solution of the system of conditional equations, obtaining numerical values of unknown displacements.

8. *Calculation of the resulting values of end-bending moments, end-shear forces at the end internal points of elements and solution of reactions, normal forces, and diagram of inner forces*

a) Calculation of the resulting end-shear forces i. e. end-bending moments and end-shear forces on individual elements by substituting the calculated numerical values of the displacement components.

b) Determination of normal forces using numerical values of resulting shear forces on elements from equilibrium conditions in individual nodes.

c) Determination of reactions can be determined from the conditions of equilibrium of shear forces in individual nodes.

d) Calculation of bending moments values and shear forces values on individual elements and calculation of their extremes.

9. *Diagram of inner forces*

Drawing of bending moments using, drawing of shear and normal forces and marking with signs.

10. *Solution control*

Carrying out control of the final solution,

a) static controls of the solution,
- static controls in joints (nodes), beams (elements), in other parts of the calculated structure,
- static control on the structure as a whole,

b) deformation controls of the solution.

III. THE SLOPE DEFLECTION METHOD IN EDUCATION PROCESS

The students, studying at the Faculty of Civil Engineering of the Technical University of Košice and they are study subject Static Analysis of Constructions, are solving three types of indeterminate structures using the slope deflection method in subject Static Analysis of Constructions:

1. continuous beams (six examples are shown in Fig. 5),
2. 2D sidesway frames (six examples are shown in Fig. 6), and
3. orthogonally loaded frames (six examples are seen shown in Fig. 7).

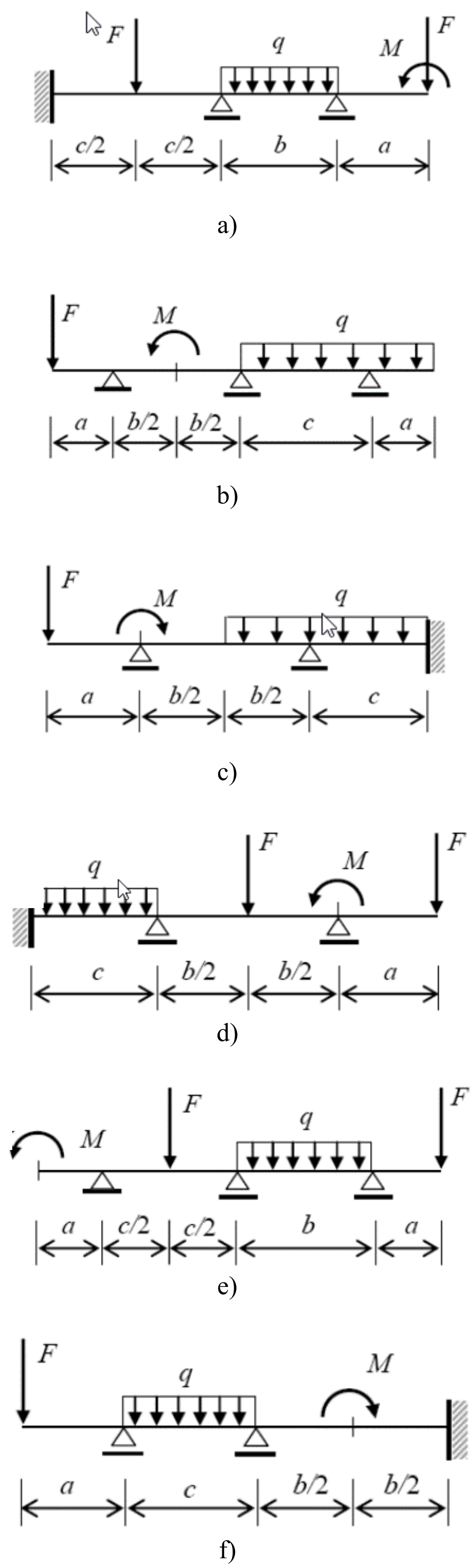
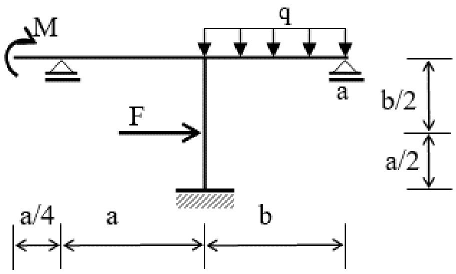


Fig. 5 Examples of structure: continuous beam

The frame structures are the structures having the combination of beams and columns to resist the gravity and other lateral loads. These structures are generally used to overcome the large forces, moments developing due to the applied loading.



