Performance study of an omnidirectional mobile robot

First A. Mkhinini maher, Second B. Knani Jilani

Abstract: In this work we introduce the kinematic model and dynamic model of an omnidirectional mobile robot with three-center directional wheels. These models allow us to simulate the behavior of the robot and evaluate its performance.

Keywords: *robot*, mobile, omnidirectionnal, modeling, kinematic, dynamic.

I. INTRODUCTION:

In the framework of robotics and unlike the industrial manipulator robots which work in many automated factories, mobile robotics plays an integral role. The space within which mobile robot should move is often very large, geometrically unknown and owning a proper dynamic

Some mobile robots, wheeled robots occupy a privileged place. The relative simplicity of their structure and mechanical energy consumption, make the vehicles most frequently encountered, both in indoor environments that use external. Wheeled vehicles have limited capacity to cross directly related to the size of their wheels. Maneuvers are often required to perform specific movements or to move in a cluttered environment.

As for robot manipulators, the control of a mobile robot requires knowledge of the kinematic model and dynamic model of vehicle. Estimating the position of a mobile robot can be derived directly from the measuring position of its joints. Each configuration of the joints is not a unique position of the platform. Connections wheel / ground are the seat of friction phenomena that induce significant inaccuracies in control and in the estimation of vehicle position.

II. KINEMATIC MODLING [7]:

The posture of a vehicle is the position of the marker linked to the vehicle in a coordinate system related to the environment. It is a non-homogeneous vector, whose first two components define the position of the reference robot in the reference mark and the latter defines the orientation of the robot reference relative to fixed frame.

Variables of the robot:

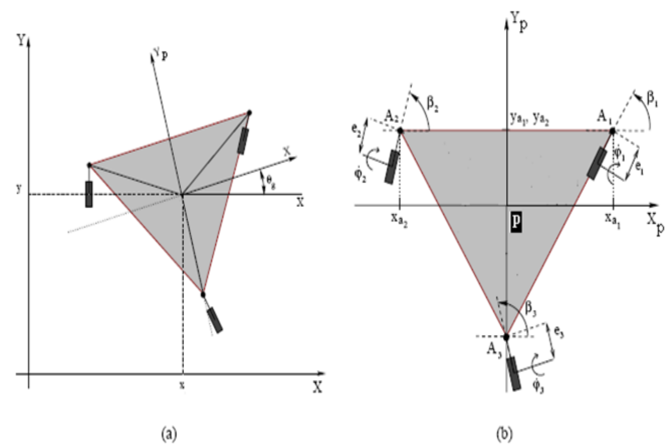


Fig. 1. variable (a) absolute position and (b) modeling of robot omni kinematics

Vector notation:

$\bar{X} = [\bar{\xi}\ \bar{\beta}\ \bar{\varphi}]^T$

$\bar{\xi} = [x\ y\ \theta]^T$ describes posture of the robot in the Galilean reference

$\bar{\beta} = [\beta_1\ \beta_2\ \beta_3\ ]^T$ describes the steering angles of wheels

$\bar{\varphi} = [\varphi_1 \varphi_2 \varphi_3]^T$ describes the angle of rotation of the wheels

$\bar{q} = [\bar{\beta}\ \bar{\varphi}]^T$ describes the joint variables

The *inversed kinematic model* links the derivate of posture vector to joint velocities vector. Knowing the speed of the vehicle in the space of postures, it allows calculating all actuators speed instructions. This design is used to command the robot. The relations between operational speeds and joint velocities are gotten from the hypothesis of wheel/ ground point contact and the non-sliding roll of vehicule wheels.
They allows the interference of wheels geometric parameters as well as the architecture of the platform.

To determine the kinematic constraint equations of the steerable wheel off center, we make two assumptions:-roll without slippage of the wheel-rolling without slippage of the wheel in the horizontal plane.

