Autotuning principles for delayed systems

Roman Prokop, Libor Pekař, Radek Matušů, Jiří Korbel

***Abstract*—**The paper brings a combination of a biased-relay feedback experiment and an algebraic control design method for time-delay systems. The combination results in a new principle of autotuning for a wide class of single input-output dynamic systems. The estimation of the controlled process is based on asymmetrical limit cycle data. Then, a stable transfer function with a dead-time term is identified. The controller is computed through solutions of Diophantine equations in the ring of stable and proper retarded quasipolynomial meromorphic functions (RMS). Controller parameters are tuned through a pole-placement problem as a desired multiple root of the characteristic closed loop equation. The controller design in this ring yields a Smith-like feedback controller with the realistic PID structure. The methodology offers a scalar tuning parameter m0 > 0 which can be adjusted by a suitable principle or further optimization. The first and second order time-delay transfer functions can sufficiently estimate systems of quite high orders. The developed principles are illustrated by examples in the Matlab + Simulink environment.

***Keywords*—**Algebraic control design, autotuning, pole-placement problem, relay experiment.

I. Introduction

THE development of various autotuning principles was started by a simple symmetrical relay feedback experiment proposed by Åström and Hägglund [1] in the year 1984. The ultimate gain and ultimate frequency are then used for adjusting of parameters by original Ziegler-Nichols rules. During the period of more than two decades, many studies have been reported to extend and improve autotuners principles; see e.g. [2], [3], [4], [8], [9]. The extension in relay utilization was performed in [2], [5], [7], [14] by an asymmetry and hysteresis of a relay. Over time, the direct estimation of transfer function parameters instead of critical values began to appear. Experiments with asymmetrical and dead-zone relay feedback are reported in [10]. Also, various control design principles and rules can be investigated in mentioned references. Nowadays, almost all commercial industrial PID controllers provide the feature of autotuning.

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Time delay systems constitute an indispensable family of industrial processes. A feedback loop is the most efficient manner how to change system properties. However, thanks to the feedback loop, time delay notably affects whole system dynamics. During recent decades various approaches and algorithms have been researched for compensating the influence of time delay in a feedback loop. In addition to that, many control design principles to obtain satisfactory loop behavior have been presented. There surely exist several classifications of control design methods for time delay systems. Nowadays, three main groups dominate. The first group contains approaches based on Smith predictor structure, or more precisely its modifications [8], [14]. These methods assume model of the controlled system in feedback loop, thus, it pertains into IMC (Internal Model Controllers). Second group consists of predictive based approaches, mainly using state-space description [15]. Last but not least, third group of algebraic approaches is assumed [12], [16] – [18]. Extension to retarded quasipolynomials utilized in this paper are studied in e.g. [19], [20].

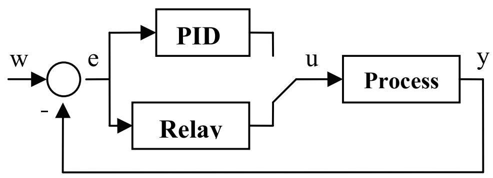

Fig. 1 Block diagram of an autotuner

This contribution brings a novel combination of identification test, made with help of biased relay with hysteresis, and algebraic controller design approach, based on solution of Diophantine equation in a special R_{MS} ring. A transfer function of the first and second order with time constant and time delay is assumed as an example for control applications giving a class of a PID like controllers with a Smith predictor structure. The pole placement problem in R_{MS} ring is formulated through a Diophantine equation and the pole is analytically tuned according to aperiodic response of the closed loop. A general basic scheme of the autotuning principle can be seen in Fig. 1.

II. Relay Feedback Tests

An auto-tuning procedure consists of a process identification experiment plus a controller design method. The traditional method was proposed by Åström and Hägglund

[1], based on a symmetrical relay feedback test when a relay of magnitude h_r is inserted in the feedback loop. The result of the original test was the critical point of the open loop Nyquist curve, see e.g. [1], [3], [4] and naturally the ultimate period and the limit cycle amplitude generated by process output. However, there are another relays used in identification experiments.

