Abstract — A study on fuzzy prime ideals in near-subtraction semigroups is already known. We have to expand the concept of prime fuzzy bi-ideals in near-subtraction semigroups and analyse some of its properties to characterize it. This will lead to learn a new type of fuzzy ideal and to develop the researcher to made their research.

Keywords — Fuzzy Ideals, Fuzzy prime ideals.

I. INTRODUCTION

In 1965, fuzzy set was first introduced by L.A. Zadeh [7]. The notion of Near-subtraction semigroup was studied by B.M. Schein. K.H. Kim et al. [2] & they established the concept of Ideals in near-subtraction semigroup & fuzzy set. Prince Williams [3] described the concept of Fuzzy ideals. Similarly, the concept such as Fuzzy bi-ideals has been described by V. Chinnadurai et. al. A detailed study on Fuzzy prime ideals was carried out by Mumtha.K and Mahalakshmi.V [6]. In this paper, we explore the concept of prime fuzzy bi-ideals in near-subtraction semigroups and discuss some of its properties.

II. PRELIMINARIES

Definition: 2.1

A right near-subtraction semigroup is a non-empty set X with “−” & “·” satisfies:

(i) (X, −) is a subtraction algebra
(ii) (X, ·) is a semigroup
(iii) For all p, q, r ∈ X, (p − q), r = p · r − q · r (right distributive law)

Definition: 2.2

If p, 0 = 0, p = 0, for all p ∈ X, then X is a zero-symmetric and is denoted by X₀. Now after, X stands for a zero-symmetric right near-subtraction semigroup (X, −, ·) with at least two elements.

Definition: 2.3

A fuzzy subset is the mapping μ from the non-empty set X into the unit interval [0,1].

Definition: 2.4

A fuzzy subset μ of X is called a fuzzy ideal of X if

(i) μ(x − y) = min(μ(x), μ(y)).
(ii) μ(xy) ≥ μ(y),
(iii) μ(xy) ≥ μ(x), for every x, y ∈ X.

Definition: 2.5

A fuzzy ideal μ is called a fuzzy prime ideal of X if σ, δ ⊆ μ ⇒ σ ⊆ μ or δ ⊆ μ, where σ & δ are any two fuzzy ideals of X.

Definition: 2.6

Let μ and λ be any two fuzzy subsets of X. Then μ ∩ λ, μ ∪ λ, μλ, λμ, μ ∗ λ are fuzzy subsets of X that are defined by,

(μ ∩ λ)(x) = min(μ(x), λ(x))
(μ ∪ λ)(x) = max(μ(x), λ(x))
(μ − λ)(x) = {sup₀{y > z} min{μ(y), λ(z)} if x = y − z
otherwise
μλ(x) = {sup₀{y = z} min{μ(y), λ(z)} if x = yz
otherwise
(μ ∗ λ)(x) = {sup₀{x = ac − a(b − c)} min{μ(a), λ(c)} if x = ac
otherwise

Definition: 2.7

For any fuzzy set μ in X and t ∈ [0,1], We define U(μ; t) = {x ∈ X/μ(x) ≥ t}, which is called a upper t-level cut of μ.

Definition: 2.8

Let I ⊆ X. Define a function fl : X → [0,1] by.

f_l(x) = { 1 if x ∈ I
0 otherwise }, for every x ∈ X.

Clearly, fl is a fuzzy subset of X and it is called the characteristic function of I.

Definition: 2.9

A fuzzy ideal μ of X is said to be normal if there exists a ∈ X such that μ(a) = 1

Definition: 2.10

A fuzzy ideal μ of X is said to be weakly complete if it is normal and there exists z ∈ X such that μ(z) < 1.

Theorem: 2.11

Let μ be a fuzzy bi-ideal of X. Then the finitely generated set, X_μ = {x ∈ X/μ(x) = μ(0)} is an bi-ideal of X.

