Optimization of Buffer-size Allocation Using Dynamic Programming

Masayoshi HASAMA, Takao ITO and Seigo MATSUNO

***Abstract*— Assembly-like Queueing systems are used in the assembly processes in production lines in the chemical industry as well as dataflow in computer systems. While many models are constructed in tandem and merge systems, assembly-like systems are known to be more complicated and difficult to analyze. These systems are not investigated in queueing theory. Whereas most research focuses on simple assembly-nodes, in this paper, we evaluate by dynamic programming using numerical analysis to propose buffer-size optimal allocation algorithm.**

***Keywords*— Assembly-like system, Buffer Allocation, Dynamic Programming, Production System**

1. INTRODUCTION

ASSEMBLY processes arise in many practical situations , including assembly-line in productions. Compared to queueing systems with a tandem and merge configurations, assembly-like queueing systems are not simple to analyze. Most research has focused on simple assembly-nodes. Research has attempted to solve an optimal algorithm in assembly-like queueing systems.

In his analysis of network models such as stationary probabilities, Harrison (1973) used infinity models to propose an approximation method in throughput systems.

Mitra (1990) and Tayur (1992) derived the heuristic rule in optimal buffer-size allocation using a tandem system. According to heuristic policy in optimal allocation, an approximation method is considered more effective.

Considering production system in many factories, we should utilize limitation space for buffer-size allocation in each station to distribute it effectively.

At the result of numerical simulation, Hemachandra and Edeupuganti (2003) used the fork-join model to discuss buffer-size allocation to minimize mean waiting time of a typical job or Work-In-Process of system. Different from previous studies on heuristic policies, in this paper we discuss multi-level assembly-like queueing systems to propose a more systematic algorithm to obtain optimal buffer-size allocations using observation and conjecture.

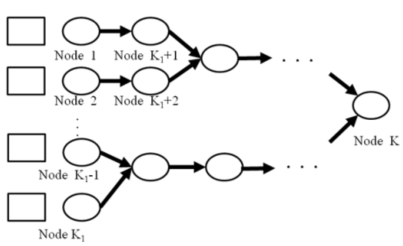

Fig1. Assembly-like Queueing System

2. Assembly-like Queueing Systems

We consider a K-node queueing system as shown in Fig 1. Each node has a single server which provides exponential service with rate μ =4.

For node 1 through K_1, there is an infinite-sized pool of customers in front of each server. Hence, these nodes cannot be idle. Upon service completion at a node, a customer enters a downstream node and receives his service there.

Customers depart from the system after service completion at the last node, Node *K*.

Suppose the input of Node *k* ($k>K_1$) is the output of Node $u_1(k)$ and u_2(k). Buffers $k_{(1)}$ and $k_{(2)}$ are prepared in Node *k* for input from each node individually (Fig 2).

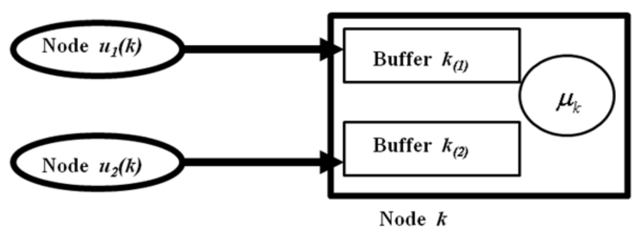

Fig 2. An Assembly-Node

In front of the server, a position is also prepared for each input node individually. The server may begin assembly service for customers in the positions only in the case that there is a customer in each position. Node *k* cannot begin a service if there is any vacant position in front of the server.

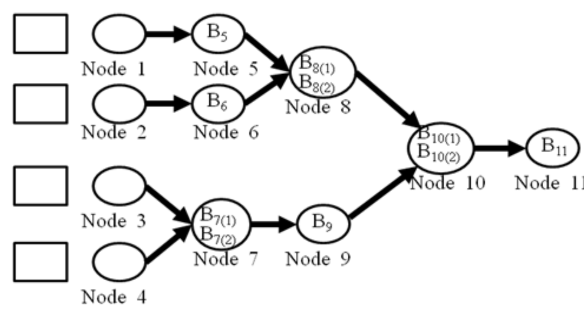

Fig3. An 11-Node Model

We define Buffer $k_{(i)}$-1 as the size of Buffer $k_{(i)}$. The server at Node $u_1(k)$ checks state of Buffer $k_{(i)}$ before starting at a new customer service. If Buffer $k_{(i)}$ is full, the customer occupies the server and blocks it.

Service is begun only after a vacant space appears in Buffer $k_{(i)}$. This is called *communicating blocking*. In Fig 3, Buffer depending on Group 1 starts at Buffer 1 lower side set Buffer 2, Buffer 3 and Buffer 4.

