Two-Phase Flow of Air and Soybeans during a Silo Discharge Process

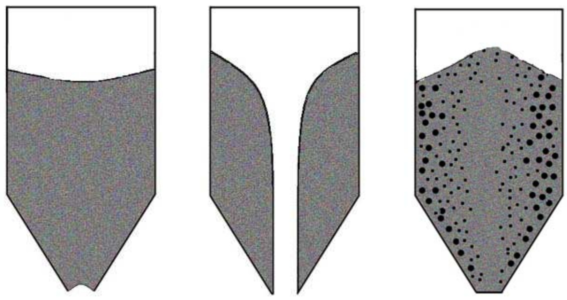
W. Chuayjan, W. Jumpen, P. Boonkrong and B. Wiwatanapataphee

Abstract**—** The paper aims to present mathematical model and numerical simulation of a granular material flow during a silo discharge process. The material flow in the silo is a form of two-phase flow consisting of particulates and an interstitial fluid. These two phases are soybeans and air. The homogeneous flow is assumed. The effect of the bottom design of the silo on the two-phase flow is investigated. The bottom shape including flat shape and cone shape and the diameter of outlet width including 0.08 m and 0.12 m are chosen for this investigation. The results show that the mathematical model can capture the granular material flow in the silo. The bottom design has significant effect on the velocity, pressure and shear rate in the granular material during the silo discharge process.

Keywords**—** cone-bottomed silo, flat-bottomed silo, finite element method, granular material, mixture model, two-phase flow.

I. INTRODUCTION

GENERALLY, silos have been used to store granular material such as rice, nuts, coffee, soybean, corn flakes and fertilizer in agricultural industries. During filling and discharging processes, arching, piping, segregation and silo collapse often cause problems for those industries [1], [2]. As presented in Fig. 1(a), a stable arch is formed above the outlet. This causes a blockage of any further discharge. Fig. 1(b) shows a formation of pipe which occurs when only the bulk solid above the outlet is flowing out, and the remaining bulk solid which is at the dead zones stays in place. Fig. 1(c) shows segregation occurring when particles have different physical properties such as particle size or shape. During the filling process, large particles accumulate near the silo walls while the small particles remain in the middle. This makes a bulk of small particles flow out before the large particles during discharging process. Moreover, a silo structure collapse due to the poor design of the silo has led to a massive damage as shown in Fig. 2.



(a) (b) (c)

Fig. 1. Flow problems: (a) Arching; (b) Piping; (c) Segregation

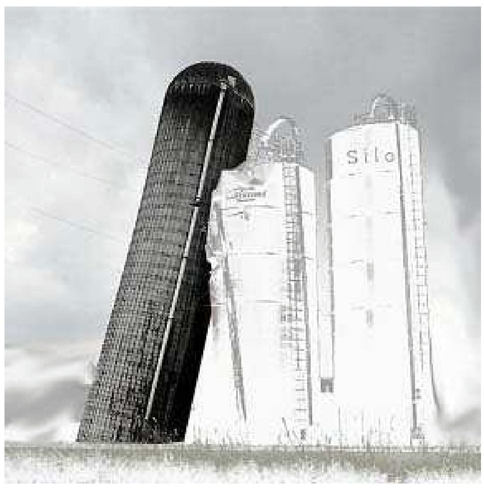


Fig. 2. Collapse of a silo due to a poor design.

To overcome the above problems, investigation of the physics of the granular material flow in silo is significantly necessary. Over the last few decades, a number of researches based on mathematical model and experimental model have been carried out to elucidate the characteristics of granular flow in silos. At an early state of this research field, the pressure distribution in silo was studied and presented by Janssen (1895) [3]. Jenike (1895) investigated how the flow of granular material has an influence on the wall pressure distribution in a silo during discharging process [4]. Aoki and Tsunakawa (1969) studied pressure distribution in granular material at the wall of bins and hoppers. It was found that loose granular material was

Manuscript received Jun 20, 2012. This work was supported by Centre of Excellence in Mathematics, PERDO, CHE, Ministry of Education, THAILAND.

W. Chuayjan is with Department of Mathematics and Statistics, Faculty of Science, Thaksin University, Phatthalung 93110, Centre of Excellence in Mathematics, PERDO, CHE, Bangkok 10400 THAILAND (email: cwariam@tsu.ac.th)

W. Jumpen is with Department of Mathematics, Faculty of Science, Mahidol University, Centre of Excellence in Mathematics, PERDO, CHE, Bangkok 10400 THAILAND (Corresponding author: wannika.jum@mahidol.ac.th)

P. Boonkrong is with Department of Mathematics, Faculty of Science, Mahidol University, Centre of Excellence in Mathematics, PERDO, CHE, Bangkok 10400 THAILAND (email: pijiito@gmail.com)

