A Method to Fit a Nonlinear Curve to NDVI, SST and LST

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***Abstract*—The sensors of Terra/MODIS send us various data since 2000 year, such as the normalized difference vegetation index (NDVI), the land surface temperature (LST) and the sea surface temperature (SST). These data usually change periodically in a year. We apply a nonlinear curve to the data and give a practical method to evaluate the coefficients and period of the curve. A criterion to determine the period of the curve is given.**

***Index Terms*—NDVI, SST, LST, Periodical curve, Relative Error**

I. Introduction

The NDVI is a simple numerical indicator that can be used to estimate the vigor of plant growth. The value is computed as follows [1]:

$$\mathrm{NDVI} = \frac{\mathrm{IR}-\mathrm{R}}{\mathrm{IR}+\mathrm{R}} \tag{1}$$

where IR and R are the reflectance factors of infrared rays and red rays respectively as shown Fig. 1.

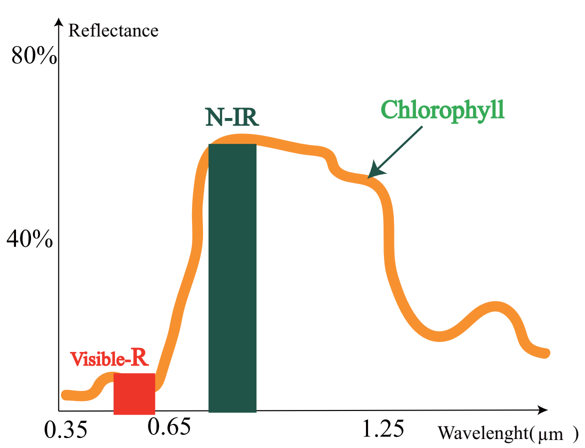

Fig. 1. Photosynthesis wants Visible-R, so the reflectance of it is low

The index range is between −1 and 1. If the values are around 0 or lower, the vegetation is seemed to be in poor condition or bare surfaces of the observing area.

The change of NDVI is expected gradually and periodically in a year if the particular phenomenon, such as a forest fire or a disaster, do not happen at the observation area. Hence we apply a nonlinear curve model to NDVI.

The land surface temperature (LST) and the sea surface temperature (SST) are also expected periodically changes in a year, hence we also apply the curve to the data. We show that the curve well fits to LST and SST. We propose a practical method to find the coefficients a_i of the curve and the period n by using Mathematica.

The values LST, SST and NDVI, especially NDVI are strongly influence by a cloud and haze, but we ignore the influence and do not correct NDVI data [2].

II. Materials

In this section, we apply the nonlinear curve model to NDVI, LST and SST sending by Noaa/AVHRR, Terra/MODIS and Aqua/MODIS. We also propose a criterion to determine the periodical parameter n contained in the model by using the built-in function `FindFit` of Mathematica.

A. NDVI, LST and SST

We apply the following 3 kinds of data to the numerical examples.

1) NDVI, LST, SST: Observed around Australia by Noaa/AVHRR from 1982 to 1999. The data structure is $\{$ 151, 153, 156, 158,$\cdots\cdots\}$
2) NDVI: Observed around Tokyo by Terra/MODIS from 2000 to 2009. The data structure is $\{$ 8201, 7295, 7335, 7468,$\cdots\cdots\}$
3) NDVI: Observed a Kyusyu by Terra/MODIS from January(2003) to November(2003) The data structure is $\{$ 0.824701, 0.830808, 0.894993,$\cdots\cdots\}$

B. Model

The proposing model to the data is as follows:

$$f(x) = a_1 + a_2 x + (a_3 + a_4 x)\sin\left(a_5 + \frac{\pi x}{n}\right) \tag{2}$$

where n is a fixed integer between $10 \le n \le 30$. If the NDVI is downloaded every week, then x changes like $x = 1, 2, 3, \cdots, 52$ in a year. The least square method and Newton method are applied to find the coefficients a_i for fixed n. The program is written by Mathematica and `FindFit` function is utilized as below.

```
Do[{Clear[f],
f[x_]=
a1+a2x+(a3+a4)\sin(\dfrac{\pi}{n}x+a5)/.
FindFit[NDVI,
a1+a2x+(a3+a4)\sin(\dfrac{\pi}{n}x+a5),
{a1,a2,a3,a4,a5},{x}}],{n,10,30}]
```

C. A criterion to determine n

We can get 31 solutions in the above Mathematica program. Here we introduce a method how to find the best integer n between them.

