Computational Mechanics Analysis of Enneper Surface in Tensioned Fabric Structures

Hooi Min Yee and Mohd Nasir Abdul Hadi

Abstract— Nonlinear analysis method is one of the earliest methods proposed for form-finding analysis of tensioned fabric structures. However due to some inherent weaknesses, the method has not been fully developed. Computational strategies for form-finding analysis of tensioned fabric structure using the nonlinear analysis method has been proposed in 2011. The computational procedures based on nonlinear analysis method in combination with properly formulated computational strategies can be presented as a general tool capable of form-finding of TFS. It has the added advantage that the program used to solve the initial equilibrium problem can also be used for further stress analysis. Numerical and experimental form-finding of tensioned fabric structure in the form of Enneper minimal surface with variables $u=v=0.4$ and $u=v=1.0$ have been investigated. In this study, form-finding using nonlinear analysis method and soap film is used to determine the surface form corresponding to a boundary shape defined by the mathematical equation for Enneper minimal surface with variables $u=v=0.4$ and $u=v=1.0$. It provides an alternative choice for civil engineer to consider the Enneper minimal surface, $u=v=0.4$ and $u=v=1.0$ to be applied in tensioned fabric structure. Such in-sight will lead to improvement of rural basic infrastructure, economic gains, sustainability of built environment and green technology initiative.

Keywords— Nonlinear analysis method, Form-finding and minimal surface

I. INTRODUCTION

Tensioned Fabric Structure (TFS) is a suitable structure to be used for long span application. TFS also known as tensioned membrane structure. [1] have mentioned that structures are built to last a long time. Actually, TFS has been used over the past 50 years ago. [2] have stated there is some particular information about the development of a new sort of a temporary modular steel footbridge for pedestrians and cyclist, which was designed as a truss system with the deck below the supports and with the closed cross-section. [3] have stated researches in course of development in the field of

Hooi Min Yee is with the Faculty of Civil Engineering, Universiti Teknologi MARA, 13500 Permatang Pauh, Pulau Pinang, Malaysia (e-mail: yhooimin@yahoo.com).
Mohd Nasir Abdul Hadi is with the Faculty of Civil Engineering, Universiti Teknologi MARA, 13500 Permatang Pauh, Pulau Pinang, Malaysia (e-mail: nasayz@yahoo.com).

protection of new and existing structures subject to dynamic events. [4] has stated the structure of TFS can be composed of fabric structural as structural members. The material is usually used is fabric, fabric is used to joined together with the structure. Fabric is tensioned through with the cable to a rigid supporting system to typically and to provide a roofing structure.

Enneper minimal surface is suitable used for TFS. The first step for structural analysis in TFS is form-finding. Form-finding is to determine the initial equilibrium shape under pre-stress pattern and boundary condition. [4] has proposed nonlinear analysis method for form-finding of tensioned fabric structures in the form of Enneper minimal surface. Applicability of the computational strategies proposed by [4] has been verified by form-finding carried out models of tensioned fabric structures in the one of the form of Enneper minimal surface [5]. Convergence was able to achieve in form-finding of Enneper TFS models. [4] showed that when $u = v \leq 0.87$, the converged shape is found to be similar to the mathematical shape. For Enneper TFS model ($u = v \geq 0.88$), the shape obtained after form-finding are found to be different from the form of Enneper surface defined by mathematical shape equation. [4], [5], [6], [7] and [8] has carried out form-finding using nonlinear analysis method also in Catenoid, Helicoid, Scherk, Enneper, Oval and Moebius Strip TFS models. References [9]-[12] have studied the problem of mathematical modeling in the industry.

In this study, only form-finding using nonlinear analysis method of Enneper minimal surface with variables $u = v = 0.4$ and $u = v = 1.0$ have been carried out.