- No skating:

$$[\cos\beta \ \sin\beta \ x_a \sin\beta - y_a \cos\beta]R(\theta).\dot{\xi} + r\dot{\varphi} = 0 \quad (1)$$

- No slippage:

$$[\sin\beta - \cos\beta \ e - x_a \cos\beta - y_a \sin\beta]R(\theta).\dot{\xi} + e\dot{\beta} = 0 \quad (2)$$

$$\text{With } R(\theta) = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (3)$$

III. MOBILITY OF THE ROBOT :

The robots are classified according to their mobility and number of independently steerable wheels centered [9]. For this, we consider a general mobile robot equipped with wheels N_f fixed, centered orientable wheel N_c, N_d -center directional wheels or $N= N_f+ N_c+ N_d$ the total number of wheels.

Vector of orientation variables

$\bar{\beta} = \begin{bmatrix} \beta_c \\ \beta_d \end{bmatrix}$ Vector of orientation variables for the centered adjustable wheels and the decentered adjustable wheels

Vector of rotation variables

$\bar{\varphi} = \begin{bmatrix} \varphi_f \\ \varphi_c \\ \varphi_d \end{bmatrix}$ Vector of rotation variables for different wheels

The kinematic constraints of the robot are written as the following matrix:

$$\begin{cases} J_1(\beta_c, \beta_d)R(\theta)\dot{\xi} + r\dot{\bar{\varphi}} = 0 \\ C_1(\beta_c, \beta_d)R(\theta)\dot{\xi} + e\dot{\bar{\beta}} = 0 \end{cases} \quad (4)$$

with

$$J_1 = \begin{bmatrix} J_{1f} \\ J_{1c} \\ J_{1d} \end{bmatrix}, C_1 = \begin{bmatrix} C_{1f} \\ C_{1c} \\ C_{1d} \end{bmatrix}$$

Or

J_{1f} ,C_{1f} are matrices of (N_f x 3) dimension

J_{1c} ,C_{1c} are matrices (N_c x 3) dimension

J_{1d} ,C_{1d} are matrices (N_d x 3) dimension

Differents types of constraints

the equations of links between solids are of 2 types:

* holonomic equations, f (q, t) = 0.

Holonomic equations reflect geometric links between solids.

* non-holonomic équations, f(q,$\dot{q}$,t) = 0.

Non-holonomic equations reflect kinematic links between solids.

We call pseudo-holonomic equations, equations of non-holonomic type that could be reduced to holonomic equations through integration.

mobility Degree –Degree de directionality [7]

the expressions of kinematic constraints of fixed wheels and centered orientable wheels are written as follows:

$$C_{1f}R(\theta)\dot{\xi} = 0$$

$$C_{1f}R(\theta)\dot{\xi} = 0 \quad (5)$$

At each moment, the robot motion is equivalent to a pure rotation around the instantaneous center of rotation (ICR) the position of which varies. The instantaneous velocity vector of each point of the robot is orthogonal to the straight line joining this point to RIC. Consequently, all the axes of fixed and orientable centered wheels are concurrent to ICR. $R(\theta)\dot{\xi}$ belongs to the core C_1^* defined as follows :

$$C_1^* = \begin{pmatrix} C_{1f} \\ C_{1c} \end{pmatrix}$$

The rank of C_1^* expresses motion possibilities of the robot. the mobility degree of the robot δ_m is defined from the rank of C_1^*:

$$\delta m = \dim(\ker(C_1^*)) = 3 - Rang(C_1^*) \quad (6)$$

if$Rang(C_1^*) = 3$, $\delta_m = 0$, no motion is possible

if$Rang(C_1^*) = 0$, $\delta_m = 3$, all motion are possible

if the robot is mobile $1 \leq \delta_m \leq 3$:

if the robot possesses at least 2 fixed wheels, their axes of rotation should be confused otherwise robot displacement is limited to rotations around competition point of these axes, that to say rank (C_1^f)≤ 1.

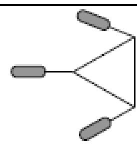
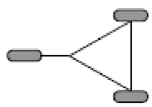
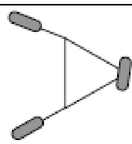
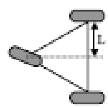
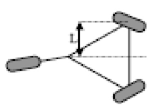
the degree of mobile robots directionality is expressed as follows : δ_s=Rang(C_{1c})

with $0 \leq \delta_s \leq 2$

δ_s presenting the number of centered wheels possible to orient the ones from the others independently, in a way that the instantaneous centre of rotation exists we can orient only centered wheels ; the extra wheels will necessarily have a movement coordinated with the two first ones.

