Computation of planetary atmospheres by action mechanics using temperature gradients consistent with the virial theorem

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Abstract— Computations of atmospheric profiles based on action mechanics – coupled with a novel version of the virial theorem – question the current paradigm proposing a common mechanism of surface warming for all greenhouse gases. A development of the virial theorem suggests a new basis for establishing temperature gradients from equilibrium between molecular mechanics and gravity, modified by work-heat transfers resulting from convection or changes in molecular phase such as condensation or evaporation. The virial-action model suggests that recent global surface warming could have been caused by increased water vapor in the atmosphere affecting its morphology or variations in local frictional coefficients of the Earth's urban and rural landscapes with compressive heating, rather than changes in greenhouse gas content alone. The virial-action hypothesis is logically tested in a computer model using data from Earth and other planets developing a novel algorithm for calculating atmospheric profiles recalling Lagrange's calculus of variations and least action. All planets with an atmosphere are shown to exhibit surface warming compared to their black body temperatures, irrespective of greenhouse gas contents.

Keywords— Action mechanics, lapse rates, virial theorem, entropy, Gibbs energy

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INTRODUCTION

A timely revision of the basic physics related to global warming is proposed. This revision is based on a recent application of the virial theorem to planetary atmospheres [1], providing more functional and precise estimates of the troposphere's lapse rate of temperature with altitude than possible with the adiabatic lapse rate. Despite both Maxwell and Boltzmann rejecting as inconsistent with the second law Loschmidt's proposal that a temperature gradient should exist for any substance including solids erected vertically in the Earth's gravitational field, this paper reintroduces his proposal, but based on a quantum mechanical effect that should apply to all materials.

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Action mechanics [2, 3], a simplified physical version of statistical mechanics recently extended to studies of the atmosphere [1, 2], enables an alternative model of atmospheric processes to the general circulation models (GCMs) based on the governing Eulerian differential equations of fluid motion. Action (@), a macroscopic property of state for a system of molecules or oscillators, was used by Planck [4] to define the minimum quantum of action *h* for radiation. Its magnitude is given by the product of generalized space coordinates (*r* or $r\delta\Theta$, cm) and the corresponding impulse coordinates (*mv*, g.cm/sec) defining the microscopic action state of the molecule ($mvr\delta\Theta = mvs$) as a multiple of this minimum value for molecules. In its combination with the virial theorem, the virial-action hypothesis has the advantage of being predictive from clear scientific principles.

This ease of intellectual application is achieved by simplifying the complex statistical mechanical functions for entropy, such as that per molecule for diatomic gases (e.g. Hill [5] eqn. 8.37), ignoring excited electron states not occurring significantly in the troposphere:

$$S/N = k\{\ln[(2\pi mkT/h^2)^{3/2}Ve^{5/2}/N] + \ln[(8\pi^2 kTI_r h^2)e/\sigma] + [(h\nu/kT)/(e^{h\nu/kT}-1) - \ln(1-e^{h\nu/kT})]\}. \quad (1)$$

into an action-based form much more amenable for modeling; this is achieved by partitioning molecular entropy into its translational, rotational and vibrational forms given in reference [3].

$$S/N = k\{\ln[e^{7/2}(@_t/\hbar)^3(@_r/\hbar)^2 Q_e/(\sigma_r z_t)] + x/e^x - 1) - \ln(1-e^x)\} \quad (2)$$

Here the translational action $@_t$ per molecule was formerly given by us [2] as $(3kTI_t)^{1/2}$, proposed to be equal to mvr, where r is half the mean molecular separation and I_t is mr_t^2, the translational moment of inertia for each molecule. However, since z_t has the value of $(2.17)^3$ for all molecules [2] this factor can be reduced to 2^3 x $(1.085)^3$, exactly equivalent to the cube of a symmetry factor of 2 preventing double counting of the translational action multiplied by the ratio of the root mean square velocity to the mean velocity of molecules in the Maxwellian distribution, the translational action $@_t$ or mvr must be given by $(3kTI_t)^{1/2}/(z_t)^{1/3}$ rather than $(3kTI_t)^{1/2}$. The fact that this correction now gives an exact result for translational entropy in equation (1) is a remarkable confirmation of the validity of action mechanics as a simpler and concise version of statistical mechanics. The rotational action $@_r$ equals $(2kTI_r/\sigma_r)^{1/2}$, where σ_r is a rotational symmetry factor avoiding excess counting of indistinguishable conformations that would require more energy for action and x a fractional number equal to $h\nu/kT$ indicating the extent to which vibrational energies are activated to higher levels of action at a given temperature [2].

Thus, the corrected version of equation (2) is given by the modified formula lacking z_t following, since the translational action $@_t$ is now considered to be $(3kTI_t)^{1/2}/(z_t)^{1/3}$.

$$S/N = k\{\ln[e^{7/2}(@_t/\hbar)^3(@_r/\hbar)^2 Q_e/\sigma_r] + x/e^x - 1) - \ln(1-e^x)\} \quad (2)$$

With rare exceptions, as for oxygen and nitric oxide, because of their asymmetric electron spins requiring more energy, the electronic multiplicity Q_e is usually 1.0 and can be omitted from equation (2). Furthermore, for water content though not for carbon dioxide [2], the vibrational entropy can practically be omitted so that an adequate description of Earth’s atmospheric molecular entropy is given by equation (3).

$$S/N = s = k\ln[e^{7/2}(@_t/\hbar)^3(@_r/\hbar)^2 Q_e/\sigma_r] \quad (3)$$

By using the ratio $(@/\hbar)$ as indicating the scale of the density of action states and assuming Q_e is 1, equation (2) may be rendered even more concisely for translation and rotation in the following dimensionless set of numbers, where n_t is equal to $(3kTI_t)^{1/2}/(z_t^{1/3}\hbar)$ and n_r is $(2kTI_r)/(\sigma_r^{1/2}\hbar)$ for linear molecules

$$S/N = s = k\{\ln[e^{7/2}n_t^3 n_r^2/\sigma_r] + x/e^x - 1) - \ln(1-e^x)\} \quad (4)$$

At sufficiently high temperature, this expression for the entropy of linear molecules simplifies further to approach equation (5),

$$s = k\ln[e^{9/2}n_t^3 n_r^2/\sigma_r] \quad (5)$$

as bonds reach breaking point (ca. 4,000 K for H_2). At very cold temperature (<80 K), the rotation is no longer classical for H_2 or HD as quantum effects are exerted and the entropy becomes even simpler, as a function of translational action only, the diatomic or linear gas (e.g. H_2, N_2, O_2, CO_2) even behaving as a monatomic gas like helium or argon, if sufficiently cold (i.e. < 80 K for H_2, <3.0 K for N_2, <2.1 for O_2, <0.56 K for CO_2).

$$s = k\ln[e^{9/2}n_t^3] \quad (6)$$

Expressed in this fashion, thermodynamic computation and development of models for numerical methods are greatly simplified and less error prone (*cf*. equation (1) versus (4)). One can then simply obtain the total thermal energy required in terms of the kinetic or enthalpic energy indicated by the exponential term and the quantized statistical energy required for a molecule to reach a given temperature T, by multiplying entropy per molecule s by temperature T. While some advisors have expressed to the author a preference to retain the more classical differential calculus for atmospheric thermodynamics without partitioning entropy and free energy, the author has now found that using these integrated but action-partitioned equations offers too many advantages for conception of ideas and in calculation to be so conservative. Rest assured your productivity will increase many-fold, should you do so too.

Employing the virial theorem together with these equations, each more relevant to the particular environmental conditions on different planets, provides exact solutions to atmospheric profiles, enabling density (N per cm^3), temperature and pressure at each altitude for steady state equilibrium sustained by heat flow to be obtained [1]. The theorem also indicates that most of the potential energy to drive heating processes on the surface from the atmosphere when the surface is colder is thermodynamic – consisting of both kinetic energy and much more statistical energy as the multiple components of entropy (equation 4) for molecular complexes. It is surprising to the author that such an obvious source of possible local warming by release of statistical work as latent heat is ignored.

This new virial-action hypothesis is tested here with published observational data from the atmospheres of Earth, Venus, Mars, the giant planets and Saturn's large moon, Titan, comparing and contrasting these data for consistency with the adiabatic theory for convective cooling with height.

METHODOLOGY

Virial lapse rates

Clausius' virial theorem [6] states that the mean kinetic energy ($+mv^2/2$) of particles in a gravitationally bound system should be equal in magnitude to half their mean potential energy ($-mv^2$). Usually, the virial theorem has been employed to explain the evolution of stars made up of hydrogen atoms, their eventual collapse into heavier atoms and ultimate explosion. However surprising at this late stage, for reasons explained by Kennedy [1], the virial theorem can also be applied to the atmosphere assuming transient reversible states related to molecular equilibrium between gravity and thermodynamics. This automatically requires a temperature gradient with altitude independently of effects of convective expansion or compression. This version of the virial theorem proposes that the decreasing kinetic energy of air molecules with altitude indicates only half the potential energy required for gravitational work of the change in altitude ($mg\delta h/2$) and that the remaining increased potential energy is provided by the release of thermodynamic quanta (also $mg\delta h/2$) as variations in Gibbs energy allowed by their degrees of freedom of action. The same correspondence between changes in half the change in potential energy and the negative change in kinetic energy is seen between particles in different gravitational orbits although molecules in the atmosphere are clearly not in orbital motion [3].