Let y_i and $\widehat{y}_i$ be the observation and the estimated values respectively. The value $|\widehat{y}_i - y_i|/|y_i|$ or $|\widehat{y}_i - y_i|/|\widehat{y}_i|$ is called relative error between the observation (true) value and the estimated (approximation) value [3].

Here we introduce the following quasi-relative error to avoid the zero divide.

$$\frac{|\widehat{y}_i - y_i|}{\max(|\widehat{y}_i|, |y_i|)}$$

The magnitude of the value is nearly equal to the relative error if $\widehat{y}_i$ is the good approximation of y_i.

For the fixed n of (2), we introduce the following CR_n value where m is the size of NDVI.

$$\mathrm{CR}_n = \frac{\sum_{i=1}^{m} \frac{|\widehat{y}_i - y_i|}{\max(|\widehat{y}_i|, |y_i|)}}{m} \tag{3}$$

CR_n is similar to the mean value of the relative errors between the observation values NDVI_x and the estimated values $f(x)$. If CR_n is small, $f(x)$ is well fit to the observation values NDVI_x. On the other hand if CR_n is large, $f(x)$ is not well fit to the observation values NDVI_x. Hence n should be chosen as the value CR_n is the smallest.

III. Results

We calculate the value CR and n is so chosen as to minimize the value CR. At the same time, we also calculate R^2 to illustrate the efficiency of CR-criterion.

The multiple correlation coefficients R^2 is usually used as a fittness criterion for just the linear regression case. R^2 is between $0 \leq R^2 \leq 1$ for linear regression and desired close to 1. On the other hand CR is between $0 \leq \mathrm{CR} \leq 1$ and desired closed to 0, because the value is similar to the relative error. We compare the both values for the each integer n.

A. Numerical results of the NDVI of 1982

Fig. 2, Fig. 3 and Fig. 4 show the observation points around Australia, obtained curve of NDVI of 1982 and the comparison of CR, and R^2 respectively. The obtained nonlinear curve is as follows:

$$5.128 + 0.000061x + (-0.0992 + 0.00469x)\sin\left(1.569 + \frac{\pi x}{17}\right) \tag{4}$$

Table I shows ns which give the minimum CR and the maximum R^2. We can see that the best n is almost the equal for the both criterion.

