

Numerical model of generating of the surface that borders a hidden volume

NICOLAE DOBRIȚOIU

Department of Management, University of Petroșani ROMANIA
(dobritoiu_n_2001@yahoo.com)

Abstract: – In the practical activity, especially in the knowledge activity of a deposit, one encounters the situation of determination of the form of the surface that borders a reserve unit of useful mineral substances.

The knowledge of the form of a surface that borders a reserve unit allows its geometric parameters determination and the calculation of the volume of reserve unit. These informations are used in the decision process of mine working of the deposits of the useful mineral substances.

Key-Words: – Knowledge points, contour line, surface, volume, tangentoid triangle, center of gravity.

I. AN ALGORITHM FOR DRAWING A CONTOUR LINE THAT BORDERS A PLANE SURFACE BY INCREASING THE NUMBER OF KNOWLEDGE POINTS

A. Introduction

The idea of an algorithm for drawing the contour line which borders a surface from a horizontal plane, starts from the remark below.

Remark 1 In a set of points from space, each point has a vicinity and every vicinity describes a geometric figure and each of these geometric figures has a center of gravity.

Using this remark, the new method of drawing of the contour lines that border various sections used in precise purposes, allows the drawing of these lines using the new proposed algorithm. Thus one eliminates the drawing approximations of the contour lines that are used to determine the areas of the sections or the surfaces which form the faces of a volume.

B. Problem Formulation [4]

Let M_i ($i=1,n$) be a set of points from the contour of a surface, from a plane. The distance between these points is variable.

Now, we formulate the following problem:

It is required to increase the number of these points such that the irregular polygon that results binding the points M_i can be approximated to another irregular polygon with a large number of sides, which can be considered a regular or an irregular curve.

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N. Dobrițoiu is from the Department of Management, University of Petroșani, Romania, (e-mail: dobritoiu_n_2001@yahoo.com).

C. Algorithm description [4]

The proposed algorithm has the following steps:

a) By using the coordinates, one defines the knowledge points from a plane which are on the contour that border the surface of the section;

b) The rotation of the plane in space with the slope angle of the plane, in order to be parallel with the plane xOy ;

c) Determining the binding order of the knowledge points in a contour and the determining of the advancing way of the contour, (Fig. 1), which will be clockwise.

Considering the contour types, we have the following cases:

1) When the contour line that borders a surface is closed (Fig. 1);

2) When the contour line of a surface is not closed.

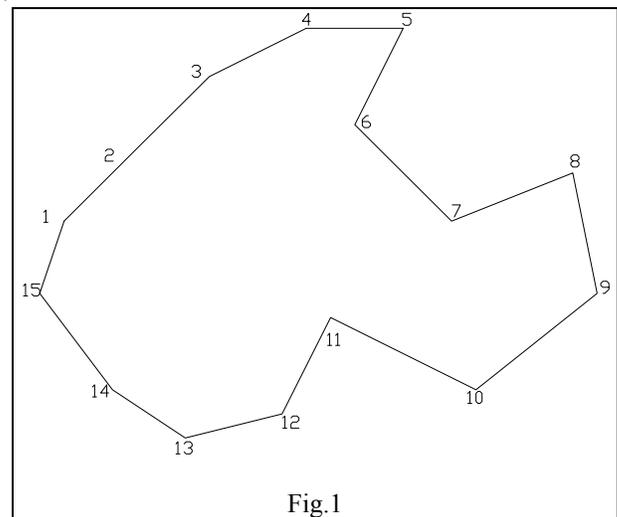


Fig.1

Remark 2 In the second case, the number of the points for the first interval $[M_1, M_2]$ and the last interval $[M_{n-1}, M_n]$ can't increase, because each of the points M_1 and M_n are influenced by just one point (M_2 for M_1 and M_{n-1} for M_n).

d) Writing the equations of the straight lines that are determined by two successive points, with the relation:

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1};$$

e) Determining the values of the angles formed with the segments which have a common knowledge point (using clockwise we will determine the angles to the right of this sense), with the relation:

$$\operatorname{tg}\theta = \frac{a_2 - a_1}{1 - a_1 a_2}$$

where a_1, a_2 are the slope of the two straight lines which form the angle:

$$y = a_1 x + b_1$$

$$y = a_2 x + b_2$$

Considering the values of these angles, we have the following situations:

