Periodical character of failure near the openings on high depth

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Abstract — Rock mass failure on the high depth near the underground openings often has zonal character. The mechanism of this phenomena consist in the periodical character of stresses in surrounding rock mass and developing of tensile macrocracks at the places (zones) of maximum tangential stresses. Mathematical model of the high stressed rock mass is developed on base of the defect medium mechanics and nonequilibrium thermodynamics principals. The correspondence between the experimental research of faulted zonal structures near the high depths openings and mathematical model calculation is achieved. Relationships between the width of cracking zones and rock mass strength property have been determined.

Keywords— Zonal failure character, rock mass, mathematical model, defect medium mechanics, relationships.

I. Introduction

Failure conditions can take place in the boundary parts of the openings at the big depth of excavation and walls drilling. In some cases the failure has zonal character, where tensile macrocracks zones alternate with relative monolithic rock mass [1, 2]. Many attempts to describe zonal character of rock mass failure near the openings have been done based on classic mechanics [3, 4]. But no one could explain all the properties of zonal failure structure without introduction of new assumption to every new case.

In recent time new gauge theory have been applied to solid to describe the whirl fields of plasticity in high energy conditions [5]. Main principal of the gauge theory is unsatisfaction of the deformational compatibility conditions in solid. But it didn't apply to the zonal failure phenomena of rock mass near the openings. In this paper we demonstrate the example of the phenomena description on the way of the new gauge mathematical model. This description can be used in walls stability problem prediction in high depths conditions.

II. Mathematical model

The rock at a great depth is built by a faulted structure which is far from the state of thermodynamical equilibrium; it is in constant all-around compression. The boundary-value problem is assigned as a task concerning the stressed state of an imponderable plane which is a solid faulted structure with stresses given on a perpetuity that model a gravity field, and loosened by a round hole that models a unlined underground opening (Fig. 1). Due to the polar symmetry of the task the equilibrium equations are as follows:

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r}\left(\sigma_{rr} - \sigma_{\varphi\varphi}\right) = 0,$$
$$\sigma_{r\varphi} = 0, \quad r_0 \le r < \infty, \tag{1}$$

where σ_{rr} - normal radial stress, MPa; $\sigma_{\varphi\varphi}$ - normal tangential stress, MPa; $\sigma_{r\varphi}$ - shear stress.

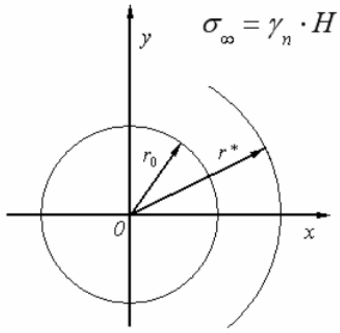

Fig. 1. Design scheme of an unlined opening task

At the opening boundary ($r = r_0$) and at infinity there are the following applied forces:

$$\sigma_{rr} = 0 \text{ at } r = r_0;$$
$$\sigma_{rr},\ \sigma_{\varphi\varphi} \to \sigma_\infty \text{ at } r \to \infty, \tag{2}$$

where $\sigma_\infty = \gamma_n \cdot H$, γ_n - rock density, H/ m^3; H – opening depth, m.

The rock mass at a great depth is modeled by the structure, where commonly the conditions of deformations compatibility ε_{ij} are not met:

$$R = \frac{\partial^2 \varepsilon_{11}}{\partial x_2^2} - 2\frac{\partial^2 \varepsilon_{12}}{\partial x_1 \partial x_2} + \frac{\partial^2 \varepsilon_{22}}{\partial x_1^2} \neq 0. \tag{3}$$

Fault parameter is according to the equation [10]:

$$\Delta^2 R - \gamma^2 R = 0, \tag{4}$$

Δ - Laplace operator; γ - model parameter.

As the task is flat and axis-symmetrical, the equation (4) in polar coordinates will become:

$$\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}\right)^2 R=\gamma^2 R \qquad (5)$$

The solution to the equation (5) decreasing at $r\to\infty$ is:

$$R(r)=aJ_0\left(\sqrt{\gamma}\, r\right)+bN_0\left(\sqrt{\gamma}\, r\right)+cK_0\left(\sqrt{\gamma}\, r\right), \qquad (6)$$

where J_0, N_0, K_0 – Bessel, Neumann, Macdonald functions of zero order, accordingly.

III. Nonclassical boundary conditions and task solution

At the opening boundary the rock mass undergoes considerable destruction, therefore fault parameter R should not equal zero. Assuming that all zones of rock destruction are equivalent and are of the same origin, we introduce the extremum of function condition in the boundary $R(r)$ and the following zones of destruction. Therefore boundary conditions for the function $R(r)$ are:

$$R^{/}(r)\Big|_{r=r_0}=0, \qquad R^{/}(r)\Big|_{r=r^*}=0, \qquad (7)$$

where r^* is determined experimentally.

