

Fig. 2. Fuzzy membership function of the linguistic scale

where  $\tilde{w}_{tj}^*$  indicates the fuzzy weight of the  $j$ th attribute assessed by the  $t$ th evaluator. The semantic of the linguistic terms in Table II is provided by triangular fuzzy numbers defined on the interval  $[0.1, 0.9]$ , which are characterized by membership functions as shown in Figure 2. For example, the linguistic variable 'VL' can be represented by the triangular fuzzy number  $(0.1, 0.1, 0.3)$ .

Step 5 To integrate all the expert opinions, Eq.(10) is adopted to aggregate the subjective judgements of  $k$  experts for obtaining the fuzzy weight  $\tilde{w}_j$  of attribute  $X_j$ .

$$\tilde{w}_j = \sum_{t=1}^k \tilde{\lambda}_t \otimes \tilde{w}_{tj}^*, j = 1, 2, \dots, n \quad (10)$$

$\tilde{\lambda}_t, t = 1, 2, \dots, k$  and  $\tilde{w}_{tj}^*, \forall t, j$  are all parameterized triangular fuzzy numbers. To ensure that the ranges of  $\tilde{w}_j, j = 1, 2, \dots, n$  belong to the interval  $[0, 1]$ , the normalized fuzzy weights of attributes can be acquired by

$$\tilde{w}_j = (w_{j1} / \max_j w_{j3}, w_{j2} / \max_j w_{j3}, w_{j3} / \max_j w_{j3}) \quad (11)$$

Step 6 The normalization of fuzzy decision matrix is performed by applying the linear scale transformation method since it preserves the property that the values of converted triangular fuzzy numbers will be scaled into  $[0, 1]$ . Hence, the normalized fuzzy decision matrix denoted by  $\tilde{\mathbf{R}}$  could be identified as

$$\tilde{\mathbf{R}} = [\tilde{r}_{ij}]_{m \times n}, i = 1, 2, \dots, m; j = 1, 2, \dots, n \quad (12)$$

$$\tilde{r}_{ij} = \begin{cases} (a_{ij}/c_j^+, b_{ij}/c_j^+, c_{ij}/c_j^+), & j \in J^+ \\ (a_j^-/c_{ij}, a_j^-/b_{ij}, a_j^-/a_{ij}), & j \in J^- \end{cases} \quad (13)$$

TABLE II. LINGUISTIC SCALE FOR THE IMPORTANT WEIGHT OF EACH ATTRIBUTE

Linguistic term	Triangular fuzzy scale
Very Low(VL)	(0.1, 0.1, 0.3)
Low(L)	(0.1, 0.3, 0.5)
Medium(M)	(0.3, 0.5, 0.7)
High(H)	(0.5, 0.7, 0.9)
Very High(VH)	(0.7, 0.9, 0.9)

where  $c_j^+ = \max_i c_{ij}, a_j^- = \min_i a_{ij}, J^+$  is associated with benefit attributes and  $J^-$  is associated with cost attributes.

Step 7 The weighted normalized fuzzy decision matrix  $\tilde{\mathbf{V}}$  can be computed by multiplying the normalized fuzzy decision element and the aggregative fuzzy weight of each attribute, which is defined as

$$\tilde{\mathbf{V}} = [\tilde{v}_{ij}]_{m \times n}, i = 1, 2, \dots, m, j = 1, 2, \dots, n \quad (14)$$

where  $\tilde{v}_{ij} = \tilde{w}_j \otimes \tilde{r}_{ij}$  and  $\tilde{v}_{ij}, \forall i, j$  are positive triangular fuzzy numbers.

