Assembling a formula for current transferring by using a summary graph and two-graph

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***Abstract*—**This paper deals with the symbolic solution of the switched current circuits. As is described, the full graph method of the solution can be used for finding relationships expressing current transfer, too. The summa MC-graph is constructed using two-graphs method in two-phase switching. By comparing the matrix form with results of the Mason's formula are derived relations for current transfers in all phases.

There are discussed various options described transistor memory cells - with loss and lossless transistors and normal transistor current mirror.

Evaluation of the graph is simplified if we consider the lossless transistors or if the y_{21}-parameter of one transistor is alpha multiple of second ones.

***Keywords*—**Switched current, two phases, two-graph, Mason's formula, relations for current transfer, summary MC-graph.

I. Introduction

A SWITCHED-CURRENT (SI) system is defined as a system using analogue sampled-data circuits in which signals are represented by a current samples. The basic building block of SI circuits is the current memory cell. This can be described by the equation of time domain or by the z-transform of the operator z-domain. Current memory cell can be represented by graph, too [3], [14].

Graph methods give results in a symbolic form [6], which makes it can be used to finding general relations. One option is to find general relations for current transfer in switched current circuits in two-phase switching. General relations are used for the calculation method of matrix calculus of final solution. This method will be demonstrated in the example. The summa MC-graph is constructed using two-graphs method in two-phase switching, two-graphs method for switched capacitor circuits is described in [4], where the resulting relationship is in the shape of the matrix.

Manuscript received July 19, 2011. This work was supported in part by the College of Polytechnics, Jihlava, Czech Republic.

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II. Calculation of the Transmissions from the Summary Graph

A. *Calculation of the Transmissions From the Summary Graph for Circuits with Less Transistors*

A circuit with a switched current has got for example the schematic wiring diagram shown in Fig.1 [7], [9], [10]. This circuit consist of two capacitors C, and three field effect transistors T_1, T_2 and T_3. Phases of the switching are marked as odd and even, not 1 and 2, which could lead to confusion with the numbering of nodes and phases.

A solution of a switched current circuit by the two-graph method of a summary MC-graph constructed on the basis of two-graphs will be shown in Fig.3.

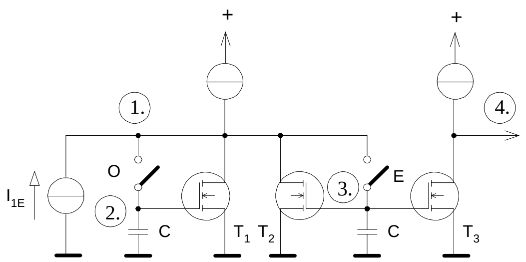


Fig.1 schematic diagram of the SI circuit from the solution

First we draw a partial diagram for the even phase and the odd phases separately by the algorithm described in [4]. These two diagrams for individual phases are in Fig.2.

Since the circuit consists of regular elements, which are described by their admittances, it is not necessary to distinguish different nodes numbering especially for voltage (V-) and current (I-) graph.

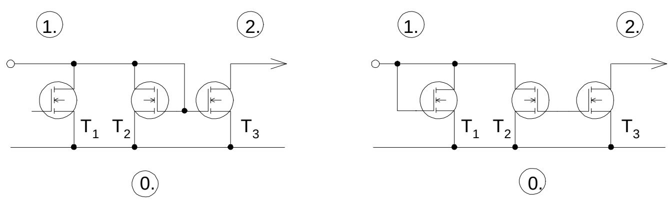


Fig.2 graphs for even and odd phases

To the diagrams for individual phases, we can assign directed graphs. For both even and odd phases it is necessary to draw a special voltage graph (ie. V-graph) and current graph (ie. I-graph). The voltage and current graphs have the same numbers of nodes as the circuit and are generated from the circuit in the following form: two-terminal passive elements of the circuit are included as appropriate branches in both current and voltage graphs, active elements, in the form of four-terminal voltage controlled current sources (VCCS), generate a branch in the voltage graph connecting the VCCS input nodes and a branch in the current graph connecting the VCCS output nodes. These graphs are shown in Fig.3.

Node numbers are in the square for the the current graph and in the triangle for the voltage graph. V-graph and I-graph of the less FET are in the Tab.1 (first row).

A summary graph is now constructed by first finding the incomplete common skeletons of the V-graph and the I-graph in the even phase and in the odd one, because the determinant Δ of the **Y**-matrix of the circuit is given by (1)

$$\Delta = \sum_{W} \pm (\text{product of addmittances}) \tag{1}$$

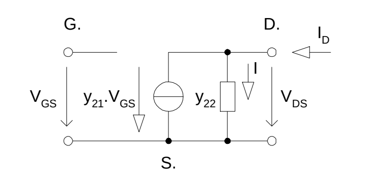
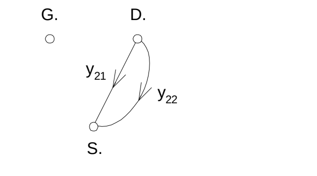
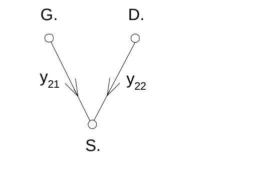
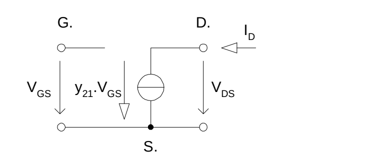
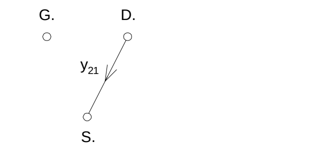
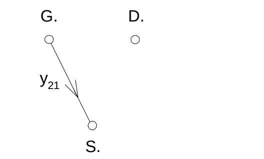
where

$$W = \{\text{set of spannig trees of V-graph}\} \cap \cap \{\text{set of spannig trees of I-graph}\} \tag{2}$$

In other words, there is a term in the expression for Δ corresponding to each spanning tree that is common to the voltage graph and current graph [12], [13].

In the even phase there is one incomplete common skeleton for example formed by the $y_{21}^{(2.)}$, $y_{22}^{(1.)}$, $y_{22}^{(2.)}$, thus obtained loop is with the transfer $y_{21}^{(2.)} + y_{22}^{(1.)} + y_{22}^{(2.)}$.