- parts of the contour that contain only the angles $< 180^\circ$, (2-3-4-5-6; 7-8-9-10-11; 11-12-13-14-15);
- parts of the contour which contain only the angles $> 180^\circ$, (5-6-7-8; 10-11-12);
- parts of the contour which contain an angle $> 180^\circ$ and another angle $< 180^\circ$, (6-7-8, 7-8-9; 10-11-12, 11-12-13);
- parts of the contour that contain an angle $< 180^\circ$ and another angle $> 180^\circ$, (4-5-6, 5-6-7; 9-10-11, 10-11-12);
- parts of the contour with the angle $= 180^\circ$, (1-2-3).

f) Using the angles formed by the segment $[M_i M_{i+1}]$ with the segments $[M_{i-1} M_i]$ and $[M_{i+2} M_{i+1}]$, we construct the tangential triangles.

Considering the values of these angles, we have the following situations:

-situation 1, (Fig. 2), one refers to the contour parts which contain only the angles $< 180^\circ$ (in Fig. 2, the angles with the vertexes in the points 3, 4, 5 are the angles $< 180^\circ$). In this case, the tangential triangles are constructed on the segments $[3-4]$ and $[4-5]$.

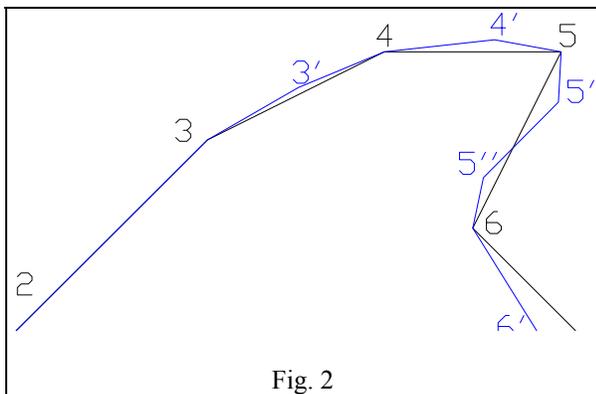


Fig. 2

A tangential triangle for a segment $[M_i M_{i+1}]$, it is constructed as follows:

- the segments $[M_{i-1} M_i]$ and $[M_{i+2} M_{i+1}]$ are extended;
- the bisectors of the angles which are resulted by the extension of the segments $[M_{i-1} M_i]$ and $[M_{i+2} M_{i+1}]$ with the segment $[M_i M_{i+1}]$, are traced; thus, it is obtain the tangential triangle;

-the coordinates of the intersection point T_i , between the two bisectors are calculated.

The equation of the bisector of an angle is:

$$\frac{A_1 x + B_1 y + C_1}{\sqrt{A_1^2 + B_1^2}} = \pm \frac{A_2 x + B_2 y + C_2}{\sqrt{A_2^2 + B_2^2}}$$

In order to determine the coordinates of the center of gravity G_i , of a tangential triangle $M_i M_{i+1} T_i$ one uses the relations:

$$x_g = (x_1 + x_2 + x_3) / 3$$

$$y_g = (y_1 + y_2 + y_3) / 3$$

$$z_g = (z_1 + z_2 + z_3) / 3$$

In the case of the tangential triangle (Fig. 2) of the segment line $[3-4]$ we obtain the point $3'$ and for the tangential triangle of the segment line $[4-5]$ we obtain the point $4'$. The contour line of the contour part 3-4-5 will contain these points.

-situation 2, one refers to the contour parts which contain only the angles $> 180^\circ$ (in Fig. 3 the angles with the vertexes in the points 6 and 7 are angles $> 180^\circ$).

