A STUDY OF CHOLERA MODEL WITH ENVIRONMENTAL FLUCTUATIONS

Gazi N. H., Das K. *, Mukandavire Z., Chiyaka C., Das P.

Abstract--The study focuses on randomly fluctuating phenomena of cholera deterministic model by incorporating white noise stochastic perturbation. For the deterministic model, stability of the equilibria and persistent aspects of population are discussed. Variances of population are evaluated for the model system at the endemic equilibrium. We conclude from the study that the inclusion of environmental fluctuation does not change substantially the dynamical behaviour of the system although it induces some initial random oscillations.

Keywords — Cholera, Stability, Persistence, Stochasticity.

I. INTRODUCTION

A recent study on cholera [1] reveals that local environmental parameters are intensely associated with cholera dynamics. In particular, increase in ocean chlorophyll concentration, sea surface temperature and river height play a significant role on the occurrence of cholera and the magnitude of the epidemic. Cholera, a man-environment disease is transmitted through drinking water which is contaminated from improper treatment of sewage. Further, it may be noted that if the degree of infectivity increases, sociological or other mechanisms which tend to saturate the effect that a large number of infectives may have often come into play [2]. Therefore we are interested in exploring the effects of environmental fluctuations by considering the saturation incidence term $kSI/(1+k'I)$. It is important to note that the form $kSI/(1+k'I)$ tends to a close approximation of the term kSI when k' is small and approaches the form kS/k' for very large value of I. In this study we revisit the carrier dependent cholera model already studied in [3] to investigate the effect of random fluctuation to the model system.
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II. DETERMINISTIC MODEL

Consider $N(t)$ be the total population density, which is divided into two subclasses: the susceptible class $S(t)$ and the infective class $I(t)$. It is assumed that all susceptible are affected by the carrier population density $C(t)$, which is governed by a general logistic law. The mathematical model is as follows:

$$\begin{aligned}\frac{dS}{dt} &= A-\frac{kSI}{1+k'I}-\lambda SC+\mu I-mS\\ \frac{dI}{dt} &= \frac{kSI}{1+k'I}+\lambda SC-(\mu+\alpha+m)I \qquad (1)\\ \frac{dC}{dt} &= (N+n_0-s_0C-s_1)C,\end{aligned}$$

where $N=S+I$. Here A is the constant immigration rate of human population from outside the region under consideration. The parameters k and λ are the transmission coefficients due to infective and carrier population respectively. Further m is the natural death rate, α is the disease related death rate and μ is the recovery rate. The constant s_1 is the death rate coefficient of carriers due to natural factors as well as by control measures. We may note that if the growth rate and death rate of carrier population are balanced then it may approach to zero. Here, k', n_0, s_0 are constant. All the parameters are assumed to be positive. The rate-coefficients of parameters have dimensions of time^{-1}. It is evident that system (1) is well-posed and bounded also.

III. LINEAR ANLYSIS

To analyze model system (1), we consider the following reduced system (since $N=S+I$):

$$\begin{aligned}\frac{dI}{dt} &= \frac{k(N-I)I}{1+k'I}+\lambda(N-I)C-(\mu+\alpha+m)I\\ \frac{dN}{dt} &= A-mN-\alpha I \qquad (2)\\ \frac{dC}{dt} &= (N+n_0)C-s_0C^2-s_1C.\end{aligned}$$

We now state the results obtained in [3] without the proof before proceeding to look at the effects of stochasticity in the cholera model system (1).

3.1. Equilibria

Equilibria of the model system are presented in the following theorem:

Theorem 1. There exists following equilibria of system (2)

(i) Disease-free equilibrium, $E_0=(0, A/m, 0)$.

(ii) Carrier-free equilibrium, $E_1=(\bar{I}, \bar{N}, 0)$,

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where $\bar{I} = \dfrac{Ak - m(\mu+\alpha+m)}{(m+\alpha)k + mk'(\mu+\alpha+m)}$,

$\overline{N} = \dfrac{Ak + (Ak'+\alpha)(\mu+\alpha+m)}{(m+\alpha)k + mk'(\mu+\alpha+m)}$, provided

$k > \dfrac{m(\mu+\alpha+m)}{A}$.