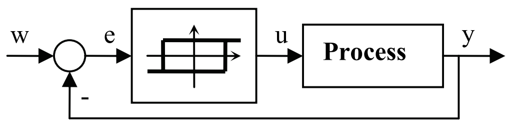

Fig. 2 Block diagram of an autotuning principle

A biased (asymmetrical) relay experiment according to Fig. 2 used for identification can give the final model transfer functions with a time delay terms. It is well known that many stable industrial processes can be adequately approximated by the model for first order (stable) systems plus dead time (FOPDT). It is supposed in the form:

$$G(s)=\frac{K}{Ts+1}\cdot e^{-\Theta s} \tag{1}$$

and the process gain can be computed by the relation [24]:

$$K=\frac{\int_0^{iT_y} y(t)dt}{\int_0^{iT_y} u(t)dt}; \quad i=1,2,3,... \tag{2}$$

The time constant and time delay terms are given by [11]:

$$T=\frac{T_u}{2\pi}\cdot\sqrt{\frac{16\cdot K^2\cdot u_0^2}{\pi^2\cdot a_r^2}-1}$$
$$\Theta=\frac{T_u}{2\pi}\cdot\left[\pi-arctg\frac{2\pi T}{T_u}-arctg\frac{\varepsilon}{\sqrt{a_r^2-\varepsilon^2}}\right] \tag{3}$$

where a_r and T_u are depicted in Fig. 3 and ε is the hysteresis.

Similarly, the second order model plus dead time (SOPDT) is assumed in the form:

$$G(s)=\frac{K}{(Ts+1)^2}\cdot e^{-\Theta s} \tag{4}$$

The gain is given by (2), the time constant and time delay term can be estimated according to [11] by the relation:

$$T=\frac{T_u}{2\pi}\cdot\sqrt{\frac{4\cdot K\cdot h_r}{\pi\cdot a_r}-1}$$
$$\Theta=\frac{T_u}{2\pi}\cdot\left[\pi-2arctg\frac{2\pi T}{T_u}-arctg\frac{\varepsilon}{\sqrt{a_r^2-\varepsilon^2}}\right] \tag{5}$$

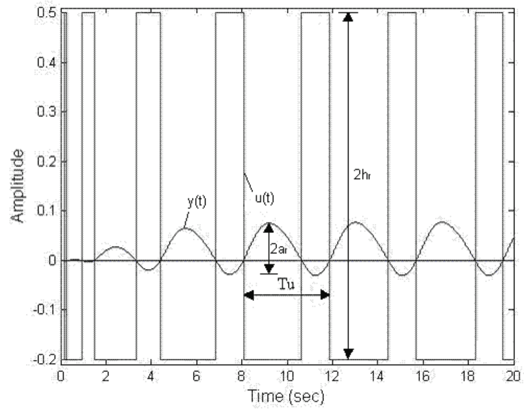

Fig. 3 Biased relay oscillation of stable processes.

III. 3 Algebraic Control Design

Rings and linear (Diophantine) equations have become common tools in modern control theory before decades. There are several rings, the ring of polynomials R_P, the ring of stable and proper rational function R_{PS} etc., see e.g. [12], [13], [17] which can be used for control syntheses. Different rings require various approximations of delay terms which reduce quality of a model. The most known is the Pade approximation, respecting the relative degree of the original transfer function. As a negative consequence, the final controllers have usually higher degrees.

This paper utilizes a ring of stable and proper meromorphic functions R_{MS} omitting any approximation which was developed especially for delay systems by Zítek and Kučera in [23]. An element of this ring is a ratio of two retarded quasipolynomials $y(s)/x(s)$.