Table 1. Two-graph of the FET

Schematic diagram	I-graph	V-graph	Matrix
			V_{GS} : V_{DS} : I_D : y_{21} y_{22}
			V_{GS} : I_D : y_{21}

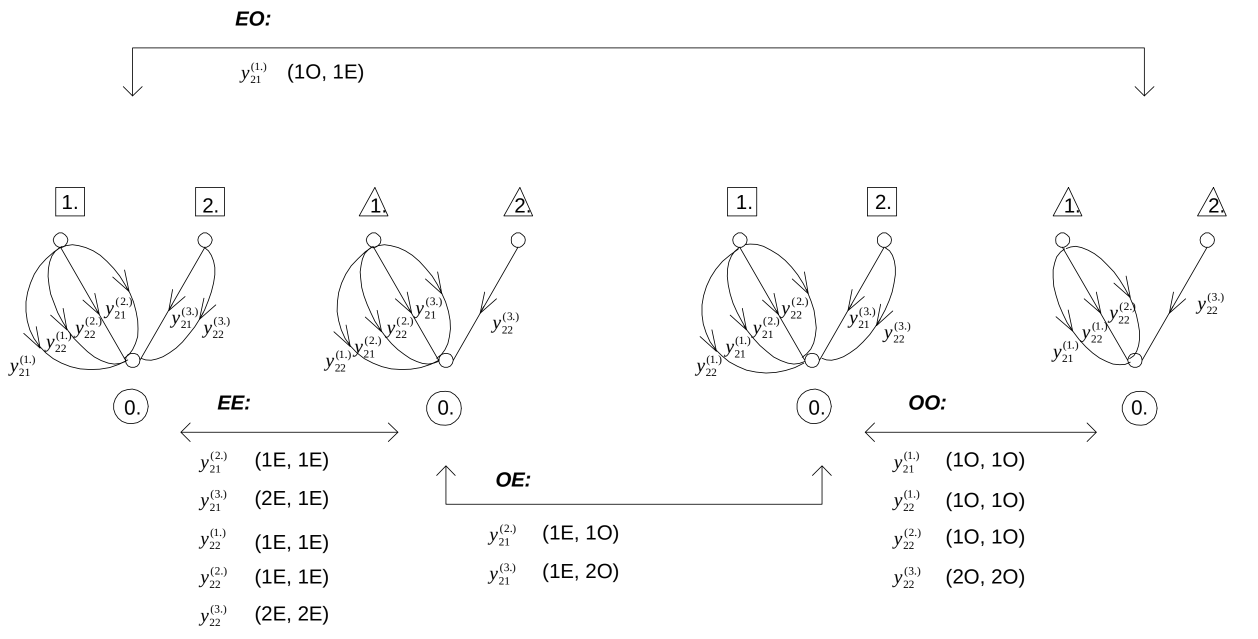


Fig.3 finding common skeletons of the V-graph and I-graph (less transistrors)

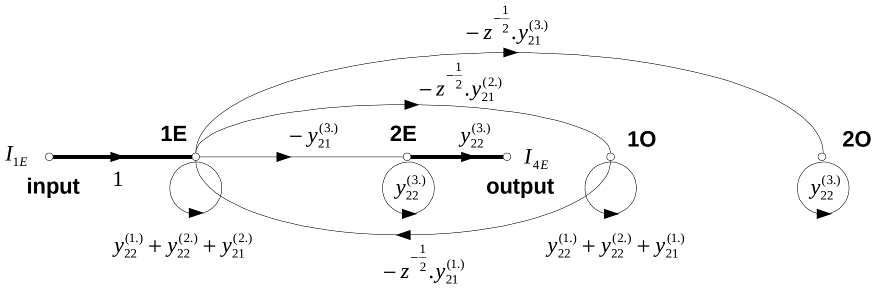


Fig.4 the summa MC-graph for transfer I_{4E}/I_{1E}

Thus obtained summary Mason-Coates graph is in Fig.4.

The current transfers [7], [10] will now be obtained from an extended graph, i.e. a graph must be extended to two branches as it is shown in Fig.4: the first branch from the input node I_{INP} to the node 1E with transfer 1 and the second branch from the node 2E to the node I_{OUT}. The transfer is equal to the transmission of its own loop at the output node. The summary graph is then evaluated by means of the Mason's rule [1], for example transfer $\frac{I_{4E}}{I_{1E}}$ is (3).

$$\frac{I_{OUT}}{I_{INP}}=\frac{I_{4E}}{I_{1E}}=\frac{\sum p_{(i)}.\Delta_{(i)}}{V-\sum S^{(K)}.V^{(K)}}=\frac{1(-y_{21}^{(3.)})y_{22}^{(3.)}(y_{22}^{(1.)}+y_{22}^{(2.)}+y_{21}^{(1.)})y_{22}^{(3.)}}{(y_{22}^{(1.)}+y_{22}^{(2.)}+y_{21}^{(2.)})y_{22}^{(3.)}(y_{22}^{(1.)}+y_{22}^{(2.)}+y_{21}^{(1.)})y_{22}^{(3.)}-(-z^{-\frac{1}{2}}y_{21}^{(2.)})(-z^{-\frac{1}{2}}y_{21}^{(1.)})y_{22}^{(3.)}y_{22}^{(3.)}}=$$

$$=\frac{-y_{21}^{(3.)}(y_{22}^{(1.)}+y_{22}^{(2.)}+y_{21}^{(1.)})}{(y_{22}^{(1.)}+y_{22}^{(2.)}+y_{21}^{(2.)})(y_{22}^{(1.)}+y_{22}^{(2.)}+y_{21}^{(1.)})-z^{-1}y_{21}^{(2.)}y_{21}^{(1.)}} \quad (3)$$

Summary Mason-Coates graph for transfer $\frac{I_{4O}}{I_{1E}}$ is shown in Fig.5, transfer of current from the Mason's rule is (4).

In Fig.6 and Fig.7 are Mason-Coates graphs for transfers $\frac{I_{4E}}{I_{1O}}$ (5) and for $\frac{I_{4O}}{I_{1O}}$ (6).