(iii) Endemic equilibrium, $E_2(I^*, N^*, C^*)$

where, $I^* = (A - mN^*)/\alpha$, provided $N^* < A/m$ and $C^* = (N^* + n_0 - s_1)/s_0$, provided $s_1 < n_0 + N^*$.

Now (a) $E_2(I^*, N^*, C^*)$ is unique if the inequalities $s_1 < \min\{n_0 + A/m, n_0 + s_0(\mu+\alpha+m)/\lambda\}$ and $\lambda < mks_0/[\alpha + k'(A + ms_1 + mA/(m+\alpha))]$ are satisfied.

(b) multiple endemic equilibria exist if either of the following conditions are satisfied:

I. $a_3 > 0$, at least one of a_1, a_2 is negative and $G^2 + 4H^3 < 0$.

II. $a_3 < 0$, $a_1 < 0, a_2 > 0$ and $G^2 + 4H^3 < 0$,

where $H = a_0 a_2 - a_1^2$, $G = a_0^2 a_3 - 3a_0 a_1 a_2 + 2a_1^3$,

$a_0 = mk'\lambda(m+\alpha)$,

$a_1 = \frac{1}{3}[\{mks_0 - \lambda(\alpha + Ak') + mk'\lambda(n_0 - s_1)\}(m+\alpha)$
$+ mk'\{ms_0(\mu+\alpha+m) - A\lambda\}]$,

$a_2 = -\frac{1}{3}[\{Aks_0 + \lambda(n_0 - s_1)(\alpha + Ak')\}(m+\alpha)$
$+ ms_0\{Ak + (\mu+\alpha+m)(\alpha + 2Ak')\}$
$- A\lambda\{\alpha + Ak' - mk'(n_0 - s_1)\}]$,

$a_3 = A[Aks_0 + \lambda(\alpha + Ak')\{n_0 - s_1 + \frac{s_0}{\lambda}(\mu+\alpha+m)\}]$.

3.2. *Local Stability*

We now state the theorem on local asymptotic stability criteria of the disease-free, carrier-free and endemic equilibrium in the following.

Theorem 2. (i) The disease-free equilibrium, E_0 is locally asymptotically stable if $k < m(\mu+\alpha+m)/A$, $s_1 - n_0 > A/m$. Again E_0 is unstable if either $k > m(\mu+\alpha+m)/A$, or $s_1 - n_0 < A/m$.

(ii) The carrier-free equilibrium, E_1 is locally asymptotically stable if $s_1 - n_0 > \overline{N}$ and is unstable if $s_1 - n_0 < \overline{N}$.

(iii) Suppose that $s_1 - n_0 < N^* < A/m$. Further assume that either (a) or (b) of Theorem 1 hold. If $\delta_i > 0,\ i = 4,5,6$ and $\delta_4\delta_5 - \delta_6 > 0$ then endemic equilibrium, E_2 is locally asymptotically stable where,

$$\delta_4 = \mu + \alpha + 2m + \lambda C^* - \frac{k(N^* - 2I^* - k'I^{*2})}{(1+k'I^*)^2} + s_1 + 2C^* s_0 - N^* - n_0,$$

$$\delta_5 = \alpha\left(\frac{kI^*}{1+k'I^*} + \lambda C^*\right) + m\left\{\mu + \alpha + m + \lambda C^* - \frac{k(N^* - 2I^* - k'I^{*2})}{(1+k'I^*)^2}\right\} + \left\{\mu + \alpha + 2m + \lambda C^* - \frac{k(N^* - 2I^* - k'I^{*2})}{(1+k'I^*)^2}\right\}\{s_1 + 2C^* s_0 - N^* - n_0\},$$

$$\delta_6 = \alpha\lambda(N^* - I^*)C^* + \left[\alpha\left(\frac{kI^*}{1+k'I^*} + \lambda C^*\right) + m\left\{\mu + \alpha + m + \lambda C^* - \frac{k(N^* - 2I^* - k'I^{*2})}{(1+k'I^*)^2}\right\}\right] \times (s_1 + 2C^* s_0 - N^* - n_0).$$

Here, the term "$s_1 - n_0$" biologically represents the negative "net rate of growth" of the carrier population, i.e. "net rate of decay" of carrier population. It is to be noted here that E_1 becomes stable when the intrinsic growth rate of the carrier population at the equilibrium density is negative otherwise it is unstable.