A retarded quasipolynomial $x(s)$ of degree n means

$$x(s)=s^n+\sum_{i=0}^{n-1}\sum_{j=1}^{h}x_{ij}s^i\exp\left(-\vartheta_{ij}s\right),\ \vartheta_{ij}\geq 0 \tag{6}$$

where *retarded* refers to the fact that the highest s-power is not affected by exponentials. A more general notion called neutral quasipolynomials also can be used in this sense, see [18]. A quasipolynomial in the form of (6) is stable when it owns no finite zero s_0 such that $\mathrm{Re}\{s_0\}\geq 0$. For stability tests, see e.g. in [23], [24].

The denominator of the ratio in R_{MS} is supposed to be stable, while the numerator $y(s)$ of an element in R_{MS} can be factorized in the form $y(s)=\tilde{y}(s)\exp(-\Theta s)$, where the term $\Theta \geq 0$ and $\tilde{y}(s)$ is any retarded quasipolynomial. The ratio $y(s)/x(s)$ is called proper when the degree of the numerator is less or equal to the degree of the denominator.

A linear time-invariant delay system can be expressed as a ratio of two elements of the R_{MS} ring. The first order system with input-output time delay can be expressed by

$$G(s)=\frac{\dfrac{K\exp(-\Theta s)}{s+m_0\exp(-\vartheta s)}}{\dfrac{Ts+1}{s+m_0\exp(-\vartheta s)}}=\frac{\dfrac{b(s)}{m(s)}}{\dfrac{a(s)}{m(s)}}=\frac{B(s)}{A(s)} \tag{7}$$
$$A(s),\ B(s)\in R_{MS},\ \ m_0>0$$

The traditional feedback loop for the control design is displayed in Fig. 4. Generally, let a model transfer function be expressed as

$$G(s)=\frac{B(s)}{A(s)},\ \ A(s),\ B(s)\in R_{MS} \tag{8}$$

and a controller be given by a ratio

$$G_R(s)=\frac{Q(s)}{P(s)},\ \ Q(s),\ P(s)\in R_{MS} \tag{9}$$

Similarly, reference and load disturbance signals can be expressed by

$$W(s)=\frac{H_W(s)}{F_W(s)},\ \ H_W(s),\ F_W(s)\in R_{MS} \tag{10}$$

$$D(s)=\frac{H_D(s)}{F_D(s)},\ \ H_D(s),\ F_D(s)\in R_{MS} \tag{11}$$

The aim of the control synthesis is to (internally) stabilize the feedback control system with asymptotic tracking and load disturbance attenuation.

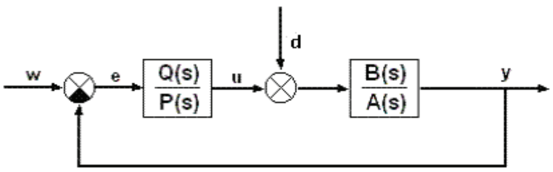

Fig.4 Feedback (1DOF) control loop

The first step of the stabilization can be formulated in an elegant way in RMS by the Diophantine equation

$$A(s)P_0(s)+B(s)Q_0(s)=1 \tag{12}$$

where $P_0(s)$ a $Q_0(s)$ is a particular solution from R_{MS}. Since for stable systems, the R_{MS} ring constitutes the Bézout domain (see [23]), the solution of (7) always exists. All stabilizing controllers can be expressed in a parametric form by

$$\frac{Q(s)}{P(s)}=\frac{Q_0(s)+A(s)Z(s)}{P_0(s)-B(s)Z(s)} \tag{13}$$
$$P_0(s)-B(s)Z(s)\neq 0$$

where $Z(s)$ is an arbitrary element of R_{MS}. The special choice of this element can ensure additional control conditions. Details and proofs can be found e.g. in [12], [13], [17], [19]. Asymptotic tracking and disturbance attenuation result from expression for $E(s)$ which reads

$$E(s)=\frac{A(s)P(s)}{A(s)P(s)+B(s)Q(s)}W(s)-\frac{B(s)P(s)}{A(s)P(s)+B(s)Q(s)}D(s) \tag{14}$$

and they lead to the condition that both $F_w(s)$ and $F_D(s)$ divide $P(s)$. Details about divisibility in R_{MS} can be found, e.g. in [13], [23].