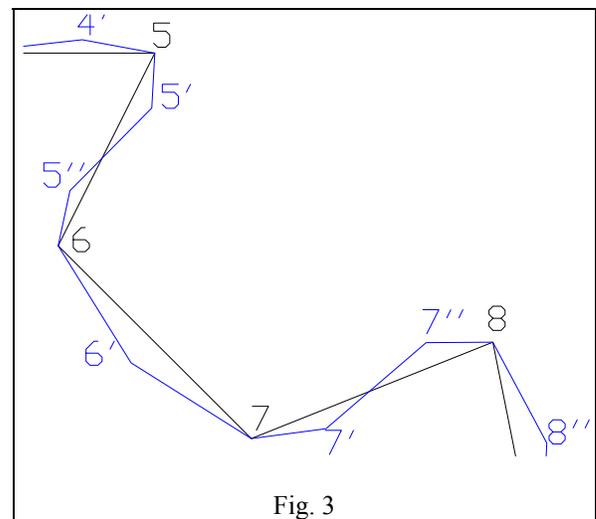


Fig. 3

In this situation, the tangential triangle is constructed in the same way as in situation 1, but it is inverse positioned with respect to the segment (if the triangle is situated at the on of the segment, in the situation 1, then will be to the right of the segment, in the situation 2). In Fig. 3 the tangential triangle is constructed to the right of the segment 6-7, and the resulted center of gravity is the point $6'$. The contour line which contains the segment $[6-7]$, also contains this point.

Remark 3 In the situation 1 the center of gravity of the tangential triangle is a point that will be situated to the left of the advancing way of the contour. In the situation 2 the center of gravity of the tangential triangle is a point which will be situated to the left of the advancing way of the contour.

-situation 3, (Fig. 4), one refers to the contour parts which have the first angle $> 180^\circ$ (the angle with the vertex in the point 7) and the second angle $< 180^\circ$ (the angle with the vertex in the point 8). In this case, the both tangential

triangles from the situations 1 and 2, will be constructed. By the intersection of the bisectors of the angles which are formed in the two points of the segment $[M_i M_{i+1}]$, a tetragon will be obtained. By drawing the second diagonal in this tetragon, four triangles are obtained, two to the left of the segment 1 and two to the right of the segment with respect to the sense of the binding of knowledge points situated on the contour line. Now, we will determine the center of gravity G_{id} for the triangle, which has a vertex in the point M_i , and is situated to the right of the segment $[M_i M_{i+1}]$. Also, we will determine the center of gravity G_{is} for the triangle which has a vertex in the point M_{i+1} , and is situated to the left of the segment $[M_i M_{i+1}]$. The first point will be situated on a convex contour, and the other point will be situated on a concave contour with respect to the segment $[M_i M_{i+1}]$. In the example from Fig. 4 it is determine the coordinates of the points $7'$ and $7''$ which are on the contour line between the points 7 and 8.

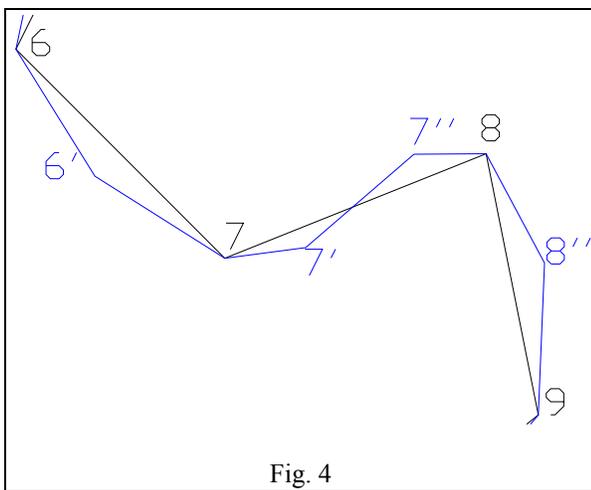


Fig. 4

Remark 4 In this case, we obtain two points that belong to the contour line that is plotted between the two research points.

–situation 4, (Fig. 5), one refers to the contour parts that have an angle $< 180^\circ$ (in Fig. 5, the angle with the vertex in the point 10) and another angle $> 180^\circ$ (the angle with the vertex in the point 11). This situation is similarly with the situation 3, but the first determined point ($10'$) will be situated on a convex curve (i.e. is situated to the left of the segment $[10-11]$), and the second determined point ($10''$) will be situated on a concave curve (i.e. will be situated to the right of the segment $[10-11]$).

