A new arrangement with time-varying capacitance for power generation

Takuya Hirata, Ichijo Hodaka, Masanori Ushimizu

Abstract—Ocean wave energy is enormous and there are many methods for power generation from the wave energy. Our method also is for power generation from the wave energy, but we utilize a capacitor for power generation instead of using the electric generator for any conventional method. Our method is one way of power generation which utilizes a phenomenon that capacitance of a capacitor is changed by wave energy.

Keywords—Wave power, Power generation, Capacitor, time-varying coefficients

I. INTRODUCTION

OCEAN wave energy is enormous, which is expected to 29,500TWh/year (106.2EJ/year) theoretically [1]. This figure is approximately twice the amount of global power demand in 2008 [1]. Utilizing wave energy for power generation leads to reduction of CO$_2$ emission, similarly to the other sustainable energy [3].

There are many possible methods for power generation with wave energy [1] [2]. Most the methods use an arrangement consisting of a turbine and an electric generator. The arrangement is extensively employed for power generation or energy conversion [4]. The heart of such arrangements is inductors which convert a type of energy into another.

This paper considers a new arrangement for power generation using wave energy: the heart of our arrangement is a capacitor which will convert kinetic energy of waves directly into electric energy stored in the capacitor. This paper also discusses how to realize the conversion based on a mathematical model of linear differential equations with time-varying coefficients.

Our arrangement has no moving parts contrarily to the usual arrangement with electric generator by inductors. This will be a remarkable advantage, since our arrangement seems inexpensive and free from maintenance. Therefore our purpose is to propose a way of power generation using a capacitor without electric generators.

II. A NEW ARRANGEMENT OF POWER GENERATION

Consider a capacitor described in Fig. 1, where $x$, $2l$, $\varepsilon_1$ and $\varepsilon_2$ denote displacement of the wave surface, length of the electrode, a relative permittivity of air and one of seawater, respectively. Suppose that the capacitor is put in the sea. The seawater intrudes in the space between two electrodes in parallel, and the water surface moves periodically as a wave.

The displacement of the surface $x[m]$ along time is written as

$$x(t) = l \left(\sin\left(\frac{2\pi}{T} t\right) + 1\right)$$

where $l[m]$, $T[s]$, and $t[s]$ are respectively half of the height of electrodes, the period of waves, and time. Under the condition, the capacitance of the capacitor is given by the following.

$$C(t) = \frac{\varepsilon_0 S}{2d} \left((\varepsilon_2 - \varepsilon_1) \sin(2\pi T) + \varepsilon_1 + \varepsilon_2\right)$$

Here we assume that $\varepsilon_0$, $\varepsilon_1$, $S$ and $d$ are the permittivity of vacuum [F/m], the relative permittivity, the area of electrodes [m$^2$] and the distance of electrodes [m], respectively.

Since the equation above involves redundant parameters, we straighten them into $A = \frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_1 + \varepsilon_2}$ and $c_1 = \frac{\varepsilon_0 S}{d(1 - A)}$. Then we have a compact form as

$$C(t) = c_1 \left(A \sin\left(\frac{2\pi t}{T}\right) + 1\right).$$

We consider the series circuit which consists of a inductor $L[H]$, the capacitor $C(t)[F]$, and a resistor $R[\Omega]$ as in Fig. 2.
The circuit is mathematically modeled by
\[
\begin{align*}
RI(t) + V_c(t) + LI'(t) &= 0 \\
I(t) &= (C(t)V_c(t))'
\end{align*}
\] (4)
where \( I[A] \) and \( V_c[V] \) are respectively the current and voltage over the capacitor. If we put \( y(t) = C(t)V_c(t) \), (4) is reduced in a second order differential equation:
\[L\ddot{y}(t) + R\dot{y}(t) + \frac{1}{C(t)}y(t) = 0. \] (5)

Notice here that \( C(t) \) is periodically time-varying, contrary to usual LCR circuit which has constant coefficients. Since the equation is stable if the capacitance is a constant, the voltage \( V_c \) of the capacitor always converges to zero and cannot be much higher than the initial voltage.

However, the equation with time-varying \( C(t) \) as in the equation (5) has a possibility of unstable behavior [5] [6]. Such an unstable situation may occur high voltage at the capacitor, and then lead to an effective electric power generation.

Therefore, a key step to power generation in the rest of this paper is to seek parameters which make the time-varying LCR circuit unstable. This is the reason why we consider such a situation that we could take electric energy effectively using unstable situation on the circuit.

### III. Result of Numerical Simulation

We simulate the behavior of the equation (5) with numerical values as in Table I. The parameter \( A \) is a constant decided by a relative permittivity of air and seawater. The other values in the table are decided by trials and errors to give an unstable situation. The variables in the circuit as time goes are illustrated in Fig 3-6. The simulation was performed with an initial condition for the equation (5) as \( y(0) = \begin{bmatrix} 10c_1 \\ 0 \end{bmatrix} \), that is \( V_c(0) = 2.8[V] \) and \( I(0) = 0[A] \).

**TABLE I. The Parameters (Type1)**

<table>
<thead>
<tr>
<th>R</th>
<th>L</th>
<th>T</th>
<th>( \epsilon_0 )</th>
<th>A</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0001</td>
<td>1.000</td>
<td>1.000</td>
<td>0.2000</td>
<td>0.9756</td>
</tr>
</tbody>
</table>

![Fig. 3. The voltage of the capacitor (Type1)](image3.png)

![Fig. 4. The current through the circuit (Type1)](image4.png)

![Fig. 5. The power of each element (Type1)](image5.png)
We also simulate the behavior of the equation 5 with numerical values as in Table.II and with the same initial condition as the previous simulation. The simulation result as Table.II is shown on Fig.7-10.

**TABLE II. THE PARAMETER (TYPE2)**

<table>
<thead>
<tr>
<th>R</th>
<th>L</th>
<th>T</th>
<th>c_i</th>
<th>A</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>0.1500</td>
<td>0.9756</td>
</tr>
</tbody>
</table>

Fig. 6. The energy of each element (Type1)

Fig. 7. The voltage of the capacitor (Type2)

Fig. 8. The current through the circuit (Type2)

Fig. 9. The power of each element (Type2)

Fig. 10. The energy of each element (Type2)
Detach the capacitor from the circuit when the capacitor energy reaches a peak and power generation is done by extracting the stocked energy from the capacitor. This is power generation by using a capacitor.

IV. CONCLUSION

In this paper, we have proposed a new arrangement for power generation especially by utilizing ocean-wave power. The feature of the arrangement is to use a capacitor for converting wave power into electric power while most other arrangements use inductors. Instead of electric generators, power generation using a capacitor has been to capture wave energy with a capacitance being changed and we have utilized the unstable LCR circuit to take electric energy effectively.

Kinetic motion of waves has been modeled as periodic variation of capacitance. Our two of numerical simulations have shown that one can obtain electric energy much larger than initially stored energy in the capacitor, which means that our arrangement certainly withdraws kinetic energy of waves. These simulations also have shown we have achieved our purpose which is to do power generation by using a capacitor instead of electric generator. Thus we have illustrated a possibility of the utility value of wave power.

REFERENCES