An important result of this unique application of the virial theorem is a simple equation relating equal changes in mean molecular kinetic energy ($nk\delta T/2$) with changes in altitude and half the change in gravitational potential energy ($-mg\delta h/2$) as follows.

$$
\begin{aligned}
-nk\delta T/2 &= mg\delta h/2 \\
-\delta T/\delta h &= mg/nk
\end{aligned}
\tag{7}
$$

Equation (7) provides a remarkably simple means of calculating the virial lapse rate of temperature with height (h), based on a steady state molecular quasi-equilibrium between thermal and gravitational states. Here m is the mean molecular weight of the gases in Daltons (28.97 for air) number-density weighted, n is the degrees of freedom of action or motion – being 3 for argon, 5 for diatomic or linear molecules like N_2, O_2 and CO_2, plus a factor for the freedom of vibrational kinetic motion for CO_2 giving n about 5.4 on the Earth's surface at 288 K.

This application of the virial theorem proposes that this second half of the change in gravitational potential energy ($mg\delta h/2$) comprises the net change in statistical field energy with height, the functional virial or potential energy of translational, rotational and vibrational motion. Previously, use of the virial theorem was restricted to the monatomic states typical of stars or gas considered as isothermal to calculate a mean value of kinetic energy, rather than the layered levels of temperature proposed for the atmosphere in the virial-action theory. Here, it is effectively extended to rotational and vibrational degrees of freedom, applying the theorem to different altitudes and temperatures as given by the lapse rate of equation (7).

Adiabatic lapse rates

By contrast, the adiabatic lapse rate theory long used in climate science, dating from the 19th century and Lord Kelvin's and Maxwell's analyses, is based on the notion of isoentropic convection of a discrete parcel of air. The validity of its mathematical and physical derivation is critiqued below but in summary its magnitude is described by the following equation.

$$\begin{aligned} -\delta T / \delta h &= mg / c_p \\ -\delta T c_p &= mg\delta h \end{aligned} \tag{8}$$

The adiabat for dry air (9.8 K/km) can be calculated using equation (8), where c_p is the molecular heat capacity at constant pressure and m the molecular mass in grams. It is more customary in climate science to use a mass-weighted heat capacity per kg C_p; but here, for consistency with the virial-action lapse rate and ease of comparsion, a molecular version is preferred. To account for the fact that this dry adiabat is rarely observed, often being less, release of heat from condensation of water in cloud formation is invoked, reducing cooling with height and thus lowering the lapse rate.

Atmospheric profiles on Earth

Applying the virial theorem to ascertain temperature gradients, action mechanics [1,2] has provided the following equation enabling the interaction between thermodynamics and gravity to be studied, assuming the virial lapse rate applies. This equation developed in [1] relates the negative Gibbs energy at altitude h_n to the variation in entropic energy (sT) at the surface h_o to the change in gravitational potential energy.

$$\begin{aligned} & kT_n \ln[(@_{tn}/\hbar)^3 Q_e] \\ &= 3.5k(T_n - T_o) + kT_o \ln[(@_{to}/\hbar)^3 Q_e] \\ &- kT_o \ln[(@_{ro}/\hbar)^2/\sigma_r] \\ &+ kT_n \ln[(@_{rn}/\hbar)^2/\sigma_r] - mgh_n \end{aligned} \tag{9}$$

By solving for the translational action ($@_{tn}$) at any altitude (h_n), this virial-action equation (9) allows pressure, number density, Gibbs energies for translation and rotation, temperature and entropy to be easily calculated using simple numerical computation [1]. The equation may also be written as follows.

$$\begin{aligned} mgh_n &= 3.5k(T_n - T_o) + \{kT_o \ln[(@_{to}/\hbar)^3 Q_e] - kT_n \ln[(@_{tn}/\hbar)^3 Q_e]\} \\ &- \{kT_o \ln[(@_{ro}/\hbar)^2/\sigma_r] - kT_n \ln[(@_{rn}/\hbar)^2/\sigma_r]\} \\ &= 3.5k(T_n - T_o) + \{kT_o \ln[n_{to}{}^3 Q_e] - kT_n \ln[n_{tn}{}^3 Q_e]\} - \{kT_o \ln[n_{ro}{}^2/\sigma_r] \\ &- kT_n \ln[n_{rn}{}^2/\sigma_r]\} \\ &= 3.5k(T_n - T_o) + \{kT_o \ln[n_{to}{}^3 Q_e] - kT_n \ln[n_{tn}{}^3 Q_e]\} \\ &- \{kT_o \ln[n_{ro}{}^2/\sigma_r] - kT_n \ln[n_{rn}{}^2/\sigma_r]\} \\ &= 3.5k\delta T + kT_n \ln[n_{rn}{}^2/n_{tn}{}^3 \sigma_r/Q_e] - kT_o \ln[n_{ro}{}^2/n_{to}{}^3 \sigma_r/Q_e] \end{aligned} \tag{10}$$

Thus, in principle we can express this gravitational, thermal and statistical configuration of energy more simply in equation (11).

$$mgh_n = 3.5k\delta T + \delta g_t - \delta g_r \tag{11}$$

Here δg_t and δg_r represent variations in mean molecular Gibbs energy required for the operation under gravity of the virial theorem and δT is (T_n - T_o). Though surprising at first that the variation in the rotational Gibbs energy (δg_r) should need to be subtracted from the variation in the translational Gibbs energy, a new principle in the interaction between gravity and thermodynamics may be recognized in this requirement.

Rotational (g_r/T) and vibrational (g_{vib}/T) Gibbs energy per degree both increase with altitude as temperature falls (i.e as their entropy declines), but translational Gibbs energy per degree g_t/T declines with altitude (i.e. statistical entropy increases) because the decrease in pressure increases radial separation ($+\delta r_t$) faster than the velocity decreases ($-\delta v$) as temperature falls giving a net increase in action. So equation (11) expresses an underlying variational process in the distribution of action and its energetic cost rather than conservation of energy – with rotational action and its quantum number state decreasing and translational action and its quantum number state increasing, offsetting each other as least action.

Since the action ratios ($@/\hbar$) and their logarithmic derivative, entropy, provide a measure of the quanta of energy required to sustain the molecular system in the macroscopic action field, we expect to find that reduced rotational action releases heat for gravitational work in increasingly greater quanta as altitude increases; by contrast, even though increasing translational action as temperature falls requires more quanta each of unit action, this need is offset by a longer radius of action compared to rotation and the quanta of lower frequency and momentum involved. Thus the yield of translational quanta for each rotational (or vibrational) quantum of energy released increases with altitude. In energy terms, translational quanta are far less expensive than rotational or vibrational quanta.

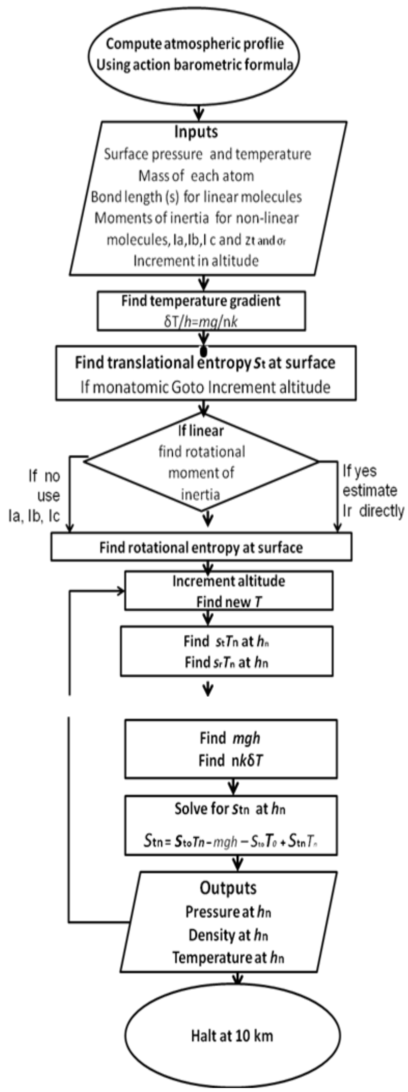


Fig. 1: Flow diagram for computation of atmospheric profiles on Earth using inputs of molecular properties (mass, bond length, symmetry factors for translation and rotation, surface temperature and pressure and altitude.