B. Wiwatanapataphee is with Department of Mathematics, Faculty of Science, Mahidol University, Centre of Excellence in Mathematics, PERDO, CHE, Bangkok 10400 THAILAND (email: benchawan.wiw@mahidol.ac.th)

present near the outlet hopper under gravity condition, and the velocity of individual particles increased rapidly during discharging process. Afterwards, effects of relevant factors including bottom angle, outlet width, inlet flow rate and particle shape have been investigated [6]-[16]. Yunus et al. (2011) proposed a discrete element model to study the flow behavior of powder inside a die shoe, which is used for die filling [17]. Chuayjan et al. (2009, 2012) proposed a mathematical model based on discrete element method (DEM) to investigate the flow pattern in three different designs of silo bottom [18], [19]. It indicate that the bottom shape has an influence on the flow pattern in the silo.

It is realized that the granular material in the silo consists of particulates and an interstitial fluid. Thus, recent researches have focused on two-phase flow of solid and liquid phases. Many mathematical models of two-phase flow in silo have been proposed [20]-[23]. Hamed (2006) proposed a two-phase flow model to elucidate the gas-particle flow in a pipe [20]. He focused on three cases of study including the case of no heat transfer (adiabatic flow), the case of heat transfer (heating flow) and the case of heat transfer (cooling flow). He compared his numerical results with published data and proved that his model adequately predicted the basic flow at low and high speed. Zarghami et al. (2005) developed a mathematical model to analyze the particles-wall contact time in fluidized beds of sand [21]. The results showed that their model was adequate to describe experimental data presented in literature. Thakurta et al. (1998) proposed a mathematical model of two-phase flow to study the thermophoretic deposition rate of small particles in the turbulent channel flow [22]. Sohn et al. (2002) investigated the radiation effects by both gas and particles on particle transport due to thermophoresis in an axisymmetric tube [23]. The results of these studies enhance our basic understanding of the characteristics of the two-phase flow of liquid and solid phase. Although these knowledge provide some guideline for the silo design, many problems still occur during the filling and discharging processes. Thus, further development of mathematical model is still required to study the two-phase flow of granular material in the silo.

In this paper, we proposed a mathematical model of homogeneous flow of granular material during the silo discharge process. The effect of the bottom design of the silo on the two-phase flow is investigated. The rest of the paper is organized as follows. Section II presents the mathematical model of the granular material flow in the silo during discharging process. Numerical investigations are presented in section III. In this section, the effect of bottom design including the bottom shape, and the diameter of the outlet width on the velocity field, pressure field and shear rate of the granular material in the silo is discussed. Some conclusions are given in section IV.

II. Mathematical Model

We deal with an assembly of a large number of discrete solid components and air inside silos. Thus, the model involves two immiscible phases including soybeans and air. A granular material is then referred to a collection of soybean particles and the air. In this section, governing equations and boundary conditions of the granular material flow during the silo discharge process are illustrated.

A. Governing Equations

The granular material flow during the silo discharge process is focused in this study. The flow is assumed to be a homogeneous flow of the granular material which consists of particulates and an interstitial fluid. The governing equations of the homogenous flow are described as follows:

$$\nabla \cdot \mathbf{u} = -\frac{(\rho_c - \rho_d)}{\rho_c \rho_d} m_{dc} \,, \tag{1}$$

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho \left(\mathbf{u} \cdot \nabla \right) \mathbf{u} + \nabla p = \nabla \cdot \left(\eta \left[\nabla \mathbf{u} + \nabla \mathbf{u}^T \right] \right) + \rho \mathbf{g} \,, \tag{2}$$

$$\rho_d \frac{\partial \phi_d}{\partial t} = -m_{dc} \,, \tag{3}$$

where two unknown $\mathbf{u}$ and ϕ_d denote the mixture velocity and the volume fraction of the solid phase, $\rho \,, \rho_c$ and ρ_d are respectively densities of the mixture, air phase and granular phase, $\mathbf{g}$ is the gravity, and m_{dc} is the mass transfer term. In equation (2), mixture density ρ is determined by [24]

$$\rho = \rho_c \left(1 - \phi_d\right) + \rho_d \phi_d \,,$$

and the dynamic viscosity η is defined by [25]

$$\eta = \eta_c \left(\frac{1 - \phi_d}{\phi_{max}} \right)^{-2.5 \phi_{max}} \,, \tag{4}$$

where η_c is the dynamic viscosity of the air phase, and ϕ_{max} is the maximum packing concentration.