The control design can be also performed in a different ring which is traditional for non-delayed systems. This ring is called the ring of proper and stable rational functions R_{PS}, see [13], [17], [19], [21]. The algebraic tools mentioned above in (12) – (14) can be adopted similarly. Any transfer function G(s) of a (continuous-time) linear system is expressed as a ratio of two elements of R_{PS}. The set R_{PS} means the ring of (Hurwitz) stable and proper rational functions. Traditional transfer functions as a ratio of two polynomials can be easily transformed into the fractional form simply by dividing, both the polynomial denominator and numerator by the same stable polynomial of the appropriate order. Then all transfer functions can be expressed by the ratio:

$$G(s)=\frac{b(s)}{a(s)}=\frac{\dfrac{b(s)}{(s+m)^n}}{\dfrac{a(s)}{(s+m)^n}}=\frac{B(s)}{A(s)} \tag{15}$$
$$n=\max(\deg(a),\deg(b)),\quad m>0$$

It is clear that fraction (15) can be considered as a special case of (7) for $\vartheta=0$. Then all feedback stabilizing controllers according to Fig. 4 are given by the same Diophantine equation (12) but in the ring $R_{PS.}$ Then all feedback controllers (13) can be utilized. In contrast of polynomial design, all controllers (13) are proper.

IV. Basic anisochronic autotuners

A. First order model

A first order delayed model (FODPT) where parameters K, T and Θ are estimated via relay identification test (2) - (3).

The model coprime factorization in the R_{MS} ring can be then expressed by

$$\tilde{G}(s)=\frac{\dfrac{K\exp(-\Theta s)}{s+m_0}}{\dfrac{Ts+1}{s+m_0}}=\frac{B(s)}{A(s)} \tag{16}$$

where $m_0 > 0$ is a free (selectable) scalar parameter.

The control loop is considered as a simple feedback system (Fig. 4) with plant and controller transfer functions (8), (9), respectively. Both external inputs (10), (11) are supposed as step functions.

The stabilizing Diophantine equation (12) reads

$$\frac{Ts+1}{s+m_0}P_0(s)+\frac{K\exp(-\Theta s)}{s+m_0}Q_0(s)=1 \tag{17}$$

Choose $Q_0(s)=1$ which yields

$$P_0(s)=\frac{s+m_0-K\exp(-\Theta s)}{Ts+1} \tag{18}$$

Obviously, this solution does not satisfy the requirements of asymptotical reference tracking and disturbance attenuation, since $P_0(0)\neq 0$, thus, the particular solution ought to be parameterized as

$$\begin{aligned} P(s)&=P_0(s)-B(s)Z(s)=\\ &=\frac{s+m_0-K\exp(-\Theta s)}{Ts+1}-\frac{K\exp(-\Theta s)}{s+m_0}Z(s) \end{aligned} \tag{19}$$

In order to have $P(s)$ in a simple form satisfying $P_0(0)=0$, choose

$$Z(s)=\frac{s+m_0}{Ts+1}\left(\frac{m_0}{K}-1\right) \tag{20}$$

which gives the controller denominator and numerator by

$$\begin{aligned} P(s)&=\frac{s+m_0(1-\exp(-\Theta s))}{Ts+1}\\ Q(s)&=\frac{m_0}{K} \end{aligned} \tag{21}$$

according to (8). Thus, the final anisochronic controller structure reads

$$G_R(s)=\frac{m_0}{K}\frac{Ts+1}{s+m_0(1-\exp(-\Theta s))}=G_{R1}(s) \tag{22}$$

where m_0 serves as a tuning parameter. The denominator in (21) has infinite number of poles. The construction of this controller is more complex than usual PI or PID controllers.

