Combination study of Fuzzy Cognitive Map

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Abstract—Multi-expert constructing Fuzzy cognitive map is a typical multi-expert knowledge combination problem. In this paper, we investigate the use of Dempster-Shafer evidence theory as a tool for multi-expert knowledge combination. In proposed method, we use each expert opinion as a evidence, the possible value of weight as frame of discernment, the expert's evaluation to a weight on frame of discernment as basic probability assignment, and Dempster-Shafe rule as combined basis of basic probability assignment m. Finally, the weight is given according to combined basic probability assignment. The strategy can gradually reduce the hypothesis sets and approach the truth with the accumulation of evidences, which make the result of decision more all –around and more scientific. The experimental result is shown that the method can keep exactitude information, reduce conflict factor and improve knowledge quality**.**

Keywords—Knowledge representation, Fuzzy cognitive map, Dempster-Shafer evidence theory, Multi-expert knowledge combination

I. INTRODUCTION

Fuzzy cognitive map (FCM) is an approach to knowledge representation and inference that are essential to any intelligent system. FCM allows experts to represent factual and evaluative concepts in an interactive framework, and can quickly draw FCM pictures or respond to questionnaires. Experts can consent or dissent to the local causal structure and perhaps the global equilibrium. Yet an FCM equally encodes the experts' knowledge or ignorance, wisdom or prejudice. Worse, different experts differ in how they assign causal strengths to edges and in which concepts they deem causally relevant. The FCM seems merely to encode its designers' biases and may not even encode them accurately. FCM combination provides a partial solution to this problem.

Multi-expert constructing FCM is a typical multi-expert knowledge combination problem. Generally, the constructing FCM process is that each expert builds individual FCM, and then combines them by weight average. However, the method cannot effectively keep exactitude information, reduce conflict factor and improve knowledge quality. There is an urgent need to develop methods for multi-expert knowledge combination. Dempster–Shafer evidence theory provides solving method for the problem. In this paper, we investigate the use of Dempster-Shafer evidence theory as a tool for multi-expert knowledge combination

The paper is organized as follows. Section 2 presents the

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formalization representation of FCM. Section 3 presents the basic concepts of evidence theory. Section 4 presents how to use evidence theory for Multi-expert opinions combination. Section 5 presents the complexity analysis of combination calculating. Section 6 applies the proposed methodology to multi-expert opinions combination. Section 7 is the conclusion and suggestions for future works.

II. FUZZY COGNITIVE MAP

FCM[1][2] is an approach to knowledge representation and inference that are essential to any intelligent system. It emphasizes the connections as basic units for storing knowledge and the structure represents the significance of system. FCM can be easily built and represent knowledge directly, And form mapped relations with the knowledge structures in the brains of the experts of this area, FCM have been used for representing knowledge and artificial inference and have found many applications, for instance, geographic information systems [3], [4], fault detection [5],, policy analysis [6], etc.

A FCM consists of nodes-concepts, each node-concept represents one of the key-factors of the system, and it is characterized by a value $C \in (0,1)$, and a causal relationship between two concepts is represented as an edge w_{ij}. w_{ij} indicates whether the relation between the two concepts is direct or inverse. The direction of causality indicates whether the concept C_i causes the concept C_j. There are three types of weights:

$w_{ij} > 0$ indicates direct causality between concepts C_i and C_j. That is, the increase (decrease) in the value of C_i leads to the increase (decrease) on the value of C_j.

$w_{ij} < 0$ indicates inverse (negative) causality between concepts C_i and C_j. That is, the increase (decrease) in the value of C_i leads to the decrease (increase) on the value of C_j.

$w_{ij} = 0$ indicates no relationship between C_i and C_j.

A FCM is a 4-tuple (V, E, C, f) where --V=$\{v_1, v_2, \dots, v_n\}$ is the set of n concepts forming the nodes of a graph.

--E:$(v_i, v_j) \rightarrow w_{ij}$ is a function $w_{ij} \in E$, v_i, $v_j \in V$, with w_{ij} denoting a weight of directed edge from v_i to v_j. Thus E $(V \times V) = (w_{ij})$ is a connection matrix.

--C: $v_i \rightarrow C_i$ is a function that at each concept vi associates the sequence of its activation degrees, such as $C_i(t)$ given its activation degree at the moment t. C(0) indicates the initial vector and specifies initial values of all concept nodes and C(t) is a state vector at iteration t.