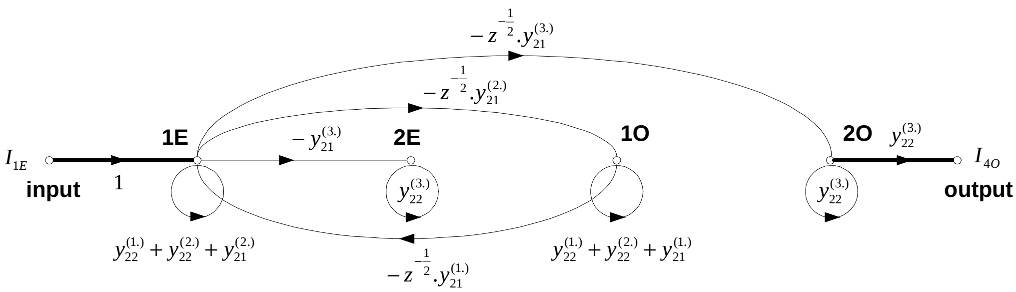


Fig.5 the summa MC-graph for transfer I_{4O}/I_{1E}

$$\frac{I_{OUT}}{I_{INP}}=\frac{I_{4O}}{I_{1E}}=\frac{\sum p_{(i)}.\Delta_{(i)}}{V-\sum S^{(K)}V^{(K)}}=\frac{1(-z^{-\frac{1}{2}}y_{21}^{(3.)})y_{22}^{(3.)}(y_{22}^{(1.)}+y_{22}^{(2.)}+y_{21}^{(1.)})y_{22}^{(3.)}}{(y_{22}^{(1.)}+y_{22}^{(2.)}+y_{21}^{(2.)})y_{22}^{(3.)}(y_{22}^{(1.)}+y_{22}^{(2.)}+y_{21}^{(1.)})y_{22}^{(3.)}-z^{-\frac{1}{2}}y_{21}^{(2.)}z^{-\frac{1}{2}}y_{21}^{(1.)}y_{22}^{(3.)}y_{22}^{(3.)}}=$$

$$= \frac{-z^{-\frac{1}{2}} y_{21}^{(3.)}(y_{22}^{(1.)} + y_{22}^{(2.)} + y_{21}^{(1.)})}{(y_{22}^{(1.)} + y_{22}^{(2.)} + y_{21}^{(2.)})(y_{22}^{(1.)} + y_{22}^{(2.)} + y_{21}^{(1.)}) - z^{-1} y_{21}^{(2.)} y_{21}^{(1.)}} \quad (4)$$

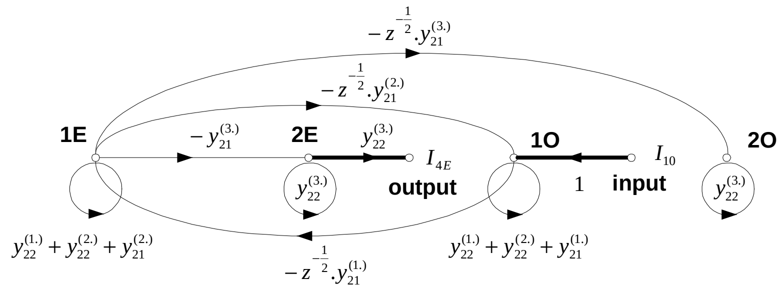


Fig.6 the summa MC-graph for transfer I_{4E}/I_{1O}

$$\frac{I_{OUT}}{I_{INP}} = \frac{I_{4E}}{I_{1O}} = \frac{\sum p_{(i)}.\Delta_{(i)}}{V - \sum S^{(K)} V^{(K)}} = \frac{1.z^{-\frac{1}{2}} y_{21}^{(1.)}(-y_{21}^{(3.)}) y_{22}^{(3.)} y_{22}^{(3.)}}{(y_{22}^{(1.)} + y_{22}^{(2.)} + y_{21}^{(2.)}) y_{22}^{(3.)} (y_{22}^{(1.)} + y_{22}^{(2.)} + y_{21}^{(1.)}) y_{22}^{(3.)} - z^{-\frac{1}{2}} y_{21}^{(2.)} z^{-\frac{1}{2}} y_{21}^{(1.)} y_{22}^{(3.)} y_{22}^{(3.)}} =$$

$$= -\frac{z^{-\frac{1}{2}} y_{21}^{(1.)} y_{21}^{(3.)}}{(y_{22}^{(1.)} + y_{22}^{(2.)} + y_{21}^{(2.)})(y_{22}^{(1.)} + y_{22}^{(2.)} + y_{21}^{(1.)}) - z^{-1} y_{21}^{(2.)} y_{21}^{(1.)}} \quad (5)$$

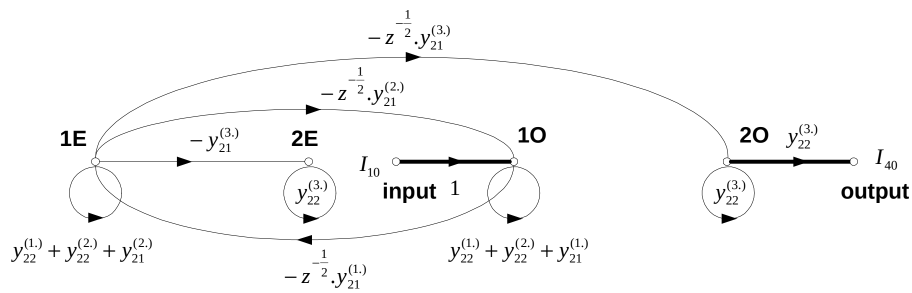


Fig.7 the summa MC-graph for transfer I_{4O}/I_{1O}

$$\frac{I_{OUT}}{I_{INP}} = \frac{I_{4O}}{I_{1O}} = \frac{\sum p_{(i)}.\Delta_{(i)}}{V - \sum S^{(K)} V^{(K)}} = \frac{1.z^{-\frac{1}{2}} y_{21}^{(1.)}(-z^{-\frac{1}{2}} y_{21}^{(3.)}) y_{22}^{(3.)} y_{22}^{(3.)}}{(y_{22}^{(1.)} + y_{22}^{(2.)} + y_{21}^{(2.)}) y_{22}^{(3.)} (y_{22}^{(1.)} + y_{22}^{(2.)} + y_{21}^{(1.)}) y_{22}^{(3.)} - z^{-\frac{1}{2}} y_{21}^{(2.)} z^{-\frac{1}{2}} y_{21}^{(1.)} y_{22}^{(3.)} y_{22}^{(3.)}} =$$

$$= -\frac{-z^{-1} y_{21}^{(1.)} y_{21}^{(3.)}}{(y_{22}^{(1.)} + y_{22}^{(2.)} + y_{21}^{(2.)})(y_{22}^{(1.)} + y_{22}^{(2.)} + y_{21}^{(1.)}) - z^{-1} y_{21}^{(2.)} y_{21}^{(1.)}} \quad (6)$$

B Circuit with Normal Current Mirror with Less Transistors

The current memory cell in Fig.1 is simply a current mirror. By normal current mirror action $\frac{y_{21}^{(3)}}{y_{21}^{(2)}} = \alpha$. We are considering $y_{21}^{(2)} = y_{21}^{(1)}$, thus obtained summary Mason-Coates graph in this cases is in Fig.8, current transfer for example $\frac{I_{4E}}{I_{1E}}$ is (7).