3.3. *Global Stability*

Global stability behaviour of the carrier-free and endemic equilibrium points are given here in the following theorem.

Theorem 3. (i) The carrier-free equilibrium E_1 is globally asymptotically stable in the I-N plane if $\overline{N} < s_1 - n_0 < A/m$.

(ii) Suppose $s_1 - n_0 < \overline{N}$. Then the endemic equilibrium E_2 is globally asymptotically stable if the inequalities

$$s_0 > \frac{1}{4m},$$

$$(4ms_0 - 1)\left(\mu + \alpha + m + \lambda C^* - \frac{kN^*}{1+k'I^*}\right) > s_0\left(\frac{k}{k'} + \alpha + \lambda C^*\right)^2 + \frac{\lambda A}{m}\left(\frac{k}{k'} + \alpha + \lambda C^*\right) + \frac{\lambda^2 A^2}{m} \quad (3)$$

are satisfied. Now from Theorems 1-3, we may observe that if E_0 is locally asymptotically stable then E_1 is not feasible whereas when E_1 is feasible then E_0 is unstable. If E_1 is unstable then E_2 is globally asymptotically stable provided

(3) holds and if E_1 is stable then E_2 may not be globally asymptotically stable.

3.4. *Persistence*
We now state a result that guarantees the survival of all populations.

Theorem 4. Suppose $s_1 - n_0 < \bar{N}$. Then system (2) is uniformly persistent.

Biologically this situation reflects that net rate of decay of carrier population is under a certain value, i.e. number of carriers within the system are under control.

IV. The Stochastic Model

We now extend our model system (2) to consider the effect of the random fluctuation. The parameters of the model equations fluctuate about their average values due to random fluctuation. We incorporate such randomness to the model equations (2), by following the method of incorporating additive white noises to the model system.

For any parameter, the white noise perturbation on it will change to p_1 of the model system (2) is given by $p_1 = p + \alpha\xi(t)$, where α is the amplitude of the random noise and $\xi(t)$ is a Gaussian white noise process at time t. With this effect model equations (2) become

$$\begin{aligned} &dI(s) = [\frac{k(N-I)I}{1+k'I} + \lambda(N-I)C - (\mu+\alpha+m)I]dt \\ &+\alpha_1(I(t)-I^*)dW_1(t), \\ &dN(t) = [A - mN - \alpha I]dt \\ &+\alpha_2(N(t)-N^*)dW_2(t), \\ &dC(t) = [(N+n_0)C - s_0C^2 - s_1C]dt \\ &+\alpha_3(C(t)-C^*)dW_3(t), \end{aligned} \tag{4}$$

so that the deterministic model and the stochastic system have the same equilibria. The above system for the randomly fluctuating driving forces on three populations when additive noise considered reduces to

$$\begin{aligned} \frac{dI}{dt} &= \frac{k(N-I)I}{1+k'I} + \lambda(N-I)C - (\mu+\alpha+m)I + \alpha_1\xi_1(t), \\ \frac{dN}{dt} &= A - mN - \alpha I + \alpha_2\xi_2(t), \\ \frac{dC}{dt} &= (N+n_0)C - s_0C^2 - s_1C + \alpha_3\xi_3(t), \end{aligned} \tag{5}$$

where, α_i, i=1,2,3, are real constants and $\xi(t) = (\xi_1(t), \xi_2(t), \xi_3(t))$ is a 3D Gaussian white noise processes satisfying $<\xi_i(t)>=0,$ and $\langle \xi_i(t), \xi_j(t) \rangle = \delta_{ij}\delta(t-t')$, $i, j = 1,2$, $< >$ denotes ensemble average, δ_{ij} is Kronecker delta and $\delta(t-t')$ is the Dirac delta function. Although the true white noise does not occur in nature, however, by studying the spectra of the white noise, thermal noise in electrical resistance, the force acting on Brownian particle and climate fluctuations, disregarding the periodicities of astronomical origin etc. are white to a very good approximation. Thus the introduction of white noise to the biological system is appropriate.