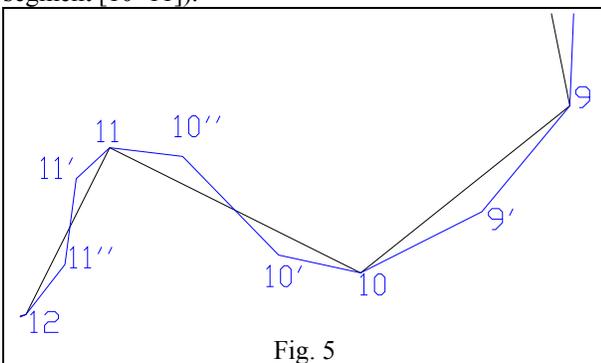


Fig. 5

g) The binding of all points from the set $P_i = M_i + G_i$, and thus results a new contour which has at least the double number of corners, Fig. 6.

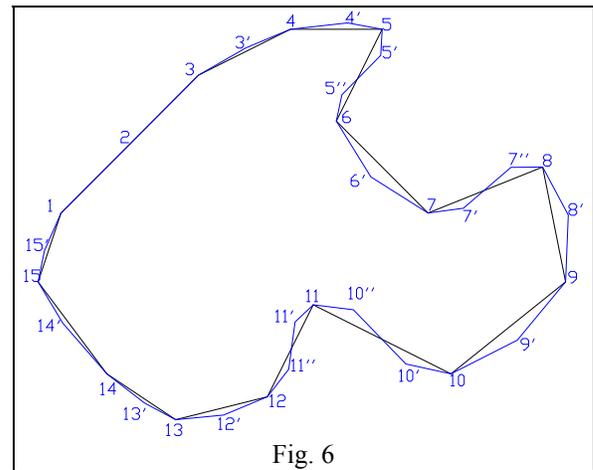


Fig. 6

The algorithm is iterative and it stops when there is obtained a proper density of points. In Fig. 7 we present iteration 3.

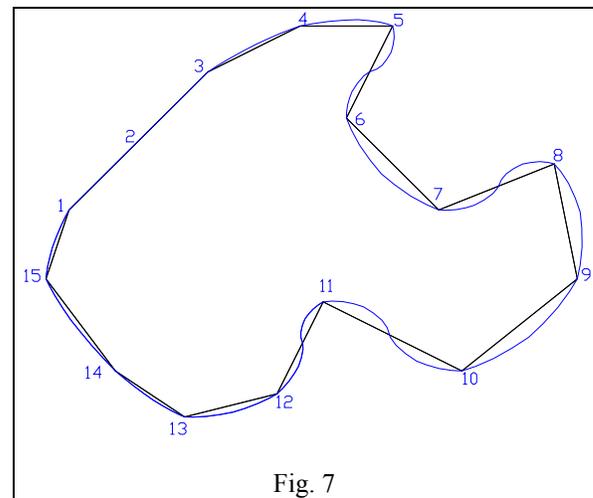


Fig. 7

II. MODEL FOR GENERATING HIDDEN SURFACES

This model is based on the method of modelling known as “the method of sections (vertical parallel, vertical unparallel, horizontal)”. The difference between the new numerical model proposed here and the two known models consists in the way of tracing the contour line setting the limits to a plane section.

Since “the algorithm for plotting a contour limiting a horizontal and irregular plane surface when a certain number of points it crosses is known” [3], which underlies the proposed numerical model, implies the plotting only of contours limiting horizontal plane surfaces, for plotting a contour limiting a certain plane surface, the following algorithms will be used:

- the algorithm for translating a system of axes in space;
- the algorithm for rotating a system of axes in space;

-the algorithm of permutation of the coordinates of a point $M(x_i, y_i, z_i)$;

-the algorithm “Determining the number of contours resulting from the intersection of a certain plane with a unit of field traced by the lines of the contours of the horizontal sections” [1];

-the algorithm for tracing the line of a contour limiting a surface situated on a vertical plane by making denser the points of knowledge which it crosses [1];

-the algorithm for determining the points of knowledge which are crossed by the contour line of a section resulting from the intersection of a reserve unit, traced by horizontal sections, with a certain plane [1];

-the algorithm for tracing the contour of a certain surface in space [1].

By combining these algorithms one can determine the shape of any contour limiting a certain section made through a unit of reserve belonging to a field.

III. APPLICATIONS OF THE MODEL FOR GENERATING HIDDEN SURFACES

As a result of geological survey carried out by means of drilling, was found a body of ore belonging to a field. The results obtained and interpreted by specialists in the field were systematized in 7 data files. A data file comprises the points of knowledge ordered in the sense of linking of the points limiting the contour of a horizontal section carried out at a certain height.