The governing virial-action equations (11), or (19) given later for vibrating systems, were the basis for the simple computer programs [1] prepared for this study able to provide exact solutions for atmospheric profiles. These logical programs of 6,000-9,000 steps were constructed (Fig. 1) in Astrocal, a machine code interpreter now in the public domain formerly marketed by Vernon Hester, Philadelphia, PA. Anyone interested in obtaining working copies of these programs operated in an emulator under Hester's Multidos 5.11 on a Windows platform should contact the corresponding author of this paper. Versions can then be easily prepared in more accessible computer codes such as the statistical program R.

RESULTS AND DISCUSSION

Theory of virial theorem and adiabatic lapse rates

Based on action mechanics as a means to simplify calculation of entropy [1-3], the virial-action hypothesis proposed that the dynamic morphology of the Earth's atmosphere tends closely to a reversible though transient equilibrium between its thermal heat content and gravity. This steady-state equilibrium in the atmosphere is constantly modulated by changes in the surface temperature from solar insolation and cooling by radiative emissions and conduction so it is never quite achieved. Given that the virial theorem requires that the time-averaged gravitational kinetic energy be half the average potential energy, atmospheric equilibration with height requires that the remaining half of the change in gravitational potential energy be equal to the changes in thermodynamic energy (Clausius' virial as potential energy) between each height. It can be considered that absorption of the quantum of the total field energy constrains or eliminates the equivalent kinetic energy, increasing the potential energy by twice this .quantum.

This result is further constrained by the need that the change in translational energy and temperature with altitude must correspond to the ideal gas law (corrected for non-ideality if necessary), allowed by changes in rotational and vibrational Gibbs energies; both these internal forms of work potential are responsive to changes in temperature only but not to changes in pressure or volume. This leads to a highly accurate definition of the lapse rate in temperature with altitude, for an ideal steady-state equilibrium atmosphere. It also allows the pressure and number density of molecules with altitude to be derived, yielding a profile for Earth consistent with this requirement [1].

The virial-action hypothesis requires that the steady-state dry lapse rate of temperature with height as given in equation (7) is equal to 6.9 K per km in Earth's well mixed atmosphere [1]. This result is given by air molecules with an average mass of 29 daltons, in contrast to the accepted dry adiabat of 9.8 K per km. Even in very dry atmospheres such as sub-arctic regions [7], such a high lapse rate is not observed. In virial-action, the lapse rate of 6.9 is regarded as an exact goal in the case of approaching steady-state equilibrium in a stable dry atmosphere. A precise adiabatic lapse in temperature with altitude caused by convection of coherent air parcels to fully account for the dry lapse in temperature of 9.8 K per km is considered here to be invalid, given the need for equilibration of thermodynamic pressure as energy per unit volume and gravitational pressure as weight per unit area at all altitudes of the troposphere. By contrast, the virial lapse rate is precisely defined as a function of thermodynamic equilibrium with gravity rather than an imprecise result of convective cooling. The temperature gradient prior to convection is a result of thermal energy reversibly driving gravitational work so that total work potentials are equal at all altitudes (equation (11)). However, convective cooling or heating by expansion or compression will serve as an adjunct to the virial-action lapse rate, increasing the the lapse rate from 6.9 K per km towards 9.8 K per km. Clearly, this is more feasible if the virial lapse rate already applies.

The anthropogenic greenhouse warming hypothesis

Since the last decades of the 20th Century, an hypothesis based on earlier work by Arrhenius has been developed explaining global warming as caused by greenhouse gases such as carbon dioxide and water [9]. Any molecule with three or more atoms able to vibrate with frequencies characteristic of infra-red radiation is a greenhouse gas, including nitrous oxide and methane. More recently, it has been recognised that under cold conditions collision processes can polarise and allow clusters of apparently symmetric molecule such as dinitrogen [10] to exist long enough to absorb far infrared absorption allowing atmospheric warming. To the extent that excessive warming has resulted from emissions of greenhouse gases through human activities, the climate change involved is called anthropogenic global warming.

Its main features are that global energy balance between absorption of solar insolation and Earth's re-emissions to space, the observed natural global warming (G) is given by the difference in radiation from the surface σT_G^4 and emission from higher in the atmosphere to space σT_E^4. Thus the surface warming factor G = σT_G^4 - σT^4_E with the Stefan-Boltzmann constant σ of 5.67 x 10^{-8} $Wm^{-2}K^{-4}$. In defining the quantum of radiation Planck [4] showed that the concentration of entropy S of black body radiation in a heated cavity was proportional to aT^3, presumably as a cubic function of the one-dimensional radiation proportional to a linear function of T as required by the Wien displacement law. Consequently the total heat radiation given by ST must be proportional to aT^4.

Clearly, the intensity of radiation emitted or escaping from the external surface of the cavity at the same temperature must also vary by the fourth power of temperature T^4. There is no mystery that the intensity of energy radiated from a heated black surface falls away so sharply as temperature falls or that the Sun radiates per unit area of its surface 160,000 times as much energy as Earth, although the density of quanta is only 8,000 times as great given the surface of the Sun is about 20 times as hot as the Earth. On Earth, 20 times as many long wave quanta are emitted to space as short wave quanta are received from the Sun, to keep the energy flow in balance, generating 20 times as much new radiative action. Each photon has action equivalent to Planck's quantum of action, *h* irrespective of frequency causing the same change in action of resonant particles.

According to Pierrehumbert and others [8,9], well established energy balance principles indicate that current increased emissions of CO_2 and other greenhouse gases such as methane and nitrous oxide to the atmosphere will further increase the greenhouse effect, thus further warming the Earth at an increased rate. Although the detailed physical nature of the greenhouse effect is difficult to convincingly demonstrate or experimentally test under field conditions, it is claimed that changes in the global infrared flux optical depth of the atmosphere force the warming effect, by restricting infrared emission to space. Increased levels of carbon dioxide in the troposphere are proposed to increase the optical depth for infrared radiation above the Earth's surface thus increasing the altitude from which the outgoing long-wave radiation is emitted to space. The lower zone in the troposphere with complete absorption and re-emission of infrared radiation by the high opacity of CO_2 at low altitudes in the atmosphere does not affect emission to space [8, 9].

However, this conclusion is based on assuming an unchanging effect of adiabatic expansion by all gases on cooling with altitude within the troposphere, in the zone where gas molecules are sufficiently dense to equilibrate thermodynamically. It is considered that significant emission to space will only occur where the atmosphere becomes sufficiently transparent; below this height, the opacity of infrared absorbing molecules prevents direct emission to space, at least of those frequencies absorbed by the gases present. So, increasing the concentration of greenhouse gases such as carbon dioxide could increase the opacity and the optical depth, raising the altitude of emission of infrared radiation to space.

There is scant evidence that radiative forcing specifically by carbon dioxide *necessarily* lowers the temperature of emission of long wave radiancy (OLR) to space, thus raising surface temperatures [8]. This is a hypothesis still needing critical testing rather than established scientific fact. Virial-action thermodynamics requires that different gases will have different influences on the lapse rates in temperature, the morphology of the atmosphere and whether net warming or cooling occurs as affected by the altitude and temperature of long wave radiation to space [8].

In fact adiabatic expansion as employed in atmospheric science predicts a temperature fall or lapse rate with altitude depending on the average mass of gas molecules doing expansive work against the pressure of the atmosphere subject to gravity (g), using the following equation, where c_p is the heat capacity per molecule of air.

$$\delta T / \delta h = -mg / c_p \qquad (12)$$

For dry air, the adiabatic lapse rate calculated is found to be 9.8 K per km.

The derivation of this equation is based on the first law of thermodynamics regarding the conservation of energy. In the steady state of thermal equilibrium with solar inputs, a column of air will have constant total energy. Thus a small parcel of air in the column this total will comprise thermal and potential energies. This is expressed as

$$dU = C_p dT + gdh \qquad (13)$$

where U is the total energy, C_p is the heat capacity of the gas at constant pressure and gh the potential energy per unit mass. At equilibrium, dU is zero and thus C_pdT and gdh are of equal magnitude. Note that if C_p varies with temperature, this lapse rate will vary accordingly.

Adiabatic expansion vertically is an approximation. Given the need to maintain hydrostatic equilibrium and equal gravitational pressure and thermodynamic pressure with height, we must have

$$p = Mg / a^2 = kT / a^3 \qquad (14)$$

where M is the mass above each area of side a at the surface, so Mg/a^2 is the weight per unit area.