Step 8 After completing the performance normalization of various attribute scales, the fuzzy positive ideal solution(FPIS,  $A^+$ ) and fuzzy negative ideal solution(FNIS,  $A^-$ ) can be defined as two referential sequences

$$\begin{aligned} A^+ &= \{(\max_i \tilde{v}_{ij} | j \in J^+), (\min_i \tilde{v}_{ij} | j \in J^-)\} \\ &= \{\tilde{v}_1^+, \tilde{v}_2^+, \dots, \tilde{v}_j^+, \dots, \tilde{v}_n^+\} \end{aligned} \quad (15)$$

$$\begin{aligned} A^- &= \{(\min_i \tilde{v}_{ij} | j \in J^+), (\max_i \tilde{v}_{ij} | j \in J^-)\} \\ &= \{\tilde{v}_1^-, \tilde{v}_2^-, \dots, \tilde{v}_j^-, \dots, \tilde{v}_n^-\} \end{aligned} \quad (16)$$

Considering that the ranges of decision elements  $\tilde{v}_{ij}, \forall i, j$  belong to the closed interval  $[0, 1]$ , it satisfies that  $\tilde{v}_{J^+}^+ = \tilde{v}_{J^-}^- = (1, 1, 1)$  and  $\tilde{v}_{J^+}^- = \tilde{v}_{J^-}^+ = (0, 0, 0)$  where  $J^+$  is associated with benefit attributes and  $J^-$  is associated with cost attributes.

Step 9 To take each of the alternatives to be the comparative sequence in order to obtain the distances between  $A_i$  and two referential sequences, which are given as Eq.(17) and Eq.(18) respectively

$$\Delta_{ij}^+ = |A^+(j) - A_i(j)| = |\tilde{v}_j^+ - \tilde{v}_{ij}| = d(\tilde{v}_j^+, \tilde{v}_{ij}) \quad (17)$$

$$\Delta_{ij}^- = |A^-(j) - A_i(j)| = |\tilde{v}_j^- - \tilde{v}_{ij}| = d(\tilde{v}_j^-, \tilde{v}_{ij}) \quad (18)$$

where  $\Delta_{ij}$  indicates the distance of the  $i$ th alternative  $A_i$  to the ideal solution with respect to the  $j$ th attribute  $X_j$ , and  $d(\tilde{v}_A, \tilde{v}_B)$  denotes the distance measurement between two triangular fuzzy numbers  $\tilde{A}$  and  $\tilde{B}$ .

Step 10 The grey relational coefficient of each alternative to the two referential sequences can be calculated as follows

$$\gamma_{ij}^+ = \frac{\min_i \min_j \Delta_{ij}^+ + \zeta \max_i \max_j \Delta_{ij}^+}{\Delta_{ij}^+ + \zeta \max_i \max_j \Delta_{ij}^+} \quad (19)$$

$$\gamma_{ij}^- = \frac{\min_i \min_j \Delta_{ij}^- + \zeta \max_i \max_j \Delta_{ij}^-}{\Delta_{ij}^- + \zeta \max_i \max_j \Delta_{ij}^-} \quad (20)$$

Grey relational coefficient is used for determining how close each alternative is to the ideal solution. Here,  $\zeta \in [0, 1]$  is the distinguishing coefficient, which generally takes 0.5.

Step 11 The grey relational grade about the  $i$ th alternative  $A_i$  and the fuzzy positive ideal solution  $A^+$  can be determined as

$$S_{i+} = \frac{1}{n} \sum_{j=1}^n \gamma_{ij}^+, i = 1, 2, \dots, m \quad (21)$$

Similarly, the grey relational grade about the  $i$ th alternative  $A_i$  and the fuzzy negative ideal solution  $A^-$  can be obtained as

$$S_{i-} = \frac{1}{n} \sum_{j=1}^n \gamma_{ij}^-, i = 1, 2, \dots, m \quad (22)$$

The grey relational grade represents the level of correlation between the referential sequence and the comparative sequence.

Step 12 Once the  $S_{i+}$  and  $S_{i-}$  of each alternative have been calculated successfully, a relative closeness coefficient is defined to determine the final ranking order of all alternatives which is calculated as

$$C_{i^*} = S_{i+} / (S_{i+} + S_{i-}), 0 < C_{i^*} < 1 \quad (23)$$

It is obvious that a greater value of  $C_{i^*}$  indicates a higher priority of the alternative FMS. Therefore, the ranking order of all alternatives can be obtained, and the best one is selected from a set of feasible alternatives, according to the  $C_{i^*}$  value.