The 7 files are rendered graphically in fig. 8 [2].

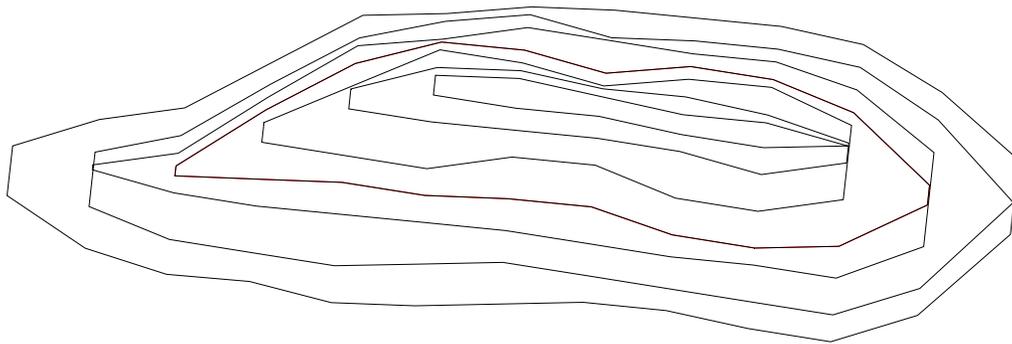


Fig. 8. The level curves obtained from the graphical representation of the data files obtained from topographical measurements

By applying “the algorithm for plotting a contour limiting a horizontal and irregular plane surface when a certain number of points which are crossed by it is known” to the 7 files, after

three iterations, 7 more files are obtained and they are shown in fig. 9.

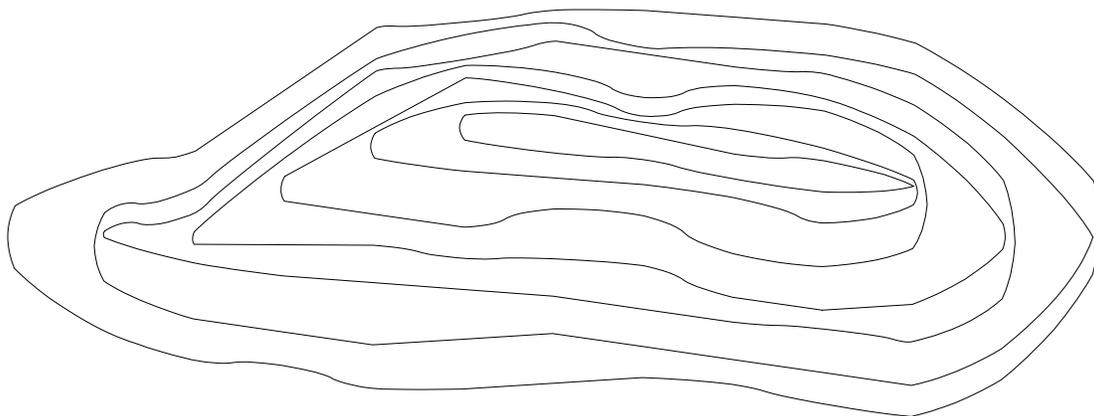


Fig. 9. The curves obtained from the graphical representation of the point files after iteration 3

Further on, in order to have a clearer view of the surface covering the unit of field, the surface of the unit of field in fig. 10 will be intersected with vertical and parallel planes to the XOZ plane of reference. The abscissas which these planes

cross are: $x = 50\text{m}$, $x = 100\text{m}$, $x = 150\text{m}$, . . . , $x = 650\text{m}$, thus resulting 13 sections.

The points of knowledge describing the contours limiting the vertical sections will be systematized in 13 data files whose graphical representation is shown in fig. 10.

