A Novel Maximum Fuzzy Entropy Thresholding of Seismic Images

Sanjay Kumar Singh

***Abstract*:** Image thresholding is very useful for keeping the significant part of an image and getting rid of the unimportant part or noise. This holds true under the assumption that a reasonable threshold value is chosen. The study of image thresholding techniques in earthquake engineering, remote sensing, geology and geophysics seems to be extremely important for recognition of certain patterns such as faults, folding, fracturing, thrusting, closure, salt domes, strong reflectors, seismic facies, channels, bright spots etc, and the identification of large zones of common signal texture which are not detectable so minutely by other techniques. This paper presents a novel maximum fuzzy entropy thresholding of seismic images. The concept of fuzzy probability and fuzzy partition is introduced first. Then, based on the conditional probabilities and fuzzy partition, a 2-level optimal thresholding is searched adaptively through the maximum entropy principle of the seismic images.

***Keywords*:** Digital Image Processing, Computer Vision, Image Thresholding, Image Segmentation, Fuzzy Probability, Fuzzy Partition, Fuzzy Entropy, Seismology, Seismic Image Processing.

I. INTRODUCTION

RECENT developments in image processing techniques have offered new opportunities in the areas of processing and interpretation of seismic sections (seismic images). Unlike natural images where both dimensions are spatial, a seismic image has one spatial and one temporal dimension. The attribute associated with a point on a seismic image is a signal amplitude which is ideally proportional to the reflection coefficient of the reflecting geological interface at a depth that corresponds to the two way travel time on the temporal dimension. Recently, because of the interest in interactive seismic interpretation, seismic sections are treated as conventional images with gray level replacing signal amplitudes. Two dimensional image enhancement, image restoration, image segmentation, image compression and computer vision is an extremely important tool in processing seismic sections, known as seismic image processing. Seismic image processing applied to seismic sections is largely motivated to process and extract information that can be used to infer a description of the sub-surface geology of the earth [1][13]-[15].

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Image thresholding is essentially a pixel classification problem. Its basic objective is to classify the pixels of the given image in to two classes: those pertaining to an object and those pertaining to the background. While one include to pixels with gray values that are below or equal to a certain threshold the other includes those with gray values above the threshold. Thresholding is popular tool for image segmentation.

Many researchers have reported the definition of fuzzy partition entropy using conditional entropy and designed the threshold selection algorithm based on maximum fuzzy entropy. Based on the conditional probability and the fuzzy partition entropy, the optimal thresholding is searched adaptively [2]-[12].

In this paper, we have adapted 2-level thresholding, the best threshold value for a given gray level image is determined by the best fuzzy 2-partition of the domain of the image. The best fuzzy partition is obtained by applying both the relationship between fuzzy partition and probability partition of the domain of the image and the maximum entropy principle, where the probability partition is directly obtained from the histogram of the image. The entropy that was defined by both probability partition and fuzzy partition is used for compatibility measure between probability partition and fuzzy partition. When the probability partition and fuzzy partition is most compatible the entropy has a maximum value.

II. BACKGROUND

Consider an image of size $M \times N$ having L gray-levels i.e.; $G = \{0,1,...,L-1\}$ and the histogram of the image is $\{h_k \mid k = 0,1,...,L-1\}$. Let p be the probability of the occurrence of gray levels i.e.; $p_k = p\{k\} = h_k$. A membership function $\mu_A(k)$ denotes the membership grade, where the gray k belongs to the fuzzy set $A = \sum_{k=0}^{L-1} \frac{\mu_A(k)}{k}$. The membership function $\mu_A(k)$ of the fuzzy set A denotes the degree of some properties, such as brightness, darkness, edginess, smoothness etc. The probability of A is computed simply by $p(A) = \sum_{k=0}^{L-1} \mu_A(k) h_k$, and the conditional probability, where the gray k is classified into A, is $p\left[\{k\}|A\right]=\mu_A(k)h_k/p(A)$ [2]-[12].

2.2 Fuzzy Partition Entropy

Definition 1: Let $X=\left[\{X_i\}\subset\Re^p \mid i=0,1,\ldots,n\right]$ (p-dimensional real space) be a random finite set and $P=\{A_i \mid i=1,2,\ldots,c\}$ be a fuzzy partition of X. B is a fuzzy event of the probability space then, the conditional entropy of P given B is defined as follows:

$$H(P|B)=-\sum_{i=1}^{c}p(A_i|B)\log p(A_i|B)$$

$$=-\sum_{i=1}^{c}\frac{p(A_iB)}{p(B)}\log\frac{p(A_iB)}{p(B)} \quad (2.2.1)$$

If a sequence of set $Q_i=\left[\{X_i\}\subset\Re^p \mid i=0,1,.,n\right]$ is constructed then, $Q=\{Q_i \mid i=1,2,\ldots\ldots,n\}$ is apparently a fuzzy partition of X, called a natural fuzzy partition of X then, according to definition1, the conditional entropy of Q on the condition that B is given is defined as