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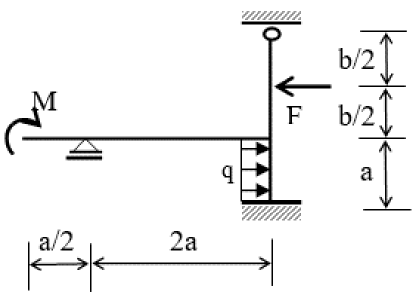

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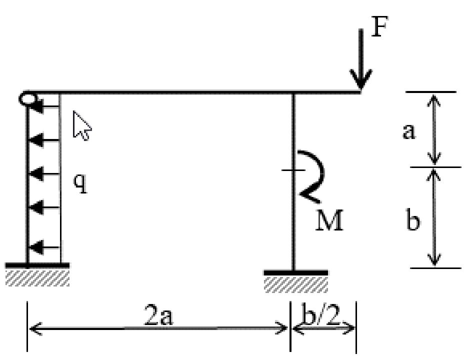

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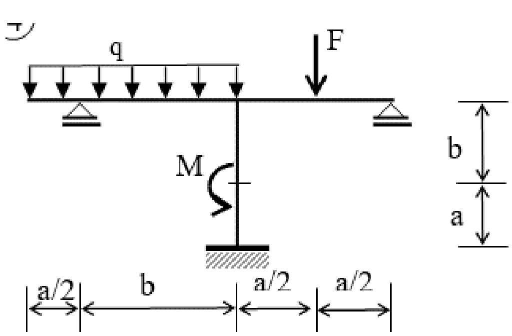

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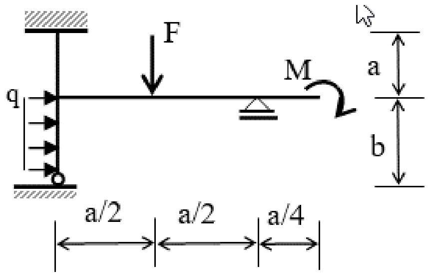

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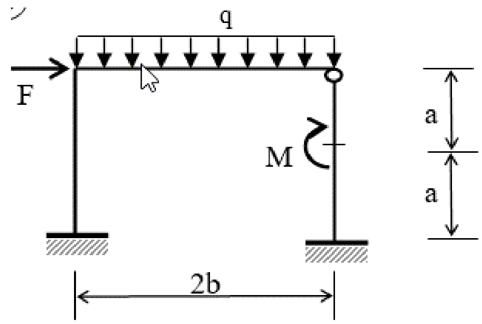
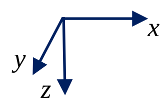


f)

Fig. 6 Examples of solving structure: 2D sidesway frame



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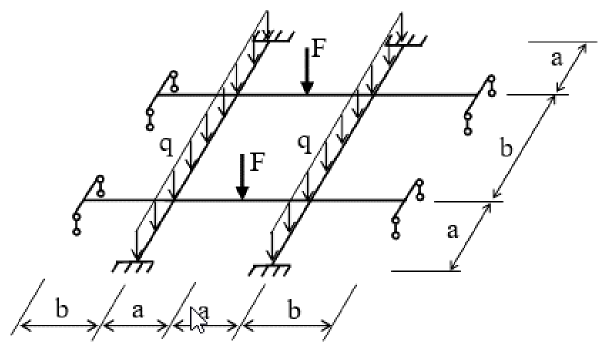

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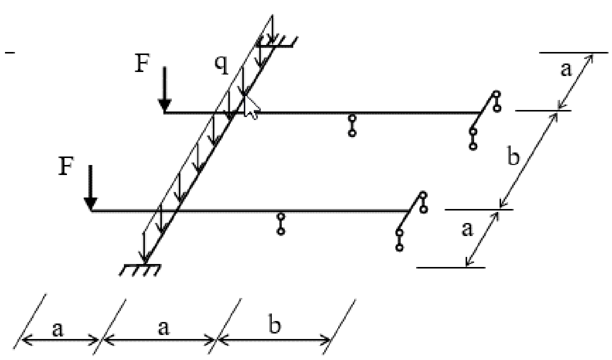

c)

