Robust control of unstable systems: algebraic approach using sensitivity functions

František Gazdoš, Petr Dostál, and Jiří Marholt

Abstract**—**This contribution proposes a methodology for robust control of unstable systems. For this purpose the algebraic approach using polynomials is utilized together with tuning some of the closed loop poles using loop sensitivity functions. The control design method is illustrated on the stabilization task of the magnetic levitation system. Complete procedure from derivation of a linearized model to controller design and tuning is described in detail. Finally the methodology proves useful for both stabilization in different operating points and output/load disturbance attenuation.

Keywords**—**Algebraic approach, Magnetic levitation system, Robust control, Sensitivity functions, Unstable systems.

I. Introduction

Many technological processes possess instable behaviour. Such systems can be represented by various types of reactors, combustion systems, crystallizers, distillation columns, etc. Besides these, lot of aviation systems in both civil and military services are naturally unstable [4], [16]. All these system need proper control since controlling unstable systems can be a real hazard, as shown many times in practice [21]. In such cases the control designer has to understand fundamental limitations that stem from the process instability [15], [19].

There are many sources devoted to the area of unstable systems control, often covering also the case of delayed and non-minimum-phase systems, e.g. [3], [5], [7], [12]-[14], [17]-[18]. In this work, the control system design is based on the algebraic approach using polynomials, e.g. [2], [10], [11]. The advantage of this approach is in its systematic and a relatively simple way of designing controllers – it provides both controller structure as well as its parameters and it allows imposing further control requirements simply. A suitable controller is then found as a solution of Diophantine equations in a given ring - polynomials in this case. A disadvantage may be seen in the fact that it can provide more complex controllers than classical PIDs but this does not seem too problematic nowadays, when controllers are usually implemented on industrial PCs or PLCs.

This paper is structured as follows: control system structure and requirements are stated first, followed by general solution using the polynomial approach. Further the system of magnetic levitation is introduced and described in detail, covering a mathematical model, its physical parameters and a suggested approximate linearized model [8]. Next section is focused on the controller design and fine-tuning of its parameters in order to provide robust - safe control. This is done by optimization of some of the closed-loop poles with the help of sensitivity functions and spectral factorization technique [9]. Control results are presented and discussed and the article concludes with some final remarks and suggestions.

II. Methodology

In this work the classical control set-up of Fig. 1 is considered where G denotes a plant to be controlled by a controller C and the signals w, e, u, y stand for the reference (set point), control error, control input (manipulated variable) and a process (controlled) variable respectively. Signals v_u and v_y represent general disturbances.

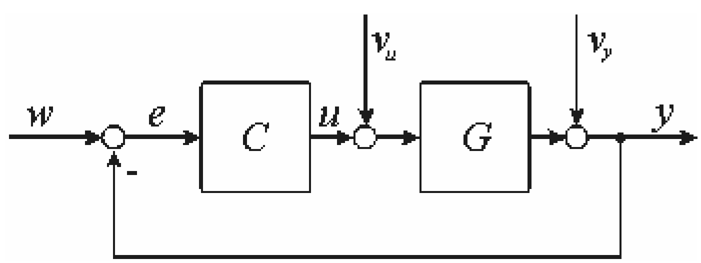

Fig. 1 Control system configuration

Let us assume that the process can be, after proper simplification and linearization, described by a linear time-invariant continuous-time model given by a transfer function

$$G(s) = \frac{b(s)}{a(s)} \tag{1}$$

where $b(s)$, $a(s)$ are coprime polynomials in the complex Laplace variable "s" satisfying the condition:

$$\deg a(s) \succ \deg b(s). \tag{2}$$

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Hence, it is a strictly proper system. Further, the controller C can be also described by a transfer function (3) with $q(s)$, $p(s)$ coprime polynomials satisfying (4), i.e. the controller is generally assumed to be proper.

$$C(s)=\frac{q(s)}{p(s)} \tag{3}$$

$$\deg\ p(s)\geq \deg\ q(s) \tag{4}$$

Requirements for the control system introduced above are formulated as stability, asymptotic tracking of the reference signal, disturbances attenuation and inner properness of all the parts of the control system. Besides these the system should be robust in order to cope with the real plant (not only with the adopted linear model) and possible disturbances. This is especially important in this case when dealing with unstable systems.

