Models for Plasma Control in Fusion Reactors

Aitor J. Garrido, Izaskun Garrido, Oscar Barambones, F. Javier Maseda and Patxi Alkorta*

Abstract— The control of plasma in nuclear fusion has revealed as a promising application of Control Engineering, with increasing interest in the control community during last years. In this paper it is outlined a control-oriented linear model for the control of plasma current. For this purpose, it is firstly provided a summary of the background necessary to deal with control problems in tokamak-based nuclear fusion reactors as it is the case of the future ITER tokamak. Besides, it is also given a review of the most used simulators and plasma models, with the aim of providing an adequate background for control engineers to derive their own control-oriented model or to choose the appropriate existing one. Finally, a simple linear model based on loop control voltage is derived.

Keywords— Fusion Control, Plasma Physics, Tokamak Modeling.

I. INTRODUCTION

A. Motivation

IN the last years, substantial effort and resources are being devoted to the development of a clean nuclear technology based upon fusion processes. This effort materializes in a large number of research papers published, specially in the field of Control Engineering applied to fusion processes (see [1] and [2]), establishing an area of novel application for Control Theory, after some timid efforts in the 50s and beginning of the 90s. Nowadays the control of plasma in fusion processes is an area of increasing interest, involved in ambitious international projects as the ITER - International Thermonuclear Experimental Reactor [3].

B. Background on nuclear fusion magnetic confinement: tokamak

When two light nuclei fuse into a heavier and more stable nucleus, the nuclear rearrangement results in a reduction in total mass and a consequent release of energy in the form of kinetic energy of the reaction products. The idea relays in heating the fuel up to a sufficiently high temperature so that the thermal velocities of the nuclei are high enough to fuse. This process that takes place continuously in the Sun and stars. In order to obtain nuclear fusion on Earth, the most suitable reaction at present takes place between the nuclei of deuterium and tritium. Nevertheless, to achieve and maintain the reaction for a substantial period of time (a pulse), temperatures of the order of $100 \cdot 10^6$ ºC (10^4[eV]) and a density of about 10^{20} m^{-3} are required. Under these conditions the fuel changes its state from gas to plasma, in which the electrons are separated from the atoms, becoming these atoms charged ions (see [4]).

This technology has nowadays reached the point in which the experimental reactors can produce almost as much energy as they consume. In this sense, the future ITER reactor is desired to generate ten times as much energy as it consumes (see [1]). For this purpose, being nuclear reactors inherently pulsed devices due to the limited main transformer magnetic flux availability (see [5] and [6]), the objective is to control the plasma so as to maintain its stability in order to achieve a pulse duration of the order of minutes instead seconds.

As indicated above, the plasma consists of two types of charged particles, ions and electrons, so that it may be contained within a region away from the vessel walls by means of magnetic fields [5-6], namely, using magnetic confinement.

The most common magnetic confinement structure is denominated tokamak, acronym of TO*roidalnaya* KA*mera i* MA*gnitnaya* K*atushka* that means “toroidal camera with magnetic coils”, which is also the design that will be used in ITER. In a tokamak, the plasma is heated in the toroidal vessel and kept away from its walls by applying two combined magnetic fields: The *toroidal field*, around the torus, which is maintained by the *toroidal field coils* surrounding the vacuum vessel (see Figure 1), providing the primary mechanism of confinement of the plasma particles. And a smaller *poloidal field* (about 10% of toroidal field), around the plasma cross section, that keeps the plasma away from the walls and contributes to maintain the plasma's shape and position. The poloidal field is induced both internally, by the current driven in the plasma, and externally, by the *outer poloidal field coils* that are positioned around the perimeter of the vessel (see Fig. 1). In turn, the main plasma current is induced in the plasma by the action of a large transformer (inductive current drive): A changing current in the primary winding formed by the *inner poloidal field coils* located around a large iron core induces a current in the plasma, which acts as the transformer secondary circuit.