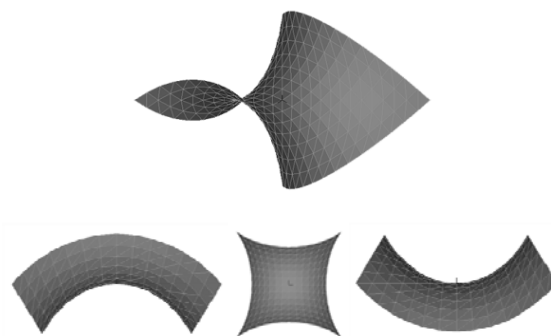## II. GENERATION OF ENNEPER MINIMAL SURFACE IN TFS

Fig. 1 Different views for Enneper minimal surface

Fig. 1 shows Enneper minimal surface. The boundary of Enneper minimal surface in Fig. 1 can be obtained using Eq. 1 [13].

$$X = u - \frac{u^3}{3} + uv^2$$

$$Y = -v + \frac{v^3}{3} - vu^2$$

$$Z = u^2 - v^2 \tag{1}$$

For u and v = variables

From this study, the software ADINA [14] has been used for the purpose of model generation. Aspect of modeling of surface of Enneper minimal surface and form as well pre-stress pattern of the resulting TFS through form-finding using nonlinear analysis method are studied.

III.   NUMERICAL METHOD USING NONLINEAR ANALYSIS METHOD

[4] stated the principle of nonlinear analysis method is based on the large displacement finite element formulation used for analysis of structural behaviour under external loads. Since the method can be used for both the initial equilibrium problem and load analysis, the approach using nonlinear analysis is quite common. The basic equation used is expressed as follows:

$$({}_0^t K_L + {}_0^t K_G)u = {}^{t+\Delta t}F - {}_0^t f \tag{2}$$

Where ${}_0^t K_L$ is linear strain incremental stiffness matrix, ${}_0^t K_G$ is nonlinear strain incremental stiffness matrix, ${}_0^t f$ is vector internal forces, ${}^{t+\Delta t}F$ is load vector and u is vector of increment in displacement.

A nonlinear finite element analysis program by [4] for the analysis of tensioned fabric structures has been used in this study. The procedure adopted is based on the work by [4]. 3-node plane stress element has been used as element to model the surface of TFS. All x, y and z translation of nodes lying along the boundary edge of the Enneper minimal surface have been restrained. The member pretension in warp and fill direction, is 2000N/m, respectively. The shear stress is zero.

Two stages of analysis were involved in the procedures of form-finding in one cycle proposed by [4]. First stage (denoted as SF1) is analysis which starts with an initial assumed shape in order to obtain an updated shape for initial equilibrium surface. The initial assumed shape can be obtained from any pre-processing software and reference [4] is chosen for this study. This is then followed by the second stage of analysis (SS1) aiming at checking the convergence of updated shape obtained at the end of stage (SF1). During stage (SF1), artificial tensioned fabric properties, E with very small values are used. Both warp and fill tensioned fabric stresses are kept constant. In the second stage of (SS1), the actual values of tensioned fabric properties are used. Resulting warp and fill tensioned fabric stresses are checked at the end of the analysis against prescribed tensioned fabric stresses. Then, iterative calculation has to be carried out in order to achieve convergence where the criteria adopted is that the average of warp and fill stress deviation should be < 0.01. The resultant shape at the end of iterative step n (SSn) is considered to be in the state of initial equilibrium under the prescribed warp and fill stresses and boundary condition if difference between the obtained and the prescribed membrane stresses relative to the prescribed stress is negligibly small. Such checking of difference in the obtained and prescribed stresses has been presented in the form of total stress deviation in warp and fill direction versus analysis step. As a first shape for the start of form-finding procedure adopted in this study, initial assumed shape is needed. For the generation of such initial assumed shape, knowledge of the requirement of anti-clastic nature of TFS is used. The incorporation of anti-clastic feature into the model will help to produce a better initial assumed shape.