From the scheme of Fig. 1 and assuming (1), (3) it is easy to derive following relationships between the controlled variable y ($Y(s)$ in the complex domain) and input signals w, v_u and v_y ($W(s)$, $V_u(s)$ and $V_y(s)$ similarly); the argument "s" is in these formulas omitted somewhere to keep them more compact and readable):

$$\begin{aligned}
Y(s)&=\frac{G\cdot C}{1+G\cdot C}\cdot W(s)+\frac{G}{1+G\cdot C}\cdot V_u(s)+\\
&+\frac{1}{1+G\cdot C}\cdot V_y(s),\\
Y(s)&=\frac{b\cdot q}{a\cdot p+b\cdot q}\cdot W(s)+\frac{b\cdot p}{a\cdot p+b\cdot q}\cdot V_u(s)+\\
&+\frac{a\cdot p}{a\cdot p+b\cdot q}\cdot V_y(s),\\
Y(s)&=\frac{b\cdot q}{d}\cdot W(s)+\frac{b\cdot p}{d}\cdot V_u(s)+\frac{a\cdot p}{d}\cdot V_y(s),\\
Y(s)&=T\cdot W(s)+S_u\cdot V_u(s)+S\cdot V_y(s).
\end{aligned} \tag{5}$$

Here, the symbol d defines a characteristic polynomial of the closed-loop and consequently it is given as:

$$a\cdot p+b\cdot q=d\,. \tag{6}$$

Symbols S, T, S_u denote important transfer functions of the loop known as the sensitivity function, complementary sensitivity function, and input sensitivity function respectively. The sensitivity functions S and S_u are further used to make the designed control system robust.

Similarly, it is straightforward to derive the formula (7) for the control error e ($E(s)$ in the complex domain).

$$E(s)=\frac{p}{d}\left[a\cdot W(s)-b\cdot V_u(s)-a\cdot V_y(s)\right]. \tag{7}$$

A. Control System Stability

From (5) it is clear that the control system of Fig. 1. will be stable if the characteristic polynomial $d(s)$ given by (6) is stable. This Diophantine equation, after a proper choice of the stable polynomial $d(s)$, is used to compute unknown controller polynomials $q(s)$, $p(s)$. Sometimes it is useful to require also so called *strong stability* which guarantees also stability of the designed controller, i.e. stability of the polynomial $p(s)$ in (3). As control of unstable systems is generally more dangerous and the suggested design methodology relies on the approximate linear model of the originally nonlinear plant only, the strong stability condition is also considered in this work for safety reasons.

B. Asymptotic Tracking of the Reference Signal and Disturbances Attenuation

Let us assume, as it is often the usual case, that the reference signal $w(t)$ is a step function, defined in the complex domain as:

$$W(s)=\frac{w_0}{s}, \tag{8}$$

and, further suppose that both disturbances $v_u(t)$, $v_y(t)$ can be also approximated by step-functions:

$$V_u(s)=\frac{v_{u0}}{s}, V_y(s)=\frac{v_{y0}}{s}. \tag{9}$$

Then substituting (8)-(9) into (7) yields:

$$E(s)=\frac{p}{d}\left(a\cdot\frac{w_0}{s}-b\cdot\frac{v_{u0}}{s}-a\cdot\frac{v_{y0}}{s}\right), \tag{10}$$

which shows that in order to guarantee zero-control error in the steady-state (despite both disturbances), the denominator polynomial of the controller $p(s)$ needs to be divisible by the "s"-term. This will be fulfilled for this polynomial in the form:

$$p(s)=s\cdot\tilde{p}(s). \tag{11}$$

Then the controller (3) can be written as

$$C(s)=\frac{q(s)}{s\cdot\tilde{p}(s)}, \tag{12}$$

and the Diophantine equation (6) defining stability will be:

$$a \cdot s \cdot \tilde{p} + b \cdot q = d \,. \tag{13}$$

C. Control System Inner Properness

Inner properness of the control system is satisfied if all its parts (transfer functions) are proper. With regard to the conditions (2) and (4) and taking into account solvability of (6) it is possible to derive following formulae for degrees of the unknown polynomials q, $\tilde{p}$ and d:

$$\begin{aligned} &\deg q(s) = \deg a(s), \\ &\deg \tilde{p}(s) \geq \deg a(s) - 1, \\ &\deg d(s) = 2 \cdot \deg a(s)\,. \end{aligned} \tag{14}$$

D. Robust Setting of the Designed Loop

In order to cope with external disturbances and with the fact that only an approximate model of a generally nonlinear unstable plant is used for the control system design, the closed loop is designed to be robust. This is done with the help of the sensitivity functions S and S_u from (5). The sensitivity function S is defined as

$$\begin{aligned} S(s) &= \frac{Y(s)}{V_y(s)} = \frac{1}{1 + G(s) \cdot C(s)} = \\ &= \frac{a(s) \cdot p(s)}{a(s) \cdot p(s) + b(s) \cdot q(s)} = \frac{a(s) \cdot p(s)}{d(s)} \end{aligned} \tag{15}$$

and it describes the impact of output disturbance v_y on the process output y; moreover, it gives the relative sensitivity of the closed-loop transfer function $T(s)$ to the relative plant model error. The peak gain of its frequency response given by the infinity norm H_∞ is a good measure of the loop robustness, e.g. [20].