Theorem: 2.12

Let A be a non-empty subset and μ_A be a fuzzy set in X defined by, μ_A(x) = {1 if x ∈ A
0 otherwise }, for every x ∈ X and s ∈ [0,1]. Then μ_A is a fuzzy bi-ideal of X iff A is an bi-ideal of X. Moreover, X_μ_A = A.
Lemma: 2.13  
Let $x_A$ be the characteristic function of a subset $A \subseteq X$. Then $x_A$ is a fuzzy bi-ideal of $X$ iff $A$ is a bi-ideal of $X$.

III. PRIME FUZZY BI-IDEALS

Definition: 3.1  
A fuzzy bi-ideal $f$ is called a prime fuzzy bi-ideal of $X$ if for any two fuzzy bi-ideals $g$ and $h$ of $X$ such that $g \cdot h \leq f \Rightarrow g \leq f$ (or) $h \leq f$.

E.g: 3.1.1  
Let $X = \{0, 1, 2, 3\}$ with "-" and "&". " are defined as,

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Let $f, g$ and $h$ be fuzzy subsets of $X$ such that,

\[
\begin{align*}
    f(0) &= 1, \quad f(1) = 0.8, \quad f(2) = 0.7, \quad f(3) = 0.5 \\
    g(0) &= 1, \quad g(1) = 0.8, \quad g(2) = 0.6, \quad g(3) = 0.3 \\
    h(0) &= 1, \quad h(1) = 0.7, \quad h(2) = 0.5, \quad h(3) = 0.2
\end{align*}
\]

Clearly, $f$ is prime fuzzy bi-ideal of $X$.

E.g: 3.1.2  
Let $X = \{0, 1, 2, 3\}$ with "-" and "&". " are defined as,

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Let $f, g$ and $h$ be fuzzy subsets of $X$ such that,

\[
\begin{align*}
    f(0) &= 1, \quad f(1) = 0.4, \quad f(2) = 0.4, \quad f(3) = 1 \\
    g(0) &= 0.8, \quad g(1) = 1, \quad g(2) = 0.8, \quad g(3) = 0 \\
    h(0) &= 0.8, \quad h(1) = 0, \quad h(2) = 0.8, \quad h(3) = 0
\end{align*}
\]

Here $g \cdot h \leq f$ but neither $g \leq f$ nor $h \leq f$, for some $x \in X$. Clearly, $f$ is not a prime fuzzy bi-ideal of $X$.

Theorem: 3.2  
Intersection of all prime fuzzy bi-ideals of $X$ is also a prime fuzzy bi-ideal of $X$.

Proof:  
Let $\{f_i | i \in \Omega\}$ be the set of all prime fuzzy bi-ideals in $X$.

To prove: $f = \bigcap_{i \in \Omega} f_i$ is also a prime fuzzy bi-ideal.

Let $g$ and $h$ be any fuzzy bi-ideals of $X$ such that $g \cdot h \leq \bigcap_{i \in \Omega} f_i$.

Since each $f_i$ is a prime fuzzy bi-ideal.

Therefore, $g \leq f_i$ (or) $h \leq f_i$, for all $i \in \Omega$.

(i.e) $g \leq \bigcap_{i \in \Omega} f_i$ (or) $h \leq \bigcap_{i \in \Omega} f_i$.

Note: 3.3  
Every fuzzy prime ideal is a prime fuzzy bi-ideal but the converse need not be true in general.

Theorem: 3.4  
If $f$ is a prime fuzzy bi-ideal of $X$ then the finitely generated set is a prime bi-ideal of $X$.

Proof:  
Assume that $f$ is a prime fuzzy bi-ideal of $X$.

By Theorem 2.11, $X_f$ is a bi-ideal of $X$.

To prove: $X_f$ is a prime bi-ideal of $X$.

Let $A$ and $B$ be any two bi-ideals in $X$ such that $AB \subseteq X_f$.

We have to prove $A \subseteq X_f$ or $B \subseteq X_f$.

Define the fuzzy subsets $g$ and $h$ of $X$ as,

\[
    g(x) = \begin{cases} 
        f(0) & \text{if } x \in A \\
        0 & \text{otherwise}
    \end{cases} \quad h(y) = \begin{cases} 
        f(0) & \text{if } y \in B \\
        0 & \text{if } y \notin B
    \end{cases}
\]

By Theorem 2.12, $g$ and $h$ are fuzzy bi-ideals.