The remaining currernt transfers (i.e. $\frac{I_{4O}}{I_{1E}}$, $\frac{I_{4E}}{I_{1O}}$ and $\frac{I_{4O}}{I_{1O}}$) can be found analogous procedure, described in the previous paragraph.

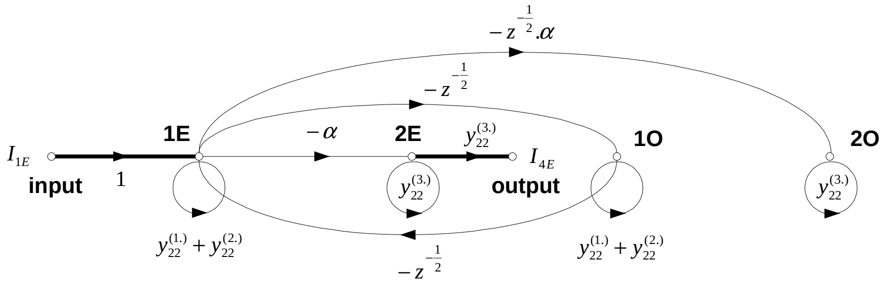


Fig.8 the summa MC-graph for transfer I_{4E}/I_{1E} for normal current mirror

$$\frac{I_{OUT}}{I_{INP}}=\frac{I_{4E}}{I_{1E}}=\frac{\sum p_{(i)}.\Delta_{(i)}}{V-\sum S^{(K)}.V^{(K)}}=\frac{1(-\alpha)y_{22}^{(3.)}(y_{22}^{(1.)}+y_{22}^{(2.)})y_{22}^{(3.)}}{(y_{22}^{(1.)}+y_{22}^{(2.)})y_{22}^{(3.)}(y_{22}^{(1.)}+y_{22}^{(2.)})y_{22}^{(3.)}-(-z^{-\frac{1}{2}})(-z^{-\frac{1}{2}})y_{22}^{(3.)}y_{22}^{(3.)}}=$$

$$=\frac{-\alpha(y_{22}^{(1.)}+y_{22}^{(2.)})}{(y_{22}^{(1.)}+y_{22}^{(2.)})(y_{22}^{(1.)}+y_{22}^{(2.)})-z^{-1}} \qquad (7)$$

C Calculation of the Transmissions From the Summary Graph for Circuits with Lossless Transistors

Voltage and current graphs are shown in Fig.9 for even and odd phases, node numbers are in square for current and in triangle for voltage graph.

V-graph and I-graph of the lossless FET are in the Tab.1 (second row).

The summary MC-graph is in Fig.9. Transfer for example $\frac{I_{4E}}{I_{1E}}$ by Mason's rule is (8). The remaining currernt transfers can be found analogous procedure.

The summary MC-graph in case normal current mirror is in Fig.10. Transfer $\frac{I_{4E}}{I_{1E}}$ by Mason's rule is (9).

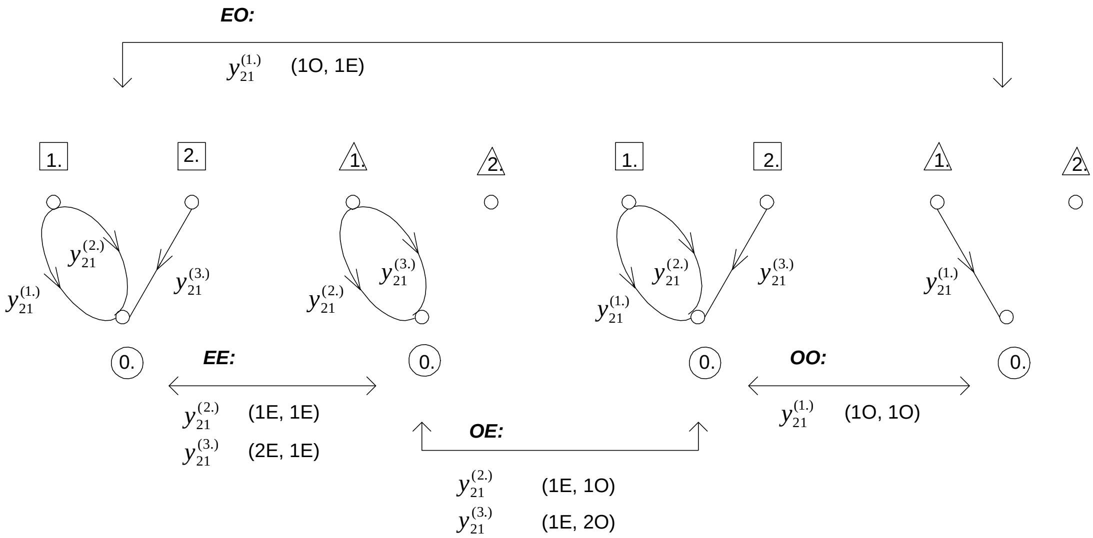


Fig.9 finding common skeletons of the V-graph and I-graph (lossless transistors)

