Subsoil damping ratio testing and computing methods

Hou Xingmin, and Shi Xiangdong

Abstract— Subsoil damping ratio measurement method of dynamic foundations was studied theoretically and experimentally based on the corresponding Chinese testing and design Codes. Results of a foundation testing show that there is a relatively large difference between the Codes method and approximate formulae. To improve the testing and analyzing precision, six-degree-freedom time histories of the center of mass of a foundation are calculated from three-component vibration curves of some points on the foundation surface, which are recommended curves as the damping ratio calculation according to the Code for Dynamic Machine Foundation Design. These studies are expected to improve damping ratio testing and analyzing methods.

Keywords—Subsoil damping ratio; motion of center of mass; time-domain; frequency-domain; window function.

I. Introduction

Design requirements of dynamic machine foundations are presented in the Chinese Code for Dynamic Machine Foundation Design (GB50040-96)[1] as rightly selecting corresponding dynamic parameters and foundation types with advanced technology, economical cost and high safety. The dynamic parameters include compressive stiffness, shear stiffness, torsional stiffness and rotational stiffness, damping ratio and mass of vibration of subsoil, where damping ratio selection can influence the foundation dynamic response especially the resonance amplitude and the resonance frequency. That is, a right damping ratio selection is important to foundation design and response calculation. And it is suggested a right damping ratio be determined based on in-situ test firstly in another Chinese Code for Measurement Method of Dynamic Properties of Subsoil (GB/T50269-97) [2]. The following vertical damping ratio calculation formulae are proposed when there is no experimental condition and has previous foundation building experience:

Clay: $$\xi_z = 0.16/\sqrt{\overline{m}} \qquad (1)$$

Sand and silt: $$\xi_z = 0.11/\sqrt{\overline{m}} \qquad (2)$$

Where $\overline{m} = m / \rho A\sqrt{A}$ is dimensionless mass ratio, ρ is subsoil density (t/m^3) and A, the bottom area of the foundation (m^2). The damping ratio of horizontal and rotational coupling vibration:

$$\zeta_{x\varphi 1} = 0.5\zeta_z \quad \zeta_{x\varphi 2} = \zeta_z \quad \zeta_{\psi} = \zeta_{x\varphi 1} \qquad (3)$$

$\zeta_{x\varphi 1}$, $\zeta_{x\varphi 2}$ are damping ratio of the first and second mode of coupling vibration and ζ_{ψ}, the torsional damping ratio. Wang presented detailed beginning and subsequent development of the damping ratio calculation method[3].

As a comparison, parameters' approximate calculating formulae for a mass-spring-damper model to represent a circular foundation on the elastic half-space are listed in Table 1, which were presented by Richard, Whitman and Lysmer and so on [4]. Song and Wolf also obtain some results based on scaled boundary finite-element method-alias consistent infinitesimal finite-element cell method [5]. The formulae have been obtained based on the frequency-dependant analytical solutions of the problem and have deep and lasting significance in dynamic machine foundation design. Even for some non-circular foundations such as rectangular foundation, stiffness formulae also have been adopted as an equivalent bottom area of the circular foundation in some engineering practice.

Table 1 Approximate formulae of equivalent mass-spring-damper model parameters of a circular foundation based on elastic half-space

	Vertical	Horizontal	Rotational	Tosional
Stiffness	$4GR/(1-\upsilon)$	$8GR/(2-\upsilon)$	$8GR^3/3(1-\upsilon)$	$16GR^3/3$
Mass ratio $\overline{m}$	$\frac{m(1-\upsilon)}{4\rho R^3}$	$\frac{m(2-\upsilon)}{8\rho R^3}$	$\frac{3I_x(1-\upsilon)}{8\rho R^5}$	$\frac{I_z}{\rho R^5}$
Damping ratio	$\frac{0.425}{\overline{m}^{1/2}}$	$\frac{0.29}{\overline{m}^{1/2}}$	$\frac{0.15}{(1+\overline{m})\overline{m}^{1/2}}$	$\frac{0.50}{1+2\overline{m}}$

m is mass of the rigid foundation, I_x and I_z are rotational inertia of the foundation around x- and z- axis (Figure 1)

In any case, approximate damping ratio formulae of (1)-(3) have the same form as what the elastic half-space ones as Table 1 shows. And basically, damping ratio value of the Code is less than these of Table 1. Some scholars in China have studied the damping ratio testing and calculating both in numerical calculating and experimental fields and drawn some conclusions. Han demonstrated that the radiation damping ratio