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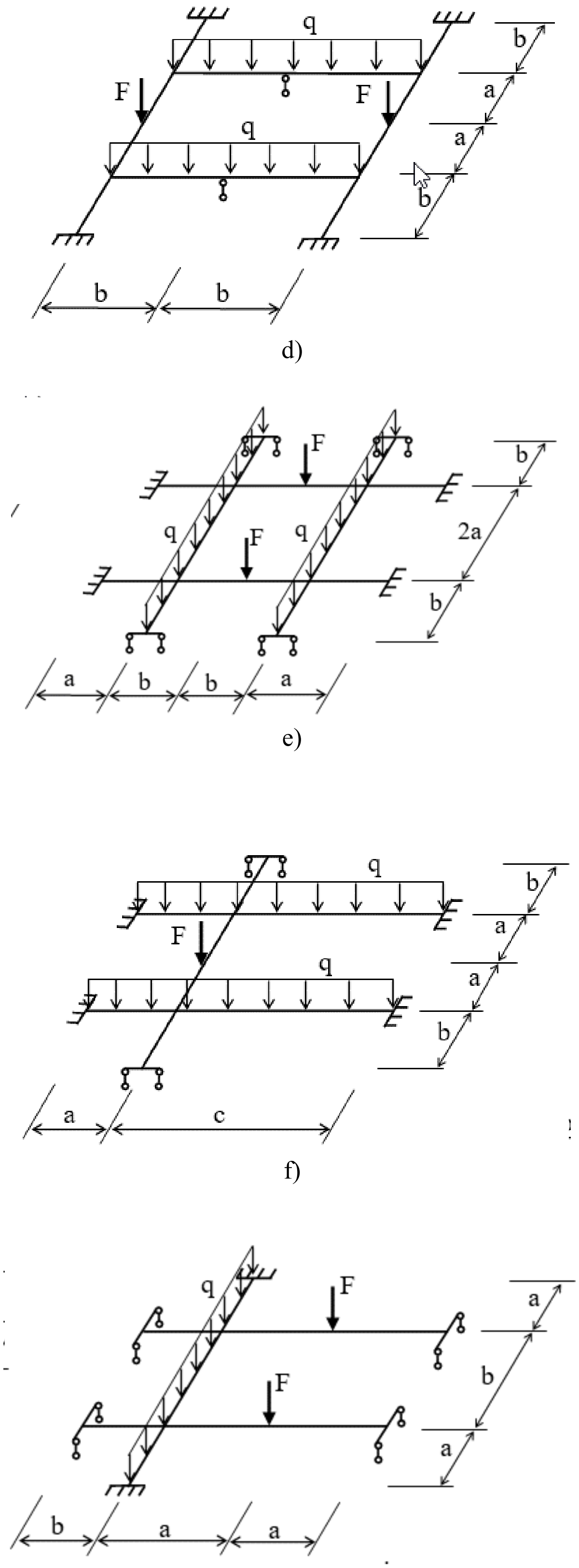

Fig. 7 a) Coordinate system, b) - g) examples of solving structure: orthogonally loaded frame

Information technologies are used in the teaching of the subject Static Analysis of Structures in order to teach students to calculate statically indeterminate structures also using the method of slope deflection method. Each student works with his own example with his own input values, i.e., each student works with an individual task, different task from other students. The input values are divided into two groups and each group has six input values. Application of information technologies in the teaching process of the slope-deflection method in the subject Static Analysis of Constructions can be explained in the example of a continuous beam. Each student is given a three-digit unique code; the first unique code number defines the shape of calculated structure. The second number of the unique code defines the numerical values of the student's task: the values of the horizontal dimension of the beam *a*, *b*, the point load force *F* and the vertical dimension of the rectangular cross-section h_c. The third number of the unique code indicates the values of the horizontal dimension of the beam *c*, the uniform load *q* and the point load moment *M* and the horizontal dimension of the rectangular cross-section b_c.

We will document the teaching procedure in more detail on the example of a continuous beam. The student's first code number of the three-digit unique code is assigned shape of indeterminate structures - continuous beam calculated by the slope deflection method, example in Fig. 5d). The second and third numbers of the unique three-digit code assign the values of the horizontal dimension of the continuous beam, lengths: a = 1.2 m, b = 2.2 m and c = 3.3 m, uniform load q = 14 kNm^{-1}, point loads F = 25 kN and M = 20 kNm, and the dimension of the rectangular cross-section: vertical dimension h_c = 0.3 m and horizontal dimension b_c = 0.22 m. The continuous beam with actual dimensions is also documented in Fig. 8a). Young modulus E defines the material of calculated structure (continuous beam). $E = 2.1\cdot10^7$kNm^{-2} is the equal value for all student tasks. Moment of inertia is given by $I = (b_c\cdot h_c^3)/12$ [m^4] using given h_c = 0.3 m and b_c = 0.22 m.