As decision making requires multiple perspectives from different people, most organizational decisions for FMS evaluation are made in groups. To make the group decision-making process as efficient and effective as possible, the voting method is employed in this paper to determine the appropriate attribute weights. Meanwhile, the definition of grey relational coefficient of GRA method is introduced to replace the definition of general distance in conventional TOPSIS under fuzzy environment. As a result, the proposed model can overcome the problem of inconsistent ranking of alternative FMSs, and it can also efficiently grasp the ambiguity in human judgments and preferences for evaluation attributes.

### V. ILLUSTRATIVE EXAMPLE FOR EVALUATING FMSs

In this section, to illustrate the feasibility and potentiality of the proposed hybrid group decision model for solving FMS evaluation problems, an empirical case of evaluating alternative FMSs(adapted from literature[12]) is considered. The aforementioned methodology is applied to solve this FMS selection problem to make the proposed group decision model more understandable.

#### A. Identify Necessary Attributes for FMS Evaluation Problem

For this FMS selection problem, the available information is adapted from Karsak and Kuzgunkaya’s research[12]. Karsak and Kuzgunkaya had presented an illustrative problem for evaluating FMSs using a fuzzy multiple objective programming approach. In this paper, the FMS selection problem consists of four attributes and eight alternative FMSs, as shown in Table III and Table IV. Among these four attributes, reduction in labor cost(RLC) and reduction in work-in-process(RWP) are benefit attributes(where higher values are desirable), whereas,

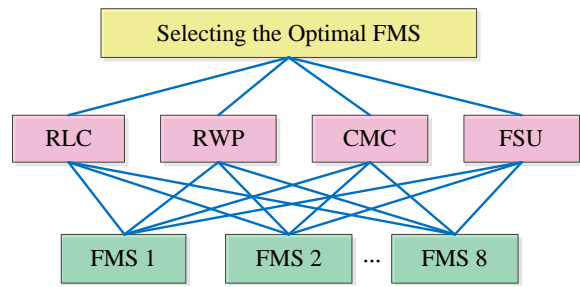


Fig. 3. Hierarchical structure for FMS selection problem

capital and maintenance cost(CMC) and floor space used(FSU) are cost attributes(where lower values are desirable).

The evaluation attributes and alternative FMSs are arranged in a hierarchical structure as depicted in Figure 3. There are three levels in the decision hierarchy for this FMS selection problem. The overall goal of the decision process defined as “Selecting the optimal FMS” is at the top level. The four attributes are at the second level and eight FMS alternatives are at the third level of the hierarchy.

#### B. Calculate the fuzzy weights of attributes using the voting method

Initially, the expert group is formed with the following members:  $\mathbf{E} = (E_1, E_2, E_3, E_4, E_5)$ . Next, the group members are asked to express their assessments of importance weights for each evaluation attribute by casting a vote according to the linguistic scales shown in Table II. The linguistic assessments for all experts and their professional titles are presented in Table V.

Then the fuzzy collective opinion matrix for all experts is constructed by converting the linguistic evaluation(shown in Table V) into triangular fuzzy numbers, as shown in Table VI. From steps 3-5 of the proposed method, an aggregative fuzzy weight value for each attribute can be obtained as

$$\tilde{w} = [(0.2909, 0.5182, 0.8182), (0.2667, 0.5182, 0.7939), (0.4606, 0.7424, 1.0000), (0.0970, 0.2091, 0.4788)]$$

#### C. Determine the ranking order of alternative FMSs based on fuzzy GRA-TOPSIS

Use step 6 to compute the fuzzy normalized fuzzy decision matrix, as shown in Table VII. The next step of the analysis is to find out the weighted normalized fuzzy decision matrix using Eq.(14), and the calculated results are listed in Table VIII. Due to the fact that triangular fuzzy numbers fall into the range of [0, 1], two referential sequences of fuzzy positive ideal solution  $A^+$  and fuzzy negative ideal solution  $A^-$  can be identified as

$$A^+ = [(1, 1, 1), (1, 1, 1), (0, 0, 0), (0, 0, 0)]$$

TABLE III. FMS EVALUATION ATTRIBUTES AND THEIR DEFINITIONS

Attribute	Definition of the Attribute
RLC	Reduction in labor cost (%)
RWP	Reduction in work-in-process(WIP) (%)
CMC	Capital and maintenance cost (\$ 1,000)
FSU	Floor space used (sq. ft.)