Correspondence to: Hou Xingmin, School of Civil Engineering, Yantai University, Yantai 264005, China. E-mail: houxm@ytu.edu.cn
Supported by: Natural Science Foundation Under Project No 51074214; Shandong Natural Science Foundation Under Project No Y2008f56.

should be taken larger to reduce the foundation vibration efficiently but not by the way of increasing the foundation mass after calculating and testing dynamic response of a 5m×5m foundation of an earthquake simulation table[6]. Wang etc. argued that neglecting the radiation damping ratio of a large dynamic machine foundation will introduce a large calculating error using mass-spring- damper model, that is, a damping ratio larger than the Code should be taken to analyze the foundation dynamic response correctly[7]. The conclusion was drawn after theoretical and experimental study on micro-vibration response calculation based on finite-element method of a large dynamic machine foundation. Chen calculated the vertical resonant damping ratio of a foundation based on distribution of base pressure of rigid and flexible foundation and different Poisson's ratio [8]. He pointed out that value of the damping ratio is nearly the same as half-space theory's result and larger than the Code's.

In this paper, a dynamic machine foundation was tested and damping ratio calculation method is studied. Damping ratio, soil-mass participating in vibration and compressive stiffness are important dynamic parameters in rigid foundation vibration analysis. Generally, measurement of these parameters and data processing method depend upon some direct time-histories of vibration sensors installed on the foundation surface. Damping ratio testing and data processing method written in Chinese Code for Dynamic Machine Foundation Design (GB50040-96) include time-domain method and frequency-domain method. The former mainly adopts free damping vibration and the latter, forced vibration. Take the vertical vibration damping testing as examples: two vertical vibration sensors (such as a velocity sensor) are installed on the two ends of the axial line of a rectangular foundation surface. Imposing a steady-state force or an impacting load on the foundation and measuring the vibration time-histories to calculate the amplitude-frequency curves through Fourier transform to obtain the resonant frequency (f_m) and amplitude value (A_m), and at least three amplitude values (A_i) and frequencies (f_i) under 0.85 f_m. The frequency-domain damping ratio can be calculated as,

$$\xi_z = \sum_{i=1}^{n} \xi_{zi} / n \tag{4}$$

$$\xi_{zi} = \left[\frac{1}{2} \left(1 - \sqrt{(\beta_i^2 - 1) / \alpha_i^4 - 2\alpha_i^2 + \beta_i^2} \right) \right]^{1/2} \tag{5}$$

$$\alpha_i = f_m / f_i , \ \beta_i = A_m / A_i \tag{6}$$

In the time domain, logarithmic decrement method often be taken to calculate the damping ratio,

$$\xi_z = \frac{1}{2\pi} \frac{1}{n} \ln \frac{A_1}{A_{n+1}} \tag{7}$$

Where A_1 (m) is the amplitude of the first period and A_{n+1} (m), amplitude of the n+1*th* period.

II. MOTION OF CENTER OF FOUNDATION

A comparison method presented in this paper with the Code is a method of calculating six-degree-of-freedom motion vector (three translational and three rotational components) of center of mass of a rigid foundation based on four group of three-component vibration records. Fig. 1 is an example of mass foundation with four groups of three-component sensors installed on the surface of the foundation.

To obtain the six-degree-of-freedom motion vector of center of mass of the rigid foundation, the following suppositions are introduced,

1 The foundation is rigid and has no deformation;

2 Motion of the foundation can be described totally by the six-degree-system as shown in Figure 1, three translational motions and three rotational motions. That is, the foundation motion state can be described exactly only by the motion vector of the mass center of the foundation and its shape,

$$\mathbf{U}(t) = \{u_x(t),\ u_y(t), u_z(t), \phi_x(t), \phi_y(t), \phi_z(t)\}^T \tag{8}$$

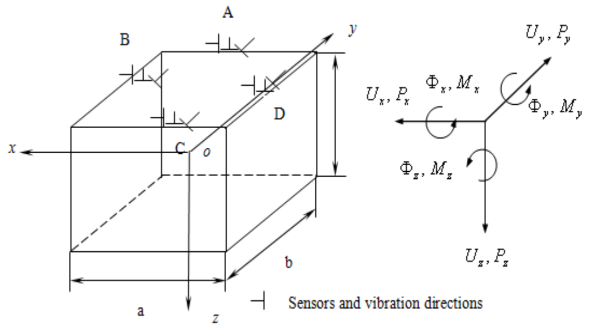

Fig. 1 Degree-of –freedom of a massive foundation vibration

Three-component vibration time-histories of point A, B, C, and D can be written as