Next we verify that $g \cdot h \leq f$.

Since $g(h(a)) = \sup_{a=bc} \min\{g(b), h(c)\}$ if $a = bc$ otherwise $g(h(c) = f(0)$.

Hence, $g(h(a)) \leq f(a)$, \forall $a \in X$. Thus $g \cdot h \leq f$.

Since $f$ is a prime fuzzy bi-ideal, $g \cdot h \leq f$.

So we have that $g \leq f$ or $h \leq f$.

Suppose $g \leq f$. If $A \subseteq X_f$, then there exists $a \in A$ such that $a \notin X_f$. This means that $f(a) \neq f(0)$. We already know that $f(0) \geq f(a)$. But $f(0) \neq f(a)$ and so $f(0) > f(a)$.

Now, $g(a) = f(0) > f(a)$.

Which is a contradiction to $g \leq f$. Hence $A \subseteq X_f$.

Similarly, If $h \leq f$, then we can show that $B \subseteq X_f$.

This shows that $X_f$ is a prime bi-ideal of $X$.

Theorem: 3.5  
Let $I$ be an bi-ideal of $X$ and $f$ be a fuzzy set in $X$ defined by, $f(x) = \begin{cases} 
        1 & \text{if } x \in I, \forall x \in X \text{ and } s \in [0,1] \\
        0 & \text{otherwise}
    \end{cases}$. If $I$ is a prime bi-ideal of $X$ then $f$ is a prime fuzzy bi-ideal of $X$.

Proof:  
Suppose $I$ is a prime ideal of $X$.

To prove: $f$ is a prime fuzzy bi-ideal of $X$.
By Theorem 2.12, \( f \) is a fuzzy bi-ideal of \( X \). Let \( g \) & \( h \) be two fuzzy ideals of \( X \) such that \( g, h \leq f \).

To prove: \( g \leq f \) or \( h \leq f \).

Suppose not, (i.e) \( g \nleq f \) and \( h \nleq f \).

Then \( g(x) > f(x) \) and \( h(y) > f(y) \), \( \forall \ x, y \in X \).

Now, \( f(x) \neq 1 \) and \( f(y) \neq 1 \) \( \Rightarrow f(x) = s \) and so \( x, y \notin I \).

Hence, \( g(h(a)) > s \). Which is a contradiction.

Hence, \( f \) is a prime fuzzy bi-ideal of \( X \).

**Corollary : 3.6**

Let \( \chi_P \) be the characteristic function of a subset \( P \subseteq X \). Then \( \chi_P \) is a prime fuzzy bi-ideal iff \( P \) is a prime bi-ideal of \( X \).

**Theorem: 3.7**

If \( f \) is a prime fuzzy bi-ideal of \( X \) then, \( f(0) = 1 \).

**Proof:**

Suppose \( f \) is a prime fuzzy bi-ideal of \( X \).

To prove: \( f(0) = 1 \).

Suppose not, (i.e) \( f(0) < 1 \).

Since \( f \) is not a constant, then there exists \( a \in X \) such that \( f(a) < f(0) \).

Define the fuzzy subsets \( g \) & \( h \) as, \( \forall x \in X \)

\[
g(x) = f(0) \quad \text{and} \quad h(x) = \begin{cases} 1 & \text{if } f(x) = f(0) \\ 0 & \text{otherwise} \end{cases}
\]

Since \( g \) is a constant function, \( g \) is a fuzzy bi-ideal.

Note that, \( h \) is the characteristics function of \( X_f \).

Now, by Theorem: 2.12, \( h \) is the fuzzy bi-ideal of \( X \).

Since \( h(0) = 1 \) \( \Rightarrow f(0) \) and \( g(a) = f(0) > f(a) \).

We have that, \( g \nleq f \) & \( h \nleq f \).

Let \( b \in X \). We know that,

\[
g.h(b) = \begin{cases} \sup_{a=cd} \left( \min \{g(c), h(d)\} \right) & \text{if } b = cd \\ 0 & \text{otherwise} \end{cases}
\]

Now, we prove, \( \min \{g(c), h(d)\} \leq f(b) \), where \( b = cd \).