The input sensitivity function $S_u(s)$ describes the impact of the input (load) disturbance on the process output and it is given as:

$$\begin{aligned} S_u(s) &= \frac{Y(s)}{V_u(s)} = \frac{G(s)}{1 + G(s) \cdot C(s)} = \\ &= \frac{b(s) \cdot p(s)}{a(s) \cdot p(s) + b(s) \cdot q(s)} = \frac{b(s) \cdot p(s)}{d(s)}\,. \end{aligned} \tag{16}$$

In this work it is suggested to use both sensitivity functions and their H_∞ norms to tune some of the closed-loop poles in order to make the designed control system more robust, i.e. safer. The procedure is shown further on the presented example of the magnetic system stabilization.

III. Magnetic Levitation System

A. Description

The magnetic levitation system CE 152 depicted in Fig. 2 represents a laboratory-scale model designed by TQ Education and Training Ltd for studying system dynamics and experimenting with control algorithms. It demonstrates control problems associated with nonlinear unstable systems. The system consists of a coil levitating a steel ball in the magnetic field with the position sensed by an inductive linear sensor connected to an A/D converter. A simplified scheme is presented in Fig. 3 where F_a, F_m, F_g denote acceleration force, electromagnetic force and gravity force respectively and i stands for the coil current.

The coil is driven by a power amplifier connected to a D/A converter. A basic control task is to control the position of the

Fig. 2 The CE 152 magnetic levitation apparatus

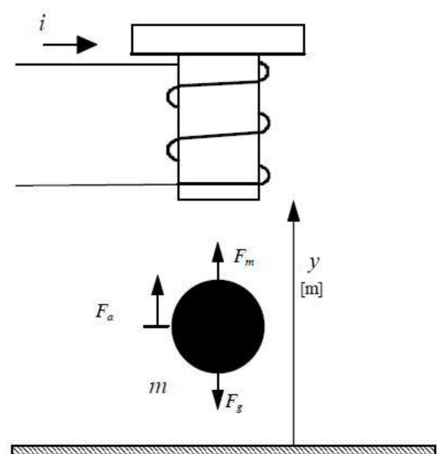

Fig. 3 The ball and coil subsystem

ball freely levitating in the magnetic field of the coil. From the control theory point of view, the magnetic levitation system is a nonlinear unstable system with one input and one output. Detailed description of the apparatus can be found in e.g. [8] and [22]-[24].

The model is connected to a standard PC via an universal data acquisition card MF614 and the Real Time Toolbox together with the Matlab environment are used for the communication.

B. Mathematical Model

A simplified mathematical model of the system including both D/A and A/D converters can be derived in the following form of a second-order nonlinear differential equation [8], [22]:

$$\frac{m_k}{k_{AD}k_x}\ddot{y}-\frac{k_{fv}}{k_{AD}k_x}\dot{y}=\frac{k_{DA}^2k_i^2u^2k_c}{\left(\frac{y-k_{AD}y_0}{k_{AD}k_x}-x_0\right)^2}-m_k g \tag{17}$$

where y denotes the controlled variable - ball position and u is the control input, proportional to the voltage from D/A converter. Other symbols used in (17) are clearly defined in Table I, together with their actual values.

State space realization of the system can be expressed in the form of (18) with x_1 corresponding to the ball position and x_2 describing its speed.