TABLE IV. EVALUATION ATTRIBUTES AND ALTERNATIVES FOR FMS SELECTION[12]

Alternatives	RLC	RWP	CMC	FSU
$A_1$	(25, 30, 35)	(20, 23, 26)	(1400, 1500, 1800)	(4000, 5000, 6000)
$A_2$	(16, 18, 20)	(7, 13, 16)	(1100, 1300, 1500)	(5500, 6000, 6500)
$A_3$	(10, 15, 20)	(10, 12, 16)	(750, 950, 1150)	(6000, 7000, 8000)
$A_4$	(23, 25, 27)	(12, 20, 22)	(800, 1200, 1300)	(3500, 4000, 4500)
$A_5$	(12, 14, 16)	(10, 18, 25)	(850, 950, 1050)	(1500, 3500, 5500)
$A_6$	(14, 17, 20)	(13, 15, 20)	(1000, 1250, 1500)	(3500, 5250, 7000)
$A_7$	(17, 23, 27)	(13, 18, 23)	(900, 1100, 1300)	(2500, 3000, 3500)
$A_8$	(12, 16, 20)	(5, 8, 12)	(1400, 1500, 1600)	(2000, 3000, 4000)

TABLE V. LINGUISTIC ASSESSMENTS GIVEN BY EXPERTS AND THEIR TITLES

Experts	RLC	RWP	CMC	FSU	Professional title
$E_1$	VL	M	VH	M	Research Fellow
$E_2$	M	L	H	L	Associate Professor
$E_3$	M	M	VH	VL	Professor
$E_4$	M	VH	M	L	Senior Research Fellow
$E_5$	H	M	H	VL	Professor

TABLE VI. FUZZY COLLECTIVE OPINION MATRIX OF DECISION-MAKERS AND THEIR VOTING WEIGHTS

Experts	RLC	RWP	CMC	FSU	Voting power weight
$E_1$	(0.1, 0.1, 0.3)	(0.3, 0.5, 0.7)	(0.7, 0.9, 0.9)	(0.3, 0.5, 0.7)	(0.2, 0.3, 0.4)
$E_2$	(0.3, 0.5, 0.7)	(0.1, 0.3, 0.5)	(0.5, 0.7, 0.9)	(0.1, 0.3, 0.5)	(0.6, 0.7, 0.8)
$E_3$	(0.3, 0.5, 0.7)	(0.3, 0.5, 0.7)	(0.7, 0.9, 0.9)	(0.1, 0.1, 0.3)	(0.8, 0.9, 1.0)
$E_4$	(0.3, 0.5, 0.7)	(0.7, 0.9, 0.9)	(0.3, 0.5, 0.7)	(0.1, 0.3, 0.5)	(0.4, 0.5, 0.6)
$E_5$	(0.5, 0.7, 0.9)	(0.3, 0.5, 0.7)	(0.5, 0.7, 0.9)	(0.1, 0.1, 0.3)	(0.8, 0.9, 1.0)

TABLE VII. THE NORMALIZED FUZZY DECISION MATRIX FOR ALTERNATIVE FMSs

FMSs	RLC	RWP	CMC	FSU
$A_1$	(0.7143,0.8571,1.0000)	(0.7692,0.8846,1.0000)	(0.4167,0.5000,0.5357)	(0.2500,0.3000,0.3750)
$A_2$	(0.4571,0.5143,0.5714)	(0.2692,0.5000,0.6154)	(0.5000,0.5769,0.6818)	(0.2308,0.2500,0.2727)
$A_3$	(0.2857,0.4286,0.5714)	(0.3846,0.4615,0.6154)	(0.6522,0.7895,1.0000)	(0.1875,0.2143,0.2500)
$A_4$	(0.6571,0.7143,0.7714)	(0.4615,0.7692,0.8462)	(0.5769,0.6250,0.9375)	(0.3333,0.3750,0.4286)
$A_5$	(0.3429,0.4000,0.4571)	(0.3846,0.6923,0.9615)	(0.7143,0.7895,0.8824)	(0.2727,0.4286,1.0000)
$A_6$	(0.4000,0.4857,0.5714)	(0.5000,0.5769,0.7692)	(0.5000,0.6000,0.7500)	(0.2143,0.2857,0.4286)
$A_7$	(0.4857,0.6571,0.7714)	(0.5000,0.6923,0.8846)	(0.5769,0.6818,0.8333)	(0.4286,0.5000,0.6000)
$A_8$	(0.3429,0.4571,0.5714)	(0.1923,0.3077,0.4615)	(0.4688,0.5000,0.5357)	(0.3750,0.5000,0.7500)