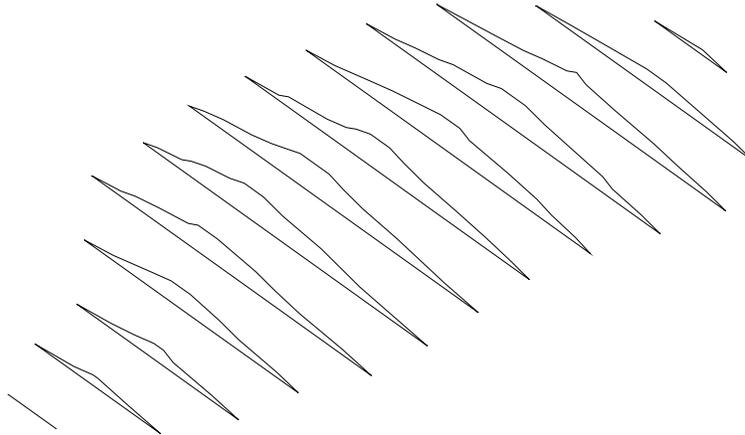


Fig. 10. The curves resulting from the intersection of the surface of the unit of field in fig. 9 with vertical planes parallel to plane XOZ

By applying the iterative algorithm i.e. “the algorithm for tracing a contour limiting a horizontal and irregular plane surface when a certain number of points which it crosses are

known”, after iteration 3 another 13 data files are obtained whose graphical representation is shown in fig. 11.

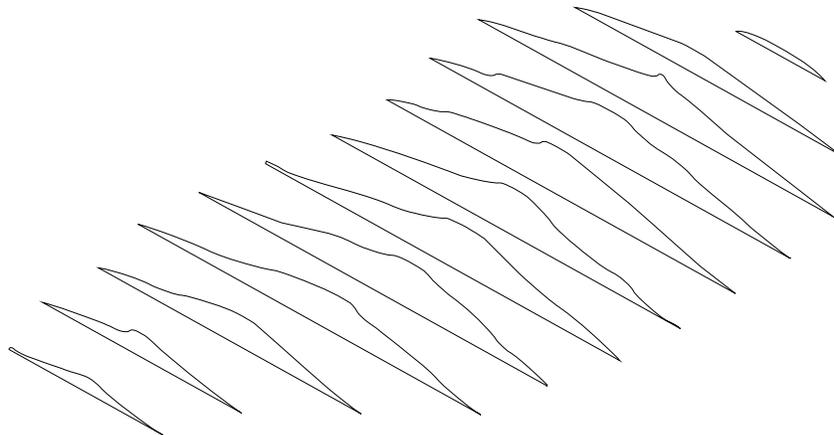


Fig. 11. The curves obtained from the graphical representation of the files of points after iteration 3

Knowing the contours of the sections obtained from the intersection of the surface covering the unit of field with the pencil of parallel planes, with the help of the “AREA”

instruction of the package of AUTOCAD IT programmes one can determine the values of the areas of these sections (table 1) [5].

Table 1. The values of the areas of the sections resulting from the intersection of the unit of field with planes parallel to the planes of reference XOY and XOZ

Section	Sections parallel to the XOY plane		Sections parallel to the XOZ plane	
	Value of area normal contour (m ²)	The values of the contour areas obtained after iteration 3 (m ²)	Normal contour section (m ²)	Contour section after iteration 3 (m ²)
1	2	3	4	5
S ₀	144373	145559,2013	0	0
S ₁	109625	110869,9209	6125,000	5997,066
S ₂	78250	79137,2270	11700,000	11698,993
S ₃	56000	56730,6667	23350,000	23626,548
S ₄	32857	33634,3920	37250,000	37314,394
S ₅	15000	15420,5457	45875,000	4601,1707
S ₆	5250	5568,8668	46250,000	46568,606
S ₇			43375,000	43099,945
S ₈			47875,000	48030,481
S ₉			49000,000	49065,673
S ₁₀			41000,000	40979,824
S ₁₁			19375,000	19450,995
S ₁₂			1875,000	2868,801

The volume of the reserves unit can be calculated using one of the following relations:

1. The formula of approximate integration (the Simpson's formula):

$$V = \frac{h}{3} (S_0 + S_n + 4 \sum S_{2n-1} + 2 \sum S_{2n} \pm \sum S_m)$$

2. The formula of the frustum of a cone:

$$V = \frac{h}{3} \sum (S_{n-1} + S_n + \sqrt{S_{n-1} * S_n} \pm S_m)$$

3. The formula of trapezium:

$$V = h \left(\frac{S_0}{2} + \sum_1^{n-1} S_n + \frac{S_n}{2} \pm \sum \frac{S_n}{3} \right)$$

where: V – represents the volume of the unit of field
h – equidistance between sections;
S₀ – surface limited by section zero;
S₁, S₂, . . . , S_n – values of the surfaces delimited by the contours of the sections;
S_m – the surface delimiting marginal recesses and prominences in the system of isolines (rises with the sign + and depressions with the sign -).
Using the three relations of calculation of the volumes, the following results are obtained (table 2):