This indicates that kT should equal Mga at each height, assuming ideality. This gives an equality in the action model between the centrifugal force of molecular translational motion $mr_t\omega^2$ and the gravitational force of 6nmg. Apparently, we can write the following equality of forces for each molecule at all heights. This expresses the equality of gravitational and thermodynamic force and pressure and the declining mean force with altitude, where M or Nm is the mass remaining in each column of air of volume a^2h of side a of molecular dimensions at its base, equal to 2r. Interestingly, one-sixth of the molecular translational inertial or centrifugal force ($mr_t\omega^2$) reflecting the temperature and pressure is opposed by the gravitational force directed vertically by the air column mass Nm, indicating the equality of gravity and the thermal forces. Gravity acts unidirectionally in this system.

$$mr_t\omega^2 / 6 = Nmg \qquad (15)$$

The current model for adiabatic cooling contains two incompatible assumptions – firstly, that an adiabatic process can truly occur in an isolated parcel of air at constant gravitational potential and second that dp/dh is actually equal to – ρ*g* in the atmosphere; as stated above this is strictly true only for isothermal systems, automatically negated if there is work performed as the altitude of the parcel increases and temperature falls as result. An adjustment is required as the change in pressure with height is only partly a result of a reduced number density per unit volume, since it is also affected by the change in temperature. Gas number density actually decreases less with height than the hydrostatic equation would predict, because pressure depends on temperature as well as number density [1]. Given that lapse rates near the dry adiabatic lapse rate are observed in radiosondes on balloons, this raises the question as to how such ideal results can be obtained. However, further cooling of the dry virial lapse rate of 6.9 K per km could readily be increased towards 10 K per km by convection when expanding or compressing.

Such flaws regarding adiabatic cooling, rather than virial theorem cooling by gravity, require persistent misunderstanding since the time of Maxwell, Boltzmann and Lord Kelvin. In Carnot's ideal cycle, adiabatic expansion of the working fluid occurs at constant gravitational potential as the heat engine is stationary with respect to altitude. This is not true in convection that occurs as a result of the atmosphere being gradually and continuously reversibly heated during expansion and cooling in contraction towards the Earth's surface.

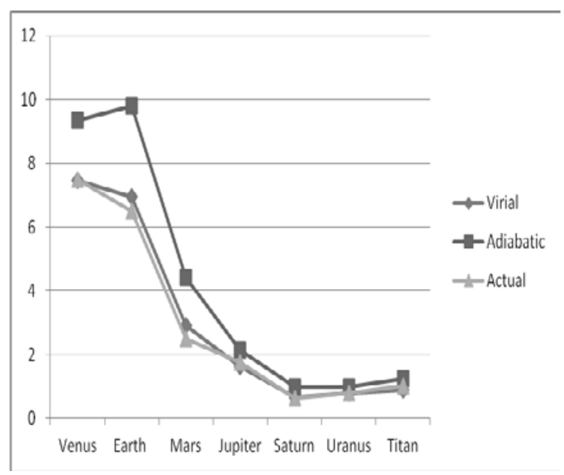
Unique features of the atmospheric heat engines

A reversible adiabatic expansion at constant pressure and gravitational potential of a working fluid as in a heat engine allows work to be performed thermally as the integral of $C_p\delta T$, including the work of elevating the atmosphere. Maximum work is obtained when the internal pressure and the external pressure remain the same during the expansion, with heat added reversibly. This allows the quantum of heat supplied to the engine to match the quantum of work done externally. However, Carnot imagined each cycle to comprise a continuous isothermal expansion at high temperature with heat provided for work followed by a continuous adiabatic expansion with no heat supplied and with declining temperature as work was done at the expense of the kinetic energy of the working fluid. In fact, both isothermal and adiabatic processes occur simultaneously in real engines, but rarely reversibly.

By contrast, the atmospheric heat engine performs reversible work on its own working fluid, given the slow rate of heat transfer by a process that can be likened to biological respiration using lungs. According to the virial theorem, the increase in gravitational potential energy is made up half as decreased kinetic energy of the working fluid as it elevates and cools and half as decreased entropic energy as increased Gibbs energy of the fluid as it rises [1]. This is expected in such a reversible process progressing by matching quantum changes between heat exchange and work produced. All work is done internally to the atmosphere.

Comparison of observed adiabatic and virial-action lapse rates

In the following section, the lapse rates observed on the planets and Saturn's largest moon Titan will be compared with those predicted by the adiabatic and virial-action hypotheses. These comparisons are shown in Table 2. Adiabatic lapse rates and virial-action temperature gradients are given, including observations for Titan. The virial-action procedure is shown by Chi square analysis to give significantly better agreement with reality, as is clear from Fig. 2 where the consistent tendency for dry adiabatic lapse rates to significantly overestimate rates is shown.



Fig, 2. Comparison of atmospheric virial, adiabatic and actual lapse rates on planets and Titan. Data is extracted from Table 2 and applies to values measured near the surface or close to 1 bar on the gas giants. The virial-action value for Earth is for dry air. A more comprehensive analysis is required as lapse rates may be affected by other dynamic factors including condensation of water. Chi square statistical analysis shows the virial theorem lapse rate is decidely more consistent with observations than adiabatic lapse rates.

Plotting virial-action profiles for atmospheres

In the following section, the virial theorem is employed to generate some thermodynamic profiles for the atmospheres of different planets. Particular characteristics of the profiles for each planet are discussed.

Earth

As shown by comparisons with US Airforce data for Earth, the virial-action hypothesis predicts the Earth's atmospheric temperature much more accurately [1] than the adiabatic hypothesis is capable.

It is remarkable how water content can affect the density of air, reducing the virial lapse rate from 6.9 with dry air to less than 6.5 (Table 2), at least near the surface. This effect is independent of convection or condensation but a function of number density and pressure on the lapase rate. The table also illustrates the well-known effect of temperature on the water content or atmospheric humidity. The water content at 100% humidity varies by a factor of more than seven between 270 K and 300 K, from 0.005 atm to 0.035 atm, even reaching almost 0.1 atm at 320 K. Obviously, this property is a valuable feature of water's function in nature.

This result directly reflects the change in the average molecular weight of air, declining as water displaces nitrogen and oxygen in the atmospheric profile at the surface, reducing the mass density but not the molecular number density of air at 1 atmosphere pressure. Thus, as humidity increases, the atmospheric profile for all other gases is elevated because of turbulent thermal mixing. Water is capable of significantly elevating the atmosphere to an extent depending strongly on the temperature. The warmer it is the greater the elevation. This property of water in the atmosphere will be explored in detail in another paper in the near future. According to the accepted climate models [8] this elevation should result in less radiation to space because the transparent zone at the top of atmosphere will be colder.

Carbon dioxide is predicted from virial-action theory to have contrasting behaviour to water. With a lapse rate of 9.59 K per km for the data in Fig. 3, when calculated as though it were present uniquely on the Earth's surface, this greenhouse gas should act to measurably increase the lapse rate of air from 6.9 in dry air with increases in its partial pressure according to relative number density. Given that water can provide more than 4% of the atmospheric pressure at the surface at 305 K, whereas carbon dioxide has just reached 0.04%, two orders of magnitude lower, the relative effect of each gas should be highly disparate on elevating or collapsing the atmosphere near the surface. However, this is not true at altitudes near or above the tropopause around 210 K, where the relative pressures of the two gases are more comparable and the maximum vapour pressure of water possible at this temperature is about 1 Pascal or 10 dynes per cm^2 or 10^{-5} atmospheres pressure. At these altitudes, the relative effects of variations in carbon dioxide or water acting from the surface on the atmosphere's morphology will be comparable.

Theoretical virial-action profiles illustrating such differences are shown in Fig. 3 for all the main constituents of air calculated at their respective pressures, including water, though at a low pressure equal to carbon dioxide. These show marked contrasts in the expected profiles, with oxygen and nitrogen being just below and above the observed lapse rate of dry air reflecting their mass and thermodynamic properties. Argon's profile is most depressed because of its high mass and low degree of freedom, but even carbon dioxide's profile is markedly lower than air. Water has only one-third the lapse rate of carbon dioxide and this is reflected in its extreme elevation, declining to the black body temperature of 254-255 K at 9-10 km compared to 3-4 km with carbon dioxide alone or 5 km in well-mixed air.

Of course this possibility is strongly influenced by the saturation vapour pressure of water. An atmosphere containing a saturating 0.016 atm of water at the surface (ca. 1620 Pa) cannot contain the full profile predicted by the virial-action model; it exceeds the saturation pressure at 1 km of altitude (1381 Pa) where the model predicts a transient equilibrium temperature for water of 285 K and 1610 Pa. Rain is predicted by the virial-action model as falling from about 500 metres with a surface temperature of 288.15. When a relatively dry atmosphere is modelled with 0.0004 atm of water (2.5% RH, 40.5 Pa, number density of 1.01835×10^{16} molecules per cm^3 of water at the surface, similar to carbon dioxide, the model provides a sustainable profile without rain, even up to 15 km where it would be about 25% relative humidity (RH) (Figure 3).