TABLE I
PARAMETERS OF THE MODEL

Symbol	Meaning	Value and SI unit
k_{AD}	A/D converter gain	0.2 MU [a]/V
k_{DA}	D/A converter gain	20 V/MU [a]
k_{fv}	damping constant	0.02 N·s /m
k_x	position sensor gain	821 V/m
k_i	power amplifier gain	0.3 A/V
k_c	coil constant	1.769×10^{-6} N·m^2/A^2
m_k	ball mass	8.27×10^{-3} kg
x_0	coil offset	7.6×10^{-3} m
g	gravity constant	9.81 m/s^2
y_0	position sensor offset	0.0183 V
y	ball position	MU [a]
u	input signal	MU [a]

[a]Voltage converted by the data acquisition card and scaled to ±1 machine unit (MU).

$$\begin{aligned}\dot{x}_1&=x_2\\ \dot{x}_2&=\frac{k_{fv}}{k_{AD}k_x}x_2+\frac{k_{AD}k_xk_{DA}^2k_i^2u^2k_c}{m_k\left(\frac{x_1-k_{AD}y_0}{k_{AD}k_x}-x_0\right)^2}-k_{AD}k_x g\end{aligned} \tag{18}$$

For the purpose of control system design, the nonlinear model (18) was linearized in the chosen operating point (equilibrium state for the magnetic force equal to the gravity), generally denoted as $\left(u^s,y^s\right)$. The resultant linear state space description is given by (19) and (20) where $\mathbf{x}$ is the vector of deviations of the originally defined states from the states in the chosen operating point $\left(u^s,y^s\right)$.

$$\begin{aligned}\mathbf{x}&=\mathbf{A}\mathbf{x}+\mathbf{B}u\\ y&=\mathbf{C}\mathbf{x}\end{aligned} \tag{19}$$

$$\mathbf{A}=\begin{pmatrix}0 & 1\\ \frac{2g}{k_{DA}k_iu^s\sqrt{\frac{k_c}{m_k g}}} & \frac{k_{fv}}{m_k}\end{pmatrix}$$
$$\mathbf{B}=\begin{pmatrix}0\\ \frac{2k_{AD}k_x g}{u^s}\end{pmatrix}\quad \mathbf{C}=\begin{pmatrix}1 & 0\end{pmatrix} \tag{20}$$

C. Model Transfer Function

A transfer function of the system used for the controller design can be easily computed from (19), (20) as

$$G(s)=\mathbf{C}\cdot\left(s\cdot\mathbf{I}-\mathbf{A}\right)^{-1}\cdot\mathbf{B} \tag{21}$$

where $\mathbf{I}$ denotes the identity matrix of the same size as $\mathbf{A}$. Consequently the transfer function takes a form of a second-order proportional system (22) with coefficients given generally according to (23).

$$G(s)=\frac{b_0}{s^2+a_1s+a_0} \tag{22}$$

$$b_0=\frac{2k_{AD}k_x g}{u^s}\quad a_1=-\frac{k_{fv}}{m_k}\quad a_0=-\frac{2g}{k_{DA}k_iu^s\sqrt{\frac{k_c}{m_k g}}} \tag{23}$$

If we choose three operating points P_1, P_2, P_3 where P_2 represents the ball levitating in the middle of the space and the others differ ±30% in the distance, then the transfer function coefficients change as presented in Table II.

TABLE II
MODEL TRANSFER FUNCTION IN DIFFERENT OPERATING POINTS

Operating point	Transfer function	Poles
P_1 (+30%)	$G_1(s)=\frac{28231}{s^2-2.418s-6134}$	p_1 = 79.54 p_2 = -77.12
P_2	$G_2(s)=\frac{18400}{s^2-2.418s-3998}$	p_1 = 64.45 p_2 = -62.03
P_3 (-30%)	$G_3(s)=\frac{13638}{s^2-2.418s-2963}$	p_1 = 55.66 p_2 = -53.24

From the table, it can be seen that the system has one stable and one unstable pole located on the real axis, nearly symmetrically with respect to the origin. It also has a constant gain independent of the chosen operating point equal to $k = \frac{b_0}{a_0} = -4.6$. Note also that the system is relatively fast with time-constants around 10-20 ms. As the system is unstable, the input signal u^s used to compute transfer functions in the operating points P1 - P3 via (23) was obtained using an auxiliary stabilizing controller.

IV. Control System Design

The task here is to design a feedback control system for the magnetic levitation apparatus described above. The control system design is based on the nominal linear model (22) (G2 in the Table II) but it must fulfill the given requirements stated in the section II of this paper not only for this model, but also for different operating points.