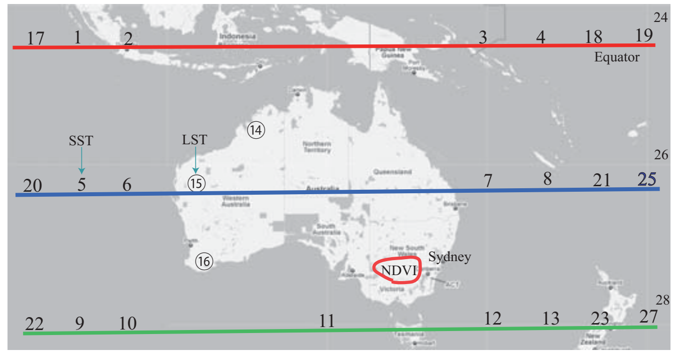

Fig. 2. Observation points around Australia of NDVI, LST and SST

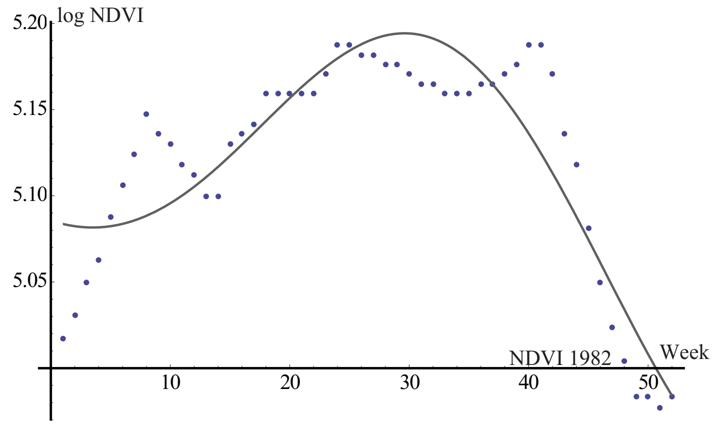

Fig. 3. NDVI and the obtained nonlinear curve of the NDVI of 1982

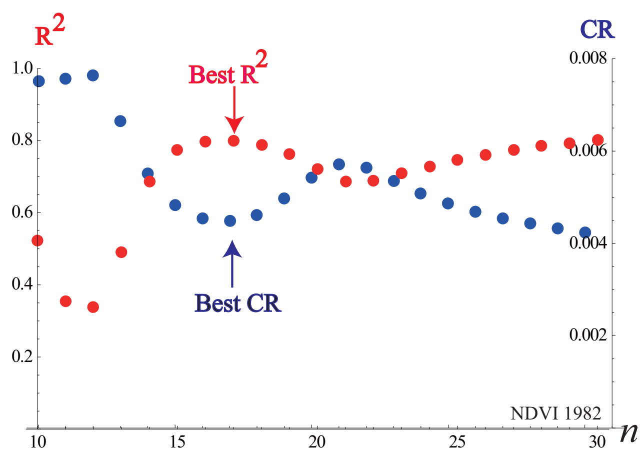

Fig. 4. The behavior of CR and R^2 of the NDVI of 1982

B. CR and R^2 of the LST of 1999

Fig. 5 shows the behavior of CR and R^2 of the LST of 1999. We can get the following coefiicients for $n = 28$

$$212.719 - 0.485x + (12.198 + 0.852x)\sin\left(2.010 + \frac{\pi x}{28}\right) \tag{5}$$

Fig. 5 shows as the value R^2 is increasing, the value CR is decresing.

C. NDVI(2000-2009) of Tokyo

The data is obtained as follows:

TABLE I
THE BEST n OF CR AND R^2 FOR NDVI(1982-1999,AUSTRALIA)

Year	82	83	84	85	86	87	88	89	90
n-CR	17	14	20	15	21	19	28	14	30
n-R^2	17	14	20	16	21	19	27	14	30

Year	91	92	93	94	95	96	97	98	99
n-CR	19	24	23	×	23	22	20	19	14
n-R^2	20	23	23	×	22	22	20	19	14

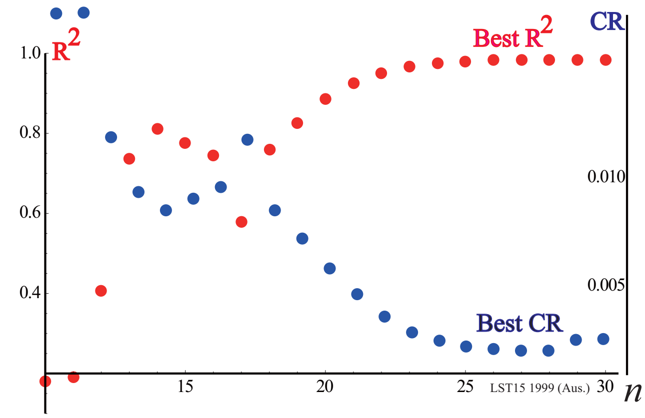

Fig. 5. The behavior of CR and R^2 of the LST of 1999

1) NDVI is observed twice a day since 2000 March by Terra/MODIS.
2) The biggest value in each 16 days interval is updated. Hence we can get 22 or 23 NDVI values par year.
3) The cloud or haze gives a strong influence to the values, sometimes the data are missing. But we do not interpolate the missing data.

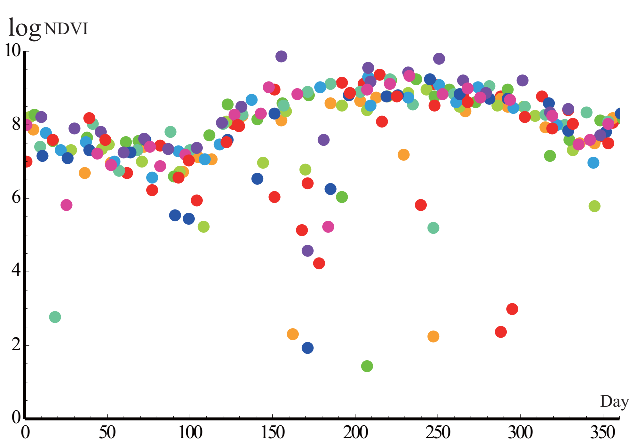

Fig. 6. The same color is the same year NDVI(2000 to 2009)

Fig. 6 shows the behavior of each year NDVI.

We choose the maximum NDVI value for each interval of every year and apply the model (2) to the data. Fig. 7 shows the behavior of CR and R^2. In this case, it is also shown that as the value R^2 is increasing, the value CR is decreasing.