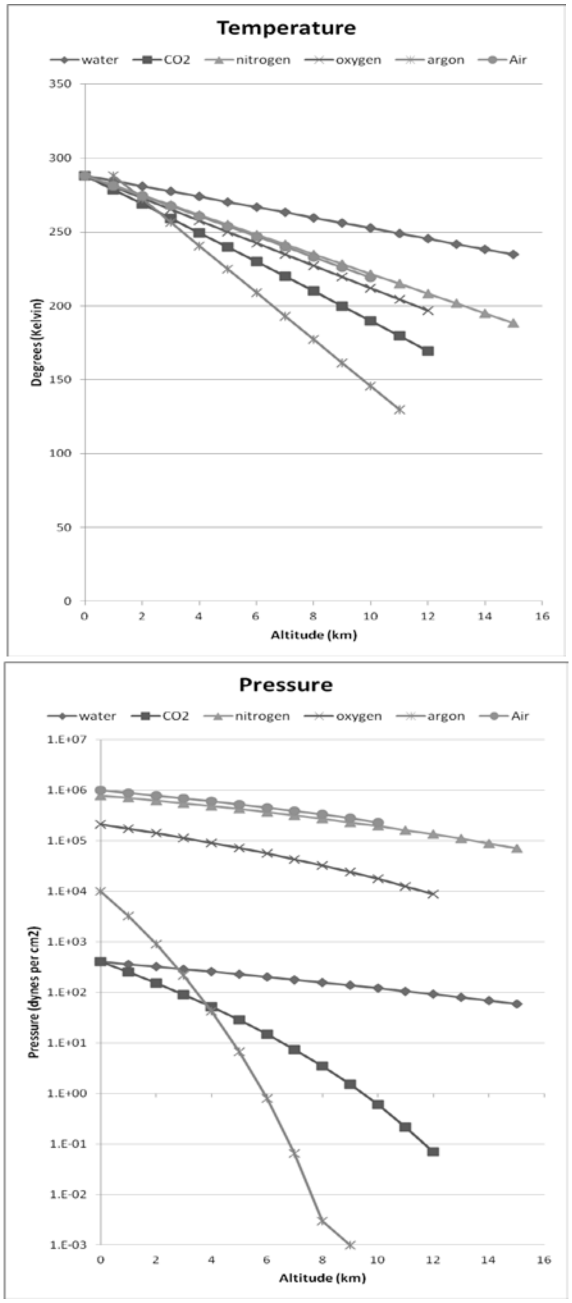


Fig. 3: Temperature and pressure plots with altitude for Earth for gases acting independently. Temperature lapse rates are calculated with the formula $\delta T/\delta h = mg/nk$, varying with mass and the degrees of freedom of action. Pressure plots with Earth's atmospheric composition are shown, with a water content equal to that of carbon dioxide of 0.0004 atmos. At this low surface pressure, no precipitation of water will occur but water maintains a relatively high pressure up to 10 km and beyond. In the real atmosphere, these gases are well mixed and therefore no gas is at gravitational equilibrium.

Even with a lapse rate of 9.8 K per km with a dry atmosphere, as predicted by the current adiabatic model, it is clear that the average temperature of the atmosphere would be much colder than the surface temperature, given the need for transient equilibrium at all heights. The height of the black body temperature of Earth's profile is currently just less than 5 km, depending on latitude. Given that significant surface warming compared to black body temperatures occurs on all planets in order with the relative weight of the atmosphere, irrespective of whether significant greenhouse gases are present (Venus, Mars, Titan) or not (Jupiter, Saturn, Uranus) as shown in Table 3, even reaching many thousands of degrees Kelvin on the gas giants Jupiter and Saturn, it seems that surface warming is primarily a thermodynamic requirement to successfully sustain the weight of the atmosphere. While carbon dioxide and water may play some part in modifying the surface temperature of Earth, by analogy with the atmospheres of the other planets without greenhouse gases, this is unlikely to be the full 33 degrees K above the black body temperature that is frequently claimed.

Venus

Unlike Earth, hot Venus has a fairly uniform surface temperature from its equator to the poles, suggesting a state of relative equilibrium. As a consequence, there is relatively little convection. Nor does Venus show evidence of significant phase changes in atmospheric gases up to 50 km altitude, unlike the condensation of water on Earth. In this zone of the troposphere carbon dioxide is the major gas (0.965) so the basal virial lapse rate should show less variation as a result of convection or phase changes. This raises the question of whether an adiabatic lapse rate is appropriate.

However, a strong effect of vibrational heat capacity for carbon dioxide on the lapse rate on Venus has already been discussed [1]. With an opposite effect to temperature below 100 K as observed on Uranus and Saturn restricting the rotational heat capacity of hydrogen – producing increased lapse rates – excitation of increased vibrational energy at the surface temperature of 735 K on Venus reduces the lapse rate compared to what would be seen for carbon dioxide alone on Earth at 288 K. The thermodynamic profile calculated for Venus using the virial-action model reproduces the thermal gradient observed by the Magellan Lander up to 60 km altitude almost exactly except where the data is irregular in an intermediate zone possibly reflecting local weather (Table 1). Some of these lapse rate values computed virial action are given in Table 2, where they can be compared with the adiabatic values.

As shown in Tables 1 and 2 and Figure 2, there is excellent agreement between the lapse rate with altitude estimated by the virial-action formulae and that measured with a shielded resistance thermometer in Venus' atmosphere by the parachuted Magellan lander. This agreement is improved compared to that when this approach was first considered, a result of overestimating the kinetic heat capacity c_{kvib} and the corresponding degrees of freedom of action. It is surmised that the vibrational heat capacity (C_{vib}) as measured experimentally is the sum of the mean vibrational kinetic energy as well as the mean potential energy, each equal to the other. So for a harmonic oscillator such as carbon dioxide, only half of the measured heat capacity is represented in its kinetic motion that is relevant to freedom of action for applying the virial theorem. Although this fact is not prominently stated in text books, heat capacity as calculated from equations must include the energy derived from multiplying entropy and temperature given in the following equations (16) and (17) [11], including the sums of all vibrations and their degeneracy, if any. The first part of equation (16) shows the experimentally measured heat capacity at constant volume, which includes both kinetic and potential energy.

$$C_{vib}T = \Sigma RT[(h\nu/kT)^2]/2[\cosh(h\nu/kT)-1]$$

$$S_{vib}T = \Sigma RT[(h\nu/kT)]/(e^{h\nu/kT}-1)-\ln(1-e^{-h\nu/kT})]\} \quad (16)$$

As a result, the true kinetic heat capacity for vibration can be taken as half the experimentally measured heat capacity.

$$C_{vib}(kinetic) = \Sigma R[(h\nu/kT)^2]/4[\cosh(h\nu/kT)-1] \quad (17)$$

Not correcting this prior error reduces the lapse rate and overestimates the elevation of the Venusian atmosphere from the correct value of 7.462 K per km to 6.213 K per km at the surface, where the pressure exceeds 90 atmospheres and the temperature is 735 K.

When this correction is made, there is a surprisingly close correspondence between the virial-action model and the actual measurements made by Magellan in both temperature and pressure (Table 1). Notwithstanding this, the small but obvious discrepancies in the values for pressure, even with the corrected lapse rate, probably results from the published number density (e.g. NASA [12] for carbon dioxide molecules being overestimated for Venus' surface, as a result of neglecting its non-ideal behaviour at high pressure, even at such high temperature where molecular attractions are prevented. The accurate Beattie-Bridgeman equation indicates that the pressure is intensified by more than 10% at Venus' surface than expected by the ideal gas law, with a lower number density producing the high pressure observed.

For Venus we have

$$\begin{aligned} mgh_n &= 3.5k(T_n - T_o) + \{kT_o \ln[(@_{to}/\hbar)^3 Q_e] \\ &- kT_n \ln[(@_{tn}/\hbar)^3 Q_e]\} \\ &- \{kT_o \ln[(@_{ro}/\hbar)^2/\sigma_r] - kT_n \ln[(@_{rn}/\hbar)^2/\sigma_r]\} \\ &+ \Sigma\{kT_o[x_o/(e^{xo}-1) - \ln(1-e^{-xo})] \\ &- kT_n[x_n/(e^{xn}-1) - \ln(1-e^{-xn})]\} \\ &+ \Sigma\{kT_o x_o^2/[4(\cosh x_o - 1)] \\ &- kT_n x_n^2/[4(\cosh x_n - 1)]\} \end{aligned} \quad (18)$$

In an earlier version of this work, equation (18) was given in a very similar form, but with the vibrational kinetic heat capacity obtained with a divisor of 2 instead of 4. It is now clear that only half the calculated vibrational heat capacity is actually required in this equation as explained above because the vibrational kinetic heat capacity is now deemed to be only half the time-averaged experimentally measured vibrational heat capacity.