B. Boundary Conditions

At the inlet of the silo, constant pressure of the mixture is set to the atmospheric pressure and insulation condition of the solid phase is used, i.e.,

$$p = p_0 \,, \tag{5}$$

$$\mathbf{u}_d \cdot \mathbf{n} = 0 \,. \tag{6}$$

At the outlet of the silo, the solid phase is assumed to flow out smoothly, and no viscous stress of the mixture is applied, i.e.,

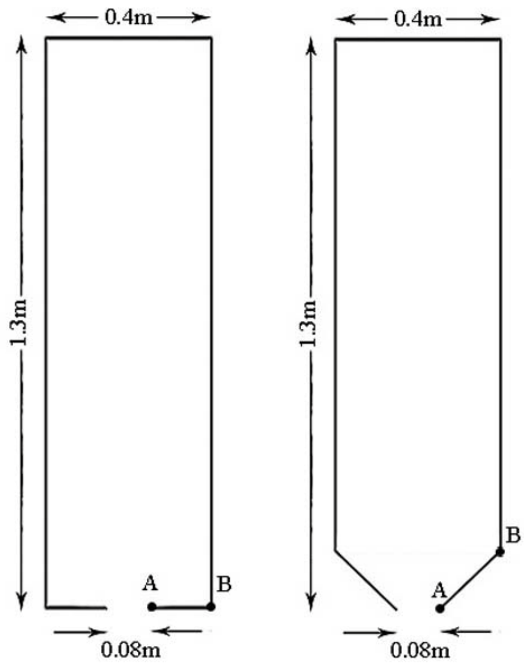
$$\left[\eta \left(\nabla \mathbf{u} + \nabla \mathbf{u}^T \right) \right] \mathbf{n} = 0 \,. \tag{7}$$

On the silo wall, slip condition of the mixture velocity, and insulation condition of the solid phase are applied.

$$\mathbf{u} \cdot \mathbf{n} = 0\,, \quad \mathbf{u}_d \cdot \mathbf{n} = 0\,. \tag{8}$$

III. Numerical Results

The numerical example in this study is the air-soybeans flow during the silo discharge process. Two types of silo including a flat-bottomed silo and a cone-bottomed silo are used in this study. Geometries of both silos and their domain meshes in two dimension are shown in Fig.3 and Fig.4, respectively.

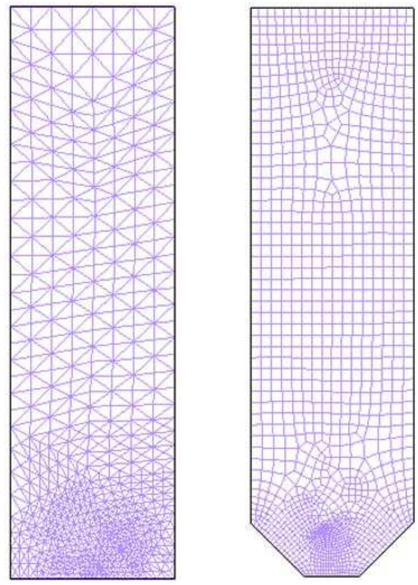


(a) (b)

Fig. 3. Geometries of the two silos having different bottom design: (a) flat shape; (b) hopper shape.

Both silos are made of steel sheet and have the same dimension, i.e., 13 m in height, 0.4 m in width and 0.08 m in outlet diameter. Soybean is assumed to be spherical and its diameter is 6 mm. The cone angle of the silo as shown in Fig. 3(b) is 45^o. There is no mass transfer between both phases because of immiscible property of the air and soybeans. The parameter values used in the simulation are listed in TABLE I.

The COMSOL MULTIPHYSICS version 4.2a. is used in this simulation. The effect of the bottom design of the silo on the two-phase flow is investigated. The bottom shape including flat shape and cone shape, and the diameter of outlet width including 0.08 m and 0.12 m are chosen for this investigation. The process starts with setting the initial volume fraction of the soybeans. In this study, we assume that at $t = 0s$ silo is full of soybeans, and the air is filled space between each particle inside the silo. The



(a) (b)

Fig. 4. Domain mesh of the two silos: (a) flat-bottomed silo; (b) cone-bottomed silo.

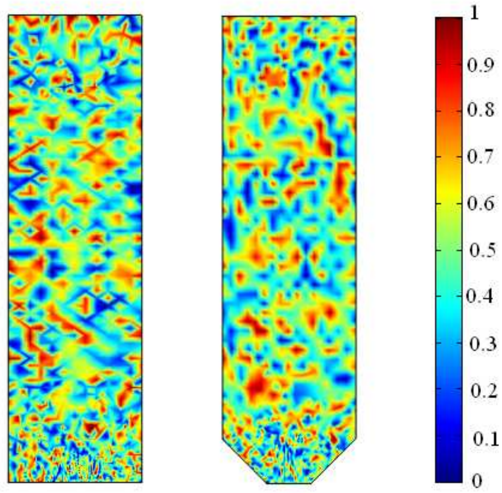


Fig. 5. Volume fraction of soybeans at initial time $t = 0s$ in the flat-bottomed silo and the cone-bottomed silo, respectively.

locations of soybeans and air are set randomly in both computational domains as shown in Fig. 5.