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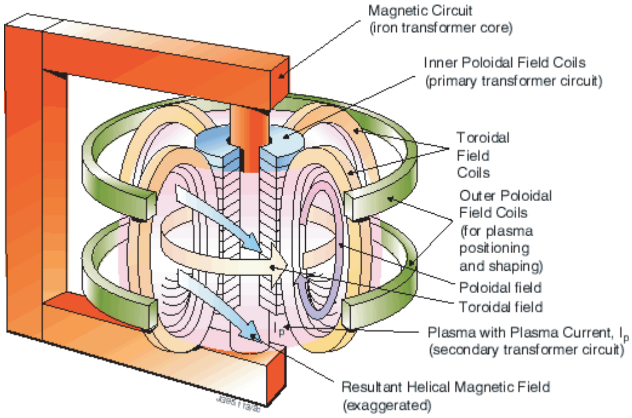

Fig. 3 Tokamak scheme

In addition to the main plasma current drive, there exist various non-inductive current drive methods, which are usually employed as actuators for control purposes (see [5]).

II. MHD EQUILIBRIUM IN TOKAMAK: GRAD-SHAFRANOV EQUATION

Magnetohydrodynamics (MHD) describes the dynamics of electrically conducting fluids. The MHD equations are given by, on the one hand the Maxwell equations jointly with Ohm's law due to electromagnetic nature of plasma, an on the other hand, the flow equation jointly with the mass conservation continuity equation due to the flow nature of plasma ([6]).

Table I. Summary of MHD equations(*)

Faraday's law $\nabla\times\mathbf{E}=-\dfrac{\partial\mathbf{B}}{\partial t}$ (1)	Ampere's law $\nabla\times\mathbf{B}=\mu_0\mathbf{j}+\mu_0\varepsilon_0\dfrac{\partial\mathbf{E}}{\partial t}$ (2)
Gauss's laws $\nabla\cdot\mathbf{E}=-\dfrac{\rho}{\varepsilon_0}$ (3) $\nabla\cdot\mathbf{B}=0$ (4)	Ohm's law $\mathbf{E}+\mathbf{v}\times\mathbf{B}=\dfrac{\mathbf{j}}{\sigma}$ (5)
Flow Equation $\rho_m\dfrac{\partial\mathbf{v}}{\partial t}=\mathbf{j}\times\mathbf{B}-\nabla p$ (6)	Continuity equation $\dfrac{\partial\rho_m}{\partial t}+\nabla\cdot(\rho_m\mathbf{v})=0$ (7)

(*) ρ_m is the mass density, ρ is the charge density, $\mathbf{v}$ is the plasma velocity, $\mathbf{j}$ is the current density, p is the plasma pressure, ε_0 is the vacuum electric permittivity, μ_0 is vacuum magnetic permeability, σ is the electrical conductivity and $\mathbf{E}$ and $\mathbf{B}$ the electric and magnetic field respectively.

Due to the quasi-static approximation, $\rho\approx 0$ can be considered for Gauss's law and the second right term of Ampere's law representing the displacement current may be neglected. Besides, under magnetohydrodynamic equilibrium conditions it is assumed that plasma velocity is zero and it is considered ideal MHD. This implies that the mass density and the magnetic field remain stationary: $\dfrac{\partial\rho_m}{\partial t}=0$; $\dfrac{\partial\mathbf{B}}{\partial t}=0$, and that the Lorentz force due to the interaction between current and magnetic field compensates the tendency of the plasma to expand due to its kinetic pressure:

$$\nabla p=\mathbf{j}\times\mathbf{B} \tag{8}$$

This expression (8), jointly with the magnetic field Gauss's law and the simplified Ampere's law $\nabla\times\mathbf{B}=\mu_0\mathbf{j}$, composes the ideal MHD equations for tokamak equilibrium conditions (see [6]).