This equation in action mechanics [2] can be rewritten in the further abbreviated form, for ($Tn - To$) equal to δT, with δg representing the variation in Gibbs energy with altitude, partitioned into translational, rotational and vibrational forms.

$$mg\delta h_n = \delta g_{tran} - \delta g_{rot} + \delta g_{vib} + [3.5k\delta T - \delta(c_{vib}T)] \quad (19)$$

32.23 = 63.80 – 39.36 + 17.09 – 18.99 + 9.69 = 32.23

For example, for the data computed in Table 1, the values for energy differences shown as ergs per molecule (times 10^{14}) were calculated between Venus' surface and the atmosphere at 50 km, an altitude where pressure and temperature approach the values found on Earth. According to the virial-action theorem, these steady state equilibrium values for each height would provide equality in total work potential at all altitudes. The increase of gravitational potential is always matched by an equivalent decrease in thermal energy involving a repartitioning of Gibbs energy. It is important to understand that this equation describes heat transfer processes with $\delta g_{trans,}$ δg_{vib}, δg_{rot} all increasing with altitude as heat is transferred between thermodynamic to gravitational states adjusting to environmental conditions of temperature and reduced pressure with height.

The varying vibrational kinetic heat capacity $\delta(c_{vib}T)$ must be subtracted from the change in translational and rotational kinetic energy, rather than added in equation (18). This requirement was originally confirmed by experimentation with signs in the Astrocal computer program for Venus, choosing the form of equation best matching known data. However, it is now clear this was logically required because of the contrasting natures of the virial for translation and rotation compared to the virial for vibration.

As explained above, vibrational potential energy and kinetic energy have the same sign and change with decreasing temperature with height in the same direction whereas for translation and rotation these vary in opposite directions in the gravitational field. I am grateful to Jacob Linder of Norway's University of Science and Technology, Trondheim, for making clear in his You-tube lecture on the virial theorem of this initially perplexing but obvious difference in the virial for vibration. These facts justify the results shown above for Venus' atmosphere between 0 and 50 km in connection with equation (17).

Mars

In contrast to Venus, the second planet possessing an atmosphere predominantly carbon dioxide, Mars, is sufficiently cold for heat of condensation to affect the lapse rate at all altitudes. Furthermore, the variation of temperature with altitude will affect the vibrational kinetic energy of this gaseous atmosphere to affect its morphology, requiring the use of equation (18) to plot this effect [1]. However, this will not be repeated here.

Carbon dioxide is a minor gas on Earth so this effect is marginal though it may have a role in climate change by its effect on the virial lapse rate at the altitude of the tropopause. According to Barth et al. [13] the troposphere of Mars shows lapse rates varying between 2 to 3 K per km as directly measured by the Viking Landers 1 and 2 compared to a calculated adiabatic lapse rate of 4.5 K per km. The effect of surface temperature varying from 212 to 400 K on the vibrational degree of freedom of motion and resultant virial lapse rates from 3.6 to 2.8 K per km is shown in Table 2. The troposphere may extend above 20 km altitude, depending on the surface temperature varying from 300 K at perihelion to 147 K at the poles where CO_2 freezes releasing heat and reducing the lapse rate. Occultation studies by the Mariner spacecraft made at the solar minimum gave lower lapse rates perhaps reflecting this effect of phase change of gaseous carbon dioxide to liquid or solid. However, from Table 2 it can be seen that the virial-action lapse rates are much closer to the general observations reported in Barth et al. [13]. The measurements made on lapse rate by the Viking Landers were confirmed as being as about half the adiabatic lapse rate during the Mars Pathfinder parachute landing in 1997 [14].

In principle, it is anticipated that heat of condensation of gaseous carbon dioxide would reduce lapse rates with altitude, but locally collapsing the atmosphere by its reduced number density, similar to the effect of condensation of water on Earth, possibly contributing to storm conditions. Evaporation of carbon dioxide under the action of sunlight should elevate Mars' atmosphere in the gravity field and convection from heat flow would be expected to increase the virial lapse rate towards the adiabatic rate under such conditions. Note that evaporation would not be expected to increase the virial lapse rate since this is independent of number density for a particular gas and carbon dioxide is the main gas at

95%. Therefore increasing its number density or pressure would not affect the virial lapse rate.

Jupiter

Based on measurements taken during a parachute descent by the Galileo probe, lapse rates significantly less than the calculated adiabatic rates all above 2.0 (see Table 2) were found [15], but more consistent with virial-action. Like Saturn, Jupiter radiates more heat than it receives from the Sun, but this effect is less than half that expected on Saturn so that distortion of the lapse rates is less apparent in Table 2. It is anticipated that an excess of heat will act to elevate the atmosphere, but still operating according to virial-action as a reversible engine.

Jupiter is considered as having a massive atmosphere of hydrogen and helium about 140,000 km in diameter extending almost to its centre (see Fig. 4). A central core about 10,000 km in diameter can be considered as providing a surface supporting the dense, superfluid-like atmosphere. The temperature at this invisible surface is considered to be about 36,000 K, providing sufficient thermodynamic back pressure to counter pressure from the weight of gravitation, a result from thermodynamic heating by compression in a gravitational field.

At the same atmospheric pressure as on Earth near Jupiter's tropopause, the ratio of temperature to the black body temperature is about 1.5, whereas at the bottom of the atmosphere at the surface it would be several hundred times greater than the black body temperature (Table 3), as a result of the heating by compression required to support the massive atmosphere for Jupiter's gravity. Jupiter is not expected to have effects on lapse rates near its surface from phase changes although there is cloud formation at higher altitudes indicating an effect on the temperature gradient in these layers of its atmosphere.

Saturn

Similar to Jupiter in structure, the case of Saturn is complicated by the fact that it emits 2-3 times more heat energy than it receives from the Sun. With a larger core of about 25,000 km in diameter, a core surface temperature of about 15,000 K is estimated (see Figure 4). It is proposed that helium is effectively "raining" through the metallic monatomic hydrogen under high pressure, releasing heat of condensation as a result. This would be expected to reduce the lapse rates with altitude, elevating the atmosphere, since a greater heat flux is involved. However, virial-action would still apply as a reversible process, as on Jupiter and Earth. It is not possible to decide whether adiabatic or virial-action lapse rates better agree with observations at the top of its atmosphere reported here (Table 2). A much more detailed analysis is required taking into account all likely factors that may affect lapse rates, such as wind speed, centrifugal effects and so on is required.

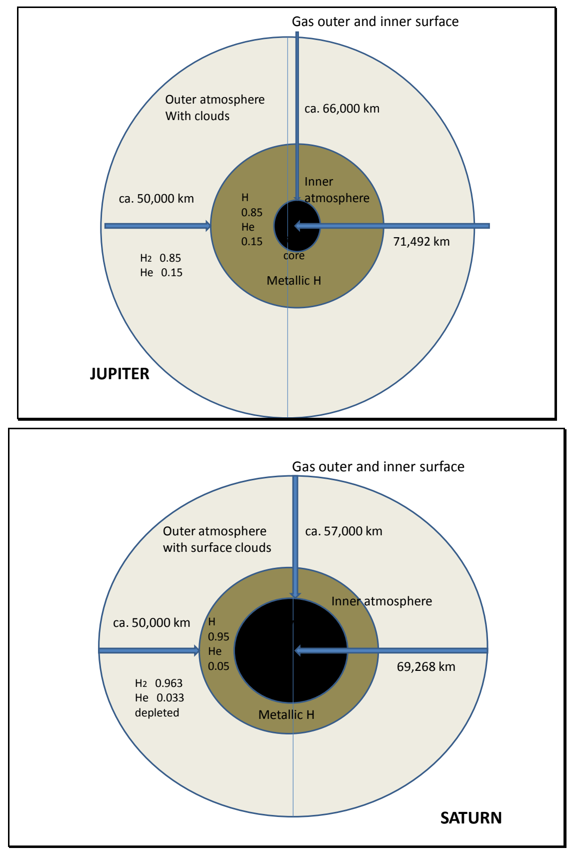


Fig. 4. Schematic diagrams of the massive atmospheres of hydrogen and helium found on Jupiter and Saturn. Temperatures of ca. 36,000 K and 15,000 K respectively for Jupiter and Saturn well above the dissociation temperature for hydrogen gas (H_2) into monatomic hydrogen (H) in the metallic atmosphere are proposed. With suitable corrections for non-ideality under such huge pressures, virial-action should be applicable to these atmospheres using equations (13) and (20), depending on environmental conditions.

Titan

Saturn's largest satellite, Titan, is claimed to have a greenhouse effect [16]. But this is not reliant on the gases we normally associate with global warming on

Earth. The surface of Titan at 93 K is far too cold for gaseous carbon dioxide to exist in significant quantities. The main constituents of Titan's atmosphere are nitrogen and methane, with the latter indicated as raining like water on Earth into surface lakes from a troposphere of temperature 70-90 K. The methane content in the atmosphere is significant, perhaps 5%, but it is not this compound's vibrational spectrum that is activated at the low temperature of Triton near 93 K, with surface infrared radiation peaking at 31.2 μm approaching microwave frequencies, whereas methane's four vibrational absorption peaks occur at 3.31, 3.43, 6.55 and 7.66 μm.