For this, we consider two cases, \( h(x) = 0 \) & \( h(x) = 1 \) in the following:

**Case - (i)**

Suppose \( h(x) = 0 \).

Then \( h(x) < h(0) \) (By definition of \( h \)). Now,

\[
\min \{g(c), h(d)\} = \min \{f(0), 0\} = 0 \leq f(xy) = f(b).
\]

**Case - (ii)**

Suppose \( h(x) = 1 \). Then \( f(x) = f(0) \).

Now, \( \min \{g(c), h(d)\} = \min \{f(0), 1\} = f(0) = f(x) \leq f(xy) = f(b) \).

From this, we conclude that,

\( g.h(b) = \min \{g(c), h(d)\} \leq f(b) \) and so \( g \leq f \).

Since, \( f \) is a prime fuzzy bi-ideal, we have \( g \leq f \) or \( h \leq f \).

Which is a contradiction to \( g \nleq f \).

Hence, \( f(0) = 1 \).

**Theorem: 3.8**

Every prime fuzzy bi-ideal is normal.

**Proof:**

By Previous Theorem 3.7, it is obviously true.

**Theorem: 3.9**

Every prime fuzzy bi-ideal is weakly completely normal.

**Proof:**

Let \( f \) be prime fuzzy bi-ideal.

Then \( f \) is normal and \( f \) lies between the values 0 & 1.

It follows that, \( f(0) = 1 \) & \( f(x) < 1 \), for all \( x \in X \).

Therefore, \( f \) is weakly completely normal.

**Theorem: 3.10**

If \( f \) is a prime fuzzy bi-ideal of \( X \) then, \( |Im(f)| = 2 \).

Moreover, \( Im(f) = \{1, s\} \), where \( 0 \leq s < 1 \).

**Proof:**

Suppose \( f \) is a prime fuzzy bi-ideal of \( X \).

To prove: \( Im(f) \) contains exactly two values.

We know that, by previous Theorem 3.7, \( f(0) = 1 \).

Let \( a \) & \( b \) be two elements of \( X \) such that,

\( f(a) < 1 \) and \( f(b) < 1 \).

Enough to prove: \( f(a) = f(b) \).

**Part-(i)**

Define the fuzzy subsets \( g \) and \( h \) as, \( \forall x \in X \) and \( a \in X \)

\[
g(x) = f(a) \quad \text{and} \quad h(x) = \begin{cases} 1 & \text{if } x \in <a > \\ 0 & \text{otherwise} \end{cases}
\]

By Theorem: 2.12, \( g \) & \( h \) are fuzzy bi-ideals of \( X \).

Since \( a \in <a > \), we have \( h(a) = 1 \) \( \Rightarrow f(a) \) and so \( g \nleq f \).

Let \( z \in X \). We know that,

\[
g.h(z) = \begin{cases} \sup_{x=xy} \left( \min \{g(x), h(y)\} \right) & \text{if } z = xy \\ 0 & \text{otherwise} \end{cases}
\]
If \( x \notin < a > \), then \( h(x) = 0 \)
\[ \Rightarrow \min \{g(x), h(y)\} = \min\{f(a), 0\} = 0 \leq f(xy) = f(z). \]
If \( x \in < a > \), then \( h(x) = 1 \)
\[ \Rightarrow \min \{g(x), h(y)\} = \min\{f(a), 1\} = f(a) \leq f(xy) = f(z). \]
We know that, \( f(x) \geq f(a) \), for all \( x \in < a > \)
It follows that, \( f(a) \leq f(x) \leq f(xy) = f(z) \).
From these, we conclude that, \( g, h \leq f \).
Since \( f \) is a prime fuzzy bi-ideal, we have \( g \leq f \) or \( h \leq f \)
Since \( h \not\leq f \). It follows that \( g \not\leq f \).
Now, \( f(b) \geq g(b) = f(a) \).