Let us start with the degrees of the unknown controller polynomials $q(s)$, $\tilde{p}(s)$ and of the characteristic polynomial $d(s)$ from (12)-(13). Assuming the nominal transfer function of the controlled system in the form of (22) the degrees will according to (14) be: $\deg q(s) = 2$, $\deg \tilde{p}(s) \geq 1$ and $\deg d(s) = 4$. Therefore the simplest controller structure according to (12) will be

$$C(s) = \frac{q(s)}{s \cdot \tilde{p}(s)} = \frac{q_2 \cdot s^2 + q_1 \cdot s + q_0}{s \cdot (\tilde{p}_1 \cdot s + \tilde{p}_0)}, \quad (24)$$

hence, it can be seen as a real (filtered) PID controller. Its coefficients are obtained by a solution of the Diophantine equation (13) for some stable characteristic polynomial $d(s)$. Therefore, the next task is to choose this polynomial which must be of the 4th order. Here it is suggested to have it in this form:

$$d(s) = n(s)(s+\alpha)^2, \quad (25)$$

where $\alpha \succ 0$ is a free tuning constant and $n(s)$ is a stable polynomial computed from the denominator polynomial of the controlled system $a(s)$ using the spectral factorization technique [9]:

$$a^*(s)a(s) = n^*(s)n(s) \quad (26)$$

(here the asterisk denotes a complex conjugate polynomial defined as $x^*(s) = x(-s)$ and the result of the factorization is a polynomial with the similar properties as the original but it is guaranteed to be stable). This choice of the characteristic polynomial will not only guarantee stability of the resultant control system but also gives connection to the original process behaviour and it will leave space enough for further possible tuning as well. Solving (26) yields $n(s)$ in the following form:

$$n(s) = s^2 + n_1 \cdot s + n_0 = s^2 + 126.483 \cdot s + 3998. \quad (27)$$

It is easy to check that whereas the original polynomial $a(s)$ has poles located at $p_1 = 64.5$ and $p_2 = -62.0$, i.e. the first one is unstable, the result of the factorization $n(s)$ (27) provides both stable poles at positions $p_1 = -64.5$ and $p_2 = -62.0$. Now the characteristic polynomial (25) can be rewritten into the form:

$$d(s) = (s^2 + 126.483 \cdot s + 3998) \cdot (s + \alpha)^2, \quad (28)$$

where the only free parameter $\alpha \succ 0$ can be easily used for further tuning of the loop. In this work this is done using the sensitivity functions of the loop $S(s)$ and $S_u(s)$ in order to make the designed control system robust, as outlined in the section II.D of this paper.

Dependence of the H_∞-norms of both sensitivity functions $S(s)$ and $S_u(s)$ on the parameter α is clearly presented in Fig. 4.

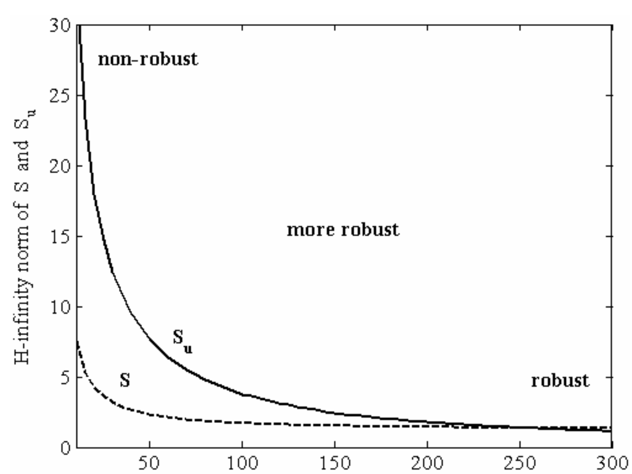

Fig. 4 Infinity-norm of the sensitivity functions S and S_u with α

From the plot it is obvious that the smaller value of the constant α the more sensitive the closed-loop system is, and vice versa – the higher value of α the more robust control system (regarding the influence of both disturbances and possible changes in the process model). Based on this information the free tuning parameter α was chosen as $\alpha = 200$. This choice will provide robust control system and approximately the same sensitivity for both disturbances. Besides this it can be seen as a trade-off between the desired robustness of the loop and limitations on the control input (higher values of α result in more aggressive control action and consequently more overshoots and oscillations of the controlled variable).

Then, the designed controller has the following form of a filtered PID:

$$C(s)_{\alpha=200} = \frac{q(s)}{s \cdot \tilde{p}(s)} = \frac{q_2 \cdot s^2 + q_1 \cdot s + q_0}{s \cdot (\tilde{p}_1 \cdot s + \tilde{p}_0)} = \\ = \frac{3.25 \cdot s^2 + 479 \cdot s + 8691}{s \cdot (s + 528.9)}, \tag{29}$$

with the coefficients computed from formulas (13) and (27),(28) as:

$$\begin{aligned} &\tilde{p}_1 = 1; \\ &\tilde{p}_0 = 2 \cdot \alpha + n_1 - a_1; \\ &q_2 = (2 \cdot \alpha \cdot n_1 + n_0 - a_1 \cdot \tilde{p}_0 - a_0)/b_0; \\ &q_1 = \left[\alpha^2 \cdot (1 + n_1) + 2 \cdot \alpha \cdot n_0 - a_0 \cdot \tilde{p}_0\right]/b_0; \\ &q_0 = \alpha^2 \cdot n_0 / b_0. \end{aligned} \tag{30}$$

It is easy to check that the strong stability condition (besides stability of the control system also stability of the controller is required – see section II.A) will be fulfilled as the coefficient $\tilde{p}_0$ is always positive for $\alpha \succ 0$.