4.1 Fourier Transform: Spectral density
In the present study we focus on the behaviour of the system at endemic equilibrium point only. Consequently we compute the population intensities of fluctuations around the endemic equilibrium point $E_2(I^*, N^*, C^*)$, according to the method introduced in [4]. The method was successfully applied in [5, 6, 7]. By changing the variables $x_1(t) = I(t) - I^*$, $x_2(t) = N(t) - N^*$, $x_3(t) = C(t) - C^*$, we centre (5) on $E_2(I^*, N^*, C^*)$ and retaining only the linear terms and the effect of linear stochastic perturbations. Hence system equations (5) reduce to

$$\begin{aligned} \frac{dx_1}{dt} &= A_s x_1(t) + B_s x_2(t) + C_s x_3(t) + \alpha_1\xi_1(t), \\ \frac{dx_2}{dt} &= D_s x_1(t) + E_s x_2(t) + \alpha_2\xi_2(t), \\ \frac{dx_3}{dt} &= F_s x_2(t) + G_s x_3(t) + \alpha_1\xi_3(t), \end{aligned} \tag{6}$$

where $A_s = \frac{k(N^* - 2I^* - k'I^{*2})}{(1+k'I)^2} - \lambda C^* - \mu - \alpha - m$,

$B_s = \frac{kI^*}{1+k'I^*} + \lambda C^*$, $C_s = \lambda(N^* - I^*)$, $D_s = -\alpha$, $E_s = -m$, $F_s = C^*$, $G_s = N^* + n_0 - 2s_0C^* - s_1$.

Taking Fourier transform of the equations (6), we obtain

$$\begin{aligned} \alpha_1\tilde{\xi}_1(\omega) &= (i\omega - A_s)\tilde{x}_1(\omega) - B_s\tilde{x}_2(\omega) - C_s\tilde{x}_3(\omega), \\ \alpha_2\tilde{\xi}_2(\omega) &= -D_s\tilde{x}_1(\omega) + (i\omega - E_s)\tilde{x}_2(\omega), \\ \alpha_3\tilde{\xi}_3(\omega) &= -F_s\tilde{x}_2(\omega) + (i\omega - G_s)\tilde{x}_3(\omega), \end{aligned}$$

where $\tilde{X}(\omega) = \int_{-\infty}^{\infty} X(t)e^{-i\omega t}dt$ is the Fourier transform of the function $X(t)$. The above algebraic system can be written in the matrix form

$$M(\omega)\tilde{x}(\omega) = \tilde{\xi}(\omega) \tag{7}$$

where

$$M(\omega)=\begin{bmatrix} i\omega-A_s & -B_s & -C_s \\ -D_s & i\omega-E_s & 0 \\ 0 & -F_s & i\omega-G_s \end{bmatrix},$$

$$\tilde{x}(\omega)=(\tilde{x}_1(\omega),\tilde{x}_2(\omega),\tilde{x}_3(\omega))^T,$$

$$\tilde{\xi}(\omega)=(\alpha_1\tilde{\xi}_1(\omega),\alpha_2\tilde{\xi}_2(\omega),\alpha_3\tilde{\xi}_3(\omega))^T.$$

Therefore, the solution of the equation (7) is given by

$$\tilde{x}(\omega)=(M(\omega))^{-1}\tilde{\xi}(\omega)=P(\omega)\tilde{\xi}(\omega) \qquad (8)$$

where $P(\omega)=(M(\omega))^{-1}$, inverse of the matrix $M(\omega)$. We have assumed that the matrix $M(\omega)$ is non-singular so that the inverse exists. The components of the solution (8) are

$$\tilde{x}_i(\omega)=\sum_{j=1}^{3}P_{ij}(\omega)\alpha_j\xi_j(\omega),\ \text{i=1,2,3},$$

where $P_{ij}(\omega)$ are the elements of the matrix $P(\omega)$. These quantities $\tilde{x}_i(\omega)$ are the mean values of the populations.