$$A^- = [(0, 0, 0), (0, 0, 0), (1, 1, 1), (1, 1, 1)]$$

Next, using Eq.(17) and Eq.(18), the distance of each candidate from two referential sequences  $A^+$  and  $A^-$  with respect to each attribute can be calculated. The result is depicted in Table IX. Then the grey relational coefficient and grey relational grade of each FMS alternative can be derived by using Eqs.(19)-(22), as shown in Table X. Here, this example used the distinguishing coefficient  $\zeta = 0.5$  to calculate the grey relational coefficient. Once the grey relational grades are determined, the relative closeness coefficient can be computed by Eq.(23), with the final results being listed in Table XI. According to the relative closeness coefficient, the ranking of all alternative FMSs in descending order is given as

$$A_1 \succ A_4 \succ A_2 \succ A_7 \succ A_6 \succ A_3 \succ A_8 \succ A_5$$

To validate the results obtained using the fuzzy GRA-TOPSIS model, fuzzy SAW[17] and fuzzy TOPSIS[6] are applied to solve the same numerical example as two comparable methods. In addition, the weights of FMS evaluation attributes are considered the same for these two comparable methods, which is calculated using the voting method for the realistic comparison of the results of all the methods. The final ranking results derived using the methodology of fuzzy SAW, fuzzy TOPSIS and fuzzy GRA-TOPSIS are shown in

Figure 4. A comparison result indicates that the ranking results obtained using fuzzy TOPSIS method and fuzzy GRA-TOPSIS method are more or less the same, while fuzzy SAW method has a great discrimination in the ranking of the same FMS alternatives. All of the three methods suggest  $A_1$  is the best choice and  $A_4$  is the second best choice. But the ranking results for the other FMS alternatives using fuzzy SAW method is significantly different from the other two methods. Fuzzy SAW method is believed to be less reliable because different ranking results will be obtained when different ranking methods are applied for ranking fuzzy numbers. Although the results of fuzzy TOPSIS method almost corroborate with those derived by the fuzzy GRA-TOPSIS method, the relative importance of the distance from  $A^+$  and  $A^-$  is not considered in fuzzy TOPSIS method. This shortcoming is overcome through the grey relational coefficient of the fuzzy GRA-TOPSIS model. Therefore, the proposed fuzzy GRA-TOPSIS model is proved to be more effective for the ranking and selection of FMS alternatives.

The group decision-making process will be finished if the group of experts accept the evaluation results. Otherwise, the group experts have to modify their linguistic assessments for each attribute until the final decision is considered as consistent and acceptable. After detailed analysis of this case study, FMS  $A_1$  is recommended by the group as the best performer among

TABLE VIII. THE WEIGHTED NORMALIZED FUZZY DECISION MATRIX FOR ALTERNATIVE FMSs

FMSs	RLC	RWP	CMC	FSU
$A_1$	(0.2078,0.4442,0.8182)	(0.2051,0.4584,0.7939)	(0.1919,0.3712,0.5357)	(0.0242,0.0627,0.1795)
$A_2$	(0.1330,0.2665,0.4675)	(0.0718,0.2591,0.4886)	(0.2303,0.4283,0.6818)	(0.0224,0.0523,0.1306)
$A_3$	(0.0831,0.2221,0.4675)	(0.1026,0.2392,0.4886)	(0.3004,0.5861,1.0000)	(0.0182,0.0448,0.1197)
$A_4$	(0.1912,0.3701,0.6312)	(0.1231,0.3986,0.6718)	(0.2657,0.4640,0.9375)	(0.0323,0.0784,0.2052)
$A_5$	(0.0997,0.2073,0.3740)	(0.1026,0.3587,0.7634)	(0.3290,0.5861,0.8824)	(0.0264,0.0896,0.4788)
$A_6$	(0.1164,0.2517,0.4675)	(0.1333,0.2990,0.6107)	(0.2303,0.4455,0.7500)	(0.0208,0.0597,0.2052)
$A_7$	(0.1413,0.3405,0.6312)	(0.1333,0.3587,0.7023)	(0.2657,0.5062,0.8333)	(0.0416,0.1045,0.2873)
$A_8$	(0.0997,0.2369,0.4675)	(0.0513,0.1594,0.3664)	(0.2159,0.3712,0.5357)	(0.0364,0.1045,0.3591)