Instead of molecular vibration, infrared opacity on Titan is provided by collision-induced absorption [16, 17] from the pairs N_2-N_2, N_2-CH_4, CH_4-CH_4 and N_2-H_2, with molecular masses of 56, 44, 32 and 30 daltons respectively. Even larger clusters active in infrared regions are known to exist. Such molecular clusters facilitated by the cold conditions and higher pressure at the surface have the capacity to absorb and emit infrared radiation, despite their existence being transient. The clustered pairs may also cause increases in the virial-action per particle as well as the lapse rates by increasing the mass and degrees of freedom and reducing the number density of particles.

Despite claims in several papers that the lapse rate at the surface is consistent with the "dry" adiabatic lapse rates, with rain in the form of droplets of methane being possible, most measurements above the surface gave values significantly less than these, but with little explanation of these observations. In fact, the virial-action lapse rates are significantly closer to actual values observed (Table 2, Figure 2).

Uranus

Uranus' atmosphere is largely hydrogen, at 85%. However, 15% of helium significantly increases the lapse rates, whether adiabatic (0.976) or from virial-action (0.799). The calculated data shown in Table 2 favour the virial-action lapse rate as more consistent with observations (0.696-0.794). However, other factors including wind speed may also be affecting these results. Uranus does not produce its own heat like Saturn and Jupiter.

The data in Table 3 suggest that the huge reserves of heat stored in the non-greenhouse gases hydrogen and helium on Jupiter and Saturn are mimics for the extra heat stored on Earth above the surface giving a 33 K increase in surface temperature. Jupiter and Saturn are shown in the table to be heated at the surface of the core by factors of 327 and 185 respectively, with Venus intermediate at four times elevation above the black body temperature and Earth only 1.133 times as heated. The general heating effect is clearly partly correlated with the pressures exerted by the weight of the respective atmospheres at the surface and not the gaseous compositions of the atmosphere. Indeed, it seems incredible that radiative forcing alone could be responsible for such an intense heating at the postulated rocky interior surface on the giant planets (see Figure 4) and a thermodynamic explanation based on the huge compressive pressures at the surface is theoretically more convincing.

Role of radiative fluxes

Apart from providing an obvious radiative pathway for emission of infrared and microwave radiation to space, infrared absorbing or greenhouse gases have an important role in radiative transfers within the atmosphere and in the retention of heat needed to sustain the higher surface temperatures. The main route for heat to leave the Earth's surface is by radiation [9], although back radiation provides a blanketing effect for the surface though not providing an explanation for raising its temperature. The steady state solutions given by equations (11) and (18) assume instantaneous equilibrium or quasi-equilibrium of molecules in the gravity field driven by thermal forces. However the very existence of weather shows that equilibrium is incompletely achieved and the atmosphere continually seeks to reach equilibrium, probably from both directions. However, the fact that these equations give good approximations to actual atmospheric profiles [1] (see Tables) shows that equilibrium is not distant, despite the frequent turbulence of the Earth's atmosphere.

By comparison, Venus is far closer to equilibrium than Earth, with similar surface temperatures at the poles as its equator, with an even distribution of the atmosphere, although the polar vortexes acting as anticyclones are still strong. On Earth, the surface atmosphere is denser nearer the poles and less elevated, although the gravitational surface pressure is the same.

Smaller temperature gradients in Polar Regions on Earth indicate a lack of equilibrium of the atmosphere with the colder surface; from this viewpoint, the remnant polar ice indicates poor equilibration between the colder surface and troposphere, consistent with lower radiative heat transport.

The thermodynamics of the atmosphere is facilitated by radiation at the speed of light, particularly in the Earth's warmer zones. The fact that the lapse rate predicted by virial action is observed in the tropics but not in Polar Regions probably reflects this fact, accentuated by the time lag for loss of ice.

But these can be considered as more likely a result of thermodynamic needs rather than as a direct forcing of surface warming. Convection is facilitated by the transfer of infrared radiation, whatever the mechanism. As a result we may need to reconsider the role of greenhouse gases such as water, carbon dioxide, methane and nitrous oxide on Earth and their role in climate change.

IV. CONCLUSION

The advantages of virial-action *vis-a-vis* the adiabatic model normally employed have been illustrated in numerous ways in this paper. Virial action is given a basic role in determining atmospheric morphology, with adiabatic processes and phase changes providing supplementary effects. It is anticipated that this combination of causes will provide better options for accurate descriptions of meteorological processes.

Using a simple theoretical algorithm relating gradients in gravity to kinetic energy, virial-action more correctly predicts atmospheric temperature gradients or lapse rates for all for planetary atmospheres examined. This suggests that adiabatic processes are inadequate for this purpose, particularly on Venus and the gas planets where modification of the adiabats by convection or phase changes are less obvious. On Earth a significant role for water's low lapse rate compared to air in warming by elevating the atmosphere is proposed. In contrast, by increasing the lapse rate and lowering the atmosphere, carbon dioxide may have an opposite effect under conditions when water does not dominate, potentially releasing more heat to space near the tropopause and cooling the surface under some conditions. This hypothesis will be investigated further in our subsequent analyses which will also pay more attention to effects on radiative forcing.

The virial-action approach provides an underlying theory for a general description of planetary atmospheres as self-organising systems. The basis of this theory is the reversible interaction between thermodynamics and gravity that allows temperature, pressure and number density to be determined in a simple computer program (see equations (11) and (19).

It is important to understand that virial-action lapse rates and adiabatic lapse rates are not regraded here as interchangeable. Rather, they are complementary, with each representing part of reality. Virial lapse rates provide the basis of the interaction between thermodynamics and gravity, whereas adiabatic convective processes represent pressure-volume work that can modify temperature gradients. On planets like Venus virial lapse rates predominate in the absence of major convective or advective processes. On turbulent Earth we see their combined effects more prominently.

There would be a strong tendency for equality of gravitational pressure and thermodynamic pressure to be maintained at all altitudes and latitudes, providing stationary solutions for action allowing heat to be absorbed or released as required. The fact this equality is never quite achieved because of varying solar insolation and varying albedo explains weather. However, the correspondence between calculated and actual atmospheric profiles shown for Venus in Table 1 shows that equilibrium must be close, perhaps approached even daily on Earth at temperature reversals.

Since decreasing heat as kinetic energy and increasing statistical Gibbs energy are equally able to supply or consume gravitational work as related components of entropic energy at each altitude, a latent source for local surface warming is revealed. It is noteworthy that the statistical energy quantified as the logarithm of multidimensional action contains most of the energy of atmospheric gases [2]. Thus gravitational work stored in air as well as the work-energy stored in rotating air masses, either on vertical or horizontal axes, can all heat the atmosphere by compression or friction promoting turbulence.

The benefit of using simple computer programs with appropriate sensitivity to reinforce theory has clearly been demonstrated. The governing equations ((11) and (19) providing highly accurate solutions to atmospheric profiles on Earth, Venus and Mars could not have been discovered without computer output feedback, revealing emergent properties as results corresponding with reality.

Meteorologists are encouraged to incorporate the physical principles shown here into climate models.

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REFERENCES

[1] I.R. Kennedy, (2015) A revised version of the atmospheric barometric formula relevant to global atmospheric warming http://arxiv.org/abs/1504.05265

[2] I.R. Kennedy, H. Geering, M.T. Rose and A.N. Crossan (2015) A simple method to estimate entropy and free energy of atmospheric gases from their action. http://arxiv.org/abs/1504.03866

[3] I.R. Kennedy (2001) *Action in Ecosystems: Biothermodynamics for Sustainability*. Research Studies Press, Baldock, John Wiley UK.

[4] M. Planck *The Theory of Heat Radiation*. Dover Publications p. 65, 1959 New York, 1913.

[5] T.L. Hill (1986) *An Introduction to Statistical Thermodynamics.* Dover Publications New York.

[6] R.J.E. Clausius (1870) On a mechanical theorem applicable to heat. *Philosophical Magazine*, Series 4, 40: 122–127.

[7] G.P. Anderson, S.A. Clough, F.X. Kneizys, J.H. Chetwynd and E.P. Shettle (1986) AFGL *Atmospheric Constituent Profiles*, (0-120 km), Report of Air Force Geophysics Laboratory, Project 7670, Hanscom, USA.

[8] R.T. Pierrehumbert (2010) *Principles of Planetary Climate.* Cambridge University Press, Cambridge United Kingdom.

[9] J.T. Kiehl and K.E. Trenberth (1997) Earth's annual global mean energy budget. *Bulletin American Meteorological Society,* 78, 197-208.

[10] D.A. McQuarrie (1963) *Statistical Mechanics*, Viva University Science Books, New Delhi.

[11] W.J. Moore (1963) *Physical Chemistry.* 4^{th} edition Longmans, London.