For a random function $f(t)$ with zero mean, the fluctuation intensity (variance) of $f(t)$ within the frequency interval $[\omega,\omega+d\omega]$ is given by $S_f(\omega)d\omega$ where $S_f(\omega)$ is the spectral density defined by [4],

$$S_f(\omega)=\lim_{T\to+\infty}\frac{\left|\tilde{f}(\omega)\right|^2}{T}.$$

The inverse transform of $S_f(\omega)$ is the autocovariance function

$$C_f(\tau')=\frac{1}{2\pi}\int_{-\infty}^{\infty}S_f(\omega)e^{i\omega\tau'}d\omega,\ \tau'=t-t'$$

The corresponding variance of fluctuation of $f(t)$ is given

$$\sigma_f^2=C_f(0)=\frac{1}{2\pi}\int_{-\infty}^{\infty}S_f(\omega)d\omega.$$

The spectral densities of $x_i,(i=1,2,3)$ are given as follows:

$$S_{x_i}(\omega)$$

$$=\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}\int_{-T/2}^{T/2}<x_i(t)x_i(t')>\exp\{i\omega(t'-t)\}dtdt'$$

$$=\sum_{j=1}^{3}\alpha_j\left|P_{ij}(\omega)\right|^2S_{\xi_j}(\omega),(i=1,2,3).$$

The variances of fluctuations of x_i ($i=1,\ 2$,3), are given by

$$\sigma_{x_i}^2=\frac{1}{2\pi}\int_{-\infty}^{\infty}S_{x_i}d\omega=\frac{1}{2\pi}\sum_{j=1}^{3}\int_{-\infty}^{\infty}\alpha_j\left|P_{ij}(\omega)\right|^2d\omega.$$

Using (8), we have three variances of x_i ($i=1,\ 2$, 3) of the model system (5) as follows:

$$\sigma^2_{x_i}=\frac{1}{2\pi}\left[\begin{aligned}&\alpha_1\int_{-\infty}^{\infty}\left|\frac{G_{i1}(\omega)}{\det M(\omega)}\right|^2d\omega+\alpha_2\int_{-\infty}^{\infty}\left|\frac{G_{i2}(\omega)}{\det M(\omega)}\right|^2d\omega\\&+\alpha_3\int_{-\infty}^{\infty}\left|\frac{G_{i3}(\omega)}{\det M(\omega)}\right|^2d\omega\end{aligned}\right]$$

$$=\frac{1}{2\pi}\left[\begin{aligned}&\alpha_1\int_{-\infty}^{\infty}\frac{\left|G_{i1}(\omega)\right|^2}{\left|M(\omega)\right|^2}d\omega+\alpha_2\int_{-\infty}^{\infty}\frac{\left|G_{i2}(\omega)\right|^2}{\left|M(\omega)\right|^2}d\omega\\&+\alpha_3\int_{-\infty}^{\infty}\frac{\left|G_{i3}(\omega)\right|^2}{\left|M(\omega)\right|^2}d\omega\end{aligned}\right] \qquad (9)$$

Here $P_{ij}(\omega)=\dfrac{G_{ij}(\omega)}{\det M(\omega)}$, i,j=1,2,3,

$\left|G_{11}(\omega)\right|^2=(-\omega^2+E_sG_s)^2+(E_s+G_s)^2$,

$\left|G_{12}(\omega)\right|^2=(D_sG_s)^2+(\omega D_s)^2$, $\left|G_{13}(\omega)\right|^2=(D_sF_s)^2$,

$\left|G_{21}(\omega)\right|^2=(\omega B_s)^2+(-B_sG_s+C_sF_s)^2$,

$\left|G_{22}(\omega)\right|^2=(-\omega^2+A_sG_s)^2+(A_s+G_s)^2\omega^2$,

$\left|G_{23}(\omega)\right|^2=(\omega F_s)^2+(A_sF_s)^2$,

$\left|G_{31}(\omega)\right|^2=(\omega C_s)^2+(C_sE_s)^2$, $\left|G_{32}(\omega)\right|^2=(C_sD_s)^2$,

$\left|G_{33}(\omega)\right|^2=(-\omega^2+A_sE_s-B_sD_s)^2+(A_s+E_s)^2\omega^2$.

Here $\left|M(\omega)\right|^2=\left|\det M(\omega)\right|^2=M_1^2+M_2^2$,

$M_1=-\omega^3+(E_sG_s+G_sA_s+A_sE_s)\omega$ and

$M_2=(A_s+E_s+G_s)\omega^2+B_sD_sG_s-E_sG_sA_s-C_sD_sF_s$.