TABLE IX. DISTANCES OF EACH ALTERNATIVE FMS FROM  $A^+$  AND  $A^-$  VERSUS EACH ATTRIBUTE

FMS	Distance of $A_i$ from $A^+$				Distance of $A_i$ from $A^-$			
	RLC	RWP	CMC	FSU	RLC	RWP	CMC	FSU
$A_1$	0.5685	0.5679	0.3923	0.1107	0.5507	0.5424	0.6491	0.9136
$A_2$	0.7242	0.7466	0.4835	0.0822	0.3200	0.3220	0.5832	0.9327
$A_3$	0.7592	0.7407	0.6913	0.0745	0.3027	0.3196	0.4693	0.9401
$A_4$	0.6290	0.6425	0.6231	0.1282	0.4366	0.4566	0.5261	0.8977
$A_5$	0.7812	0.6513	0.6404	0.2816	0.2535	0.4906	0.4602	0.8263
$A_6$	0.7358	0.6817	0.5209	0.1240	0.3138	0.4001	0.5664	0.9082
$A_7$	0.6604	0.6457	0.5835	0.1781	0.4220	0.4618	0.5199	0.8619
$A_8$	0.7475	0.8181	0.3964	0.2169	0.3080	0.2326	0.6392	0.8448

TABLE X. RESULTS OF GREY RELATIONAL COEFFICIENT FOR FMS EVALUATION PROBLEM

FMS	FPIS				FNIS			
	RLC	RWP	CMC	FSU	RLC	RWP	CMC	FSU
$A_1$	0.4947	0.4950	0.6035	0.9304	0.6884	0.6940	0.6279	0.5078
$A_2$	0.4267	0.4185	0.5418	0.9843	0.8893	0.8872	0.6671	0.5009
$A_3$	0.4139	0.4206	0.4395	1.0000	0.9093	0.8898	0.7480	0.4983
$A_4$	0.4659	0.4599	0.4685	0.9001	0.7750	0.7583	0.7054	0.5137
$A_5$	0.4063	0.4561	0.4608	0.7001	0.9711	0.7315	0.7553	0.5420
$A_6$	0.4224	0.4434	0.5200	0.9073	0.8964	0.8076	0.6779	0.5098
$A_7$	0.4522	0.4585	0.4872	0.8236	0.7877	0.7541	0.7098	0.5276
$A_8$	0.4181	0.3941	0.6004	0.7725	0.9031	1.0000	0.6335	0.5344

TABLE XI. RELATIVE CLOSENESS COEFFICIENT AND RANKING ORDER OF ALTERNATIVE FMSs

	$A_1$	$A_2$	$A_3$	$A_4$	$A_5$	$A_6$	$A_7$	$A_8$
$S_{i+}$	0.6309	0.5928	0.5685	0.5736	0.5058	0.5733	0.5554	0.5463
$S_{i-}$	0.6295	0.7361	0.7614	0.6881	0.7500	0.7229	0.6948	0.7677
$C_i^*$	0.5005	0.4461	0.4275	0.4546	0.4028	0.4423	0.4442	0.4157
Rank	1	3	6	2	8	5	4	7