--f is a transformation function, which includes recurring relationship between C(t+1) and C(t).

$$C_i(t+1) = f\left(\sum_{\substack{i=1 \\ j \neq i}}^{n} w_{ij} C_j(t)\right) \quad (1)$$

The transformation function is used to confine the weighted um to a certain range, which is usually set to [0, 1].

$$o_i(t+1) = \frac{1}{1 + e^{-C(t)}} \quad (2)$$

Eq. (7) describes a functional model of FCM. An FCM represents a dynamic system that evolves over time, it describes that the value of each concept is calculated by the computation of the influence of other concepts to the specific concept.

III. DEMPSTER-SHAFER EVIDENCE THEORIES

Dempster-Shafer evidence theory provides a powerfully intelligent tool for multi-expert Opinions combination. It is introduced by Dempster[7] and extended later by Shafer[8]. Dempster-Shafer theory is concerned with the question of belief in a proposition and systems of propositions. Evidence can be considered in a similar way when forming propositions, and it is concerned with evidence, weights of evidence and belief in evidence. The theory does not make any assumption concerning the way human imagination works. Simply, it describes decision-makers receiving information from different sources and evaluating to what extent the evidence that they provide is compatible or contradictory.

Dempster-Shafer evidence theory includes probability theory as a special case that obtains –among else–under the hypothesis that decision-makers are able to consider any combination of a given set of even and that no other event can be conceived. Namely a decision-maker who is envisaging two possibilities X and Y in a possibility set, according to probability theory, the complementary of X is the whole dashed area of possibility set. In fact, probability theory assumes that this decision-maker is able to conceive all possibilities in possibility set-to be honest, the very idea of envisaging only X and Y would not make much sense within probability theory.

A. Frame of Discernment

In Dempster-Shafer theory, possibility sets are mental representations of empirical evidence in an individual's mind. Shafer preferred to use another term [1]: it should not be thought that the "possibilities" that comprise [a set] Ω will be determined and meaningful independently of our knowledge. Quite to the contrary: Ω will acquire its meaning from what we know or think we know, distinctions that it embodies will be embedded within the matrix of our language and its associated conceptual structures and will depend on those structures for whatever accuracy and meaningfulness they possess. In order to emphasize this epistemic nature of the set of possibilities, we will call it the frame of discernment. In the standard probability framework, all elements in Ω are assigned a probability. And when the degree of support for an event is known, the remainder of the support is automatically assigned to the negation of the event.

B. Mass Functions, Focal Elements And Kernel Elements:

When the frame of discernment is determined, the mass function m is defined as a mapping of the power set m: $2^\Omega \rightarrow [0, 1]$

$$1、\ m(\phi) = 0 \quad (3)$$

$$2、\ \sum_{A \subset \Omega} m(A) = 1 \quad (4)$$

The mass function m is also called a basic probability assignment function. m (A) expresses the proportion of all relevant and available evidence that supports the claim that a particular element of H belongs to the set A but to no particular subset of A. In engine diagnostics, m (A) can be considered as a degree of belief held by an observer regarding a certain fault; different evidence can produce different degrees of belief with respect to a given fault. Any subset A of Ω such that m (A) > 0 is called a focal element; the union of all focal element C =∪ m (A) ≠ 0, A is called a kernel element of mass function m in the frame of discernment.

C. Belief and Plausibility Functions

The belief function Bel is defined as:

$$\text{Bel} : 2^\Omega \rightarrow [0,1] \quad \forall A \subset \Omega$$

$$PI(A) = 1 - \text{Bel}(\overline{A}) = \sum_{B \subseteq \Omega} m(B) - \sum_{B \subset A} m(B) = \sum_{B \cap A \neq \Phi} m(B) \quad (5)$$

The belief function Bel(A) measures the total amount of probability that must be distributed among the elements of A; it reflects inevitability and signifies the total degree of belief of A and constitutes a lower limit function on the probability of A.

The plausibility function Pls and double function Dou are defined as: Pl: $2^\Omega \rightarrow [0,1]$

$$PI(A) = 1 - \text{Bel}(\overline{A})$$
$$\text{Dou}(A) = \text{Bel}(\overline{A}) \quad (6)$$

The plausibility function Pl(A) measures the maximal amount of probability that can be distributed among the elements in A; it describes the total belief degree related to A and constitutes an upper limit function on the probability of A. it describes the total belief degree related to A and constitutes an upper limit function on the probability of A.