The first order systems (16) for the R_{PS} synthesis is supposed with $\Theta=0$ and the Diophantine equation (12) in this ring can be easily transformed into polynomial equation:

$$(Ts+1)p_0+Kq_0=s+m_0 \tag{23}$$

with general solution:

$$\begin{aligned} P&=\frac{1}{T}+\frac{K}{s+m_0}\cdot Z\\ Q&=\frac{Tm_0-1}{TK}-\frac{Ts+1}{s+m_0}\cdot Z \end{aligned} \tag{24}$$

where Z is free in the ring R_{PS}. Asymptotic tracking is achieved by the choice:

$$Z=-\frac{m_0}{TK} \tag{25}$$

and the resulting PI controller is in the form:

$$C(s)=\frac{Q}{P}=\frac{q_1 s+q_0}{s} \tag{26}$$

where parameters q_1 a q_0 are given by:

$$q_1=\frac{2Tm_0-1}{K}\quad q_0=\frac{Tm_0^2}{K} \tag{27}$$

It is obvious that scalar parameters m_0 also in this case is a tuning parameter.

B. *Second order model*

A second order model with dead time (SOPDT) has form (4) which can be expressed in R_{MS} as a ratio

$$\tilde{\tilde{G}}(s)=\frac{\dfrac{K\exp(-\Theta s)}{(s+m_0)^2}}{\dfrac{(Ts+1)^2}{(s+m_0)^2}}=\frac{B(s)}{A(s)} \tag{28}$$

Similarly as in (16) and (17) for a first order model, a stabilizing (non unique) particular solution of (12) can obtained as

$$Q_0(s)=1,\quad P_0(s)=\frac{(s+m_0)^2-K\exp(-\Theta s)}{(Ts+1)^2} \tag{29}$$

and the parameterization (13) enables to satisfy the reference tracking and disturbance attenuation with the option

$$Z(s)=\frac{(s+m_0)^2}{(Ts+1)^2}\left(\frac{m_0^2}{K}-1\right) \tag{30}$$

Then the numerator and denominator result in

$$P(s)=\frac{s^2+2m_0s+m_0^2\left(1-\exp\left(-\Theta s\right)\right)}{\left(Ts+1\right)^2}$$
$$Q(s)=\frac{m_0^2}{K} \tag{31}$$

The controller structure is then

$$G_R(s)=\frac{m_0^2}{K}\frac{\left(Ts+1\right)^2}{s^2+2m_0s+m_0^2\left(1-\exp\left(-\Theta s\right)\right)} \tag{32}$$

The second order synthesis in R_{PS} also supposes □$=0$ in (28) and the design equation (12) leads to the form:

$$(Ts+1)^2\cdot s\cdot p_0+K\cdot(q_2s^2+q_1s+q_0)=(s+m_0)^3 \tag{33}$$

After similar manipulations, the resulting PID controller gives the transfer function:

$$C(s)=\frac{Q}{P}=\frac{q_2s^2+q_1s+q_0}{s(s+p_0)} \tag{34}$$

with parameters:

$$p_0=\frac{1}{T^2};\ q_2=\frac{3Tm_0-2}{KT}$$
$$q_1=\frac{3T^2m_0^2-1}{KT^2};\ q_0=\frac{m_0^3}{K} \tag{35}$$

For both systems FOPDT and SOPDT the scalar parameter $m>0$ seems to be a suitable „tuning knob" influencing control behavior and various properties of the closed loop system.

Naturally, both derived controllers correspond to classical PI and PID ones. It is clear that (26), (27) represents the PI controller:

$$u(t)=K_P\cdot\left(e(t)+\frac{1}{T_I}\cdot\int e(\tau)d\tau\right) \tag{36}$$

and the conversion of parameters is trivial. Relation (17) represents a PID in the standard four-parameter form (see e.g. Åströn and Hägglung, 1995):

$$u(t)=K_P\cdot\left(e(t)+\frac{1}{T_I}\cdot\int e(\tau)d\tau+T_Dy_f(t)\right)$$
$$\tau y_f'(t)+y_f(t)=y(t) \tag{37}$$

The program realization of proposed controllers is quite easy. Fig. 5 demonstrates the Simulink scheme of the anisochronic structure of proposed controller (32) and the traditional PID controller (34) is simulated in a standard way.