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The expressions in (9) give three variances of the three populations. The integrations over the real line can be evaluated which give the variances of the populations. We can calculate the same numerically.

V NUMERICAL SIMULATION AND CONCLUSIONS

The carrier dependent infectious disease cholera is studied here by incorporating environmental fluctuations through additive white noise. The analytical results and numerical simulation of deterministic model suggest that cholera is generally endemic in nature and prevails in the society. The stable nature of the system shows this situation in Fig. 1.

Further for stochastic model system population variances characterize the stochastic stability of the system. Numerical simulations exhibit that the trajectories of the system oscillate randomly with remarkable variance of amplitude with the increasing value of the strength of white noises initially but ultimately fluctuate at an average pace (see Fig. 2, Fig. 3 and Fig. 4). This indicates that trajectories approach towards a stable equilibrium in the long run with medium amplitude fluctuations around an asymptotic level. This is shown in Fig. 5. Thus simulation results suggest that the disease cholera still remains endemic with this environmental fluctuations though the peak of the disease varying remarkably with time. Hence we conclude that inclusion of stochastic perturbation create no significance difference in dynamical feature of the system rather than unpredictable fluctuations into it. It should be noted that all these fluctuations are closely associated with the severity of the disease. Therefore by controlling the environmental fluctuations the severity of the disease can be checked.

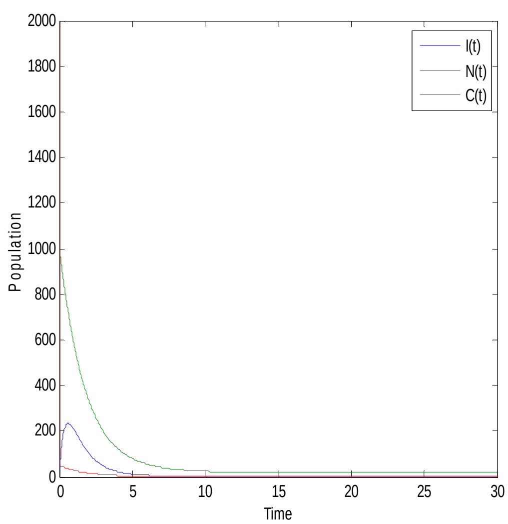

Fig 1. The figure depicts orbits of the system for $k=1\ \text{day}^{-1}, k'=10, \lambda=0.02\ \text{day}^{-1},\ \mu=1\ \text{day}^{-1}$, $\alpha=0.2\ \text{day}^{-1}, n_0=1, s_0=20, m=0.5, A=10\ \text{day}^{-1}, s_1=10\ \text{day}^{-1}$ and $I(0)=10,\ N(0)=1000,\ C(0)=2000$.

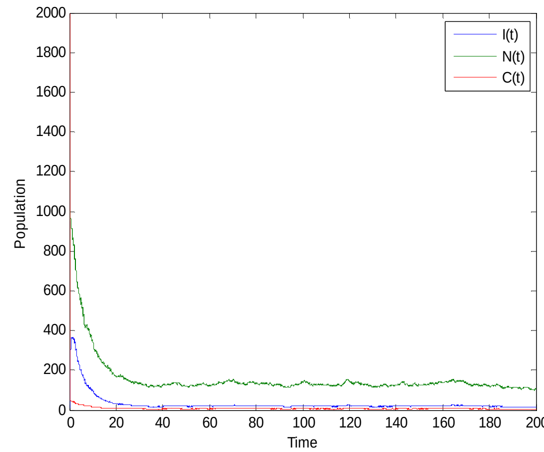

Fig 2. The figure depicts orbits of the system for $k=1\ \text{day}^{-1}, k'=10, \lambda=0.02\ \text{day}^{-1},\ \mu=1\ \text{day}^{-1}$, $\alpha=0.2\ \text{day}^{-1}, n_0=1, s_0=20, m=0.5, A=10\ \text{day}^{-1}$, $n_0=1, s_0=20, m=0.5, A=10\ \text{day}^{-1}, s_1=10\ \text{day}^{-1}$, $\alpha_1=2, \alpha_2=2, \alpha_3=2$, $I(0)=10,\ N(0)=1000, C(0)=2000$.