D. Evidence Combination

Let Bel1 and Bel2 be two belief functions in the same frame of discernment, then the corresponding basic belief assignment are m1 and m2 based on information obtained from two different information sources in the same frame of discernment Ω, focus elements are $X_1, X_2, \ldots, X_k$, and $Y_1, Y_2, \ldots, Y_k$, if $X_i \cap Y_j = A$, $X \subset \Omega$, then m1 (X_i)m2 (Y_j) is the probability assignment to A, The total belief of A

is: $\underset{X_i \cap Y_j = A}{\Sigma} m_1(X_i)m_2(Y_j), \quad A \neq \phi$

when $A = \phi$, $\sum_{X_i \cap Y_j = \phi} m_1(X_i)m_2(Y_j)$ is on the belief of

void set φ. We have the rule of evidence combination.

$$m(A) = m_1 \oplus m_2 = \begin{cases} 0 & A=\Phi \\ \dfrac{1}{1-k} \sum_{X \cap Y = A} m_1(X)m_2(Y) & A \neq \Phi \end{cases} \quad (7)$$

Where $k = \sum_{X \cap Y = \phi} m_1(X)m_2(Y)$, K represents a basic

probability mass associated with conflicts among the sources of evidence. It is determined by summing the products of mass functions of all sets where the intersection is null. K is often interpreted as a measure of conflict between the sources. The larger the value of K is, the more conflicting are the sources, and the less informative is their combination.

The produced function m =m1 $\oplus$ m2 is also a mass function in the same frame of discernment Ω, it represents the combination of m1 and m2 and carries the joint information from the two sources.

In the case of n mass functions m_1, m_2. . . ,m_n in Ω, according to rule of evidence combination:

$$m(A) = m_1 \oplus m_2 \oplus ... \oplus m_n = \begin{cases} 0 & A=\Phi \\ \dfrac{\sum_{A_1 \cap A_2 \cdots \cap A_n = A} \prod_{i=1}^{n} m_i(A_i)}{1-k} & A \neq \Phi \end{cases} \quad (8)$$

Where k= $\sum_{A_1 \cap A_2 \cdots \cap A_n = \Phi} \prod_{i=1}^{n} m_i(A_i)$

IV. MULTI-EXPERT OPINIONS COMBINATION

According to the formulized definition of FCM, experts' opinions are reflected on the estimate of the degree of the cause that is between nodes in the referred concept set, namely weight estimate. In the construction of FCM, multi-experts' opinions combination is represented as the combination of the corresponding elements in the connection matrix provided by experts. Then each expert's estimate of some cause relation can be regarded as evidence. The possible values of the affection degree of the cause relation between concepts form a frame of discernment. The combined probability assignment function is regarded as the evidence of last weight integration.

A FCM equals the code of experts' knowledge; In general, because of experts' different preferences and knowledge structures, the understandings about the problem may be different. Such as, different experts differ in how they assign causal strengths to edges and in which concepts they deem causally relevant. There is a requirement to build a selection rule of concept set and to enact a standard of cause effect degree before FCM combination.

Definition 1: Connection Matrix Standardization

Suppose there are n experts, the FCM of each expert's is established according to their own experiences and knowledge. The connection matrices of n experts' are F_1, F_2, … , F_n. The union (m) of all experts' concepts is regarded as a set of concept. The connection matrices of experts' are expanded to m ×m, and we fill the row or column absent of concept nodes with 0. The process is called the standardization of connection matrix.

The general process of combining multi-experts' FCM with evidence theory is as follows:

1) A frame of discernment is firstly defined; it translates the research of proposition into the research of a set.
2) Basic probability assignments are established according to evidence.
3) Basic probability assignment functions are combined according to the combination rule of evidence theory, and then the target type is determined by the rule of belief evaluation.
4) Applying weighted average on all elements of the frame according to the integrative basic probability assignment function

A. Building of Discernment Frame

The selection of frame of discernment depends upon our knowledge, cognition and what we know and want. In application of FCM, expert estimates the weight using linguistic weight. Their values are usually nothing, very weak, weak, medium, strong, and very strong.