$$H(Q|B)=-\sum_{i=1}^{n}p(Q_i|B)\log p(Q_i|B)$$

$$=-\sum_{i=1}^{n}\frac{p(Q_iB)}{p(B)}\log\frac{p(Q_iB)}{p(B)} \quad (2.2.2)$$

Definition 2: Let $X=\left[\{X_i\}\subset\Re^p \mid i=0,1,..,n\right]$ be a random finite set. Let $P=\{A_i \mid i=1,2,\ldots,c\}$ and $Q=\{Q_i \mid i=1,2,\ldots,n\}$ be finite partition of fuzzy set X. Then, the entropy of P is [3]-[12].

$$H(P)=\sum_{i=1}^{c}H(Q|A_i)$$

$$=-\sum_{i=1}^{c}\sum_{j=1}^{n}\frac{p(Q_jA_i)}{p(A_i)}\log\frac{p(Q_jA_i)}{p(A_i)} \quad (2.2.3)$$

III. PROPOSED METHODOLOGY

3.1 Fuzzy Partition of Gradient Image

Let us consider a gradient image $g(x,y)$ have L gray levels, i.e. $G=\{0,1,\ldots,L-1\}$ and let the histogram is denoted by $\{h_k \mid k=0,1,\ldots,L-1\}$. The sequence of set $Q_k=\{k \mid k=0,1,\ldots,L-1\}$, is constructed. $Q=\{Q_i \mid i=0,1,\ldots,L-1\}$ is apparently a fuzzy partition of G. Considering the gradient image is composed of object D_o and background D_b. Based on the definition of fuzzy partition entropy, the problem of edge detection is to find the unknown probabilistic fuzzy 2-partition of P, where $P\{D_o,D_b\}$. The probability distribution of D_o and D_b are denoted by p_o and p_b, i.e.; $p_o=p(D_o)$, $p_b=p(D_b)$. The two fuzzy partitions of D_o and D_b are characterized by two membership functions, $\mu_o(g;a,b)$ and $\mu_b(g;a,b)$ respectively. Parameter couple (a,b) are used to control shape of membership functions. $\mu_o(g;a,b)+\mu_b(g;a,b)=1$; $g=0,1,\ldots,L-1$, $a<b$. The functions are shown in fig.3.1 [3]-[12].

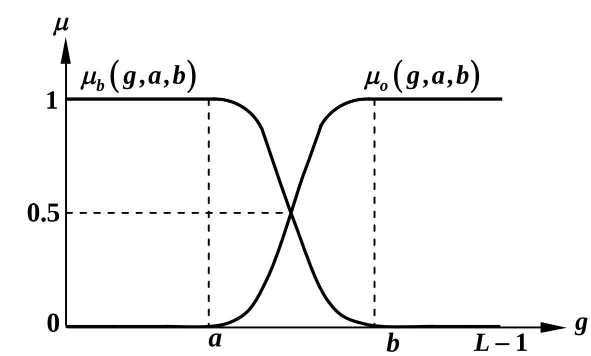


Fig.3.1: Membership Functions of Object and Background Region

In the proposed scheme, $\mu_o(g;a,b)$ is Zadeh's *S*-function, $\mu_b(g;a,b)$ is a *Z*-function.

$$\mu_b(g;a,b)=\begin{cases}1 & ;0\le g\le a\\ 1-2\left(\dfrac{g-a}{b-a}\right)^2 & ;a\le g\le\dfrac{a+b}{2}\\ 2\left(\dfrac{g-b}{b-a}\right)^2 & ;\dfrac{a+b}{2}\le g\le b\\ 0 & ;b\le g\le L\text{-}1\end{cases} \quad (3.1.1)$$

$$\mu_o(g;a,b) = 1 - \mu_b(g;a,b) \qquad (3.1.2)$$

In fact, the membership functions $\mu_o(g;a,b)$ and $\mu_b(g;a,b)$ represent the conditional probability that a pixel is classified into the object D_o and the background region D_b respectively on the condition that they have a gradient of g, i.e.;

$$\mu_o(g) = p_o|g, \quad \mu_b(g) = p_b|g \qquad (3.1.3)$$

According to Eq.(2.2.2), the conditional entropy of Q, on condition that the object region D_o is given, is determined by