Firstly, we investigate the effect of different outlet

TABLE I
Parameter values used in simulation

Parameter	Definition	Value
ρ_c	Density of gas phase	1.184 kg/m^3
η_c	Viscosity of gas phase	1.965×10^5 $Pa \cdot s$
ρ_d	Density of solid phase	1,033 kg/m^3
ρ_0	Atmospheric pressure	101,325 Pa
$\mathbf{g}$	Gravity	-9.8 m/s^2
ϕ_{max}	Maximum packing concentration	0.2
m_{dc}	Mass transfer term	0 $kg/m^3 \cdot s$

width on the pressure distribution and shear rate. Two sizes of outlet width of 0.12 m and 0.08 m are used in this investigation. High pressure and high shear rate appear near the orifice as shown in Fig. 6 and Fig. 7. The results also show that the outlet width has significant effects on the pressure and shear rate at the bottom part of the silo. The longer the outlet width, higher pressure and higher shear rate are present at the bottom part of the silo.

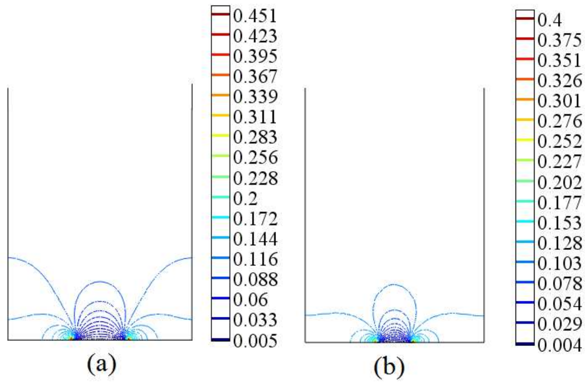


Fig. 6. Contour plot of pressure (MPa) at $t = 1s$ in the granular material at the bottom part of the flat-bottomed silo with two different outlet widths: (a) $0.012m$; (b) $0.08m$.

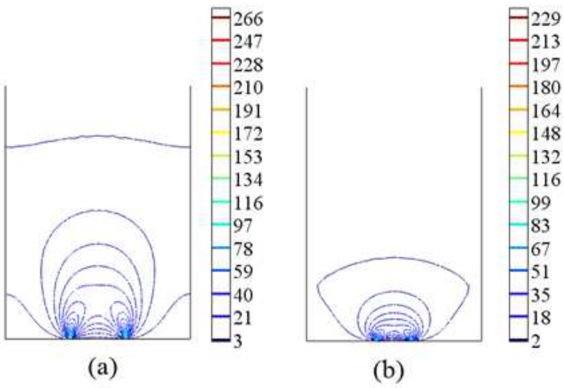


Fig. 7. Contour plot of shear rate (MPa) at $t = 1s$ in the granular material at the bottom part of the flat-bottomed silo with two different outlet widths: (a) $0.012m$; (b) $0.08m$.

Next, we investigate the influence of the bottom design on the flow behavior in the silo at $t = 1s$. Pressure distribution and shear rate at the bottom part of both silos with outlet width of 0.08 m are compared. Fig. 8 shows contour plot of pressure and shear rate of the granular material at the bottom part of the cone-bottomed silo at $t = 1s$. It is found that the highest pressure at the cone-bottom part as shown in Fig. 8(a) is approximately 0.3 MPa while the one in the flat-bottom part is about 0.4 MPa as shown in Fig. 6(b). The highest shear rate at the cone-bottomed part, as shown in Fig. 8(b), is about 359

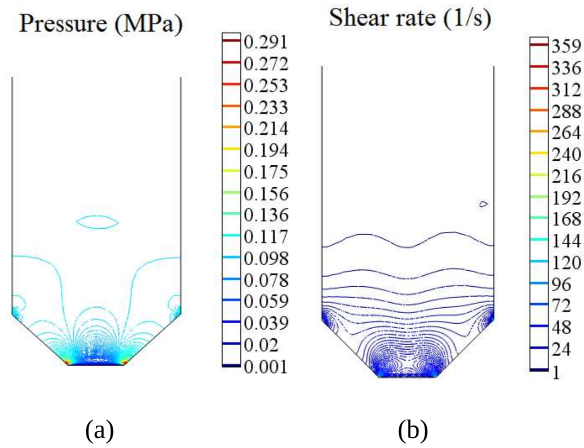


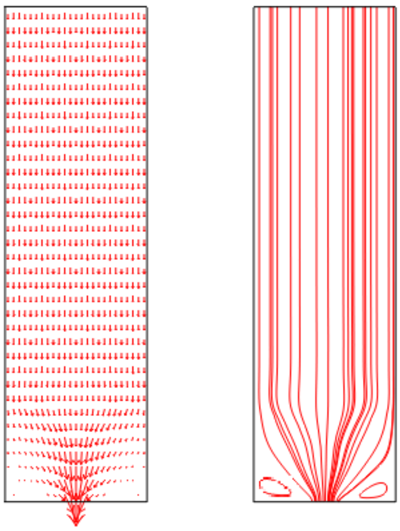
Fig. 8. Contour plot of pressure and shear rate of the granular material at the bottom part of the cone-bottomed silo at $t = 1s$. .

s^{-1} while the one in the bottom part of the flat-bottomed silo is about 229 s^{-1} as shown in Fig. 7(b). This indicates that the cone-bottomed wall induces low pressure and high shear rate.