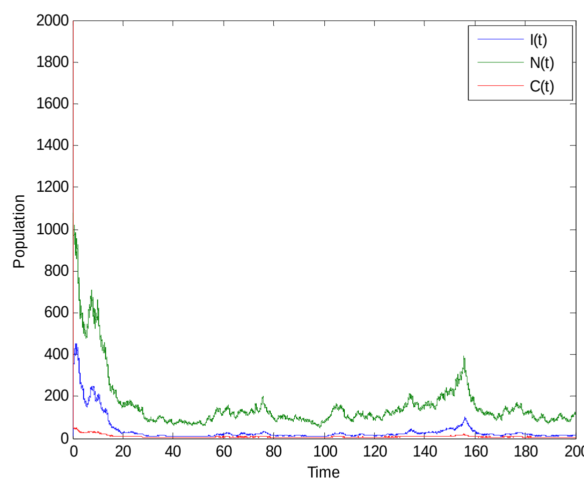

Fig 3. The figure depicts orbits of the system for $k=1\ \text{day}^{-1}, k'=10, \lambda=0.02\ \text{day}^{-1},\ \mu=1\ \text{day}^{-1}$, $\alpha=0.2\ \text{day}^{-1}, n_0=1, s_0=20, m=0.5, A=10\ \text{day}^{-1}, s_1=10\ \text{day}^{-1}$, $\alpha_1=20, \alpha_2=20, \alpha_3=20$, $I(0)=10$, $N(0)=1000, C(0)=2000.$.

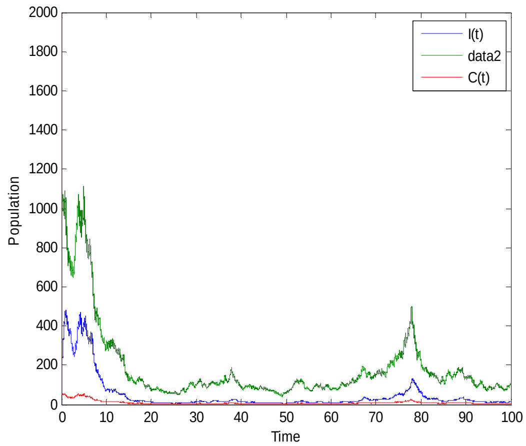

Fig 4. The figure depicts orbits of the system for $k = 1\ day^{-1}, k' = 10, \lambda = 0.02\ day^{-1},\ \mu = 1\ day^{-1},$
$\alpha = 0.2\ day^{-1}, n_0 = 1, s_0 = 20, m = 0.5, A = 10\ day^{-1}, s_1 = 10\ day^{-1},$
$\alpha_1 = 40, \alpha_2 = 40, \alpha_3 = 40, I(0) = 10,$
$N(0) = 1000, C(0) = 2000.$

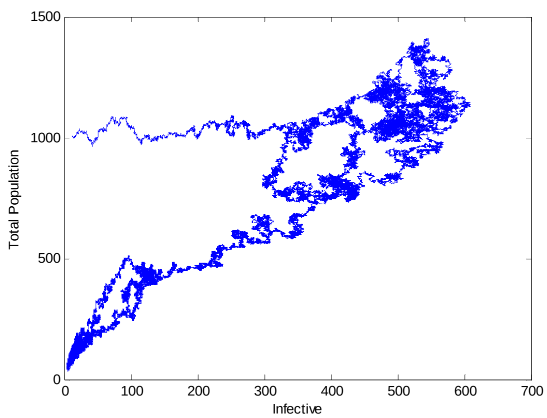

Fig 5. The figure depicts orbits of the system for $k = 1\ day^{-1}, k' = 10, \lambda = 0.02\ day^{-1},\ \mu = 1\ day^{-1},$
$\alpha = 0.2\ day^{-1}, n_0 = 1, s_0 = 20, m = 0.5, A = 10\ day^{-1}, s_1 = 10\ day^{-1}$
$\alpha_1 = 60, \alpha_2 = 60, \alpha_3 = 60\ I(0) = 10,\ N(0) = 1000, C(0) = 2000.$

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