Classification of mobile robots by their degree of mobility δ_m - δ_s degrees of directionality [9]

Table I: Classification of mobile robots by type (δ_m, δ_s)

Type	*Exemple de robot*	*Matrice Σ*	*Dim(u)*
(3, 0)		$\boldsymbol{I}_{3x3}$	*3*
(2, 0)		$\begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$	*2*
(2, 1)		$\begin{pmatrix} -\sin(\beta_{c1}) & 0 \\ \cos(\beta_{c1}) & 0 \\ 0 & 1 \end{pmatrix}$	*3*
(1, 1)		$\begin{pmatrix} 0 \\ L\sin(\beta_{c3}) \\ \cos(\beta_{c3}) \end{pmatrix}$	*2*
(1, 2)		$\begin{pmatrix} -2L\sin(\beta_{c1})\sin(\beta_{c2}) \\ L\sin(\beta_{c1}+\beta_{c2}) \\ \sin(\beta_{c2}-\beta_{c1}) \end{pmatrix}$	*3*

IKM (inverse kinematics model) allows the robot to move: Cartesian variables derived $\dot{\bar{\xi}}$ joint variables derived $\dot{\bar{q}}$ or $\dot{\bar{q}} = f(\dot{\bar{\xi}}, \bar{\xi}, \bar{\beta})$

It follows that V (CЄ Wheel/R0) = 0

So:

$\bar{V}$(CЄ Wheel /R_0)= $\bar{V}$(CЄ pl /R_0)+ $\bar{V}$(CЄ Wheel /pl) (7)

The kinematic equations are expressed in the landmark was Rp

$$\bar{V}(\text{CЄ Wheel /pl})=\begin{bmatrix} r\dot{\varphi}\overline{\iota_r} \\ -e\dot{\beta}\overline{J_r} \\ 0 \end{bmatrix} \quad (8)$$

$$\bar{V}(\text{CЄ pl }/R_0)= \bar{V}(\text{AЄ pl }/R_0)+\bar{\Omega}\ (\text{pl }/R_0)\wedge \overline{AC} \quad (9)$$

$$\bar{V}(\text{AЄ pl }/R_0)= \bar{V}(\text{PЄ pl }/R_0)+\bar{\Omega}\ (\text{pl }/R_0)\wedge \overline{PA} \quad (10)$$

$$\text{With } \bar{\Omega}\ (\text{pl }/R_0)=\begin{bmatrix} 0 \\ 0 \\ \theta\dot{\overline{Z_0}} \end{bmatrix} \quad (11)$$

In writing these equations for each wheel, we obtain the inverse kinematics model of the robot in the following matrix form:

$$\begin{cases} J_1R(\theta)\dot{\bar{\xi}} + r\dot{\bar{\varphi}} = 0 \\ C_1R(\theta)\dot{\bar{\xi}} + e\dot{\bar{\beta}} = 0 \end{cases} \quad (12)$$

With

$$J_1=\begin{bmatrix} \cos\beta_1 & \sin\beta_1 & x_{a1}\sin\beta_1 - y_{a1}\cos\beta_1 \\ \cos\beta_2 & \sin\beta_2 & x_{a2}\sin\beta_2 - y_{a1}\cos\beta_2 \\ \cos\beta_3 & \sin\beta_3 & x_{a3}\sin\beta_3 - y_{a3}\cos\beta_3 \end{bmatrix} \quad (13)$$

$$C_1=\begin{bmatrix} \sin\beta_1 & -\cos\beta_1 & e - x_{a1}\sin\beta_1 - y_{a1}\cos\beta_1 \\ \sin\beta_2 & -\cos\beta_2 & e - x_{a2}\sin\beta_2 - y_{a1}\cos\beta_2 \\ \sin\beta_3 & -\cos\beta_3 & e - x_{a3}\sin\beta_3 - y_{a3}\cos\beta_3 \end{bmatrix} \quad (14)$$