Example 1:

(none, very weak, weak, strong, very strong, extremely strong) → {0, 0.2, 0.4, 0.6, 0.8, 1}

Example 2:

(none, weak, strong, extremely strong) → {0, 0.4, 0.6, 1}

The possible values of weight form a frame of discernment, which is defined by the demand of accuracy.

We can define a frame of discernment according to the example above.

Ω ={0 , 0.2 , 0.4 , 0.6 , 0.8 , 1}

Or:

Ω = {0 , 0.4 , 0.6 , 1}

B. Building of Mass Function

According to the experience and knowledge, each expert makes a basic probability assignment function m(also called the mass function m) for every element of the connection matrix in a frame of discernment. Suppose there are n experts, we can gain n basic probability assignment functions: m1, m2, …, mn .

N experts' evaluating a weight in the frame of discernment can capture a matrix form: The matrix M is as follows:

$$M = \begin{bmatrix} m_{11} & m_{12} & \cdots & m_{1k} \\ m_{21} & m_{22} & \cdots & m_{2k} \\ & & \vdots & \\ m_{n1} & m_{n2} & \cdots & m_{nk} \end{bmatrix}$$

Where each row in matrix M represents the evaluation of ith expert; each column of matrix M represents the evaluation that n experts get after evaluating the jth element of the frame of discernment. m_{ij} denotes the ith expert's probability assignment of the jth element of the frame of discernment Ω.

The result that the ith expert estimates a weight in the frame of discernment is a fuzzy value. A basic probability assignment function mi is produced by solving membership of the fuzzy value.

For example, solving m, when a fuzzy value is 0.48, the membership function is defined as follows:

Fig.1. the six membership functions corresponding to each one of the six linguistic variables the fuzzy number

$$\mu_0(x) = \begin{cases} (0.2-x)/0.2 & 0 < x \le 0.2 \\ 0 & other \end{cases}$$

$$\mu_{0.2}(x) = \begin{cases} (0.2-x)/0.2 & 0 < x \le 0.2 \\ (0.4-x)/0.2 & 0.2 < x \le 0.4 \\ 0 & other \end{cases}$$

$$\mu_{0.4}(x) = \begin{cases} (x-0.2)/0.2 & 0.2 < x \le 0.4 \\ (0.6-x)/0.2 & 0.4 < x \le 0.6 \\ 0 & other \end{cases}$$

$$\mu_{0.6}(x) = \begin{cases} (x-0.4)/0.2 & 0.4 < x \le 0.6 \\ (0.8-x)/0.2 & 0.6 < x \le 0.8 \\ 0 & other \end{cases}$$

$$\mu_{0.8}(x) = \begin{cases} (x-0.6)/0.2 & 0.6 < x \le 0.8 \\ (1.0-x)/0.2 & 0.8 < x \le 1.0 \\ 0 & other \end{cases}$$

$$\mu_{1.0}(x) = \begin{cases} (1.0-x)/0.2 & 0.8 < x \le 1.0 \\ 0 & other \end{cases}$$

According to formula above: $\mu_{0.4}(0.48)=0.6$

$\mu_{0.6}(0.48)=0.4$

We can obtain the base probability assignment m=[0,0,0.6,0.4,0,0]

C. Evidence Combination

Firstly, the conflict between the expert's opinions is calculated with $k = \sum_{X \cap Y = \phi} m_1(X)m_2(Y)$.

If combination condition is satisfied, then the combinative base probability assignment is calculated according to formula bellow:

$$m(A) = m_1 \oplus m_2 = \begin{cases} 0 & A = \Phi \\ \dfrac{1}{1-k} \sum_{X \cap Y = A} m_1(X)m_2(Y) & A \ne \Phi \end{cases}$$

D. Calculate Integrated Weight

The integrated weight *w* is defined as:

$$w = \sum_{j=1}^{m} \frac{a_j}{\sum_{j=1}^{m} a_j} \theta_j \qquad (9)$$

Where a_j is the base probability assignment of the jth state, θ_j is the jth state value of the frame of discernment.