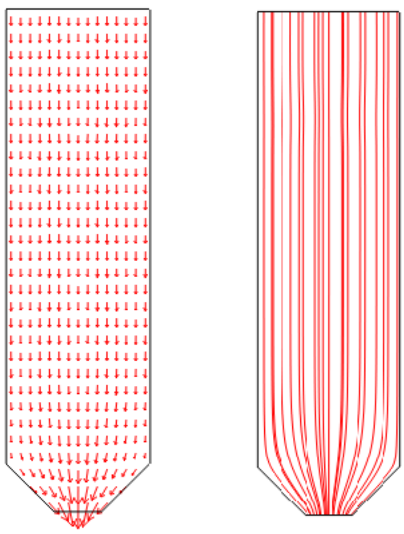
Velocity profiles in the flat-bottomed silo and the cone-bottomed silo are investigated. Fig. 9 shows velocity vector and stream line of the granular material in both silos. The results as shown in Fig. 9(a) indicate a dead zone above the flat-bottomed silo where there is no flow of the granular material. To investigate the maximal flow of the granular material, we plot the mixture velocity field at $t = 1s$ at the bottom of both silos as shown in Fig 10. The results reveal that maximal velocity of the granular material obtained from the cone-bottomed silo is 3.5 m/s whereas the one obtained from the flat-bottomed silo is only 1.65 m/s. This indicates that the cone-bottomed silo induces high flow rate.

Volume fractions of soybeans in both silos are illustrated in Fig. 11 at various times. Comparing with the flow inside the flat-bottomed silo, red shading inside the cone-bottomed silo illustrates that the granular material exits from the silo very fast. High volume fraction of the air is on the top part of the silo whereas the high volume fraction of the soybean is underneath during discharging process. The mass flow is present in the cone-bottomed silo while the funnel flow is present in the flat-bottomed silo. The granular material (bulk solid) in the cone-bottomed silo flows faster than the one in the flat-bottomed silo.

Fig. 12 and Fig. 13 show velocity vector of solid phase, mixture and air phase in the flat-bottomed silo and the cone-bottomed silo at $t = 1s$. The color bar on the left presents the volume fraction of the soybeans in the silo while the one on the right shows the volume fraction of the air phase. Fig. 12(a), Fig. 12(b) and Fig. 12(c) respectively present the flow direction of the solid phase, the mixture and the air phase. The results indicate that the soybeans flow downward as shown in Fig. 12(a) and the air flows upward as shown in Fig. 12(c). It also shows that the granular material (mixture) as shown in Fig. 12(b) flows downward and flows out at the orifice of the silo. The same flow behaviors of the soybeans, the mixture and the air are shown in Fig. 13(a), Fig. 13(b) and Fig. 13(c) respectively.

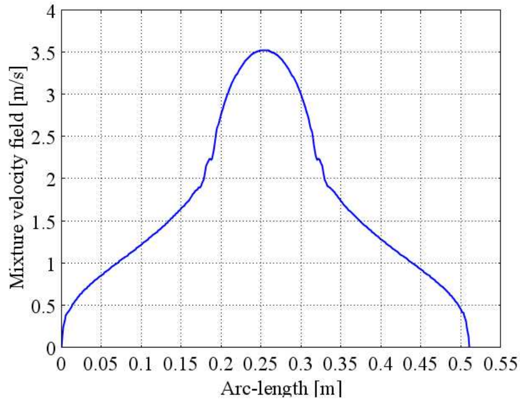


(a)

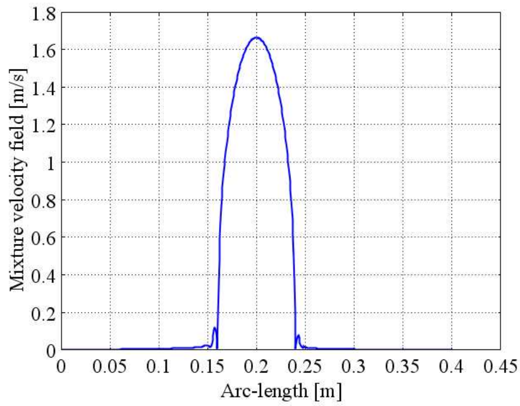


(b)

Fig. 9. Velocity vector and stream line of the granular material in both silos with the outlet width of 0.08 m: (a) the flat-bottomed silo; (b) the cone-bottomed silo.