$$\begin{cases} \dot{\bar{\varphi}} = -\frac{1}{r}J_1R(\theta)\dot{\bar{\xi}} \\ \dot{\bar{\beta}} = -\frac{1}{e}C_1R(\theta)\dot{\bar{\xi}} \end{cases} \quad (15)$$

$$\text{Or } \dot{\bar{X}} = S(\bar{X})\dot{\bar{\xi}} \quad (16)$$

$$S(\bar{X}) = \begin{bmatrix} I_{3X3} \\ -\frac{1}{e}C_1 R(\theta) \\ -\frac{1}{r}J_1 R(\theta) \end{bmatrix} \quad (17)$$

If $\dot{\bar{\xi}} = \bar{0}$, then $\dot{\bar{X}} = \bar{0}$. There is no joint movement possible without moving the platform; it can not be any internal reconfiguration of the robot. This point is important for design. Indeed if you look matrices j_1 and C_1, we see that the position of the wheels is required to calculate the joint velocities, especially at the start of the movement. Now, we can reconfigure the wheels. Simple encoder stepper requiring research tops of zero are not sufficient. It is necessary to know the absolute positions of wheels relative to the platform. The absolute position encoders are needed.

IV. DYNAMIC MODEL

The dynamic model of the robot represents the relationship between pairs of actuators and the accelerations, velocities, joint positions and external forces. They are expressed as follows:

$$\bar{\Gamma} = f\left(\bar{X}, \dot{\bar{X}}, \ddot{\bar{X}}, \bar{F}\right)$$

With the vector $\bar{\Gamma}$ couples joint

The direct dynamic model is written:

$$\ddot{\bar{X}} = f\left(\bar{X}, \dot{\bar{X}}, \bar{F}, \bar{\Gamma}\right)$$

The use of this model is very new for the synthesis of control laws for mobile robots[14],[15].
To calculate the dynamic model of wheeled mobile robots, the most common approach is the use of Lagrangian formalism. There is the advantage of providing an explicit model directly usable for the simulation model directly under the general form:

$$\bar{J} = \overline{Q_e} + \bar{Q}_i + \bar{L} \quad (18)$$

or
$\bar{J}$ represents the vector of generalized actions of inertia
$\overline{Q_e}$ represents the vector of external actions
$\bar{Q}_i$ represents the vector of internal actions
$\bar{L}$ represents the vector of generalized action liaison

The dynamic model of the robot is determined by assuming that the robot is a material system composed of non-deformable, hence $\bar{Q}_i$=0
The previous equation reduces to $\bar{J} = \overline{Q_e} + \bar{L}$ (19)

A. Shares generalized inertia

The generalized action of inertia J_i is calculated from the Lagrange formula for the q_i :

$$J_i = \frac{d}{dt}\left(\frac{\partial Ec}{\partial \dot{X}i}\right) - \left(\frac{\partial Ec}{\partial Xi}\right) \quad (20)$$

The calculation of partial derivatives of the energy Ec can, by arrangement, to obtain expression measures generalized inertia:

$$\bar{J}\left(\bar{X}, \dot{\bar{X}}, \ddot{\bar{X}}\right) = T(\bar{X})\ddot{\bar{X}} + \bar{h}\left(\bar{X}, \dot{\bar{X}}\right) \quad (21)$$

Or $T(\bar{X})$ is the matrix of inertia
$\bar{h}\left(\bar{X}, \dot{\bar{X}}\right)$ is the vector of centrifugal terms and the terms of Coriolis

B. External actions

The vector of external actions $\overline{Q_e}$ can be decomposed into three terms:

$$\overline{Q_e} = \overline{Q_g} + \overline{Q_m} + \overline{Q_d} \quad (22)$$

With $\overline{Q_g}$ actions due to gravity.
$\overline{Q_m}$ shares due to the engine.
$\overline{Q_d}$ actions dissipation due to viscous friction and dry.