Teachers prepared program in code Fortran for solution of all kinds of examples for all input values. The software output gives the decisive values, intermediate and final, respectively.

The program for solution of continues beam using slope-deflection method give next output decisive intermediate and final values:

a) decisive intermediate values:

k = 26,77k kNm^{-2}

r = + 7.925 kNm

Φ_2 = - 0.000276097 rad

The degree of deformation indeterminacy of the statically indeterminate structures – continuous beam is 1, it is just a nodal rotation in node 2: Φ_2. The system of conditional equations (45) is transformed into one equation with one unknown: Φ_2. The stiffness matrix [M] is transformed to one value k, the vector of unknown displacements {δ} is transformed to one value Φ_2 and the loading vector {R} is transformed to one value r.

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b) decisive final values:

M_{12} = -14.444 kNm
M_{21} = +9.2262 kNm
M_{23}^* = -9.2262 kNm
M_{32}^* = -10.0 kNm
V_{12} = +24.681 kN
V_{21} = -21.518 kN
V_{23}^* = -14.4 kNm
V_{32}^* = -14.4 kNm
V_f = +4.1383 kN.

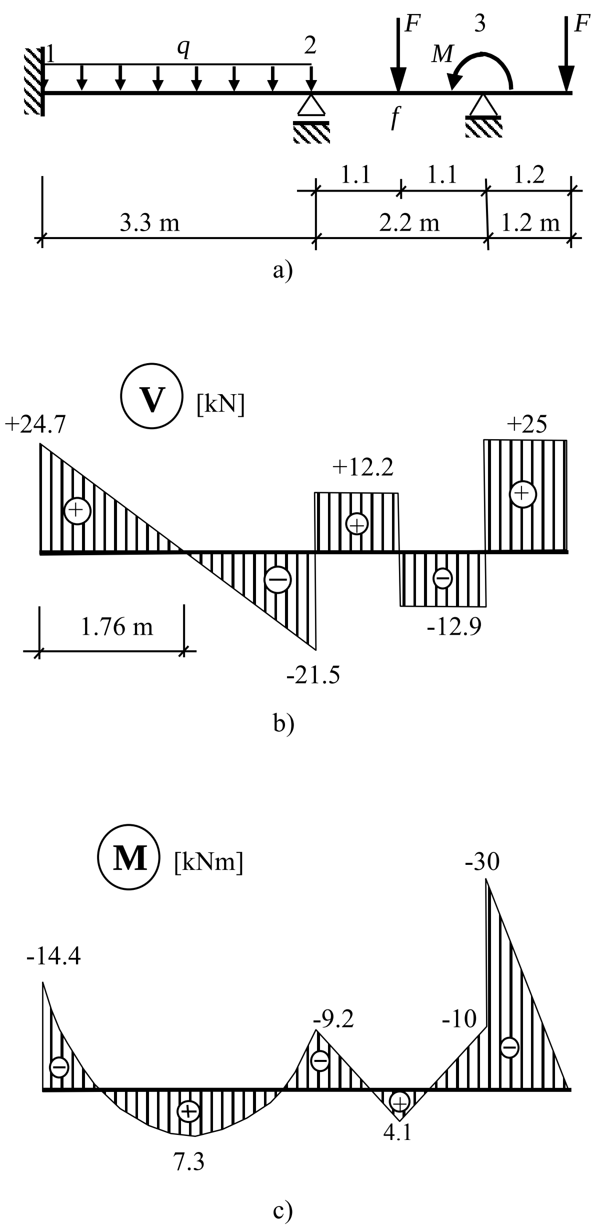

Fig. 8 a) Solving structure – continuous beam, b) diagram of shear forces, c) diagram of bending moment

M_{12} is resulting value of end bending moment in node 1 on the beam (element) 1-2, see Fig 8a) and Fig. 8c). M_{21} is resulting value of end bending moment in node 2 on the element 1-2. M_{23}^* is resulting value of end bending moment in node 2 on the element 2-3. M_{32}^* is resulting value of end bending moment in node 3 on the element 2-3.