(a)



(b)

Fig. 10. Maximal velocity (m/s) of the granular material at the bottom obtained from both silos : (a) the cone-bottomed silo; (b) the flat-bottomed silo.

IV. Conclusion

The model of two-phase flow has been proposed for studying the effect of the silo-bottom design including the outlet width and the bottom shape on the dynamic flow behavior of a mixture which is combination of air and soybeans. Governing equations with boundary conditions are solved by finite element method. Volume fraction of soybeans, velocity field, pressure distribution and shear rate are investigated. The results reveal that the way of the bottom design affects the rate of flow of the granular material out of the silos. Granular material in the cone-bottomed silo flows faster than the one in the flat-bottomed silo, and in the silo with larger outlet width, pressure distribution and shear rate in the granular material are higher. The flat-bottomed silo has the less efficiency in granular draining. The cone-bottomed silo should be selected for the discharging process.

Acknowledgment

The first author gratefully acknowledges the financial support from the Centre of Excellence in Mathematics, PERDO, CHE, Ministry of Education, THAILAND.

References

[1] C. Leppert and D. Dinkler, A viscous model for free surface granular flow in silos, *in Proc. 5th Annu. International conference on the micromechanics of granular media: powders and grains*, 2005, pp. 461-464.

[2] A. J. M Spencer and J. M. Hill, Non-dilatant double-shearing theory applied to granular funnel-flow in hoppers, *Journal of Engineering Mathematics*, 41, 2001, pp. 55–73.