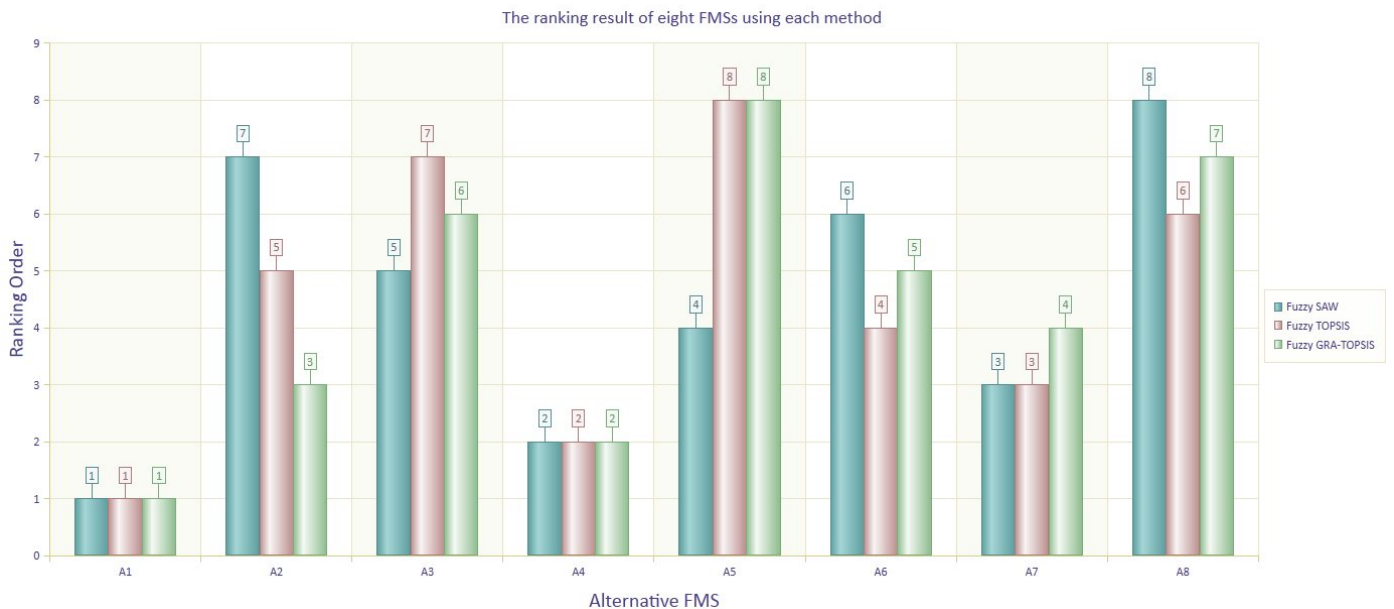


Fig. 4. The ranking results of FMS alternatives using each method

these alternatives.

#### D. Sensitivity analysis

The aim of sensitivity analysis is to determine how different values of an independent variable will carry an impact on a particular dependent variable under a set of assumptions [13], [21]. In general, a sensitivity analysis is performed on MADM problems to check the ranking reversal of the candidates by changing the assigned weights of evaluation attributes. Therefore, this study uses the concepts of sensitivity analysis to investigate the impact of distinguishing coefficient on the final ranking order of alternative FMSs obtained using the fuzzy GRA-TOPSIS method. The sensitivity of the degree of the relative closeness coefficient is analysed with the different distinguishing coefficient  $\zeta$  which varies from 0.1 to 1.0 with an interval of 0.1, and the results are shown in Figure 5 and Figure 6.

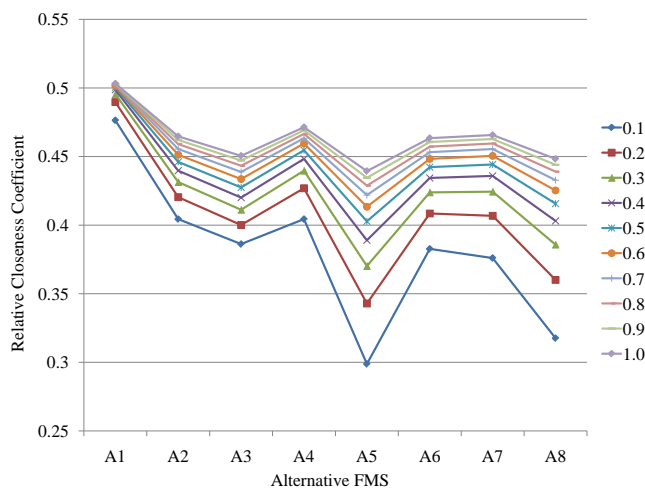


Fig. 5. Variation analysis of  $C_i^*$  value for alternative FMS with change of distinguishing coefficient