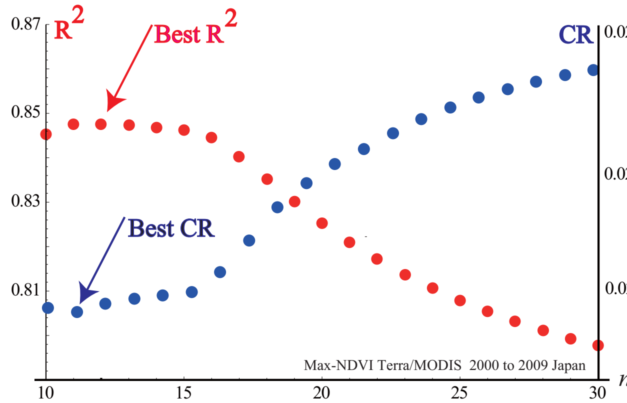

Fig. 7. The behavior of CR and R^2 of the maximum NDVI

The obtained nonlinear curve is as follows:

$$9.126 - 0.0030x + (-0.1768 + 0.005678x)\sin\left(0.2435 + \frac{\pi x}{11}\right) \quad (6)$$

D. LST(2001-2009) of Tokyo

Fig. 8 and Fig. 9 show the obtained curve and the behavior of CR and R^2 respectively for LST. The obtained model is as

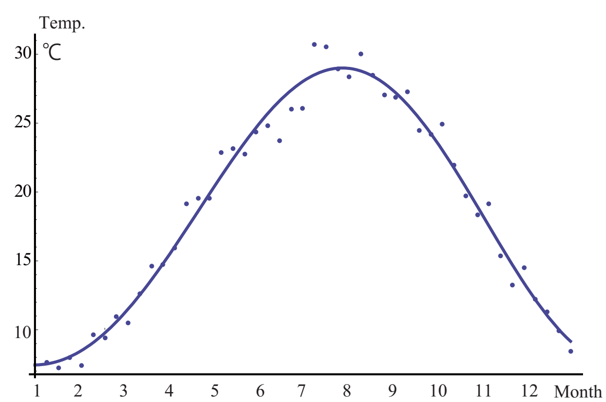

Fig. 8. LST and the fitting curve for the maximum values of each 46-interval

follows:

$$14.908 + 0.130x + (-7.462 - 0.127x)\sin\left(1.583 + \frac{\pi x}{25}\right) \quad (7)$$

IV. CONCLUSIONS

This paper describes two facts. One is a nonlinear model for the NDVI, LST and SST and another is a criterion to decide the best period n of the model (2).

The coefficients of the model are calculated by the Newton method based on the least square method. When n is taken as the unknown parameter of the model, the Newton method does not terminate. Because `FindFit` function does not correspond to such a transcendental equation and to give an appropriate termination criterion for system of nonlinear equations is very difficult [5].

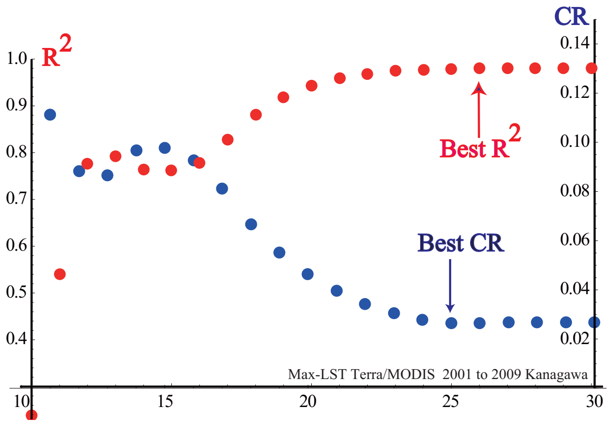

Fig. 9. The behavior of CR and R^2 of the maximum LST

Hence we give a integer n a priori, for example between $10 \leq n \leq n$ and choice the best n by introducing the criterion CR.

We compare the best n decided by the criterion CR to the best n decided by R^2, the n is almost the same as shown Table I and Fig. 4, Fig. 5, Fig. 7 and Fig. 9.

We compare the best n for the following nonlinear model, the best n is almost the same [4].

$$f(x) = a_1 + a_2 x + a_3 x^2 + (a_4 + a_5 x) \sin\left(\frac{\pi}{n} x + a_6\right) \quad (8)$$

The model and CR are also applied to the SST in the Fig. 2 and the other fields, such as the ever green forest, the paddy fiels and the rural area. The numerical results illustrate the efficiency of the model and CR [6]

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