Moreover, Group 2 allocates Buffer 5, Buffer 6, and Buffer $7_{(1)}$, Buffer $7_{(2)}$, Buffer $8_{(1)}$, Buffer $8_{(2)}$. Buffer 7 is assembly node. These nodes are shown by the number in bracket.

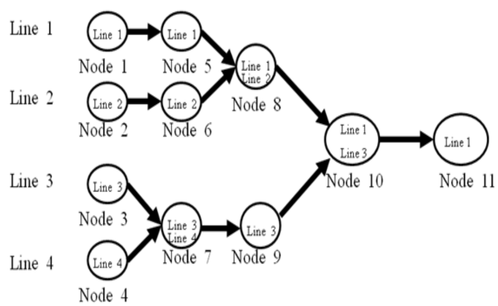

Fig 4. An example of 4 lines

3. Observation and Conjecture

This section illustrates a method to approach an optimal throughput in simulation analysis. First, four lines in this model are defined. Second, we simulate this model as buffer allocation in each line. And finally, this system will achieve the optimal buffer-size allocation.

3.1Line

We divide the model into 4 lines. Line 1 starts at Node 1 and end at Node *K*. The server in Node 1 consists of Buffer 5, Buffer $8_{(1)}$, Buffer $10_{(1)}$ and Buffer 11.

Line 2 starts at Node 2 and ends at Buffer $8_{(2)}$. Line 3 and Line 4 start from Node 3 and 4 respectively, and end at Buffer $10_{(2)}$ and Buffer $7_{(2)}$ respectively.

An each line does not duplicate (i.e. there is the model existing three assembly-nodes and we need to classify it: Independency of line).

For example, Buffer 8 locating by Node 8 in this system stands for Buffer $8_{(1)}$ and Buffer $8_{(2)}$.

The system in this model defines that the number of buffer and buffer allocation cannot make the adjustment, but buffer-size can make it.

And we consider that achieve the max throughput how to allocate in each buffer, if it is given total buffer-size in this system.

We derive optimal buffer in the following steps.

Step1) Simulated by using assembly-like queuing system
Step2) Searching for regularity of buffer
Step3) Select the best throughput value in getting regularity of buffer

3.2Independency of Buffer-size Allocations

For general assembly-like queueing systems, we define the following notations for buffer-size and allocations.

Notations:

N: Total buffer size in the system
L_j: Line j $(j = 1,2,3,\cdots,K_1)$
n_j: Total buffer size of L_j $(j = 1,2,3,\cdots,K_1)$
$\boldsymbol{B}_j(n_j)$: Buffer-size allocation vector of L_j given that the total buffer-size of L_j is n_j
$\boldsymbol{B}'_j(n_j)$: Buffer-size allocation vector other than $\boldsymbol{B}_j(n_j)$
$\overline{\boldsymbol{B}}_j(n_j)$: Buffer-size allocation vector of the system other than L_j given $n_j(j = 1,2,3,\cdots,K_1)$
$\mathrm{T}\left(\boldsymbol{B}_j(n_j),\overline{\boldsymbol{B}}_j(n_j)\right)$: Throughput of the system given $\boldsymbol{B}_j(n_j)$ and $\overline{\boldsymbol{B}}_j(n_j)$ $(j = 1,2,3,\cdots,K_1)$

Based on the result of simulation, we get the following observation.

Observation: Given the total buffer size in L_j is n_j, if

$$^{\forall}\overline{B}_j(n_j){:}T\left(\boldsymbol{B}_j(n_j),\overline{\boldsymbol{B}}_j(n_j)\right) > T\left(\boldsymbol{B}'_j(n_j),\boldsymbol{B}_j(n_j)\right),$$

then

$$^{\forall}B'_j(n_j){:}T\left(\boldsymbol{B}_j(n_j),\overline{\boldsymbol{B}}'_j(n_j)\right) > T\left(\boldsymbol{B}'_j(n_j),\overline{\boldsymbol{B}}'_j(n_j)\right).$$

As an example, in the 11-node model, let N=13 and $n_1 =$ 5.Table 1 enumerates 4 cases of $B_1(5)$, and Table 2 lists some case for $\overline{B_1}(5)$.

In Fig5, we can see that the line for Case① never cross with Case② or other cases. This is what the above observation implies. And the observation can be regarded as a kind of independency among the lines of the system.

To derive the maximum $\left(\boldsymbol{B}_j(n_j),\overline{\boldsymbol{B}}_j(n_j)\right)$, buffer allocation depend on (1, 1,2,1) in L_1.