- Review $\overline{Q_g}$: the dynamic model is established on a ground plane and horizontal. Shares of zero gravity are:
  $\overline{Q_g}$ =0 (23)
- Review $\overline{Q_m}$: actions engines are due only to torque provided by motor-driven actuators in the joints:
  $\overline{Q_m} = B\bar{\tau}_{mot}$ (24)
  Or B (9x9) is a matrix selection motorized joints.
- Review $\overline{Q_d}$: the dissipation due to viscous and dry friction in wheel movement in tension and guidance:
  $\overline{Q_d} = -\bar{\tau}_{rés}$ (25)

These frictions preclude couples directly applied on the joints.
The external actions are as follows:

$$\overline{Q_e} = B\bar{\tau}_{mot} - \bar{\tau}_{rés} \quad (26)$$

C. Shares Generalized Relation

Shares generalized liaison representing the forces and torques due to bonds of contacts. For wheeled mobile robots, this condition reflects the rolling without slipping. These actions are the product of the kinematic constraint matrix by a vector of Lagrange multiplier.

$$\bar{L} = A^T(\bar{X}).\bar{\lambda} \quad (27)$$

With $A = \begin{bmatrix} C_1 & e.I_{3X3} & 0_{3X3} \\ J_1 & 0_{3X3} & r.I_{3X3} \end{bmatrix}$, $\bar{\lambda}$ (6x1) vector of Lagrange multipliers

D. Equation of dynamics

$$T(\bar{X})\ddot{\bar{X}} + \bar{h}\left(\bar{X}, \dot{\bar{X}}\right) = B\bar{\tau}_{mot} - \bar{\tau}_{rés} + A^T(\bar{X}).\bar{\lambda} \quad (28)$$

E. Kinetic Energy

The total kinetic energy is equal to the sum of kinetic energies of the different bodies constituting the system.

❖ Kinetic energy of the platform (pl)

Gpl is the centroid of the platform, we can then write speed Gpl:

$$\bar{V}(G_{pl} \in pl /R_0) = \bar{V}(P\in pl /R_0) + \bar{\Omega}(pl /R_0) \wedge \overline{PG}_{pl} \quad (29)$$

avec $\overline{PG}_{pl} = \begin{bmatrix} x_g \\ y_g \end{bmatrix}_{Rp}$

kinetic energy of the platform reads:

$$Ec_{pl} = \frac{1}{2} m_{pl} V_{Gpl}^2 + \frac{1}{2} I_{pl} \dot{\theta}^2 \quad (30)$$

With m_{pl} is the mass of the platform

I_{pl} denotes the moment of inertia of a platform relative to vertical axis passing through Gpl

❖ Kinetic energy of the wheels

Writing the absolute velocity of center of gravity G_{pl} the rotating part:

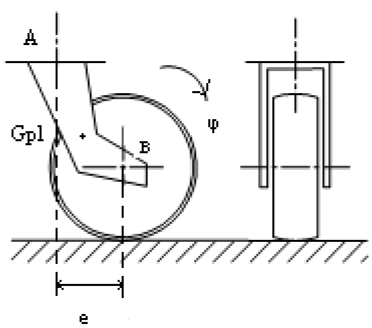


Fig. 2 center of gravity G_{pl}

$$\bar{V}(G_{pl} /R_0) = \bar{V}(P\in pl /R_0) + \dot{\theta}.\overline{Z_0} \wedge \overline{PG}_{pl} + \dot{\beta}.\overline{Z_0} \wedge \overline{A_iG_{pl}}$$
(28)

Calculating x_{Gpl} :

We assume that the center of gravity of the yoke FHAG is at a distance e / 2 point A_i anchorage on X_b.