The ideal MHD equations for tokamak equilibrium conditions can be expressed as a nonlinear elliptic partial differential equation obtained from the two-dimensional reduction (R, z) of the ideal MHD equations given above. To do that, it is considered that any variation with respect to ϕ is zero; $\dfrac{\partial}{\partial\phi}=0$, accordingly with the axisymmetric geometry of the toroidal vessel (see Figure 2).

In this way, the equilibrium equation in ideal MHD for a 2D plasma can be expressed as (see [4], [6] and [7]):

$$\Delta^*\Psi=-\mu_0R^2\frac{dp(\Psi)}{d\Psi}-F(\Psi)\frac{dF(\Psi)}{d\Psi} \tag{9}$$

known as Grad-Shafranov equation, where Δ^* represents the elliptic-like operator:

$$\Delta^*=R\frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial}{\partial R}\right)+\frac{\partial^2}{\partial z^2} \tag{10}$$

and where the toroidal flux function $F(\Psi)$ and the pressure function $p(\Psi)$ are solely dependent of the poloidal flux function Ψ, which is both a dependent and independent variable in the above equation (9).

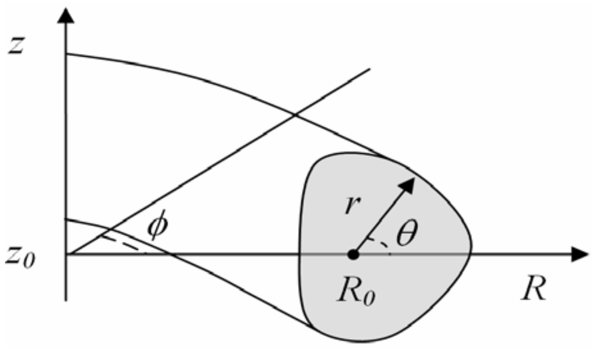

Fig. 2 Cylindrical coordinates (R, ϕ, z) for the plasma and polar coordinates (r, θ) of the poloidal section referred to the magnetic axis (R0, z0)

The Grad-Shafranov equation is usually solved numerically using the set of transport equations considered which define the time evolution of the plasma in the tokamak (see [8] and [9]). The iterative calculation procedure philosophy is as follows: For the first equilibrium calculation, initial profiles of functions Ψ, p and F are used, it is computed its time evolution using the transport equations (presented below) which use in turn the results of the equilibrium calculation, and then a new equilibrium can be calculated. This scheme is used by the nonlinear code DINA (see [10] and [11]).

At this point it is necessary to choose a set of appropriate transport equations. This is not a trivial issue, since the problem of transport in tokamaks composes still an open matter (see [12]), with intense efforts and resources employed on it [13-15]. In fact, in the absence of a theoretical understanding of the so-called anomalous transport, is not uncommon to use empirical techniques in order to identify the behaviour of plasma from experimental data of the tokamak. Nevertheless, usually the plasma transport phenomenon may be modelled using a set of known transport equations. This technique provides only approximated results, and the transport calculation time interval between two equilibrium points must be small enough in order to keep the error bounded and to provide an adequate input to calculate the new equilibrium.

A set of transport equations used in some simulation codes as DINA are the following ([6], [16] and [17]):

The magnetic field diffusion equation:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta_0 \nabla^2 \mathbf{B}\,, \qquad (11)$$

which describes the magnetic field evolution in a conducting fluid, being $\eta_0 = 1/\mu_0\sigma$ the magnetic diffusion factor.

The density equation:

$$\frac{\partial n_p}{\partial t} + \nabla \cdot (n_p \mathbf{v}_p) = S_p\,, \qquad (12)$$

where the subindex p (particles) may be substituted by i (ions) or e (electrons), being S_p the source term and n_p the density.

The energy balance equation

$$\frac{3}{2}\frac{\partial p_p}{\partial t} + \nabla \cdot (\frac{3}{2} p_p \mathbf{v}_p) + p_p \nabla \cdot \mathbf{v}_p + \nabla \cdot \mathbf{q}_p = Q_p\,, \quad (13)$$

where $\mathbf{q}_p$ is the heat flux, Q_p represents the heat generated in the plasma and the rest of the parameters have been previously defined.