Remark. Applying to observation, suppose we consider buffer allocation in $L_1 \sim L_4$when total buffer size is 13. Now, the buffer allocation feasible in L_1 enumerate with total buffer size. In this case, L_1 allocate range of between$\boldsymbol{B}_1(5)$ and $\boldsymbol{B}_1(8)$. Based on simulation result, the highest throughput in L_1 is $\boldsymbol{B}_1(5) =$ (1,1,2,1). Allocation vector in $\boldsymbol{B}_1(5)$ note(1,1,1,2), (1,1,2,1), (1,2,1,1) and (2,1,1,1). Next, buffer allocation with $L_2 \sim L_4$ besides L_1 buffer allocation in L_1subtract from total buffer size. In figure 3, it noted buffer allocation type that four cases remain the buffer size in $L_2 \sim L_4$. Compared to $T\left(\boldsymbol{B}_j(n_j),\overline{\boldsymbol{B}}_j(n_j)\right)$ with $T\left(\boldsymbol{B}'_j(n_j)|\overline{\boldsymbol{B}}_j(n_j)\right)$ in$\boldsymbol{B}_1(5)$. In $\boldsymbol{B}_1(5)$, suppose $\boldsymbol{B}_1(5) = (1,1,2,1), \mathbf{B}'_1(5) =$ (2,1,1,1) ,$\boldsymbol{B}_1(5) = (1,1,2,1)$ is the highest throughput in Case Ⅱ$\left(\text{i. e. } \mathrm{T}\left(\boldsymbol{B}_j(n_j)|\overline{\boldsymbol{B}}_j(n_j)\right) > T\left(\boldsymbol{B}'_j(n_j)|\overline{\boldsymbol{B}}_j(n_j)\right)\right)$ independent of Case Ⅰ, Ⅲ, Ⅳ$\left(\text{i. e. } T\left(\boldsymbol{B}_j(n_j)|\boldsymbol{B}'_j(n_j)\right) > T\left(\boldsymbol{B}'_j(n_j)|\overline{\boldsymbol{B}}_j(n_j)\right)\right)$.Hence, $\boldsymbol{B}_1(5) = (1,1,2,1)$ is the highest throughput in(1,1,1,2), (1,1,2,1), (1,2,1,1), (2,1,1,1).

Moreover, in $\overline{\boldsymbol{B}}_{\boldsymbol{j}}(n_j)$, to maximize $T\left(\boldsymbol{B}_{\boldsymbol{j}}(n_j)|\overline{\boldsymbol{B}}_{\boldsymbol{j}}(n_j)\right)$, it is essential to buffer allocation (1,1,2,1)in $L_1$with $\boldsymbol{B}_1(5)$ (Fig.5).

Finding of our study based on Observation derived the highest value of $T\left(\mathbf{B_j}(n_j)|\overline{\mathbf{B}}_{\mathbf{j}}(n_j)\right)$, thus, subject to hold for buffer allocation until $L_j$and considered maximizing $T\left(\mathbf{B_j}(n_j)|\overline{\mathbf{B}}_{\mathbf{j}}(n_j)\right)$ the addional allocation in $L_j$using Dynamic Programming.

We search for buffer size feasible in $L_1$and derive allocation feasible remain $L_j$with total buffer size.

It means that location of buffer can be existing in each L_j and this algorithm applies to derive buffer size in feasible allocation with L_j and remain L_j.

Table1. Buffer-size Allocation in L_1

Case	$B_1(5)$
a	(1,1,1,2)
b	(1,1,2,1)
c	(1,2,1,1)
d	(2,1,1,1)

Table 2.Buffer-size Allocation in L_2～L_4

	B_6	$B_{8(2)}$	$B_{7(1)}$	B_9	$B_{10(2)}$	$B_{7(2)}$
①	1	1	1	1	2	2
②	1	1	2	1	1	2
③	1	2	1	1	2	1
④	1	1	1	1	1	3

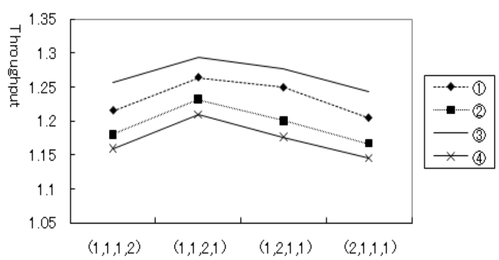

Fig 5.Value of $T\left(B_1(5),\overline{B}_1(5)\ \right)$

3.3 Optimal Buffer Allocation

Using the result of the above observation, we derive the optimal buffer allocation until L_1.