The equation for the first invariant of stresses $\sigma=\sigma_{zz}+\sigma_{rr}+\sigma_{\varphi\varphi}$ is:

$$\Delta\sigma=\frac{E}{2(1-\nu)}R,\ \sigma\to 2(1+\nu)\sigma_\infty,\ r\to\infty, \qquad (8)$$

with determined function R, where E – is modulus of elasticity, ν - Poisson ratio.

Solving the task (8) gives the formulas for stress components:

$$\sigma_{rr}=\sigma_\infty\left(1-\frac{r_0^2}{r^2}\right)-\frac{E}{2\left(1-\nu^2\right)\gamma^{3/2}}\cdot \quad ;$$
$$\bullet\frac{1}{r}\left[aJ_1\left(\sqrt{\gamma}\cdot r\right)+bN_1\left(\sqrt{\gamma}\cdot r\right)+cK_1\left(\sqrt{\gamma}\cdot r\right)\ \right]$$

$$\sigma_{\varphi\varphi}=\sigma_\infty\left(1+\frac{r_0^2}{r^2}\right)-\frac{E}{2\left(1-\nu^2\right)\gamma}\bullet$$
$$\bullet\left[aJ_0\left(\sqrt{\gamma}\cdot r\right)+bN_0\left(\sqrt{\gamma}\cdot r\right)-cK_0\left(\sqrt{\gamma}\cdot r\right)\ \right]+$$
$$+\frac{E}{2\left(1-\nu^2\right)\gamma^{3/2}}\cdot \quad , \qquad (9)$$
$$\bullet\frac{1}{r}\left[aJ_1\left(\sqrt{\gamma}\cdot r\right)+bN_1\left(\sqrt{\gamma}\cdot r\right)+cK_1\left(\sqrt{\gamma}\cdot r\right)\right]$$

where r - distance from the center of the opening to a selected point in the rock mass.

IV. Stress calculation and criteria of failure

Stresses surrounding the opening are of oscillating character (Fig. 2).

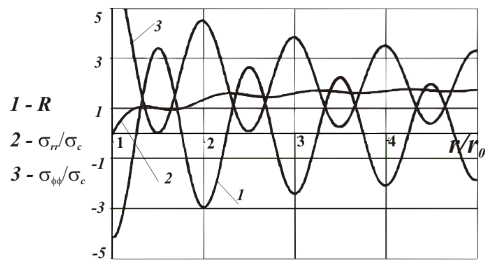

Fig. 2. Oscillating character of the stresses and R - functions in the rock mass surrounding the underground opening

Failure zones appear in the areas where the conditions of cracking at compression are met:

$$K_I=\left(\pi\cdot l\right)^{1/2}\cdot\left(\gamma_1\cdot\sigma_1^0-\gamma_3\cdot\sigma_3^0\right)\ \geq\ K_{Ic}, \qquad (10)$$

where l -is half the length of fracture faults of the rock mass which is assumed equal to the minimum half length of unstable in stress conditions tensile macrocrack, m; σ_1^0, σ_3^0 -- - max and min of major stresses, accordingly, MPa; γ_1, γ_3 - empirical factors; K_I – coefficient of stress intensity, MPa $*$m$^{1/2}$; K_{Ic} - fracture toughness of rock material, MPa $*$m$^{1/2}$.

As a criterion function the following dependence is taken:

$$K(r)=K_I/K_{Ic} \qquad (11)$$

At K_I/K_{Ic}<1 there is no failure around the opening; at K_I/K_{Ic}>1 the fracture start to appear. Criterion function as well as stress function and R have an oscillating character (Fig. 3). The comparison of the results of the analytical and experimental studies shows their good convergence (Table 1).

The research of zonal failure of rock mass was carried out for an unlined opening. In order to run calculations the algorithms and programs were developed that included formulas for calculating the formulas of defectiveness $R(r)$, stress and criterion function $K(r)$.