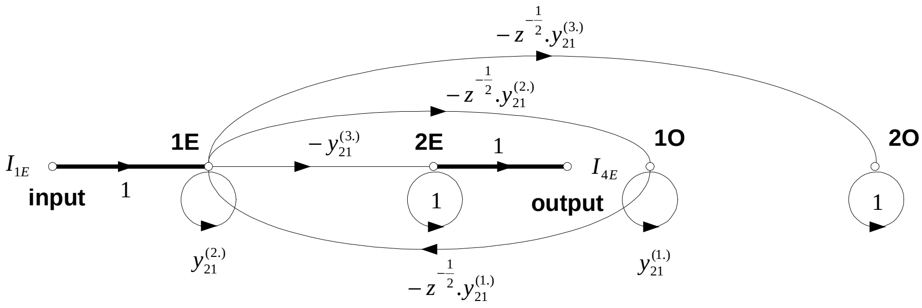


Fig.10 the summa MC-graph for transfer I_{4E}/I_{1E} (lossless transistors)

$$\frac{I_{OUT}}{I_{INP}}=\frac{I_{4E}}{I_{1E}}=\frac{\sum p_{(i)}.\Delta_{(i)}}{V-\sum S^{(K)}.V^{(K)}}=\frac{1(-y_{21}^{(3.)}).1}{y_{21}^{(2.)}y_{21}^{(1.)}.1.1-(-z^{-\frac{1}{2}}y_{21}^{(2.)})(-z^{-\frac{1}{2}}y_{21}^{(1.)})}=\frac{-y_{21}^{(3.)}}{y_{21}^{(2.)}y_{21}^{(1.)}-z^{-1}y_{21}^{(2.)}y_{21}^{(1.)}} \tag{8}$$

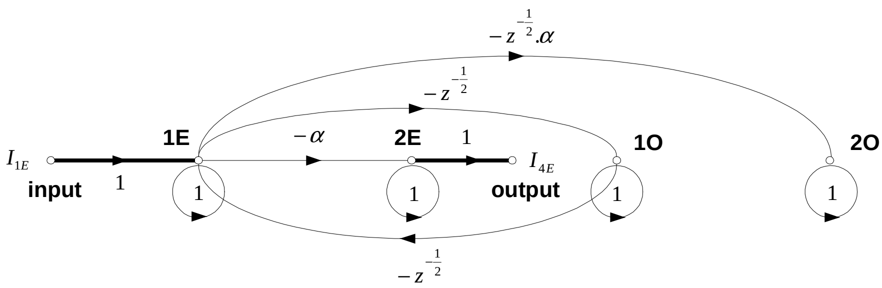


Fig.11 the summa MC-graph for transfer I_{4E}/I_{1E} (lossless transistors, normal current mirror)

$$\frac{I_{OUT}}{I_{INP}}=\frac{I_{4E}}{I_{1E}}=\frac{\sum p_{(i)}.\Delta_{(i)}}{V-\sum S^{(K)}.V^{(K)}}=\frac{1.(-\alpha).1}{1.1.1.1-(-z^{-\frac{1}{2}})(-z^{-\frac{1}{2}})}=-\frac{\alpha}{1-z^{-1}} \tag{9}$$

III. Calculation of the Transmissions from the Model of the Circuits

A Description of the CircuitUsing a Matrix

Relations for transfers of currents (1) to (4) have been expressed as fractions in which conductivities $y_{21}^{(i)}$ and $y_{22}^{(i)}$ are found both in the numerator and in the denominator. That shows a theoretical possibility to express these transfers as ratios determinants of certain algebraic complements of matrices constructed from the mentioned conductivities $y_{21}^{(i)}$ and $y_{22}^{(i)}$.

In order to find such relations, a linearized diagram of the circuit from the Fig. 1. is drawn in Fig. 12.

When the switches are on, this circuit has two nodes, so it can be described by two equations for currents I_1, I_2 constructed by means of the Kirchhoff's law. However, as the currents can occur in even and odd phases, the total of four equations will be constructed (10). The transistors are modelled by the VCT elements with the control voltages V_{1E}, V_{1O}. Next there is the voltage V_2 in the circuit, again in both phases, i.e. V_{2E} and V_{2O}.

The corresponding equation system (10)

$$\begin{aligned}
I_{1E} &= z^{-\frac{1}{2}}.y_{21}^{(1.)}.V_{1O}+y_{22}^{(1.)}.V_{1E}+y_{22}^{(2.)}.V_{1E}+y_{21}^{(2.)}.V_{1E}\\
I_{1O} &= y_{21}^{(1.)}.V_{1O}+y_{22}^{(1.)}.V_{1O}+y_{22}^{(2.)}.V_{1O}+z^{-\frac{1}{2}}.y_{21}^{(2.)}.V_{1E}\\
I_{2E} &= y_{21}^{(3.)}.V_{1E}+y_{22}^{(3.)}.V_{2E}\\
I_{2O} &= z^{-\frac{1}{2}}.y_{21}^{(3.)}.V_{1E}+y_{22}^{(3.)}.V_{2O}
\end{aligned} \tag{10}$$

can be rewritten to a matrix form (11).

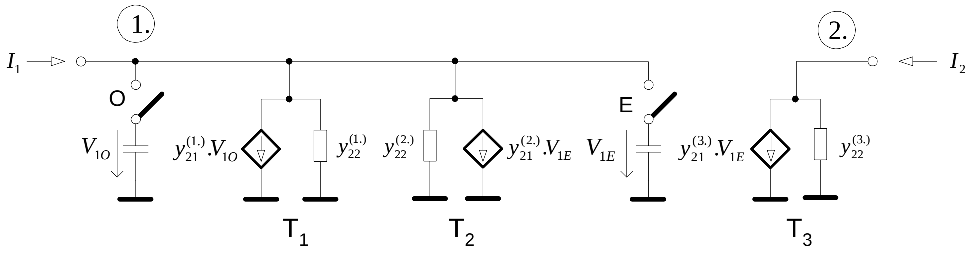


Fig.12 linearized diagram of the circuit from the Fig. 1

$$\begin{array}{c} \\ I_{1E}: \\ I_{2E}: \\ I_{1O}: \\ I_{2O}: \end{array}\begin{array}{cccc} V_{1E}: & V_{2E}: & V_{1O}: & V_{2O}: \end{array}$$

$$\begin{bmatrix} y_{22}^{(1.)}+y_{22}^{(2.)}+y_{21}^{(2.)} & 0 & z^{-\frac{1}{2}}.y_{21}^{(1.)} & 0 \\ y_{21}^{(3.)} & y_{22}^{(3.)} & 0 & 0 \\ z^{-\frac{1}{2}}.y_{21}^{(2.)} & 0 & y_{22}^{(1.)}+y_{22}^{(2.)}+y_{21}^{(1.)} & 0 \\ z^{-\frac{1}{2}}.y_{21}^{(3.)} & 0 & 0 & y_{22}^{(3.)} \end{bmatrix} \quad (11)$$

B Obtaining General Relations by Comparing the Results

In the next step, the elements occurring in the numerator and denominator of the relation (3) are written into the corresponding positions in which they are in the matrix (11).