In a control-oriented linear model as the one presented in the next section, there may be considered the electrical circuit equations to compute the evolution of the current in the different elements ([16] and [26]).

III. Control oriented tokamak plasma models: Simple linear model

A. *Tokamak simulation codes and models*

In order to model the combined plasma, vessel and poloidal field coil system, many approaches have been considered. From initial simple models [18] which considered the plasma as a filament or a non-deformable plasma [19-20], to DINA, a nonlinear free boundary resistive MHD and transport-modeling plasma simulation code, [11]. On the one hand, it may be considered several nonlinear codes based on nonlinear models as PET, ASTRA, TSC, EFIT, PROTEUS, CREATE or the aforesaid DINA, which are useful to perform simulations including nonlinear behaviours as, for example, large amplitude non-linearities (e.g. large vertical position displacement) or non-time-invariant nonlinearities, but that in turn, possess a quite complicated structure which generally makes them unsuitable for controller design purposes. And, on the other hand, it may be considered linear models as RZIP, an enhanced rigid current displacement model considering changes to the plasma current and to its radial position (see [21], [22] and [23]) or CREATE-L which considers the plasma deformation by conserving an equilibrium of the plasma current distribution (see [24-25]).

B. *Linear tokamak model*

In general, controllers are designed by means of linear models. Thus, the desired stability and performance of the closed loop system with respect to the full nonlinear tokamak is only valid when the states remain in the neighbourhood of the equilibrium. Nevertheless, in practice it turns out that controllers based on linear models are robust enough to ensure stability and acceptable performance, even if the tokamak plant follows nonlinear dynamics, i.e. during the ramp-up and ramp-down phases. Thus, if a controller is designed during the flat-top phase they usually work well for the whole discharge (see [4], [16] and [17]).

A tokamak with non-inductive current generation may be described as an electric circuit with distributed parameters, whose equivalent circuit is represented in Figure 3, consisting of a mutual inductance between the plasma and the toroidal measuring coil parallel to the plasma, M, a non-inductive current represented by either an ideal voltage source $\tilde{V}$ or an ideal voltage current $\tilde{I}$, and where the plasma is represented by a resistance R and an inductance L.

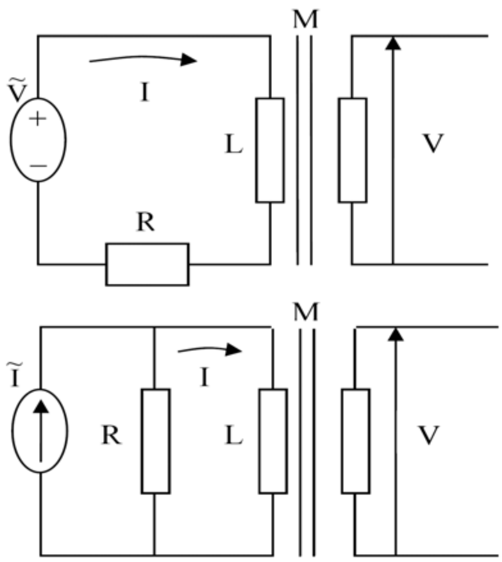

Fig. 3 Equivalent circuit model

In this way, the dynamics of the elements of the equivalent circuit determine de dynamics of the loop voltage, which correspond to changes in the resistance, inductance and Ohmic current according to the following expression (see Fig. 3):

$$V = R\left(I - \tilde{I}\right) + \frac{\partial}{\partial t}\left((L - M)I\right) \qquad (14)$$

Loop voltage is one of the most useful and easy to measure parameters of the plasma since it may be determined by measuring the voltage around the toroidal wire parallel to the plasma. Although the physical interpretation of the loop voltage is not always simple, basically responds to changes in the main plasma current [6].