$$\mathbf{U}_A(t) = \{u_{xA}(t),\ u_{yA}(t), u_{zA}(t)\}^T \tag{9}$$

$$\mathbf{U}_B(t) = \{u_{xB}(t),\ u_{yB}(t), u_{zB}(t)\}^T \tag{10}$$

$$\mathbf{U}_C(t) = \{u_{xC}(t),\ u_{yC}(t), u_{zC}(t)\}^T \tag{11}$$

$$\mathbf{U}_D(t) = \{u_{xD}(t),\ u_{yD}(t), u_{zD}(t)\}^T \tag{12}$$

Therefore, the mass center motion with the motions of the four points can be related as the following formula

$$\mathbf{U}_i(t) = \mathbf{T}_i \mathbf{U}(t) \ (i\text{=A, B, C, D}) \tag{13}$$

Where $\mathbf{T}_i$ is a transform matrix of motions between mass center and four points' time-history measured by vibration sensors,

$$\mathbf{T}_A = \begin{bmatrix} 1 & 0 & 0 & 0 & -l_1 \cos\theta_1 & 0 \\ 0 & 1 & 0 & a/2 & 0 & -a/2 \\ 0 & 0 & 1 & 0 & 0 & l_1 \sin\theta_1 \end{bmatrix} \tag{14}$$

$$\mathbf{T}_B = \begin{bmatrix} 1 & 0 & 0 & 0 & -h/2 & -b/2 \\ 0 & 1 & 0 & l_2 \cos\theta_2 & 0 & 0 \\ 0 & 0 & 1 & l_2 \sin\theta_2 & 0 & 0 \end{bmatrix} \tag{15}$$

$$\mathbf{T}_C = \begin{bmatrix} 1 & 0 & 0 & 0 & l_1 \cos\theta_1 & 0 \\ 0 & 1 & 0 & -a/2 & 0 & a/2 \\ 0 & 0 & 1 & 0 & 0 & -l_1 \sin\theta_1 \end{bmatrix} \tag{16}$$

$$\mathbf{T}_D = \begin{bmatrix} 1 & 0 & 0 & 0 & h/2 & b/2 \\ 0 & 1 & 0 & -l_2 \cos\theta_2 & 0 & 0 \\ 0 & 0 & 1 & -l_2 \sin\theta_2 & 0 & 0 \end{bmatrix} \tag{17}$$

And,

$$l_1 = \sqrt{\left(\frac{h}{2}\right)^2 + \left(\frac{a}{2}\right)^2}, \ \sin\theta_1 = \frac{a}{\sqrt{a^2 + h^2}}$$

$$\cos\theta_1 = \frac{h}{\sqrt{a^2 + h^2}} \tag{18}$$

$$l_2 = \sqrt{\left(\frac{h}{2}\right)^2 + \left(\frac{b}{2}\right)^2}, \ \sin\theta_2 = \frac{b}{\sqrt{b^2 + h^2}},$$

$$\cos\theta_2 = \frac{h}{\sqrt{b^2 + h^2}} \tag{19}$$

Equations (13) - (19) are 12 linear functions, only 6 unknown variables which can be solved by the least-square method as follows

$$\mathbf{D}\mathbf{U}(t) = \mathbf{w}(t) \tag{20}$$

$$\mathbf{D} = \begin{bmatrix} 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{a^2}{2} + 2l_2^2 & 0 & -\frac{a^2}{2} \\ 0 & 0 & 0 & 0 & \frac{h^2}{2} + 2(l_1 \cos\theta_1)^2 & \frac{bh}{2} \\ 0 & 0 & 0 & -\frac{a^2}{2} & \frac{bh}{2} & \frac{a^2}{2} + 2(l_1 \sin\theta_1)^2 + \frac{b^2}{2} \end{bmatrix} \tag{21}$$

And,

$$\mathbf{w}(t) = \begin{Bmatrix} x_1 + x_2 + x_3 + x_4 \\ y_1 + y_2 + y_3 + y_4 \\ z_1 + z_2 + z_3 + z_4 \\ \frac{a}{2} y_1 + l_2 \cos\theta_2 y_2 - \frac{a}{2} y_3 - l_2 \cos\theta_2 y_4 + l_2 \sin\theta_2 z_2 - l_2 \sin\theta_2 z_4 \\ -l_1 \cos\theta_1 x_1 - \frac{h}{2} x_2 + l_1 \cos\theta_1 x_3 + \frac{h}{2} x_4 \\ -\frac{b}{2} x_2 + \frac{b}{2} x_4 - \frac{a}{2} y_1 + \frac{a}{2} y_3 + l_1 \sin\theta_1 z_1 - l_1 \sin\theta_1 z_3 \end{Bmatrix} \tag{22}$$

Where the subscript i of x_i, y_i, z_i (i=1, 2, 3, 4) is the ith testing point and x_i, y_i, z_i are the corresponding time-histories.