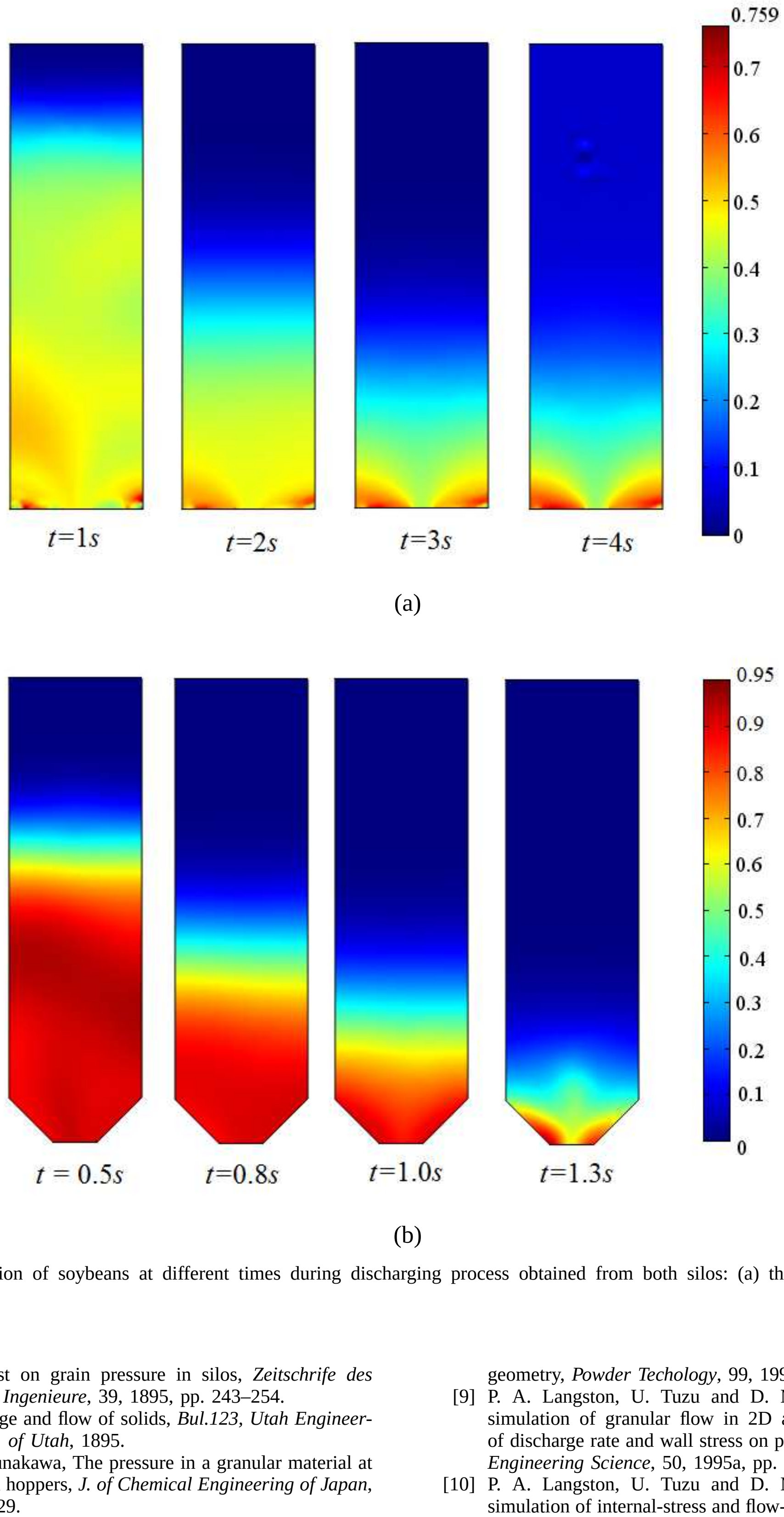


Fig. 11. Volume fraction of soybeans at different times during discharging process obtained from both silos: (a) the flat-bottomed silo; (b) the cone-bottomed silo.

[3] H. A. Janssen, Test on grain pressure in silos, *Zeitschrife des Vereines Deutscher Ingenieure*, 39, 1895, pp. 243–254.
[4] A. W. Jenike, Storage and flow of solids, *Bul.123, Utah Engineering Expt. Sta. Univ. of Utah*, 1895.
[5] R. Aoki and H. Tsunakawa, The pressure in a granular material at the wall of bins and hoppers, *J. of Chemical Engineering of Japan*, 2, 1969, pp. 126–129.
[6] P. A. Cundall and O. D. L. Strack, Discrete numerical model for granular assemblies, *Geotechnique*, 29, 1979, pp. 47–65.
[7] R. J. Goodey, C. J. Brown and J. M. Rotter, Predicted patterns of filling predures in thin-walled square silos, *Engineering Structures*, 28, 2006, pp. 109–119.
[8] T. Karlsson, M. Klisinski and K. Runesson, Finite element simulation of granular material flow in plane silos with complicated geometry, *Powder Techology*, 99, 1998, pp. 29–39.
[9] P. A. Langston, U. Tuzu and D. M. Heyes, Discrete element simulation of granular flow in 2D and 3D hoppers dependence of discharge rate and wall stress on particle interactions, *Chemical Engineering Science*, 50, 1995a, pp. 967–979.
[10] P. A. Langston, U. Tuzu and D. M. Heyes, Discrete element simulation of internal-stress and flow-fileds in funnel flow hoppers, *Powder Technology*, 85, 1995b, pp. 153–169.
[11] J. Li, P. A. Langston, C. Webb and T. Dyakowski, Flow of sherodisc particles in rectangular hoppers a DEM and experimental comparision in 3D, *Chemical Engineering Science*, 59, pp. 5917–5929.
[12] J. M. Rotter, C. J. Brown and E. H. Lahlouh, Patterns of wall pressure on filling a square planform steel silo, *Engineering Structures*,

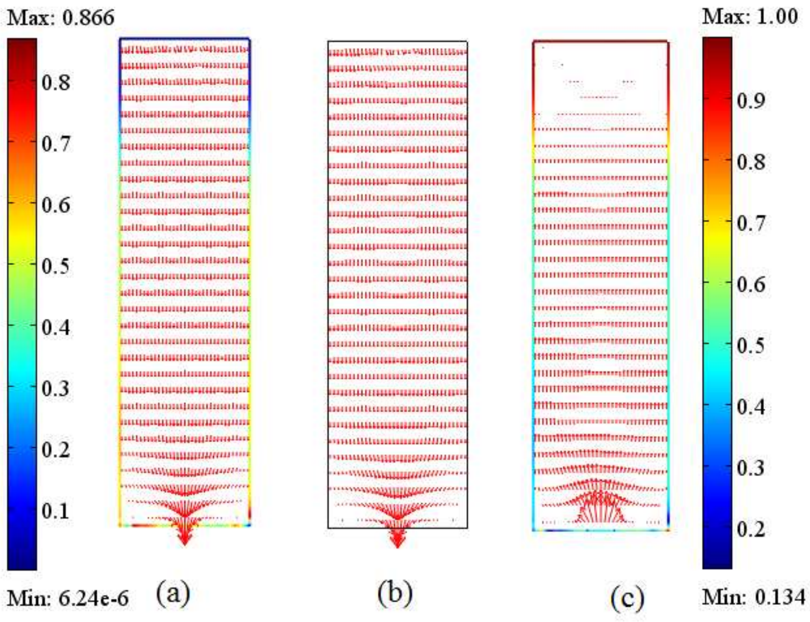


Fig. 12. Velocity vector of solid phase, mixture and air phase in the flat-bottomed silo at $t = 1s$.