V. Experiments

Several experiments were performed on the magnetic system in order to test the designed control loop. First, control in different operating points were analysed for two settings of the tuning parameter α - robust one ($\alpha = 200$), resulting in the controller (29) and, non-robust ($\alpha = 50$) with the non-robust controller of the form:

$$C(s)_{\alpha=50} = \frac{q(s)}{s \cdot \tilde{p}(s)} = \frac{q_2 \cdot s^2 + q_1 \cdot s + q_0}{s \cdot (\tilde{p}_1 \cdot s + \tilde{p}_0)} = \\ = \frac{1.15 \cdot s^2 + 89 \cdot s + 543}{s \cdot (s + 228.9)}. \tag{31}$$

Some of the control responses are presented further in Fig. 5 - Fig. 8.

From the graphs it is obvious that the suggested robust setting for ($\alpha = 200$) provides more stable response and better tracking of the reference signal. It gives relatively big overshoots but this can be improved by e.g. different control configuration, as shown in [8].

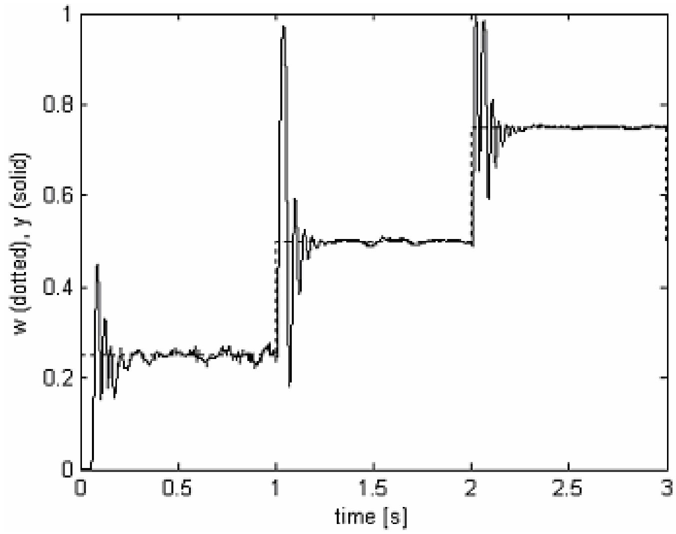

Fig. 5 Control response in different operating points: robust setting ($\alpha = 200$)

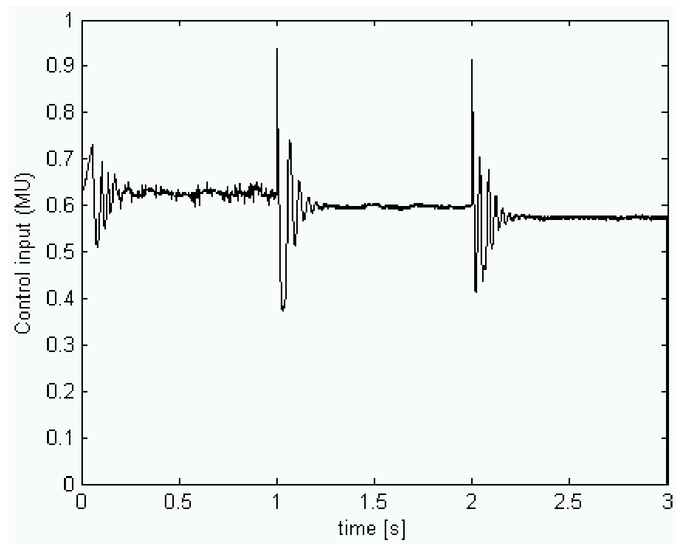

Fig. 6 Control action for different operating points: robust setting ($\alpha = 200$)