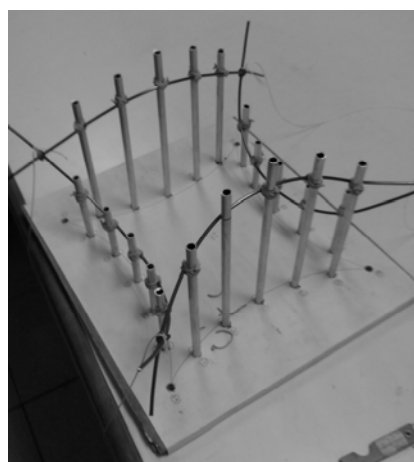### IV.   EXPERIMENTAL METHOD USING SOAP FILM

Fig. 2 Soap film model

The experimental soap film model as shown in Fig. 2 is based on [4] work. The material and equipment have been used in the project is steel, aluminium wires, plywood, super glue, rubber band, wire, glycerin, concentrated car detergent, distilled water, theodolite, plumb bob and camera.

The boundary frame on the surface has been built based on the coordinate. The coordinates has been calculated using mathematical equations. Plywood has been used as x-coordinate and y-coordinate of the boundary, standing steel rods has been used to support the wire frame at the desired height corresponding to z coordinate of the wire frame. Aluminum wires have been used to build the boundary of models. Super glue has been used to fix the steel and plywood. The rubber band has been used to secure the wire to steel rods. The wire has been tied to the steel rod at specified

coordinates with rubber bands in order to produce the desired boundary defined by equations Enneper minimal surface. Then, glycerin, concentrated car detergent and distilled water used to prepare a soap solution. The preparation of soap solution has been contained the composition of water, detergent and glycerin. The composition used is 25.7% of glycerin, 22.8% of concentrated car detergent and 51.1% of distilled water.

Fig. 3 Experiment Setup

Experimental setup form-finding using soap film model has been carried out as shown in Fig. 3. In this experimental setup contain theodolite, hydraulic jack, container with soap solution, steel frame and physical model. Soap film model has been used to determine the surface form corresponding to the boundary shape defined by mathematical equations for Enneper minimal surface in detail.