V. THE COMPLEXITY ANALYSIS

The frame of discernment is set as: $\Omega = \{\Theta_1, \Theta_2, \ldots \Theta_n\}$, there are k evidences that k experts offer. In the extreme, each group of evidences has 2^n-1 mass function values: $m(\{\Theta_1\})$, $m(\{\Theta_2\}), \ldots, m(\{\Theta_1, \Theta_2\}), \ldots, m(\{\Theta\})$. On this condition, the complexity degree of the information is $O(k*2^n)$. Then we'll discuss the complexity degree of using the combination formula of 2 evidences and the combination formula of k evidences to combine k experts' knowledge. For using the combination formula of two evidences, the main calculation is the multiplication of two mass functions, so the complexity degree is $(2^n-1)*(2^n-1)=O(2^{2n})$. And for the knowledge combination of k experts, the complexity degree of information is $k*O(2^{2n})=O(k*2^{2n})$. For the combination formula of multi-evidences, the main calculation is the multiplication of k mass functions. So the complexity degree is $(2^n-1)^k$, namely $O(2^{kn})$.

For the problem of combining many experts' knowledge on FCM, when there are n values of mass function in each evidence, the complexity degree of the information is- $O(kn)$. Using the combination formula of two evidences to calculate k evidences, the complexity degree of the information is $O(n^2)$. And for the knowledge combination of k experts', the complexity degree of information is $k*O(n^2)=O(k*n^2)$. For the combination formula of multi-evidences, the complexity degree is $O(n^k)$.

Based on the analysis above, when the knowledge of k experts' are being combined in the same frame of discernment, the complexity degree of two evidences combination relates linearly to the number of evidences, and may form a exponential relation with the number of possible results in the frame of discernment. For the multi-evidences combinations, the complexity degree has an exponential relationship with the number of evidences and the number of possible results in frame of discernment.

VI. APPLICATION

To demonstrate the feasibility of the proposed method, we applied the proposed method to the combination of three experts' opinions.

We define a frame of discernment:

$\Omega = \{0, 0.2, 0.4, 0.6, 0.8, 1\}$

Three experts give judgment to the cause affection degree of C_i and C_j in concept set $\{C_1, C_2, \ldots C_n\}$, see table 1.

TABLE1

EXPERT KNOWLEDGE

state	0	0.2	0.4	0.6	0.8	1
expert						
Expert 1	0	0	0.7	0.3	0	0
Expert 2	0	0.1	0.8	0.	0	0
xpert 3	0	0	0.9	0.1	0	0

According to formula $k = \sum_{X \cap Y = \phi} m_1(X) m_2(Y)$, we get

k=0.41.

The combinative result of Expert 1 and expert 2 according to Eq (7) is shown in table 2.

TABLE 2
EXPERTS KNOWLEDGE COMBINATION (1)

state	0	0.2	0.4	0.6	0.8	1
expert						
Expert 1	0	0	0.7	0.3	0	0
Expert 2	0	0.1	0.8	0.1	0	0
Combination result 1	0	0	0.95	0.05	0	0

Again, according to $k = \sum_{X \cap Y = \phi} m_1(X) m_2(Y)$, we get

k=0.14

The combinative result of expert 1, expert 2 and expert 3 according to Eq (7) is shown in table 3.

TABLE 3
EXPERTS KNOWLEDGE COMBINATION (2)

State	0	0.2	0.4	0.6	0.8	1
Expert						
Result 1	0	0	0.95	0.05	0	0
Expert 3	0	0	0.9	0.1	0	0
Combination result	0	0	0.994	0.0058	0	0

The result of three experts' combination can be seen from table 3. The base probability assignment is 0.994 when state value is 0.4 and m (0.6) is 0.0058.

Using Eq(9) to solve the integrated weight according to the combined base probability assignment function m.

Based on the example above, we get w_{ij};

w_{ij}=0.4*0.994+0.6*0.0058=0.40108

VII. CONCLUSION

We have developed a method for Multi-Expert Opinions Combination Based on Evidence Theory. . In the method, we use multi-expert knowledge as evidence, the possible value of weight as frame of discernment, expert's evaluation to a weight on frame of discernment as basic probability assignment, and Dempster-Shafe rule as combined basis of basic probability assignment m. Finally, the weight is given according to combined basic probability assignment. The strategy can gradually reduce the hypothesis sets and approach the truth with the accumulation of evidences, which make the result of decision more all–around and more scientific. Consummating the proposed method and exploring the applying area are the direction of our future work.

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