III. COMPARISON OF VERTICAL DAMPING RATIO CALCULATING METHOD

A. Vertical damping ratio

Forced and decrement vibrations of a massive concrete foundation was measured to calculate the damping ratio of different methods above. The foundation is 8.0 meters long, 4.0 meters wide and 2.0 meters deep, about 155 tons (Fig. 2). The subsoil consists of no more than 1 meter miscellaneous fills and silty clay layer. The bottom of the foundation lies on silty clay. Static and dynamic experiments of some soil samples at 2.0 meters' deep from the ground surface have been done with water content about 14.7%, density 1.95t/m^3 and void ratio about 0.78.

Besides the 12 sensors as shown in Fig.1, a group of three-dimensional sensors also are installed at the machine pedestal to record velocity time-histories of the vibration resource. The sampling frequency is 1024 times per second, which satisfies sampling theorem. Type of the exciting pneumatic equipment is QJQ3-80, with gas resource is a potable air compression (Fig. 3). Totally 24 group of steady-state exciting, 12 group of decrement, and 3 group of tremor experiments have been tested. Time-histories of an exciting and a decrement test are shown in Figure 4 and Figure 5. Damping ratio results using *Code for Measurement Method of Dynamic Properties of Subsoil* (GB/T50269-97) and method of section 2 are listed in Table 2.

Fig. 2 Foundation appearance

Fig.3 Input system of forcing vibration

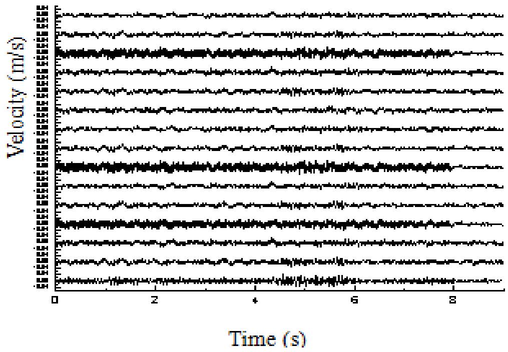

Fig. 4 Time-histories (Forcing vibration)

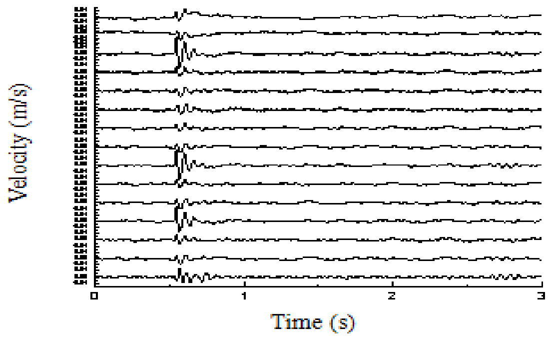

Fig. 5 Time histories (Free vibration)

Table 2 Comparison of vertical damping ration of subsoil

	The Code method (Eq. 1)	Approximate formulae (Table 1)	The code method results Eq. (4)-(6)	The code method Eq. (7)	Method of Section 2 Eq. (8)-(22)	
					Steady-state force	Impact force
Forced No. 1	0.241	0.641	0.008		0.015	
Forced No. 2	0.241	0.641	0.008		0.008	
Forced No. 3	0.241	0.641	0.010		0.015	
Decrement No. 1	0.241	0.641		0.174		0.179
Decrement No. 2	0.241	0.641		0.177		0.197
Decrement No. 3	0.241	0.641		0.170		0.166

From Table 2, the damping ratio is only about 0.179 using decrement force experimental results in the time domain and also, only about 0.01 using steady-state vibrations in the frequency domain, which if far less than that of the approximate formulae method based on the Chinese Code for Dynamic Machine Foundation Design (GB50040-96) (0.241) of Equations (1)-(3) and half-space approximate formulae result of Table 1(0.641). Using method of section 2, the results is a little larger.

It should be mentioned that the foundation size of this paper is larger than that of the Code requirement (2.0m×1.5m×1.0m), but another foundation's (1.4m×1.4m×0.8m) calculating results shown the same results as Table 2 lists.