Part-(ii)

Now, we construct fuzzy bi-ideals \( \rho, \theta \) of \( X \),
\[ \rho(x) = f(b) \quad \text{and} \quad \theta(x) = \begin{cases} 1 & \text{if } x \in < b > \\ 0 & \text{otherwise} \end{cases}, \quad \forall x \in X \]
As in part-(i), we can verify that \( f(a) \geq f(b) \).
Thus from parts-(i) & (ii), it follows that \( f(a) = f(b) \).
Hence the proof.

**Theorem: 3.11**

Let \( f \) be fuzzy bi-ideal in \( X \). Then \( f \) is a first prime fuzzy bi-ideal of \( X \) iff each level subset \( f_t, t \in \text{Im}(f) \) of \( f \) is a prime bi-ideal of \( X \).

**Proof:**

Assume that \( f \) is a prime fuzzy bi-ideal of \( X \).

By Theorem 3.7, \( f_t \) is an bi-ideal of \( X \).

To prove: \( f_t \) is a prime bi-ideal of \( X \).

Let \( A \) & \( B \) be two ideals in \( X \) such that \( AB \subseteq f_t \).

Define the fuzzy subsets \( g & h \) of \( X \) as,
\[ g(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad h(x) = \begin{cases} 1 & \text{if } x \in B \\ 0 & \text{otherwise} \end{cases} \]

By Theorem 2.12, \( g & h \) are fuzzy bi-ideals of \( X \).

Next we verify that, \( g, h \leq f \).

Since, \( g, h \) assume exactly two values.
\[ g(h(a)) = \sup_{a \in A} \min \{g(b), h(c)\} \]
We conclude that, \( g(b) = h(c) \geq t \). So \( b \in A \cap c \in B \).

Now, \( \alpha = bc \in AB \subseteq f_t \). (i.e, \( \alpha \in f_t \)) \( \Rightarrow f(a) \geq t \).

Hence \( g, h \leq f \), \( \forall \alpha \in A \). Thus \( g, h \leq f \).

Since \( f \) is prime fuzzy bi-ideal, we have \( g \leq f \) or \( h \leq f \).

Suppose \( g \leq f \). If \( A \not\subseteq f_t \), then there exists \( \alpha \in A \) such that \( \alpha \not\in f_t \). This means that \( f(a) \geq t \). (i.e, \( f(a) < t \).

Now, \( g(a) \geq t > f(a) \). Which is a contradiction to \( g \leq f \).

Similarly, if \( h \leq f \), then we can show that \( B \subseteq f_t \).
This shows that \( f_t \) is a prime bi-ideal of \( X \).

Conversely,

Assume that \( f_t, t \in \text{Im}(f) \) is a prime bi-ideal of \( X \).

To prove: \( f \) is a prime fuzzy bi-ideal of \( X \).

Let \( f \) be a fuzzy subset of \( X \) defined by,
\[ f(x) = \begin{cases} 1 & \text{if } x \in f_t \\ s & \text{otherwise} \end{cases} \]

By Theorem 2.12, \( f \) is an fuzzy bi-ideal of \( X \).

To prove: \( f \) is prime.

Let \( g & h \) be two fuzzy bi-ideals of \( X \) such that \( g, h \leq f \).

Enough To prove: \( g \leq f \) or \( h \leq f \).

Suppose \( g \not\leq f \) and \( h \not\leq f \).

Then \( g(x) > f(x) \) and \( h(y) > f(y), \forall x \in X \).

Now, \( f(x) \neq 1 \) and \( f(y) \neq 1 \)
\[ \Rightarrow f(x) = f(y) = s \]
Since \( f_t \) is a prime ideal, we have that \( (x)(y) \not\in f_t \).
Then \( f(a) = s \) and hence \( g, h(a) \leq f(a) = s \).

Since \( a = cd, c = < x > \) and \( d = < y > \). Then \( s = f(a) \geq g, h(a) \).

Now, \( g, h(a) = \min \{g(c), h(d)\} \geq \min \{g(\alpha), h(\beta)\} \geq \min(f(x), f(y)) = s \).