Table 1.
Comparison of the theoretical and experimental [1] results (unlined opening)

Parameter	Method	Elements of zonal structure			
		1st zone of failure	2nd zone of failure	3rd zone of failure	4th zone of failure
1. Location of the further zone boundary, r/r_0	Experiment	1,03	2,23	3,40	4,54
	Theory	1,28	2.17	3,09	3,97
	Deviation, %	24,3	-2,7	-9,1	-12,6
2. Relative critical zone stresses, σ/σ_c	Experiment	1,1	2,2	2,7	-
	Theory	0,95	2,1	3,1	-
	Deviation, %	-13,9	-4,5	14,8	-

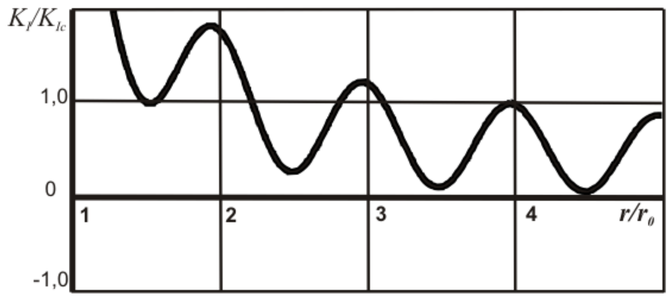

Fig. 3. Character of criterion function

The research of zonal failure of rock mass was carried out for an unlined opening. In order to run calculations the algorithms and programs were developed that included formulas for calculating the defectiveness function *R(r)*, stresses and criterion function $K_r(r)$. On the basis of the developed programs the experiment was carried out, the parameters of three types being analyzed. The first type included parameters of model γ, c, which are determined from the experiment taking into account the all-around compression at a high depth. The second type includes parameters that characterize mechanical properties of rock mass: E - modulus of elasticity, MPa; **ν**- Poisson ratio; σ_c - uniaxial compression strength, MPa; and also the value of gravitational stresses in the rock massσ_∞, MPa. The third type includes parameters in the formula (10) which characterize the cracking structure of rock mass: half length of rock fracture faults - l; fracture toughness of rock material - K_{Ic}, and also coefficients γ_1, γ_3 (further the dependence γ_3/γ_1 is used).

In order to select the parameters stated above six programs were developed. They are: model parameters selection programs (Al, A2); programs for determination of the last destruction zone (for lined and unlined openings — B1, B2); programs for calculation of radial length of fracture zones (for lined and unlined openings Cl, C2).

Program charts with a brief description were developed. The patterns of the changes of zonal structure of rock mass failure depending from various factors were obtained. The main parameters of zone structure were singled out: number of zones of failure, location of a furthest from the opening boundary fracture zone (the last zone of failure); relative critical stresses of failure zones creation, and also the value of radial length of failure zones.

V. Results of research

As a result of the modeling experiment on the basis of the adopted mathematical model we determined that parameters of zonal structure depend slightly on the values of the elastic modulus of rocks E and Poison ratio v. This conclusion corresponds to the data obtained during laboratory studies (at variation of elasticity for 10 times critical stresses of zone creation change for 2-5 % on average). The research of fracture zones was carried out for rather solid rock (σ_c =150 MPa) and for weak rock (σ_c = 15 MPa).

It was determined that the basic factor that influences the parameters of zonal failure structure is the value of stresses that act within the rock mass (opening depth). With the increase of stresses the number of failure zones raises, their radial length increases until it reaches neighboring zones. At that as closer the zone is located to the opening boundary as faster the process. The boundary of the last zone of failure moves further into the rock mass (Fig. 4, left).

Parameters of fracture structure of the rock mass also influence the character of zone failure. Radial length of the zones of failure decreases if the rock fracture toughness rises (Fig. 4, right). When the rock fracture toughness decreases zones of failure appear at lower relative stresses and the distance of the last failure zone from the opening boundary increases. The activity of rock destruction also influences greatly the parameters of zone failure structure. At the increase of the fracture faults length the radial length of failure zones increases. This parameter decreases if the

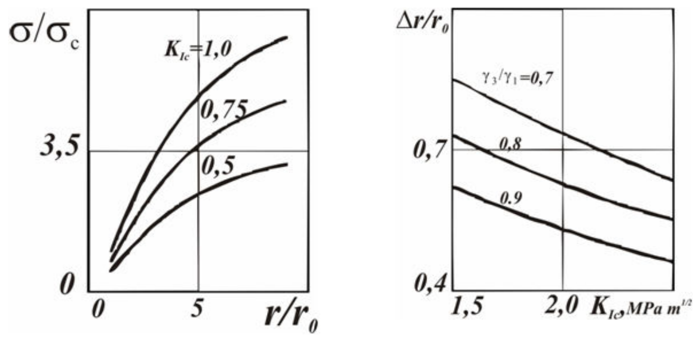

Fig. 4. Dependence of the last failure zone position from the stresses acting within the rock mass (left) and the radial length one of the first zone of failure from the rock fracture toughness (right)

dependence γ_3 / γ_1 rises. Regularities determined for weak rock mentioned above are true for solid rock also.

VI. Conclusion

Thus, the research conducted shows that as the depth of the opening rises the zonal character of rock failure becomes more expectable which should be taken into consideration when designing a lining for such conditions.

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VII. References

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