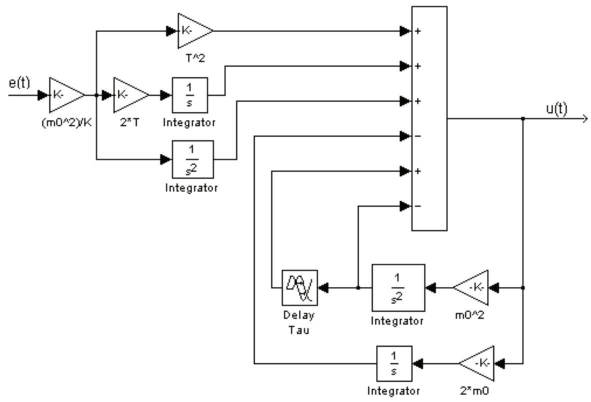

Fig. 5 Matlab-Simulink scheme of controller (26)

A program system for design, tuning and simulation of introduced autotuning and control method was developed in the Matlab-Simulink environment. The Main menu of this program can be seen in Fig. 6. The program system is designed in a user–friendly philosophy.

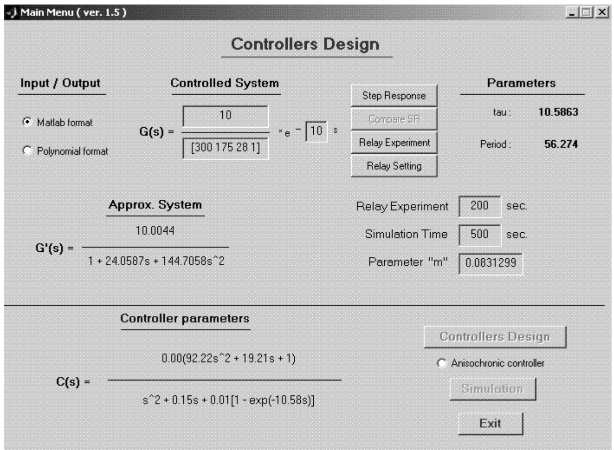

Fig. 6 Main menu of program system

At the beginning of the simulation, the controlled transfer function is defined and parameters for the relay experiment must be entered. Then, the experiment is performed and it can be repeated with modified parameters if necessary. After the experiment, parameters of the estimated transfer function are calculated automatically and controller parameters are generated after pushing of the appropriate button. During the simulation routine, a standard Simulink scheme is performed and required outputs are displayed. The simulation horizon can be prescribed as well as tuning parameter m_0, other simulation parameters can be specified in the Simulink environment. In all simulation a change of the step reference is performed in the second third of the simulation horizon and a step change in the load is injected in the last third.

V. EXAMPLES AND SIMULATIONS

As an example, a stable system with time delay governed by the transfer function of the third order was chosen

$$G(s)=\frac{10\exp(-10s)}{(20s+1)(5s+1)(3s+1)} \tag{38}$$

The estimation was performed by the relay feedback experiment where asymmetric relay with hysteresis was used with adjusted parameters: h_r = 0.225 (0.2 when on, -0.25 when off), ε = 0.05. Limit cycles result in a_r = 1.022, T_u = 56.28.