Because the nodes of Fig.1 are renumbered in Fig.2, when $4 \to 2$. Transfers of the currents after renumbered are following: $\frac{I_{4E}}{I_{1E}} \to \frac{I_{2E}}{I_{1E}}$, $\frac{I_{4O}}{I_{1E}} \to \frac{I_{2O}}{I_{1E}}$, $\frac{I_{4O}}{I_{1O}} \to \frac{I_{2O}}{I_{1O}}$, $\frac{I_{4E}}{I_{1O}} \to \frac{I_{2E}}{I_{1O}}$.

So will :

$$\frac{I_{2E}}{I_{1E}} = \frac{-y_{21}^{(3.)}(y_{22}^{(1.)}+y_{22}^{(2.)}+y_{21}^{(1.)})}{(y_{22}^{(1.)}+y_{22}^{(2.)}+y_{21}^{(2.)})(y_{22}^{(1.)}+y_{22}^{(2.)}+y_{21}^{(1.)}) - z^{-1}y_{21}^{(2.)}y_{21}^{(1.)}} =$$

$$= \frac{\begin{array}{c|cc|} & 2E: & 2O: \\ 1E: & & \\ & y_{21}^{(3.)} & \\ & & y_{22}^{(1.)}+y_{22}^{(2.)}+y_{21}^{(1.)} \\ 2O: & & \end{array}.(-1)}{\begin{array}{c|cc|} & 2E: & 2O: \\ & y_{22}^{(1.)}+y_{22}^{(2.)}+y_{21}^{(2.)} & z^{-\frac{1}{2}}.y_{21}^{(1.)} \\ 2E: & & \\ & z^{-\frac{1}{2}}.y_{21}^{(2.)} & y_{22}^{(1.)}+y_{22}^{(2.)}+y_{21}^{(1.)} \\ 2O: & & \end{array}} =$$

$$= \frac{\Delta_{1E,2O:2E,2O}}{\Delta_{2E,2O:2E,2O}} \quad (12)$$

By comparing with the matrix (11) it is now apparent that in the numerator there is an algebraic complement of this matrix (11) created out of this matrix by leaving out the rows 1E and 2O and the columns 2E and 2O, symbolically written $\Delta_{1E,2O:2E,2O}$. In the denominator, there is then the algebraic complement of the matrix (11) created out of this matrix by leaving out the rows 2E and 2O and the columns 2E and 2O, symbolically written $\Delta_{2E,2O:2E,2O}$.

$$\frac{\Delta_{1E,2O:2E,2O}}{\Delta_{2E,2O:2E,2O}} = \frac{-1.\begin{vmatrix} y_{21}^{(3.)} & 0 \\ 0 & y_{22}^{(1.)}+y_{22}^{(2.)}+y_{21}^{(1.)} \end{vmatrix}}{\begin{vmatrix} y_{22}^{(1.)}+y_{22}^{(2.)}+y_{21}^{(2.)} & z^{-\frac{1}{2}}.y_{21}^{(1.)} \\ z^{-\frac{1}{2}}.y_{21}^{(2.)} & y_{22}^{(1.)}+y_{22}^{(2.)}+y_{21}^{(1.)} \end{vmatrix}} =$$

$$= \frac{-y_{21}^{(3.)}(y_{22}^{(1.)}+y_{22}^{(2.)}+y_{21}^{(1.)})}{(y_{22}^{(1.)}+y_{22}^{(2.)}+y_{21}^{(2.)})(y_{22}^{(1.)}+y_{22}^{(2.)}+y_{21}^{(1.)}) - z^{-1}y_{21}^{(2.)}y_{21}^{(1.)}} \quad (13)$$

According to the theory of multiple algebraic complements, the sign (-1) in the numerator gives the number of omitted odd indices, in the numerator the row 1E is omitted, which is the first (odd index) row in the matrix, while the remaining row 2O is even in the row as well as the omitted column 2E, which is the second column in the matrix, and the column 2O, which is the fourth column in the matrix.

In the denominator the omitted rows 2E and 2O and columns 2E and 2O are the second and fourth rows and columns in the matrix, i.e. they always have even indices, which then corresponds to a positive sign.

From the graph solution it is also possible to derive relations for the remaining current transfers. If the numerator of the relation (4) ie. $-z^{-\frac{1}{2}}y_{21}^{(3.)}.(y_{22}^{(1.)}+y_{22}^{(2.)}+y_{21}^{(1.)})$, i.e. only the elements $-z^{-\frac{1}{2}}y_{21}^{(3.)}$ and $y_{22}^{(1.)}+y_{22}^{(2.)}+y_{21}^{(1.)}$, are written into the corresponding position in the conductivity matrix (11) the relation (14) will hold.

$$-z^{-\frac{1}{2}}y_{21}^{(3.)}.(y_{22}^{(1.)}+y_{22}^{(2.)}+y_{21}^{(1.)}) =$$

$$\begin{array}{c} \\ 1E: \\ 2E: \\ \\ \\ \end{array} \begin{array}{c} 2E: \qquad\qquad\qquad\qquad 2O: \\ = \left| \begin{array}{cc|cc} & & & \\ \hline & & y_{22}^{(1.)}+y_{22}^{(2.)}+y_{21}^{(1.)} & \\ z^{-\frac{1}{2}}.y_{21}^{(3.)} & & & \end{array} \right| = \end{array}$$

$$= \Delta_{1E,2E:2E,2O} = \begin{vmatrix} 0 & y_{22}^{(1.)}+y_{22}^{(2.)}+y_{21}^{(1.)} \\ z^{-\frac{1}{2}}.y_{21}^{(3.)} & 0 \end{vmatrix} \qquad (14)$$

For the transfer of the currents $\frac{I_{2O}}{I_{1E}}$ we can write the relation (15) if the numerator stayed the same.

$$\frac{I_{2O}}{I_{1E}} = \frac{\Delta_{1E,2E:2E,2O}}{\Delta_{2E,2O:2E,2O}} \qquad (15)$$