$$H(Q|D_o) = -\sum_{k=0}^{L-1} \frac{p(Q_k D_o)}{p(D_o)} \log \frac{p(Q_k D_o)}{p(D_o)} = -\sum_{k=0}^{L-1} \frac{\mu_b(k)h_k}{p(D_b)} \log \frac{\mu_b(k)h_k}{p(D_b)} \qquad (3.1.4)$$

where, $$p(D_o) = \sum_{k=0}^{L-1} \mu_o(k)h_k \qquad (3.1.5)$$

The conditional entropy of Q, on the condition that the non-edge region D_b is given, is determined by

$$H(Q|D_b) = -\sum_{k=0}^{L-1} \frac{p(Q_k D_b)}{p(D_b)} \log \frac{p(Q_k D_b)}{p(D_b)} = -\sum_{k=0}^{L-1} \frac{\mu_b(k)h_k}{p(D_b)} \log \frac{\mu_b(k)h_k}{p(D_b)} \qquad (3.1.6)$$

where, $$p(D_b) = \sum_{k=0}^{L-1} \mu_b(k)h_k \qquad (3.1.7)$$

According to Eqs. (3.2.4) and (3.2.6), the fuzzy partition entropy $H(P)$ is given by

$$H(P) = H(Q|D_o) + H(Q|D_b) = -\sum_{k=0}^{L-1} \left\{ \frac{\mu_o(k)h_k}{p(D_o)} \log \frac{\mu_o(k)h_k}{p(D_o)} + \frac{\mu_b(k)h_k}{p(D_b)} \log \frac{\mu_b(k)h_k}{p(D_b)} \right\} \qquad (3.1.8)$$

3.2 Maximum Fuzzy Entropy Criterion

Necessary Condition for Maximum Fuzzy Entropy

The necessary conditions for the entropy function $H(a,b)$ to reach a maximum value is derived as

$$\frac{\partial H(a,b)}{\partial a} = 0 = -\left\{ \frac{\partial p_o}{\partial a} \log\left(\frac{p_o}{1-p_o} \right) \right\} \qquad (3.2.1)$$

and similarly,

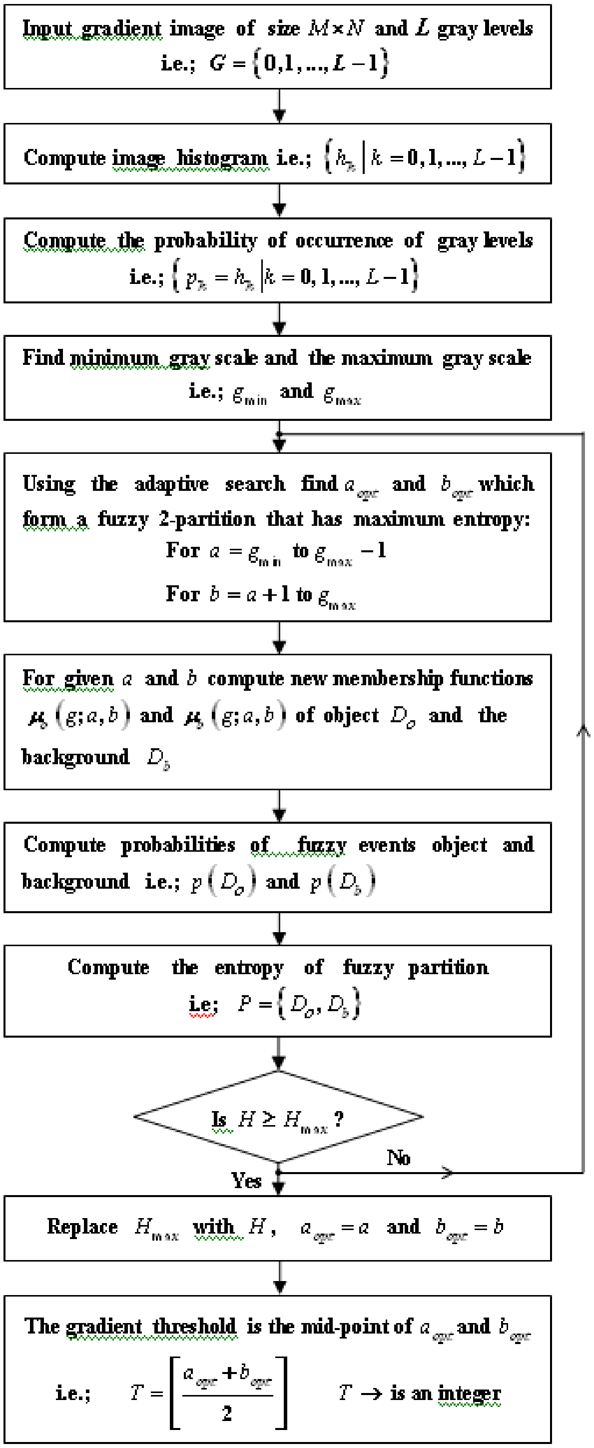
$$\frac{\partial H(a,b)}{\partial b} = 0 = -\left\{ \frac{\partial p_o}{\partial b} \log\left(\frac{p_o}{1-p_o} \right) \right\} \qquad (3.2.2)$$

It is thus clear that the condition for entropy to reach a maximum value is $p_o = p_b = \frac{1}{2}$ since, $\frac{\partial p_o}{\partial a}$ and $\frac{\partial p_o}{\partial b}$ cannot be zero.