The first order model (1) was obtained by the experiment approximation using (2) - (3) is

$$\tilde{G}_1(s)=\frac{10}{23.45s+1}\exp(-16.91s) \tag{39}$$

The second order model (3) can be obtained similarly, by relations (2) and (5) in the form

$$\tilde{G}_2(s)=\frac{10}{(12.03s+1)^2}\exp(-11.04s) \tag{40}$$

The step responses of (38) - (40) are pictured and compared in Fig. 7. Estimated models (39), (40) were used for the algebraic controller design in the sense of transfer functions (7), (28). The methodology mentioned in part III in the ring R_{MS} then results in the first and second order anisochronic controllers, respectively. The final first order controller has the form

$$G_{R1}(s)= \\ =2.85\cdot 10^{-3}\frac{23.45s+1}{s+2.85\cdot 10^{-2}\left(1-\exp(-16.91s)\right)} \tag{41}$$

where parameter m_0 = $2.85\cdot 10^{-2}$ was tuned by the "equalization" principle.

The second order model for the tuning parameter m_0 = $7.37\cdot 10^{-2}$ takes the transfer function

$$G_{R2}(s)= \\ =\frac{5.43\cdot 10^{-4}(12.03s+1)^2}{s^2+14.74\cdot 10^{-2}s+5.43\cdot 10^{-3}\left(1-\exp(-11.04s)\right)} \tag{42}$$

The tuning parameter m_0 can be adjusted by various principles, one of them is the "equalization" principle proposed in [14]. In both cases (30), (31), comparing the appropriate parameters defined the final value of tuning parameters.

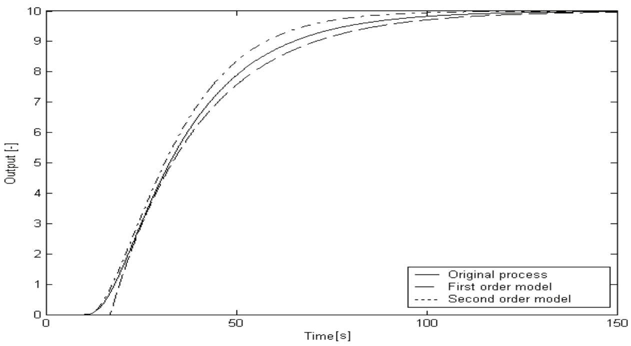

Fig. 7 Step responses of (38)-(40)

Control responses for both models and controllers are compared in Fig. 8 (control variable) and in Fig. 9 (controlled variable). Reference signal $w(t)$ is changed from 1 to 2 in time $t=200$ and the step load disturbance $d(t)$ = -0.1 is injected at t = 400 s.

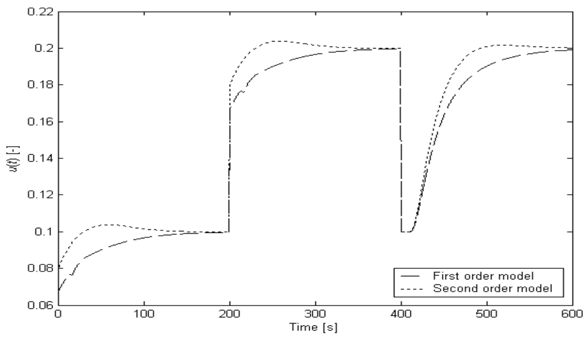

Fig. 8 Control input – first and second order synthesis in R_{MS}

Control responses are rather slow; however, without abrupt changes of control signals (except instants of step changes of the reference signal). This result agrees with the philosophy of the "equalization" method which suggests a compromise between a suitable control response and carefulness to actuators. Generally, higher m_0 gives faster but more oscillating control responses, and vice-versa. Naturally, second order approximation as well as control responses exhibit better and more acceptable behavior.