Table 2. The values of the volumes of the unit of field obtained in the case of the sections parallel to the XOY and XOZ planes calculated for the normal situation and after iteration 3 of increasing the density of the points of knowledge

Volume (m ³)	Relation of volume calculation	Sections parallel to XOY		Sections parallel to XOZ	
		Normal contour section	Contour section after iteration 3	Normal contour section	Contour section after iteration 3
1	2	3	4	5	
V ₀	Simpson's formula	18238950			
	The formula of the frustum of a cone	18194610			
	The formula of trapezium	18327180			
V ₀₃	Simpson's formula		18479263,99		
	The formula of the frustum of a cone		18435757,49		
	The formula of trapezium		18567839,32		
V _v	Simpson's formula			1864042	
	The formula of the frustum of a cone			1839897	
	The formula of trapezium			1860563	
V _{v3}	Simpson's formula				1868435
	The formula of the frustum of a cone				1847273
	The formula of trapezium				1866393

IV. CONCLUSIONS

The accuracy of the calculations of the values of a section area increases together with the increase in the number of points of knowledge which the contour limiting the section crosses: “it is an increase achieved iteratively by applying the algorithm for tracing a contour limiting a horizontal and irregular plane surface, when a certain number of points which this crosses is known”.

It has been found that there is a dynamic of the increase in the accuracy of the calculations for the first iterations which later on decreases along with the increase in the number of iterations.

Consequently, it may be remarked that the dynamics of the increase in accuracy is inversely proportional to the increase in the number of iterations meant to improve the points on the contour of a section.

The accuracy of the calculation of the volume of the field unit increases directly proportionally to the number of sections.

The numerical model proposed can be programmed in an IT application.

In practical activity, the numerical model of calculation allows calculation in a short time of the values of the quantities of reserves, expressed in terms of volumes or tons, which exist in a unit of field.

It also allows a quick update of database.

Errors of calculation are eliminated by using this type of model.

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Brief Biography of the Author:

a) Studies

Educational background:

1976–1981 – Mining Institute of Petroșani, Mining Faculty.
1990–1998 – Ph.D. on Science Department “Technics”, specialization “Production Processes Engineering”

1993–1994 – Refresher training “CESTEMIN” within “TEMPUS” program at “ÉCOLE NATIONALE SUPÉRIEURE DES MINES DE NANCY”.

1998–1999 – Post-university courses “The Management of Human Resources” organized by the Faculty of Science, Petroșani University.

January 2006–The course “Specialist in quality management

systems” organized according to the requirements of nr. V10 ale TÜV Rheinland InterCert, of the department of personnel certification of TÜV Akademie Rheinland GmbH in Köln.

Jun.2006–Sept.2006 – The course “The Assessment and the Financing of the Real Estates” organized by ANEVAR (National Assessors Association from Romania) in Tg-Jiu.

Professional experience:

– sept.1981–dec.1983 – trainee engineer for E.M. Paroșeni
– jan.1984–jul.1985 – technologist engineer as part of the Endorsement Bureau of the Development Service of C.M. Jiu Valley

b) Academic Positions

–jul.1985–oct.1990 – assistant at Management Department
–oct.1990–oct.2001 – lecturer at Management Department
–oct.2001–present – associate professor at Management Department

I conducted the seminars for Mathematical modelling for mining design, Economic analysis and informational systems, The management and the organization of companies.

c) Scientific Activities (research, publications, projects, etc....)

– 70 articles published in speciality magazines and volumes of national and international scientific symposia;
– 25 papers done relying on the contracts with the production lines or the research – design divisions
– 3 didactic books and books of problems.
– 10 manuals and courses
– 1 treatise: “The management and the quality assurance”.

d) WSEAS Activities (papers, sessions, organization of sessions, organization of conferences, books, special issues in the journals etc... within WSEAS)*

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WSEAS International Conference on Mathematical and Computational Methods in Science and Engineering)

e) Others:

– Permanent member of “The managers and economist–engineers association from Romania”.