B. An advantage of using motions of center of mass

Figure 6 is a direct record of a vertical sensor in an impacting experiment. It is difficult to determine number of decrement period and calculate the damping ratio using (7). However, based on method of (8)-(22), the time history of the same test (Fig. 7) is very suitable for calculating damping ratio in the time domain.

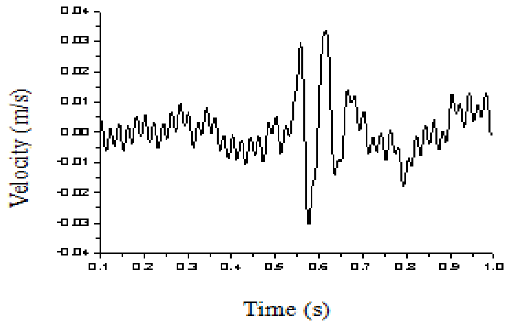

Fig. 6 Time-history of free vibration of original record

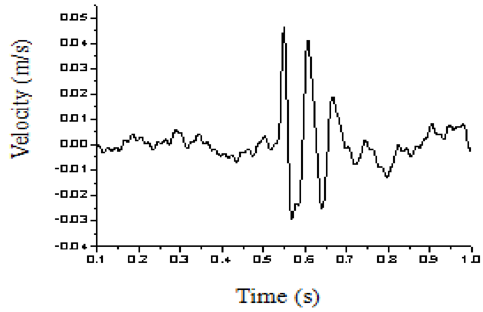

Fig. 7 Time-history of center of mass

IV. Conclusions

Vertical damping ratio testing method of a dynamic machine foundation is studied theoretically and experimentally based on the Chinese Code for Dynamic Machine Foundation Design (GB50040-96) and Code for Measurement Method of Dynamic Properties of Subsoil (GB/T50269-97). The damping ratio value of Code methods, both in the time domain method and the frequency domain, are far from that of the approximate formulae and elastic half-space analytical method. A method of calculating six-degree-of-freedom motion vector (three translational and three rotational components) of center of mass of a rigid foundation based on four three-component vibration records is presented to improve the calculating precision. The method is verified to be suitable for damping ratio analysis.

Furthermore theoretical and experimental research on subsoil damping ratio should be carried out to really resolve the problem of selecting a reasonable smoothing schedule to test and calculate subsoil damping ratio correctly.

References

[1] Chinese Building Code, GB50040-96, Code for dynamic machine foundation design, Chinese Plan Publisher, Beijing, 1996. (in Chinese)
[2] Chinese Building Code, (GB/T50269-97), Code for measurement method of dynamic properties of subsoil, Chinese Plan Publisher, Beijing, 1997. (in Chinese)
[3] Wang Xikang, "Development of discipline for vibration of machinery foundation in China and some problems being studied," Industrial Construction, vol. 37, no.6, pp. 83-90, 2007. (in Chinese)

[4] Richard, F. E. and Whitman, R. V., “Comparison of footing vibration tests with theory,” Journal of the Soil Mechanics and Foundations Division, ASCE, vol. 93, no.6, pp. 143~168, 1967.
[5] Chongmin Song, Wolf, John P, “The scaled boundary finite-element method-alias consistent infinitesimal finite-element cell method- for elastrodynamics,” Computer Methods in Applied Mechanics and Engineering, vol. 147, pp. 329-355, 1997
[6] Han Yingcai, “Study on the dynamic foundation design,” Journal of building structure, vol5, pp. 61-69, 1986. (in Chinese)
[7] Wang Zhenyu, Liu Jingbo, Pei Yuxiao, Zhang Kefeng, “Theoretical and experimental study on micro2vibration reduction of large dynamic machine foundation,” Chinese Journal of Geotechnical Engineering, vol. 24, no.3, pp. 363-366, 2002. (in Chinese)
[8] Chen Jiong, “The vertical vibration of dynamically loaded foundation,” China Civil Engineering Journal, vol. 35, no. 4, pp. 70-76, 2002. (in Chinese)

Hou Xingmin, born in Shandong Province, China in January, 1970. Ph. D and Professor of school of civil engineering, Yantai University. Prof. Hou received his doctoral degree in disaster prevention and mitigation engineering from Institute of Engineering Mechanics, China Earthquake Administration in July, 2002. Published more than 30 papers in civil engineering, technological engineering field.

Shi Xiangdong, born in Henan Province, China in November, 1967. Senior engineer of school of civil engineering, Yantai University. Received her Master’s degree in geotechnical engineering from Xi’an University of Technology in April, 1994.