We have $\overline{A_iG_{pl}} = \begin{bmatrix} x_{Gpl} \\ 0 \end{bmatrix}_{Rb}$ with $x_{Gpl} = \frac{e}{2} + \frac{mr.\frac{e}{2}}{m_{Wheel\_i}}$ (31)

The kinetic energy of the wheel i:

$$Ec_{Wheel\_i} = \frac{1}{2} m_{Wheel\_i} \bar{V}_{Gpl}^T \bar{V}_{Gpl} + \frac{1}{2} I_{\varphi i} \dot{\varphi}_i^2 + \frac{1}{2} I_{\beta i} (\dot{\theta} + \dot{\beta}_i)^2$$
(32)

With m_{Wheel_i} is the mass of the rotating part of the wheel i (wheel tread)

$I_{\varphi i}$ moment of inertia of the wheel i about the axis of traction (horizontal rotation)

$I_{\beta i}$ moment of inertia of the wheel tread i around the axis direction (vertical rotation)

❖ Total kinetic energy

Total kinetic energy of the robot is written in the quadratic form:

$$Ec = Ec_{pl} + \sum_{i=1}^{3} Ec_{Wheel\_i} = \frac{1}{2} \dot{\bar{X}}^T T(\bar{X}) \dot{\bar{X}} \quad (33)$$

with $T(\bar{X})$the matrix of inertia symmetric positive definite of dimension 9x9

$$T(\bar{X}) = \begin{bmatrix} R^T(\theta).F.R(\theta) & R^T(\theta).G & 0_{3x3} \\ G^T R(\theta) & I_\beta & 0_{3x3} \\ 0_{3x3} & 0_{3x3} & I_\varphi \end{bmatrix} \quad (34)$$

F. Action Motors

They are represented by the vector $\bar{\tau}_{mot}$ (dim = 9x1) or assuming all joints are motorized.

$$\bar{\tau}_{mot} = \begin{bmatrix} 0 & 0 & 0 & \tau_{\beta 1} & \tau_{\beta 2} & \tau_{\beta 3} & \tau_{\varphi 1} & \tau_{\varphi 2} & \tau_{\varphi 3} \end{bmatrix}^T \quad (35)$$

The selection of the engine is made by the selection matrix B appropriate.

G. Scrubbing action

For the dynamic model of the robot, it is necessary to take into account friction. These phenomena are in practice difficult to model. For axis traction, we used a simplified model consisting of dry Coulomb friction and viscous friction as a function of speed. These frictions are taken into account in the actuators and transmission. They are written as follows:

$$Ci = fs.sign(\dot{q}_i) + fv.\dot{q}_i \quad (36)$$

With fs: dry friction coefficient of coulomb
fv: coefficient of viscous friction

when changing direction of the robot, the steering axis must overcome a resisting torque of friction of the wheel on the ground. contact with the wheel on the ground is not reduced to a point but is a surface due to deformation of the tread under the load of the robot.

If we consider that the wheel is cylindrical and non-ring, the contact surface wheel-ground is a rectangle of length L and width W. The pressure distribution on the ground is parabolic[12].

Different methods are used to express the pressure. We have chosen the term of the pressure given by Nikravesh:

$$P = 4\frac{Pmax}{l} x \left(l - \frac{x}{l}\right) \quad (37)$$

With Pmax=1,5 Fz/WL (38)

Fz is the force of gravity on the wheel.

By integrating the pressure on the surface, we obtain the torque required to pivot the wheel to stop:

$$Mpa = \int_0^1 (x - \tfrac{1}{2})\sigma_n w dx \qquad (39)$$

With $\sigma_n = \mu.P$ tangential coefficient
μ the coefficient of friction.

The coefficient of friction depends on the nature of the tread of the wheel and the soil. When the wheel is in motion, the equation can no longer calculate the torque for pivoting the contact surface and the coefficient friction change. Pacejka proposes to model the pair of swinging motion as follows [13]:

$$M_{pm} = K_{rés}.\varphi \qquad (40)$$

With Krés factor proportionality torque pivot
The amount of pivoting φ is defined as the ratio of the speed guidance on the linear speed of the wheel, r is radius of the wheel.