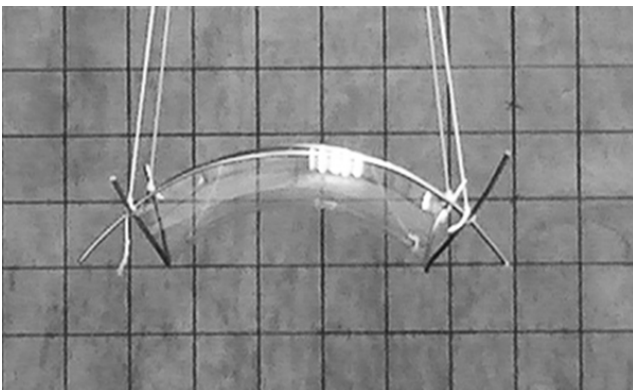## V. COMPARISON BETWEEN COMPUTATIONAL AND EXPERIMENTAL RESULTS

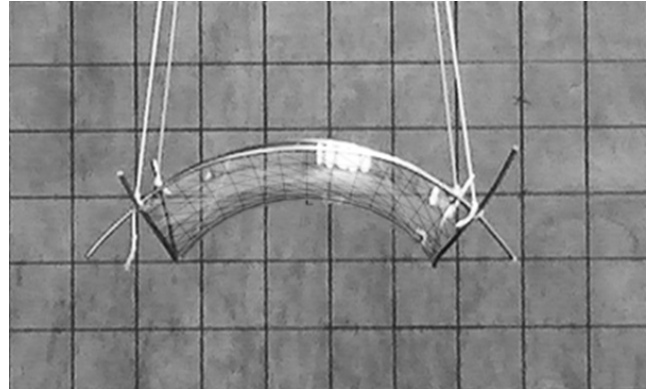Fig, 4   Soap film model, $u=v=0.4$

Fig. 5 Comparison of experimental and computational results of Enneper minimal surface, $u=v=0.4$
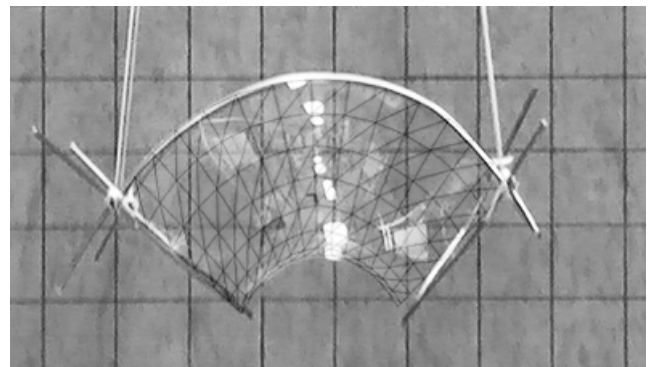

Fig. 6 Comparison of experimental and computational results of Enneper minimal surface, $u=v=1.0$

Fig. 4 shows the soap film model of Enneper minimal surface, $u=v=0.4$. Fig. 5 and Fig. 6 show the comparison between experimental and computational result of Enneper minimal surface, $u=v=0.4$ and $u=v=1.0$. The result is found to be in close agreement. The geometry between experimental and computational result has been found to match very closely.

VI. CONCLUSION

Soap film model with surface shape in close agreement with mathematically defined Enneper surfaces, $u=v=0.4$ and $u=v=1.0$ have been produced. Enneper surfaces, $u=v=0.4$ and $u=v=1.0$ provide an alternative choice for civil engineer to consider the tensioned fabric green structure in the form of Enneper minimal surface. The Enneper surfaces, $u=v=0.4$ and $u=v=1.0$ would enhance the understanding on the suitable choice of Enneper minimal surface for TFS among structural designer. The proposed computational strategies in 2011 can be presented as a general tool capable of form-finding analysis of tensioned fabric structures for structural engineer.

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Hooi Min YEE borned in Sungai Petani, Kedah, Malaysia. She was awarded a Universiti Sains Malaysia Fellowship to study for doctorate degree in Structural Engineering in Universiti Sains Malaysia. She graduated with a Structural Engineering in 2011, Master of Science Engineering in 2007 and Bachelor of Engineering (Honours) Civil Engineering in 2006 from the Universiti Sains Malaysia, Pulau Pinang, Malaysia.

During her PhD study, she worked as fellow at Universiti Sains Malaysia. After graduating, she worked as Assistant Professor at Faculty of Civil Engineering, Universiti Teknologi MARA, Pulau Pinang, Malaysia. Her research interest is Computational Mechanics, Computational Analysis of Shell and Spatial Structures, Nonlinear Analysis and Architectural Engineering.

Ir. Dr. Yee is a Professional Engineer of Board of Engineers Malaysia, Institution of Engineers Malaysia, Construction Industry Development Board Malaysia and Concrete Society of Malaysia. Ir. Dr. Yee is a committee member in The Institution of Engineers, Malaysia (Penang Branch) and also as a member of evaluation panel of Engineering Accreditation Council. Ir. Dr. Yee has obtained 9 awards in Invention, Innovation and Research Design Platform and has published 49 up-to-date publications.

Mohd Nasir Abdul HADI borned in Kuala Berang, Terengganu, Malaysia, graduated in Bachelor of Engineering (Honours) Civil Engineering in 2014 from the Universiti Teknologi MARA, Pulau Pinang, Malaysia. He is now further his master postgraduate study in the Universiti Teknologi MARA, Pulau Pinang, Malaysia. During his master study, he worked as research assistant at Universiti Teknologi MARA, Pulau Pinang, Malaysia.