Analogically for the transfer $\frac{I_{2E}}{I_{1O}}$ the relation (17) holds if the numerator of the relation (3) $z^{-\frac{1}{2}} y_{21}^{(1.)}.y_{21}^{(3.)}$, i.e. the elements $z^{-\frac{1}{2}} y_{21}^{(1.)}$ and $y_{21}^{(3.)}$, are written into their positions given by the matrix (11),

$$z^{-\frac{1}{2}} y_{21}^{(1.)}.y_{21}^{(3.)} = \begin{array}{c} \\ \\ 1O: \\ 2O: \end{array} \begin{array}{c} 2E: \qquad\qquad\qquad 2O: \\ \left| \begin{array}{cc|cc} & & z^{-\frac{1}{2}}.y_{21}^{(1.)} & \\ y_{21}^{(3.)} & & & \\ \hline & & & \\ & & & \end{array} \right| = \end{array}$$

$$= \Delta_{1O,2O:2E,2O} = \begin{vmatrix} 0 & z^{-\frac{1}{2}}.y_{21}^{(1.)} \\ y_{21}^{(3.)} & 0 \end{vmatrix} \qquad (16)$$

$$\frac{I_{2E}}{I_{1O}} = \frac{\Delta_{1O,2O:2E,2O}}{\Delta_{2E,2O:2E,2O}} \qquad (17)$$

and for $\frac{I_{2O}}{I_{1O}}$ relation (19) from (18). The numerators stay unchanged.

$$-z^{-1} y_{21}^{(1.)} y_{21}^{(3.)} = \begin{array}{c} \\ 2E: \\ 1O: \\ \\ \end{array} \begin{array}{c} 2E: \qquad\qquad\qquad 2O: \\ \left| \begin{array}{cc|cc} & & z^{-\frac{1}{2}}.y_{21}^{(1.)} & \\ \hline & & & \\ z^{-\frac{1}{2}}.y_{21}^{(3.)} & & & \end{array} \right| = \end{array}$$

$$= \Delta_{2E,1O:2E,2O} = \begin{vmatrix} 0 & z^{-\frac{1}{2}}.y_{21}^{(1.)} \\ z^{-\frac{1}{2}}.y_{21}^{(3.)} & 0 \end{vmatrix} \qquad (18)$$

$$\frac{I_{2O}}{I_{1O}} = \frac{\Delta_{2E,1O:2E,2O}}{\Delta_{2E,2O:2E,2O}} \qquad (19)$$

Another possible way of writing the above mentioned derived relations for current transfers by means of multiple algebraic complements in individual phases using the symbols stated in [1] is then (18)

$$\frac{I_{2E}}{I_{1E}} = \frac{\Delta_{1E>0,2O>0:2E>0,2O>0}}{\Delta_{2E>0,2O>0:2E>0,2O>0}} \quad \frac{I_{2O}}{I_{1E}} = \frac{\Delta_{1E>0,2E>0:2E>0,2O>0}}{\Delta_{2E>0,2O>0:2E>0,2O>0}}$$
$$\frac{I_{2E}}{I_{1O}} = \frac{\Delta_{1O>0,2O>0:2E>0,2O>0}}{\Delta_{2E>0,2O>0:2E>0,2O>0}} \quad \frac{I_{2O}}{I_{1O}} = \frac{\Delta_{1O>0,2E>0:2E>0,2O>0}}{\Delta_{2E>0,2O>0:2E>0,2O>0}} \qquad (20)$$

where the symbol > 0 means adding a given row (or column) to the zero row (or column) of the matrix.

Now we are considering normal current mirror [2]. In this case the matrix (11) will have the form (21).

$$\begin{array}{c} \\ I_{1E}: \\ I_{2E}: \\ I_{1O}: \\ I_{2O}: \end{array} \begin{array}{c} V_{1E}: \quad V_{2E}: \qquad V_{1O}: \qquad V_{2O}: \\ \left[\begin{array}{cc|cc} y_{22}^{(1.)}+y_{22}^{(2.)} & 0 & z^{-\frac{1}{2}} & 0 \\ \alpha & y_{22}^{(3.)} & 0 & 0 \\ \hline z^{-\frac{1}{2}} & 0 & y_{22}^{(1.)}+y_{22}^{(2.)} & 0 \\ z^{-\frac{1}{2}}.\alpha & 0 & 0 & y_{22}^{(3.)} \end{array} \right] \end{array} \qquad (21)$$

The elements occurring in the numerator and denominator of the relation (7) are written into the corresponding positions in which they are in the matrix (21).

Thus

$$\frac{I_{2E}}{I_{1E}} = \frac{-\alpha(y_{22}^{(1.)}+y_{22}^{(2.)})}{(y_{22}^{(1.)}+y_{22}^{(2.)})(y_{22}^{(1.)}+y_{22}^{(2.)}) - z^{-\frac{1}{2}}.z^{-\frac{1}{2}}} =$$

$$= \frac{\begin{array}{c} \\ 1E: \\ \\ 2O: \end{array} \begin{array}{c} 2E: \qquad\qquad 2O: \\ \left| \begin{array}{cc|cc} & & & \\ \alpha & & & \\ \hline & & y_{22}^{(1.)}+y_{22}^{(2.)} & \\ & & & \end{array} \right| .(-1) \end{array}}{\begin{array}{c} \\ 2E: \\ 2O: \end{array} \begin{array}{c} 2E: \qquad\qquad 2O: \\ \left| \begin{array}{cc|cc} y_{22}^{(1.)}+y_{22}^{(2.)} & & z^{-\frac{1}{2}} & \\ \hline z^{-\frac{1}{2}} & & y_{22}^{(1.)}+y_{22}^{(2.)} & \\ & & & \end{array} \right| \end{array}} = \frac{\Delta_{1E,2O:2E,2O}}{\Delta_{2E,2O:2E,2O}} \qquad (22)$$

The results (12), (15), (17), (19) and (22) are identical with the results (8.49), (8.50), (8.51), (8.52) published in [2]. But finding relations (8.49), (8.50), (8.51), (8.52) in [2] requires the use of multiple (i.e. double and triple) algebraic complements.

This calculation is somewhat more difficult than described graphs method.

C Obtaining General Relations for Lossless Transistor

We are considering circuit with the lossless transistors. The linearized circuit diagram of the circuit from the Fig.1. is drawn in this case in Fig.13.