Expressions for L, L-M, $\tilde{V}$ and $\tilde{I}$ may be obtained applying the MHD equations of the previous section and the Poynting theorem for the poloidal magnetic field:

$$\frac{\partial}{\partial t}\frac{\mathbf{B}_\theta^{\;2}}{2\mu_0}+\mathbf{E}_\phi \mathbf{j}_\phi = -\frac{1}{\mu_0}\nabla\cdot\left(\mathbf{E}_\phi \times \mathbf{B}_\theta\right) \tag{15}$$

where $\mathbf{B}_\theta$, $\mathbf{E}_\phi$ and $\mathbf{j}_\phi$ represent the poloidal $(\cdot)_\theta$ and toroidal $(\cdot)_\phi$ components of the magnetic field, electric field and current density, respectively (see Fig. 2). Note that, since the poloidal magnetic field component is dominant, this equation is given only for $\mathbf{B}_\theta$.

Taking the integral form of the Poyntin's theorem considering a volume Ω defined for a certain contour around the plasma, it is possible to derive adequate expressions for the parameters of the model. In this way it is obtained that:

$$R=\frac{\int_\Omega \eta\, \mathbf{j}_\phi^{\;2} dV}{I^2}; \qquad \tilde{V}=\frac{\int_\Omega \eta\, \mathbf{j}_\phi \tilde{\mathbf{j}}_\phi dV}{I} \tag{16}$$

$$L-M=\mu_0 \frac{r_0}{2}(l_i+l_e+l_0) \tag{17}$$

where η denotes the plasma resistivity, the internal inductance $l_i=\frac{2}{\mu_0^{\;2} I^2 2r_0}\int_\Omega \mathbf{B}_\phi^{\;2} dV$, the external inductance l_e may be approximated by $l_e=\ln\frac{b}{a}$, being a the minor radius of the plasma (considering the major radius of the plasma $r_0 >> a$), b the distance between the centre of the plasma current and the measurement coil, and where $l_0 \cong \frac{1}{I}\int_0^{t_r}(l_i+l_e)\frac{dI}{dt}dt$, being t_r the stationary (*flat-top*) reaching time of the discharge [26-27].

And then, from the Norton and Thevenin forms of the equivalent circuit of Fig. 3:

$$\tilde{I}=I\,\frac{\int_\Omega \eta\, \mathbf{j}_\phi \tilde{\mathbf{j}}_\phi dV}{\int_\Omega \eta\, \mathbf{j}_\phi^{\;2} dV} \tag{18}$$

At this point, it may be considered for the flat-top phase a space-state first order linear model as follows:

$$\begin{aligned} \dot{x}_i &= -\frac{1}{\tau_i}\left(x_i - x_i\big|_{t=0}\right)+\frac{k_i}{\tau_i}u \\ y &= f\left(x_i\right) \end{aligned} \tag{19}$$

where the output y represents the loop voltage, which may be expressed as a bilinear system $f(x_i)$ from eq. (14). The input u represents the power injection to the plasma (usually performed by Lower Hybrid Current Drive, but also by means of other non-inductive plasma heating methods (see [5] and [6])). And where the system of differential equations describing the dynamics of the plasma resistance x_1, inductance x_2, and Ohmic current x_3, have been approximated by first order linear equations in a neighbourhood of the operation point from expressions (16-18). For this purpose, each gain k_i is estimated from the rate of change of the state x_i with the input u, and the time constant τ_i can be computed from both transport eqs. (11-13) and experimental results [26].

IV. Conclusion

In this paper a control-oriented linear model for plasma current control in tokamak-based fusion reactors has been derived. To do so, some necessary background on nuclear fusion process and technology has been given. Also, a review of the most used simulators and plasma models has been provided, presenting a linear model of the plasma valid to loop voltage control in tokamak reactors. The aim of the paper is to establish low-order linear plasma equilibrium response models to be used for control design purposes. In fact, future work will address the implementation of these low-order tokamak models over robust controllers in order to investigate its accuracy and effectiveness.

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