To find a fuzzy partition of the maximum information (or, maximum uncertainty), i.e.; to search optimal parameters in the image space to maximize Eq.(3.1.8). Hence, the parameter's values that correspond to the maximum entropy are selected and then the best selected parameters, $\tilde{a}$ and $\tilde{b}$, are the ones satisfying:

$$H(\tilde{a},\tilde{b}) = \max_{g=0,1,\ldots,L-1} \left[H\{a(g), b(g)\} \right] \qquad (3.2.3)$$

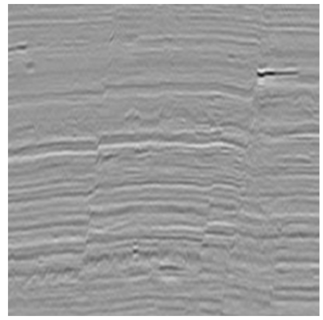
Eq.(3.1.8) can be regarded as the total information measurement on the basis of fuzzy partition P, i.e. $P = \{D_o, D_b\}$. In order to search fuzzy partition which has maximum information entropy, one encounters the problem of parameter optimization, the algorithm is as Follows [2]-[12];

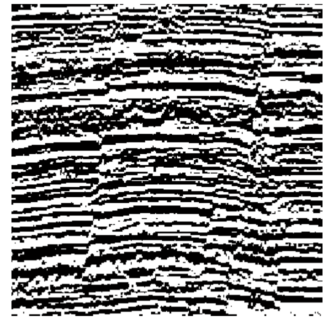


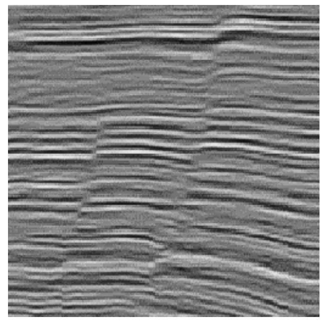
Flow Chart 3.2: Algorithm for Maximum Entropy Thresholding

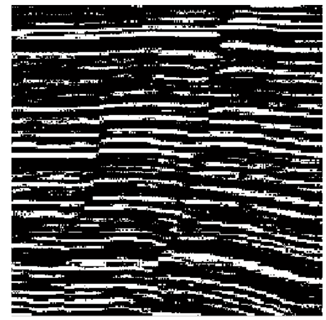
Entropy Thresholding is a means of thresholding an image that selects an optimum threshold value by choosing the pixel intensity from the image's histogram that exhibits the maximum entropy over the entire image.

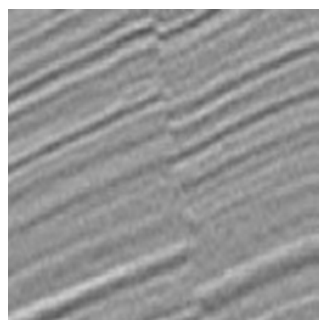
IV. SIMULATION

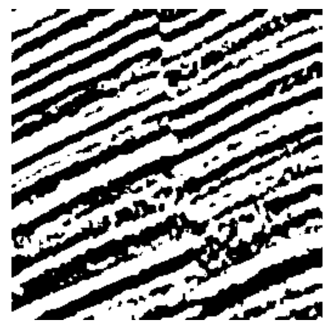

4.1.(a)

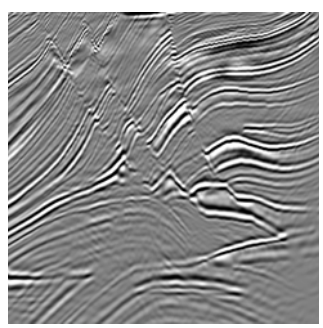

4.1.(b) Resulted Image


4.2.(a)


4.2.(b) Resulted Image


4.3.(a)


4.3.(b) Resulted Image


4.4.(a)


4.4.(b) Resulted Image

Figures	Threshold
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V. CONCLUSION

The excellent performance of the proposed technique has been exercised through simulation using Matlab on a set of Seismic test images. The experimental results verifies that the proposed method performs well and motivates for further research by combining fuzzy logic and fuzzy probability statistics with the higher-level of thresholding.

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