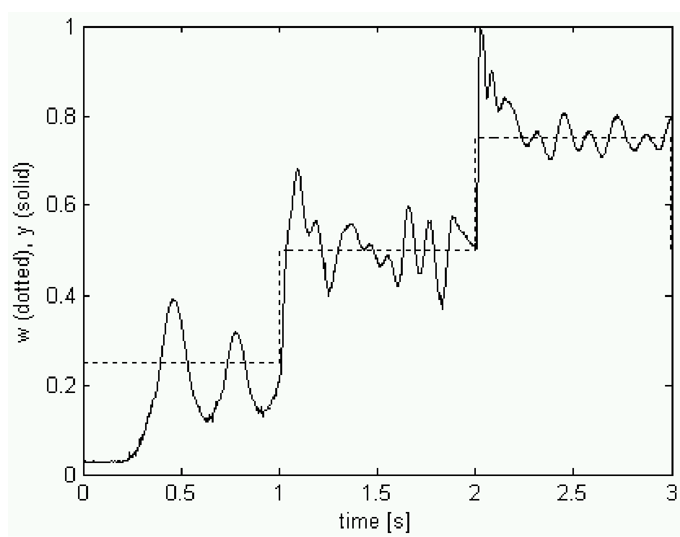

Fig. 7 Control response in different operating points: non-robust setting ($\alpha = 50$)

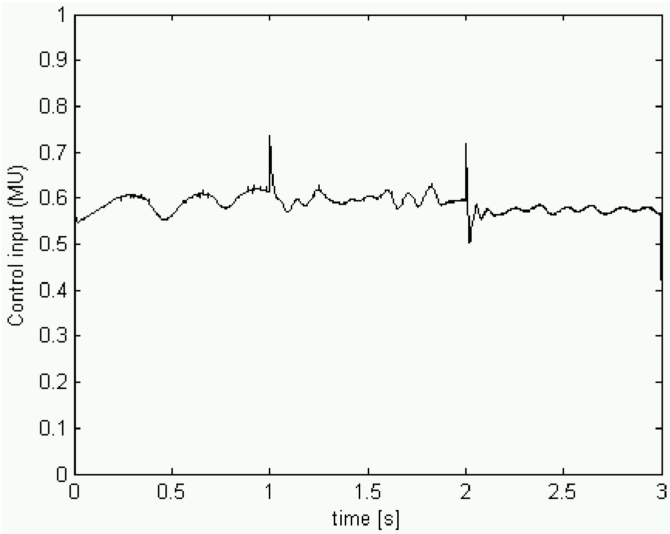

Fig. 8 Control action for different operating points:
non-robust setting ($\alpha = 50$)

Further attention was focused on the disturbance attenuation. During the control both disturbances (affecting control input at the time 1 sec. and controlled output at the time 2 sec.) were injected into the loop and the response was analysed. Both disturbances were step-functions as assumed in the section II.B of this paper and their amplitude was 10% of the set-point signal. The figures below (Fig. 9 - Fig. 12) show some of the achieved responses.

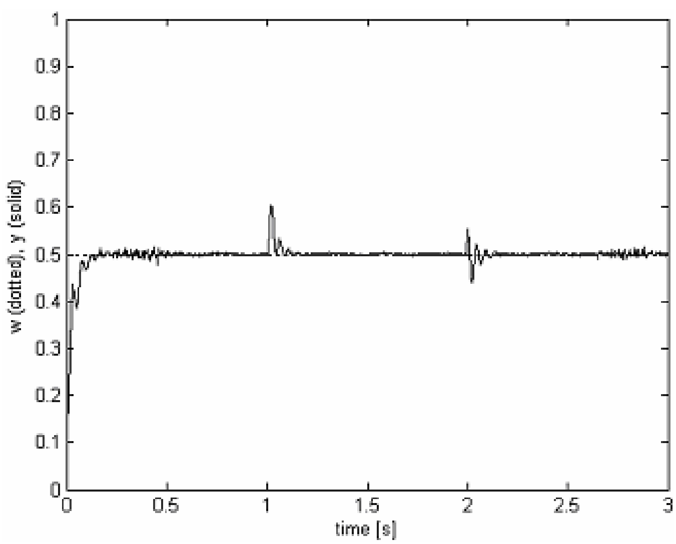

Fig. 9 Disturbance attenuation:
robust setting ($\alpha = 200$)

As can be clearly seen from the graphs, the robust setting of the tuning parameter α provides better responses to both disturbances.

Different approach for the control of the magnetic levitation system can be found in e.g. [1] and [6] where the state space approach and iterative procedure were successfully utilised respectively.