V_{12} is resulting value of end shear force in node 1 on the beam (element) 1-2, see Fig 8a) and Fig. 8b). V_{21} is resulting value of end shear force in node 2 on the element 1-2. V_{23}^* is resulting value of shear force in node 2 on the element 2-3. V_{32}^* is resulting value of end shear force in node 3 on the element 2-3. The resulting value of end shear force V_f is end shear force in node f on the element 2-3.

The normal forces are zero, so the diagram of normal forces is not documented.

The students have to document final of their tasks: drawing the diagrams of inner forces. For drawing of the diagrams of inner forces, the students are using calculated resulted values given from calculating of statically indeterminate structure – continuous beam using of the slope deflection method: M_{12}, M_{21}, M_{23}^*, M_{32}^*, V_{12}, V_{21}, V_{23}^*, and V_{32}^*. The students know draw the diagrams of normal forces, shear forces and bending moment based on previously study of statically determined constructions. On based of previously study of statically determined constructions, the students eventually know calculate the intermediate values of inner forces that are necessary fir drawing of the diagrams of inner forces. These are the values V_3, M_3, and M_f in example, see Fig. 8a), 8b), 8c).

Teacher used programming software in Fortran code to writing program for solving of indeterminate structures - continuous beams using the slope deflection method in teaching process of subject Static Analysis of Constructions. The became results from this program: continuous and finally results serve the purpose for control the student tasks of solving of continuous beam. There are currently many commercial programs using Finite element method to solve of statical determinate structures as well as statical indeterminate structures and these results are only the final results: diagrams of in ner forces and teacher using commercial programs can control only final diagrams of inner forces.

The prepared program by teachers gives the decisive values: intermediate and final, respectively. Teachers are using the decisive output to check the correctness of the student's tasks. When the student does not have the correct results, the program outputs are helping the teachers to identify errors, and then the teachers know, whether the errors are the result of misunderstanding of the problem, of misunderstanding of the method and the calculation procedure of solution of indeterminate structures, the continuous beams using the slope deflection method (see Fig. 5), or it is just a numerical error. Then the teachers can guide the students to eliminate of errors.

Teachers prepared programs in code Fortran for solution of all selected examples and for all giving input values, and for all kinds of indeterminate structures by using the slope deflection method.

The indeterminate structures 2D sidesway frames documented in Fig. 6 are the statically indeterminate structures with 2x degree of deformation indeterminacy. The unknows are a nodal rotation Φ and horizontal 'floor' displacement u. The system of conditional equations (45) is transformed into two equations with two unknowns Φ, u. The stiffness matrix $[M]$ is 2 x 2, the vector of unknown displacements $\{\delta\}$ is vector with two unknowns Φ, u, and the loading vector $\{R\}$ is vector with two members.

The indeterminate structures orthogonally loaded frames documented in Fig. 7b)-Fig.7g) are the statically indeterminate structures. Depends on structure the Degree of deformation indeterminacy is 6 – 14. Using of symmetry of the structures and idealisations of the slope deflection method, the unknows of orthogonally loaded frames in Fig. 7b)-Fig.7g) are only three the nodal rotations Φ_x, Φ_y (rotations about axis x and y), and vertical displacement w (in direction of axis z). The system of conditional equations (45) is transformed into three equations with three unknowns Φ_x, Φ_y, w. The stiffness matrix $[M]$ is 3 x 3, the vector of unknown displacements $\{\delta\}$ is vector with three unknowns Φ_x, Φ_y, w, and the loading vector $\{R\}$ is vector with three members.

IV. Conclusion

The Static analysis of constructions is an important and demanding engineering subject. Given its importance, due attention should be paid to his teaching. The presented paper deals with the teaching process of the slope-deflection method in the subject Static Analysis of Buildings at the Faculty of Civil Engineering of the Technical University in Košice. The slope-deflection method is the method for the analysis of statically indeterminate structures and serves to obtain inner forces and deformations. Students are documenting their knowledges and skills on three kinds of structures.

Teachers prepared programs in code Fortran for solution of all kinds of examples. The program outputs give the decisive values, intermediate and final results, respectively. Teachers are using the output decisive final values to check the correctness of the student's tasks and by using of decisive intermediate values can guide students to work to correct of mistakes in their work.

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