$$\phi = \frac{\dot{\beta}}{r.\dot{\varphi}} \qquad (41)$$

Pacejka determine experimentally the proportionality coefficient K_0 of a wheel supporting a mass m_0. the coefficient of proportionality Kres is in the following form:

$$K_{rés} = K_0 \left(\frac{m}{m_0}\right)^{5/2} \qquad (42)$$

It expresses Mpi, the pair of pivoting by:
If $|M_{pm}| > |M_{pa}| \Rightarrow M_{pi} = sign(\dot{\beta}).|M_{pa}|$
Otherwise $M_{pi} = sign(\dot{\beta}).|M_{pm}|$
The friction for the guiding i written:

$$C_i = fs.sign(\dot{q}) + fv.\dot{q}_i + M_{pi} \qquad (43)$$

The vector of shares of friction is written:

$$\bar{\tau}_{rés} = [0\ \ 0\ \ 0\ \ C_1\ \ C_2\ \ C_3\ \ C_4\ \ C_5\ \ C_6]^T \qquad (44)$$

V. DYNAMIC MODEL

Given the various terms of kinetic energy, engine friction and the equation of the Lagrange formalism is written as follows:

$$\frac{d}{dt}\left(\frac{\partial\left(\frac{1}{2}\dot{\bar{X}}^T T(\bar{X})\dot{\bar{X}}\right)}{\partial\dot{\bar{X}}} - \left(\frac{\partial\left(\frac{1}{2}\dot{\bar{X}}^T T(\bar{X})\dot{\bar{X}}\right)}{\partial\bar{X}}\right)\right) = B\bar{\tau}_{mot} - \bar{\tau}_{rés} + A^T(\bar{X}).\bar{\lambda} \qquad (45)$$

The elimination of Lagrange multipliers is done using the kinematic constraint matrix $A^T(\bar{X})$and relation $\dot{\bar{X}} = S(\bar{X})\dot{\bar{\xi}}$.

Since $S(\bar{X})$belongs to the kernel of $A^T(\bar{X})$, we multiply the above equation by $S^T(\bar{X})$ [3].

We obtain a new form for the equation:

$$M(\bar{X})\ddot{\bar{X}} + \bar{H}(\bar{X}, \dot{\bar{X}}) = \bar{\Gamma}_{mot} - \bar{\Gamma}_{rés} \qquad (46)$$

$$\text{With } M(\bar{X}) = S^T(\bar{X}).T(\bar{X}) \qquad (47)$$

$$\bar{H}(\bar{X}, \dot{\bar{X}}) = S^T(\bar{X}).\left[\frac{dT(\bar{X})}{dt}\dot{\bar{X}} - \frac{1}{2}\frac{\partial\left(\dot{\bar{X}}^T T(\bar{X})\dot{\bar{X}}\right)}{\partial\dot{\bar{X}}}\right] = S^T(\bar{X}).\bar{h}(\bar{X}, \dot{\bar{X}}) \qquad (48)$$

$$\bar{\Gamma}_{mot} = S^T(\bar{X}).B.\bar{\tau}_{mot} \qquad (49)$$

$$\bar{\Gamma}_{rés} = S^T(\bar{X}).\bar{\tau}_{rés} \qquad (50)$$

The kinematic and dynamic models of robot 3-center directional wheels allow us to simulate the kinematic model and dynamic model.
The inverse dynamic model of the platform incorporating the terms of Coriolis, centrifugal terms and the terms of friction allows for a realistic design of the platform and its components. For a motion $\dot{\bar{\xi}}$ desired platform, the simulator provides couples to apply to the wheels.

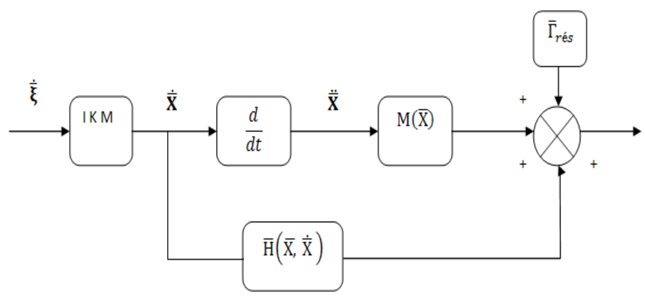


Fig. 3. Open loop simulator

Parameters and values of friction:

Dry friction and viscous friction on the axes :

The coefficients of dry and viscous friction were drawn from the results obtained for the robot MELODY of l'IRCYN (Institute of Research on Cybermetric of Nante) The coefficients of dry and viscous friction were drawn from the results obtained for the robot during the identification phase dynamic parameters [14].