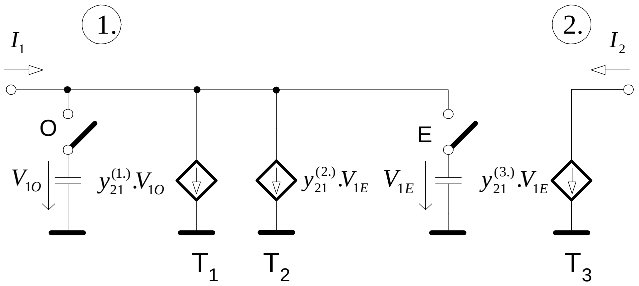


Fig.13 linearized diagram of the circuit with lossless FETs

The corresponding equation system is in this case (23) and can be rewritten to a matrix form (24).

$$\begin{aligned} I_{1E} &= z^{-\frac{1}{2}}.y_{21}^{(1.)}.V_{1O} + y_{21}^{(2.)}.V_{1E} \\ I_{1O} &= y_{21}^{(1.)}.V_{1O} + z^{-\frac{1}{2}}.y_{21}^{(2.)}.V_{1E} \\ I_{2E} &= y_{21}^{(3.)}.V_{1E} \\ I_{2O} &= z^{-\frac{1}{2}}.y_{21}^{(3.)}.V_{1E} \end{aligned} \quad (23)$$

$$\begin{array}{c} \\ I_{1E}: \\ I_{2E}: \\ I_{1O}: \\ I_{2O}: \end{array} \begin{array}{c} \begin{array}{cccc} V_{1E}: & V_{2E}: & V_{1O}: & V_{2O}: \end{array} \\ \left[\begin{array}{cc|cc} y_{21}^{(2.)} & 0 & z^{-\frac{1}{2}}.y_{21}^{(1.)} & 0 \\ y_{21}^{(3.)} & 0 & 0 & 0 \\ \hline z^{-\frac{1}{2}}.y_{21}^{(2.)} & 0 & y_{21}^{(1.)} & 0 \\ z^{-\frac{1}{2}}.y_{21}^{(3.)} & 0 & 0 & 0 \end{array}\right] \end{array} \quad (24)$$

The network elements $y_{ij}^{(k)}$ in the relation (8) are in next step written into the corresponding positions in the previous matrix (24).

Thus

$$\frac{I_{2E}}{I_{1E}} = \frac{-y_{21}^{(3.)}.y_{21}^{(1.)}}{y_{21}^{(2.)}.y_{21}^{(1.)} - z^{-1}y_{21}^{(2.)}y_{21}^{(1.)}} =$$

$$= \frac{\begin{array}{c|cc|} & 2E: & 2O: \\ 1E: & & \\ & y_{21}^{(3.)} & \\ & & y_{21}^{(1.)} \\ 2O: & & \end{array}.(-1)}{\begin{array}{c|cc|} & 2E: & 2O: \\ 2E: & y_{21}^{(2.)} & z^{-\frac{1}{2}}.y_{21}^{(1.)} \\ 2O: & z^{-\frac{1}{2}}.y_{21}^{(2.)} & y_{21}^{(1.)} \end{array}} =$$

$$= \frac{\Delta_{1E,2O:2E,2O}}{\Delta_{2E,2O:2E,2O}} \quad (25)$$

According to the theory of multiple algebraic complements, the sign (-1) in the numerator gives the number of omitted odd indices, in the numerator the row 1E is omitted, which is the first (odd index) row in the matrix, while the remaining row 2O is even in the row as well as the omitted column 2E, which is the second column in the matrix, and the column 2O, which is the fourth column in the matrix.

The result (12), (25) and (8.49) in [2] are identical.

Now we are considering normal current mirror [2].

In this case the corresponding system of equations have the form (26).

$$\begin{aligned} I_{1E} &= z^{-\frac{1}{2}}.1.V_{1O} + \alpha.V_{1E} \\ I_{1O} &= 1.V_{1O} + z^{-\frac{1}{2}}.1.V_{1E} \\ I_{2E} &= \alpha.V_{1E} \\ I_{2O} &= z^{-\frac{1}{2}}.\alpha.V_{1E} \end{aligned} \quad (26)$$

and can be rewritten to a matrix form (27).

$$\begin{array}{c} \\ I_{1E}: \\ I_{2E}: \\ I_{1O}: \\ I_{2O}: \end{array} \begin{array}{c} \begin{array}{cccc} V_{1E}: & V_{2E}: & V_{1O}: & V_{2O}: \end{array} \\ \left[\begin{array}{cc|cc} 1 & 0 & z^{-\frac{1}{2}} & 0 \\ \alpha & 0 & 0 & 0 \\ \hline z^{-\frac{1}{2}} & 0 & 1 & 0 \\ z^{-\frac{1}{2}}.\alpha & 0 & 0 & 0 \end{array}\right] \end{array} \quad (27)$$

The elements in the relation (9) are written into the corresponding positions in the matrix (27). According to the theory of multiple algebraic complements, the sign (-1) in the numerator gives the number of omitted odd indices.

$$\frac{I_{2E}}{I_{1E}} = \frac{-\alpha.1.1}{1.1.1.1 - z^{-\frac{1}{2}}.z^{-\frac{1}{2}}} =$$

$$= \frac{\begin{array}{c|cc|} & 2E: & 2O: \\ 1E: & \alpha & \\ 2O: & & 1 \end{array}.(-1)}{\begin{array}{c|cc|} & 2E: & 2O: \\ 2E: & y_{21}^{(2.)} & z^{-\frac{1}{2}} \\ 2O: & z^{-\frac{1}{2}} & y_{21}^{(1.)} \end{array}} = \frac{\Delta_{1E,2O:2E,2O}}{\Delta_{2E,2O:2E,2O}} \qquad (28)$$

As we can see, the result (28) is identical to the previous resulsts, too.

CONCLUSION

A unified method in analyzing switched current circuits is presented. The advantages of this approach are in its uniformity in deriving results from graph. The two-graph method is applied to assembly of the MC-graph. A clearly arranged set of graphs derived for different types of switching circuits can by used for finding a formula for current transfering. By comparing the results from the matrix and results obtained from the Mason's formula are derived general relations for current transfers.

As can be seen, the description of different types of memory cells can be used for the assembling of the general relations. The more elements the description contains, the more matrix is full. Finding algebraic complements is then more easy and most arranged, too. The relationship can be found from the simplest description, of course.

The described method can be used for solving and understanding of simple circuits.

ACKNOWLEDGMENT

This work was supported by the Department of Electronics and Informatics of the College of Polytechnics, Jihlava, Czech Republic.

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