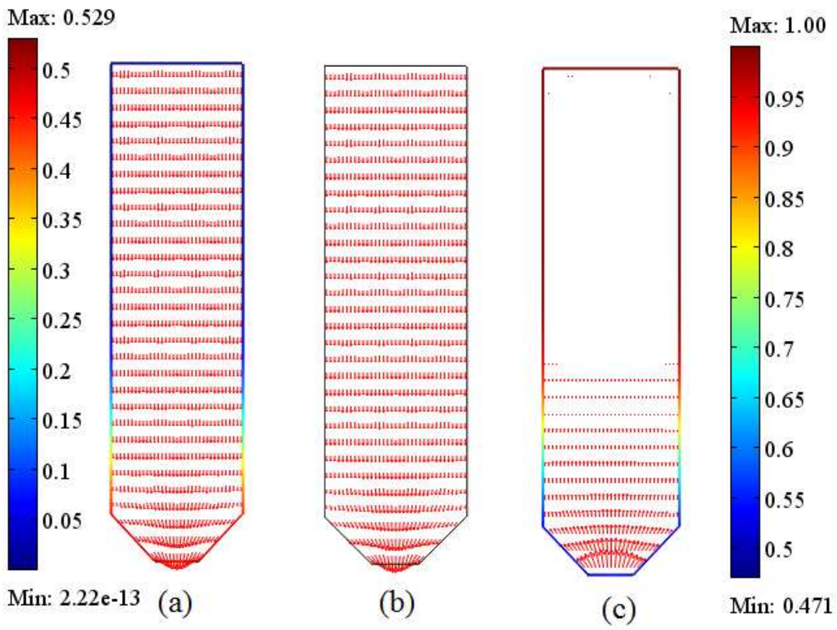


Fig. 13. Velocity vector of solid phase, mixture and air phase in the cone-bottomed silo at $t = 1s$.

24, 2002, pp. 135–150.

[13] G. H. Ristow, Pattern formation in granular material, *Springer - New York*, 2000.

[14] S. Masson and J. Martinez, Effect of particle properties and silo geometry on stresses predicted by discrete simulations of bulk material, *15th ASCE Engineering Mechanics Conference*, 2-5, 2002, pp. 1–8.

[15] J. Li, P. A. Langston, C. Webb and T. Dyakowski, Flow of sherodisc particles in rectangular hoppers a DEM and experimental comparision in 3D, *Chemical Engineering Science*, 59, 2004, pp. 5917–5929.

[16] J. Wu, J. Binbo, J. Chen and Y. Yang, Multi-scale study of particle flow in silos, *Advanced Power Technology*, 20, 2009, pp. 62–73.

[17] M. Radzim Yunus, D. T. Gethin and R. Hashim, Simulation of Powder Flow in a Die Shoe, *Recent Researches in Educational Technologies*, 2011, pp. 235–239.

[18] W. Chuayjan, S. Pothiphan, B. Wiwatanapataphee and Y. H. Wu, Numerical simulation of granular flow during filling and discharging of a silo, *International Journal of Pure and Applied Mathematics*, 62(3), 2009, pp. 347–364.

[19] W. Chuayjan, P. Boonkrong and B. Wiwatanapataphee, Effect of the silo-bottom design on the granular behavior during discharging process, *Proceedings of the 11th WSEAS International Conference on System Science and Simulation in Engineering (ICOSSSE'12), Singapore, May*, 2012, pp. 68–73.

[20] M. H. Hamed, Subsonic gas-particle two-phase flow in pipes,

Proceeding of the 2006 WSEAS/IASME International Conference on Fluid Mechanics, Miami, Florida, USA, January 18-20, 2006, pp. 95–100.

[21] R. Zarghami, N. Mostoufi and R. Sotudeh-Gharebagh, Particle-Wall Contact Time in Fluidized Beds, *Proceeding of the 3rd IASME/WSEAS Int. Conf. on HEAT TRANSFER, THERMAL ENGINEERING AND ENVIRONMENT, Corfu, Greece, August 20-22*, 2005, pp. 85–90.

[22] D. G. Thakurta, M. Chen, J. B. McLaughlin and K. Kontomaris, Thermophoretic deposition of small particles in a direct numerical simulation of turbulent channel flow, *International Journal of Heat and Mass Transfer*, 41, 1998, pp. 4167–4182.

[23] Y. M. Sohn, S. W. Baek and D. Y. Kim, Thermophoresis of particles in gas-particle two-phase flow with radiation effect, *Numerical Heat Transfer, Part A*, 41, 2002, pp. 165–181.

[24] C. Y. Wang and C. Beckermann, A two-phase mixture model of liquid-gas flow and heat transfer in capillary porous media- I. Formulation, *International Journal of Mass Transfer*, 36, 1993, pp. 2747–2758.

[25] M. Manninen and V. Taivassalo, On the mixture model for multiphase flow, *Valtion teknillinan tutkimuskeskus (VTT)*, 1996, pp.1–21.