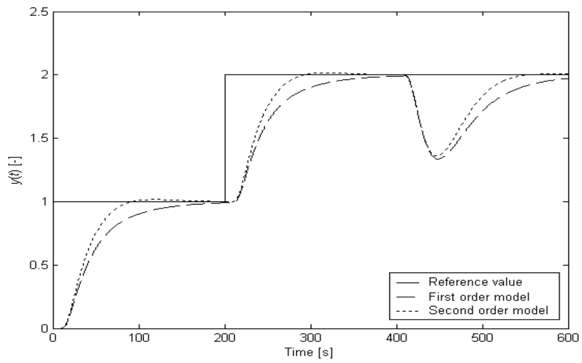

Fig. 9 Controlled output – first and second order synthesis in R_{MS}

Further, PI and PID controllers were derived according to the methodology described in the ring R_{PS}. Naturally, transfer functions (39), (40) were utilized for the first and second order syntheses, respectively. The tuning parameter m_0 was chosen from the requirement of aperiodic control behavior, see [20]. The final values for the first and second orders were m_0 = 0.026 and m_0 = 0.062, respectively. The final controllers have the transfer functions

$$G_{R1}(s) = \frac{1.7 \cdot 10^{-3} s + 6.7 \cdot 10^{-5}}{3.5 \cdot 10^{-2} s} \quad (43)$$

$$G_{R2}(s) = \frac{25.67 \cdot 10^{-5} s^2 + 3.90 \cdot 10^{-5} s + 0.15 \cdot 10^{-5}}{6.9 \cdot 10^{-3} s^2 + 0.6 \cdot 10^{-3} s} \quad (44)$$

Control responses for both models and controllers are compared in Fig. 10 (control variable) and in Fig. 11 (controlled variable). It is obvious that the controllers derived in the R_{MS} synthesis exhibit better responses that classical PI and PID ones, especially in responses of load disturbance attenuation.

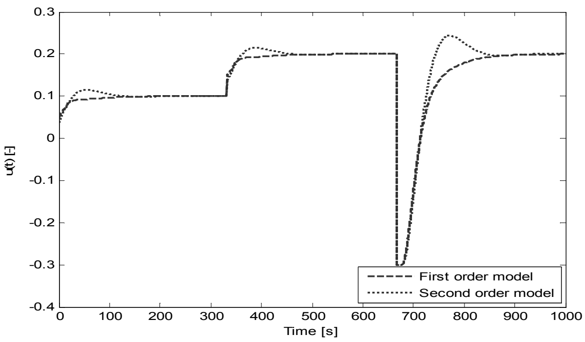

Fig. 10 Control input – first and second order synthesis in R_{PS}

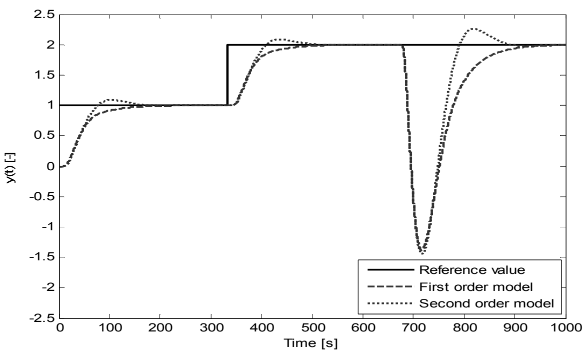

Fig. 11 Controlled output – first and second order synthesis in R_{PS}

VI. Conclusions

The contribution brings a novel principle of autotuners with controller design based on a special ring of meromorphic functions. A transfer function with time delay is estimated from asymmetric limit cycle data by a biased relay with hysteresis. The control synthesis is then performed through a solution of the Diophantine equation in the ring of proper and stable RQ-meromorphic functions. For first and second order models the methodology generates a class of generalized PI or PID controllers in the sense of the Smith predictor. The design method brings a scalar tuning parameter $m_0 > 0$ that can be adjusted by various strategies. The paper also presents the traditional control design in the ring of stable and proper rational functions. This approach generates for the first and second order systems PI and PID controllers. The illustrative example shows the application of the proposed methodology in the ring R_{MS} (first and second order) to control of a higher order system with time delay. The same system is also controlled by traditional controllers derived in the ring R_{PS} for comparison. Both methodologies bring a scalar positive parameters which can be used as a "tuning knob" for influencing of control responses. Naturally, this parameter can be subjected to further optimization for obtaining the optimal behavior in a given sense.

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