Therefore, \( g, h(a) > s \). Which is a contradiction.
Hence \( f \) is a prime fuzzy bi-ideal of \( X \).

**Theorem: 3.12**

Let \( P \) be a prime bi-ideal of \( X \) and \( a \) be a prime element of \( L, L \in [0,1] \). Let \( f \) be a fuzzy subset of \( X \) defined by,
\[ f(x) = \begin{cases} \begin{cases} 1 & \text{if } x \in I \\ s & \text{otherwise} \end{cases} & \text{iff } f \text{ is a prime fuzzy bi-ideal of } X \end{cases} \]

**Proof:**

Clearly, \( f \) is a non-constant fuzzy bi-ideal.

To prove: \( f \) is prime.

Let \( g & h \) be two fuzzy bi-ideals such that, \( g \not\leq f \) and \( h \not\leq f \).
Then there exists \( x, y \in X \) such that \( g(x) \not\leq f(x) \) and \( h(y) \not\leq f(y) \).

This implies that \( f(x) = f(y) = s \) and hence \( x, y \notin I \).
Since \( I \) is prime, then there exists an element \( r \in X \) such that \( xry \notin I \).

Now, we have \( f(x) \neq s \) or \( f(x) \neq s \) (otherwise \( h(y) \neq a \) and since \( a \) is prime, \( g(x) \neq f(x) \).

This implies that \( g(x) \neq h(y) \) and \( h(x) \neq h(y) \) and hence \( f(x) \neq f(y) \).

Hence \( f \) is prime fuzzy bi-ideal.

Conversely,

Let \( f \) be a prime fuzzy bi-ideal. Then, \( f(0) = 1 \).
Next we observe that \( f \) assumes exactly two values.
Let \( a & b \) be elements of \( X \) such that \( f(a) < 1 \) \& \( f(b) < 1 \).
Define \( g \) & \( h \) as, 
\[
g(x) = \begin{cases} 
1 & \text{if } x \in (a) \\
o & \text{otherwise} 
\end{cases} \\
h(x) = f(a), \forall x \in X.
\]

By Theorem: 2.12, \( g \) & \( h \) are fuzzy bi-ideals.

And also we have, \( g(x), h(y) \leq f(xy), \forall x, y \in X. \)

And hence \( g \cdot h \leq f \). Put \( g \equiv f \).

Since \( f \) is prime fuzzy bi-ideal and so \( h \leq f \) so that \( h(b) \leq f(b) \) hence \( f(a) \leq f(b) \). Thus \( f \) assumes only one value, say \( a \) other than 1.

Let \( I = \{ x \in X/f(x) = 1 \} \). Then clearly, \( I \) is a proper bi-ideal of \( X \) and for \( x \in X, f(x) = \begin{cases} 
1 & \text{if } x \in I \\
a & \text{otherwise} 
\end{cases}. 
\)

Now, to prove: \( I \) is a prime bi-ideal of \( X \) & \( a \) is a prime element in \( L \).

That \( a \) is prime follows that the fact that for any \( a \in L \) & for the constant map \( \bar{a} \leq f \) iff \( a \leq a \). Let \( J \) & \( K \) be ideals of \( X \) such that \( JK \subseteq I \). Then \( \chi_J \chi_K = \chi_{JK} \subseteq \chi_I \subseteq f \) so that \( \chi_J \subseteq f \) or \( \chi_K \subseteq f \). Which implies that \( f \leq I \) or \( K \subseteq I \).

**Corollary: 3.13**

Let \( L \) be a complete chain and \( P \) is an bi-ideal of \( X \). Then \( P \) is a prime bi-ideal of \( X \) iff \( \chi_P \) is a prime fuzzy bi-ideal of \( X \).

**IV. CONCLUSION**

We have analyse the concept of prime fuzzy bi-ideal \( f \) in near-subtraction semigroups and investigated some of its properties. We find

- \( f(0) = 1 \)
- \( Im(f) = \{1, s\} \), where \( 0 \leq s < 1 \).
- Prime fuzzy bi-ideal iff each level subset is prime fuzzy bi-ideal.

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**REFERENCES**


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