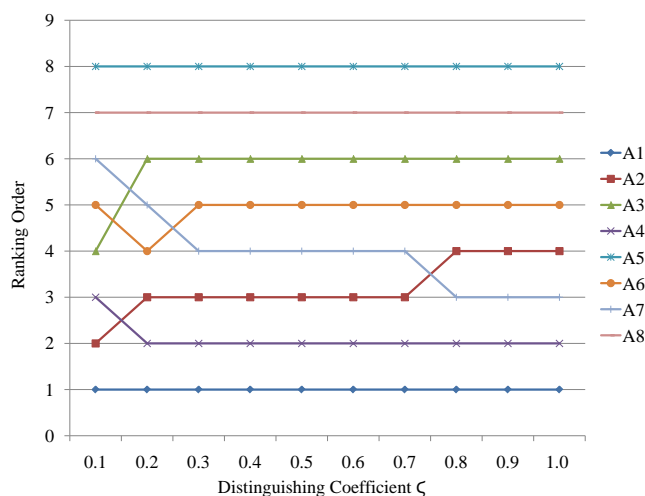


Fig. 6. An effect of distinguishing coefficient on ranking order of FMS alternatives

The results of sensitivity analysis depicted in Figure 5 and Figure 6 indicate that the effect of distinguishing coefficient

$\zeta$  on final ranking order of FMS alternatives using fuzzy GRA-TOPSIS is minor. It can be observed that the ranking sequence of FMS alternatives  $A_1$ ,  $A_5$  and  $A_8$  remain the same, no matter what value the distinguishing coefficient is. There is a slight change in ranking order of the other alternatives when the distinguishing coefficient varies greatly. However, FMS alternative  $A_1$  is ranked first and  $A_5$  is ranked last for every value of distinguishing coefficient. In addition, a similar ranking order is obtained for distinguishing coefficient values from 0.3 to 0.7, which indicates that the results obtained using the fuzzy GRA-TOPSIS is non-sensitive within a certain range. This allows us to draw a logical conclusion that the developed hybrid decision model is robust.

#### VI. CONCLUSION AND FUTURE RESEARCH

The expanding competitiveness due to the globalization has dramatically increased the need for manufacturers to produce high-quality products efficiently and respond to changes quickly. Flexible manufacturing systems provide the means to arrive at a solution consistent with industrial goals and objectives. To help address the issue of evaluation and selection of alternative FMSs where the information available is subjective and imprecise, an effective fuzzy GRA-TOPSIS method applied in the group decision-making model is developed. The proposed model is intended to enhance group decision-making, promote consensus and provide invaluable analysis aids.

In this paper, the voting method is integrated into the group decision-making model to obtain the appropriate attribute weights by aggregating multiple fuzzy linguistic preferences of a group of experts. The assessments of a group of experts are considered to be more objective and unbiased than those individually evaluated. Then the fuzzy GRA-TOPSIS method is employed to determine the final ranking order of alternative FMSs. We also present a case study to illustrate the applicability and potentiality of the proposed group decision model. It has been shown that the proposed model could help a group of decision makers to think comprehensively and systematically about FMS selection problem and improve the quality of decision-making process. In addition, a comparative study is used to examine the rationality of the results of the proposed method. Moreover, a sensitivity analysis is performed to demonstrate the robustness of the proposed model.

In future research, various MADM techniques such as ELECTRE, PROMETHEE and VIKOR could be applied comparatively along with fuzzy set theory to select the best FMS alternative. And our work will also focus on the application of the proposed hybrid model in the similar decision problems, such as material selection, weapon system selection, and location selection.

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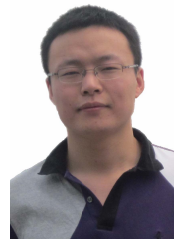
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