[12] National Aeronautics and Space Administration Fact Sheet , National Aeronautics and Space Agency, accessed December 1, 2014. http://nssdc.nasa.gov./planetary/factsheet.html;

[13] C.A. Barth, I.F. Stewart, S.W. Burgher, P.M. Henten, S.J. Bauer and A.F. Nagy (1992) Aeronomy of the current Martian atmosphere, . In *Mars*, Edited by H.H. Kieffer, B.M. Jakosky, D.W. Snyder and M.Ss Mathews, University of Arizona Press.

[14] J.A. Magalhaes J.T. Schofield, A. Seiff (1999) Results of the Mars pathfinder atmospheric structure investigation. *Journal Geophysical Research* 104, 8943-8055.

[15] A. Seiff, D.B. Kirk, T.C.D. Knight, J.D. Mihalov, R.C. Blanchard, R.E. Young, G. Schubert,U. von Zahn,, G. Lehmacher G, F.S.J Milos and J. Wang (1996) Structure of the atmosphere of Jupiter: Galileo probe measurements. *Science*, 272, 844-845.

[16] A. Borysow and C. Tang (1993) Far-infrared CIA spectra of N_2-CH_4 pairs for modelling of Titan's atmosphere. *Icarus*, 105, 175-183.

[17] T.D. Robinson and D.C. Catling (2013) Common 0.1 bar tropopause in thick atmospheres set by pressure-dependent infrared transparency. *Nature Geoscience,* 7, 12-15. (supplementary information p.15).

[18] J.M. Jenkins,P.G. Steffes, D.P. Hinson, J.D. Twicken and G.L.Tyler (1994) Radio occultation studies of the Venus atmosphere with the Magellan spacecraft. *Icarus,* 110, 79-94.

[19] G.F. Lindal, G.E. Wood, H.B. Hotz and D.N. Sweetnam (1983) The atmosphere of Titan: An analysis of the Voyager 1 radio occultation measurements. *Icarus,* 53, 348-363.

[20] G.F. Lindal, D.N. Sweetnam and V.R. Eshleman. (1985) The atmosphere of Saturn: An analysis of the Voyager radio occultation measurements. *Astronomical Journal,* 1136-1146.

[21] D. Saumon and T. Guillot (2004) Shock compression of deuterium and the interiors of Jupiter and Saturn. *Astrophysical Journal,* 609, 1170-1180.

[22] H.B. Niemann, S.K. Atreya, S.J. Bauer, G.R. Carignan, J.E. Demick, R.I. Frost, D. Gautier , J.A. Haberman, D.N. Harplod, D.M. Hunten, G. Israel, J.I. Linine, D. Rasprzak, T.C. Owen, M. Paulkovich, F. Raulin, E. Raaen. and S.H. Way (2005) The abundance of constituents of Titan's atmosphere from the GCMS instrument on the Huygens probe. *Nature,* 438, 779-784.

Table 1: Comparison of virial-action data with the Magellan spacecraft data for Venus

Altitude km	δT/δh x10[5] K/km	Virial temperature data K	Magellan temperature data[1] K	Virial pressure data x10[6] Pascals	Magellan pressure data[1] x10[6] Pascals	Virial density data x10[-20] cm[-3]	Magellan Density data[1] $p/kT = 1/a^3$ x10[-20]
0	7.462	735.0	735	8.9918	9.330	8.8161	9.1938
5	7.706	697.7	697	5.7274	6.752	5.9459	7.0162
10	7.746	659.2	658	4.1810	4.800	4.5942	5.2834
15	7.788	620.4	621	2.9895	3.347	3.4900	3.9036
20	7.834	581.5	579	2.0868	2.281	2.5994	2.8553
25	7.884	542.3	537	1.4161	1.647	1.8913	2.2214
30	7.940	502.9	495	0.9289	1.087	1.3379	1.5905
35	8.003	463.2	453	0.5844	0.599	0.9139	0.9577
40	8.075	423.2	416	0.3489	0.355	0.5972	0.6181
45	8.160	382.8	383	0.1946	0.200	0.3682	0.3782
50	8.264	342.0	348	0.0991	0.108	0.2098	0.2248
55	8.390	300.7	300	0.0445	0.054	0.1071	0.1304
60	8.547	258.7	263	0.0166	0.024	0.0465	0.0661

[1]Jenkins et al., [18]

Table 2: Atmospheric lapse rates of the planets and selected satellites

Planet	Solar Irradiance Watts/m[2]	Surface gravity	Compo-sition	MW g/mol	Pressure mbar	Temp K	Black Body Temp K	Virial lapse rate, K/ km	Adiabatic lapse rate	Actual lapse rate K/km
Mercury	9126.6	3.70	0	0	0	440	440.1	0	0	0
Venus	2613.9	8.87	CO_2	43.45	92,000	737	184.2	7.462 0 km	9.344	7.5
			0.965			586		7.834 20 km		
						466		8.003 30 km		
						337		8.264 50 km		~8
Earth	1367.6	9.80	N_2 0.78	28.97	1,014	288	254.3	6.898 (RH0)	9.8	6.5[2]
			O_2 0.21			306		6.660 (RH1.0)		
			A 0.01							
Moon	1367.6	1.62			0	260	270.7	0	0	0
Mars	589.2	3.71	CO_2 0.957	43.34	8.7	212	210.1	3.588	5.264	2-3[3]
			N_2 0.027			300		2.922	4.411	2-3
			A 0.016			400		2.840	3.974	2-3
Jupiter	50.50	24.79	H_2 0.843	2.22		100	110.0	1.954	2.287	
			He 0.157			200		1.618	2.139	1.750[4]
						300		1.534	2.026	
Jupiter core	74,953		H_2O		$2x10^6$	~36,0		~0.6		~0.5 est.
Saturn	14.90	10.4 1b	H_2 0.963	2.07	1 bar	134 1b	81.1	0.654 1b	0.976	0.626[5] 400 mb
			He 0.033					0.685		0.795[5] 501 mb
Saturn core	666.6				ca. 10^6	~15,0		0.5 calc.		ca. 0.3 est.
Titan	14.90	1.354	N_2 0.984	28	0.146	93.3		0.892	1.242	1.030[a6] 0.950[b6]
			CH_4 0.014						10-20 km	0.842[a6] 0.750[b6]
Uranus	3.71	8.9 at	H_2	2.64	>>10	76 1b	58.2	0.779	0.976	0.794

Data from NASA (2014)
[a]Radio wave refraction; [b]Huygens lander [22]
[1]Kennedy [1]; [2]Various; [3][14]; Barth et al., [13]; [4][15]; [5]Lindal et al. [19]; [6][20]

Table 3: Thermal properties of the planets and satellite with atmospheres

Planet	Solar irradiance Watts per square m	Visible radius km	Surface gravity, m sec^{-2}	Bond albedo	Black body temp K	Surface temp K	Virial lapse rate K per km	Mass at surface kg m^{-2}	Density kg m^{-3}	Pressure mbar	Surface Temp/ Black body T
Mercury	9126.6	2,440	3.70	0.068	440.1	440	0	~0	0	~0	~1.000
Venus	2613.9	6,052	8.87	0.90	184.2	737	7.8	1.043×10^6	~65	92,000	4.002
Earth	1367.6	6,371	9.80	0.306	254.3	288	6.9 dry	1.000×10^4	1.217	1,014	1.133
Moon	1367.6	1,737	1.62	0.11	270.7	260	0	~0	~0	~0	~0.960
Mars	589.2	3,390	3.71	0.250	210.1	212	2.90	1.732×10^2	~0.02	8.7	1.009
Jupiter	50.50	71,492	24.79 1b	0.343	110.0	165 at 1b	1.95 at 1b		0.16 at 1b	1,000	1.500 at 1b
Jupiter core	74,953	5,000[1]	1.73			~36,000	0.6 vlr 0.5 mean[1]	2.77×10^{10}	~100	~2,000,000	~327 at core
Saturn	14.90	69,268	10.4 at 1b	0.342	81.1	134 at 1b	0.654 at 1b		0.19 at 1b	1,000	1.652 at 1b
Saturn core	666.6	12,500[1]	1.88			~15,000	0.5 vlr 0.3 mean[1]	1.07×10^{10}	~100	~1,000,000	~185 at core
Titan	14.90	2,575	1.352	0.220	82	93.3	0.892	1.100×10^5		1,600	1.14 at 1.6b
Uranus	3.71		8.9 at 1b	0.300	58.2	76 at 1b	0.779 at 1b	5.0×10^9	0.42 at 1b	>>1000b	1.306 at 1b
Neptune	1.51	24,622	11.2 at 1b	0.290	46.6	72 at 1b	0.45 at 1b		0.45 at 1b	>>1000b	1.545 at 1b
Pluto	0.89	1,195	0.58	0.5	37.5	50			~0	~0	1.333

[1]Mean is average fall per km; virial lapse rate (vlr) is corrected for average *g* value proportional to radius