The couple of dry friction on the axe of MELODY is about 8 Nm, and the viscous friction coefficient de 1.5 Nm/rad/s. the mass of MELODY is 410 Kg while the mass of our platform is about 250 Kg. the dry friction has been estimated in the masses report, that to say a couple of dry friction of 5 Nm reported to the wheel. The viscous friction coefficient is taken to the identical.

Couple of pivot, H.Pacejka model

To entirely determine the model of H.Pacejka, we should evaluate the pure pivot couple M_{pa} and the parameter $K_{rés}$ of pivot couple in motion of M_{pm} .

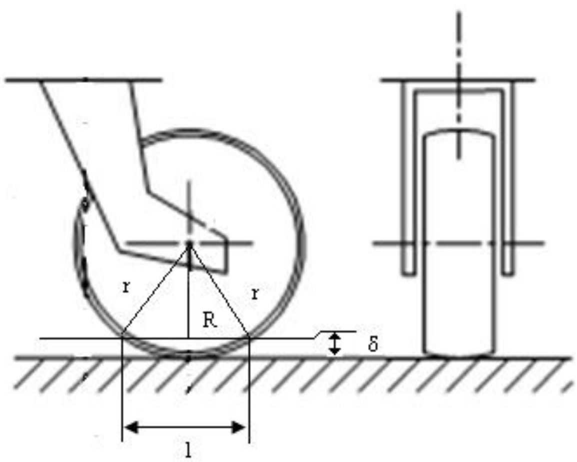
Couple of pure pivot

Returning to the equations (39) et (40), the couple of pure pivot is expressed as follows :

$$M_{pa} = \int_0^l \left(x - \frac{l}{2}\right) x \left(l - \frac{x}{l}\right) \left[4\mu \frac{P_{max}}{l} w\right] dx \quad (51)$$

with $P_{max} = 1.5 \frac{Fz}{wl}$

To evaluate the characteristics of the wheel print on the ground we estimate its vertical crushing. For a hard rubber bandage, the vertical rigidity is equal to 340.000 N/m. The vertical deformation δ for a charge of 250 Kg distributed on 3 wheels is on the order of 2.4 mm.



For a radius of 100 mm, the length of the print l is of 28.2 mm. the wheel width is of 40 mm. the value of the static friction coefficient μ is equal to 0.35 a rubber wheel on a plastic covered ground .after the integration of equation (39) we find the de pivot couple when stopping, M_{pa}= 6 Nm

Pivot Couple of the wheel in movement

Recall that the couple required to pivot is expressed as follows:

$$M_{pm} = K_{rés} \frac{\dot{\beta}}{r.\dot{\varphi}} \quad (52)$$

with $K_{rés} = K_0 \left(\frac{m}{m_0}\right)^{\frac{5}{2}}$ (53)

For a wheel supporting a vertical charge of 75 Kg, K_0= 0.80Ncm² [13]. Applying formula (36), we find

$$K_{rés} = 0.80 \times \left(\frac{250/3}{75}\right)^{\frac{5}{2}} = 1.09\, Ncm^2$$

Hence the following overall model of pivot:

If $|M_{pm}| > |M_{pa}| \Rightarrow M_{pi} = sign(\dot{\beta}).|M_{pa}|$

Otherwise $M_{pi} = 1.09\, sign(\dot{\beta}).\left|\frac{\dot{\beta}}{r.\dot{\varphi}}\right|$

VI. CONCLUSION:

We took the kinematic modeling of a robot with three-center directional wheels. The analysis of this type of robot provides the following information:
This model of robot must be powered on to avoid singular configurations.
The robot can configure its wheels without moving so it is necessary to consider an absolute position sensor on each axis direction.

We have presented the different conventions allowing the study the mobile robots of conventional wheels as well as characterizing them by their degree of mobility and directionality we have done a ranking, in term of mobility, of the different structures of the robot using the conventional wheels. The only way to obtain an omni-directional robot of conventional wheels is through the only use of decentred orientable wheels.

We have afterwards described the dynamic model of the robot taking into consideration a model of wheel friction on the ground.

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