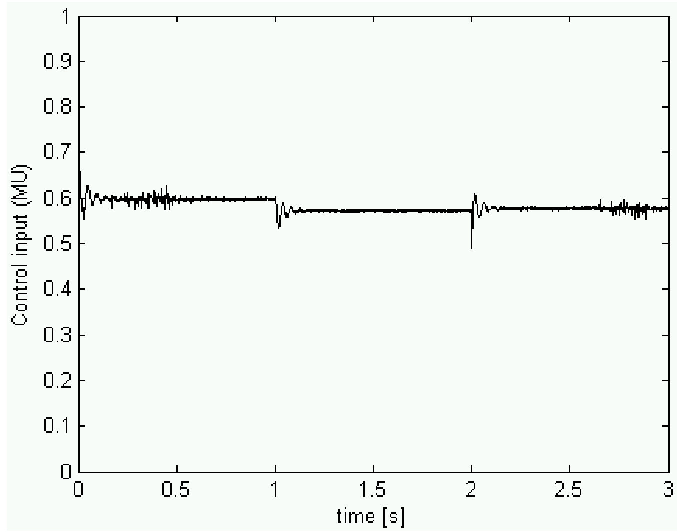

Fig. 10 Control action for disturbance attenuation:
robust setting ($\alpha = 200$)

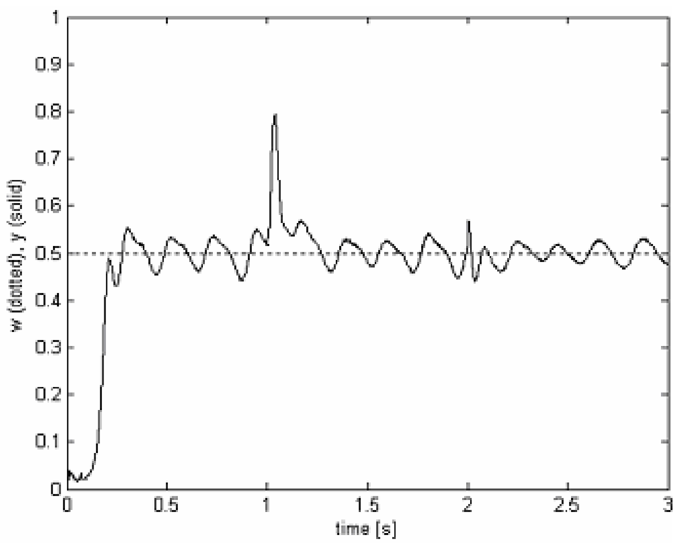

Fig. 11 Disturbance attenuation:
non-robust setting ($\alpha = 50$)

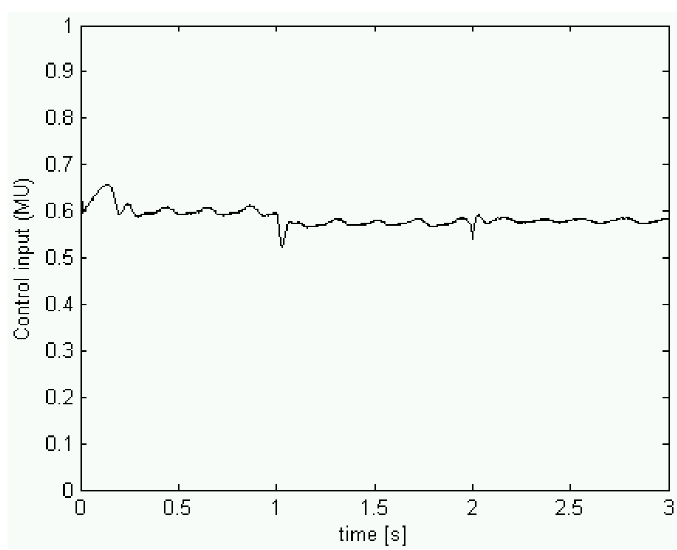

Fig. 12 Control action for disturbance attenuation:
non-robust setting ($\alpha = 50$)

VI. Conclusion

This paper presented a relatively simple framework for control of unstable single input – single output (SISO) processes. It exploited the advantages of systematic algebraic approach together with some useful tools from robust control theory, namely sensitivity functions and their H-infinity norms. The resultant controllers are designed to be robust with respect to both, changes in the operating point (adopted model) and disturbances affecting manipulated or controlled variables. The presented experimental results on the magnetic levitation system show applicability of the approach to safer control of unstable processes. Further, the presented methodology can be extended to cover also multi input – multi output (MIMO) processes which is the field for possible future research directions.

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