Moreover, total buffer-size until L_j denote s_j, Buffer-size allocation vector until L_j denote $\boldsymbol{\beta}_j(s_j)$. To derive optimal buffer allocation, we define the below.

Notations

$\boldsymbol{\beta}_j(s_j)$: Buffer-size vector from L_1 to L_j given s_j $(j = 1,2,3,\cdots,K_1)$

$\overline{\boldsymbol{\beta}}_j(s_j)$: Buffer-size vector behind L_{j+1} given s_j $(j = 1,2,3,\cdots,K_1-1)$

$\boldsymbol{\beta}_j^*(s_j)$: Optimal Buffer allocation vector until L_j $(j = 1,2,3,\cdots,K_1)$

$T\left(\beta_j(s_j),\overline{\beta}_j(s_j)\right)$: Throughput of the system given $\boldsymbol{\beta}_j(s_j)$ and $\overline{\boldsymbol{\beta}}_j(s_j)$ $(j = 1,2,3,\cdots,K_1)$

$\boldsymbol{B}_{\boldsymbol{j}}^*(n_j)$: Optimal Buffer-size allocation L_j (j=1, 2, 3,･･･, K_1)

$s_j = \sum_{j=1}^{K_1} n_j$: Total buffer-size until L_j (j=1, 2, 3,･･･, K_1)

Conjecture: Assume that optimal buffer allocation $\boldsymbol{\beta}_j^*(s_j)$

$$\boldsymbol{\beta}_{\boldsymbol{j+1}}^*(s_{j+1}) = T\left(\beta_j^*(s_j), B_{j+1}^*(s_{j+1}-s_j)\right)$$

Proof: we enumerate four possibilities for $\boldsymbol{\beta}_{j+1}^*(s_j)$, where $\beta_j'(s_j) \neq \beta_j^*(s_j)$, $B_j'(n_j) \neq B_j^*(n_j)$

$$\boldsymbol{\beta}_{\boldsymbol{j+1}}^*(s_{j+1}) = T\left(\beta_j^*(s_j), B_{j+1}^*(s_{j+1}-s_j)\right) \quad (1)$$

$$\boldsymbol{\beta}_{\boldsymbol{j+1}}^*(s_{j+1}) = T\left(\beta_j^*(s_j), B_{j+1}'(s_{j+1}-s_j)\right) \quad (2)$$

$$\boldsymbol{\beta}_{\boldsymbol{j+1}}^*(s_{j+1}) = T\left(\beta_j'(s_j), B_{j+1}^*(s_{j+1}-s_j)\right) \quad (3)$$

$$\boldsymbol{\beta}_{\boldsymbol{j+1}}^*(s_{j+1}) = T\left(\beta_j'(s_j), B_{j+1}'(s_{j+1}-s_j)\right) \quad (4)$$

Showing that (2),(3) and (4) do not hold, the conjecture is proved.

3.4 Proof of an inconsistent

(i)Proof of not holding (2)

Assume that

$$\exists B_j^*(s_{j+1}-s_j) \neq B_{j+1}^*(s_{j+1}-s_j), \beta_{j+1}^*(s_{j+1}) = T\left(\beta_j^*(s_j), B_j'(s_{j+1}-s_j)\right) \quad (5)$$

$\beta_{j+1}^*(s_{j+1})$ is the optimal buffer allocation.

$$T\left(\beta_j^*(s_j), B_{j+1}^*(s_{j+1}-s_j), \beta_{j+1}(s_{j+1})\right) < T\left(\beta_{j+1}^*(s_{j+1}), \overline{\beta}_{j+1}(s_{j+1})\right) \quad (6)$$

Using the suppose,

$$T\left(\beta_{j+1}^*(s_{j+1}), \overline{\beta}_{j+1}(s_{j+1})\right) = T\left(\beta_j^*(s_j), B_{j+1}'(s_{j+1}-s_j), \overline{\beta}_{j+1}(s_{j+1})\right) \quad (7)$$

Moreover, $B_{j+1}^*(s_{j+1}-s_j)$ is the optimal buffer allocation of L_{j+1}. Using the result of observation, $^\forall\overline{\beta}_{j+1}$ is

$$T\left(\beta_j^*(s_j), B_{j+1}'(s_{j+1}-s_j), \overline{\beta}_{j+1}(s_{j+1})\right) < T\left(\beta_j^*(s_j), B_{j+1}^*(s_{j+1}-s_j), \overline{\beta}_{j+1}(s_{j+1})\right) \quad (8)$$

Hence,

$$T\left(\beta_j^*(s_j), B_{j+1}^*(s_{j+1}-s_j), \beta_{j+1}(s_{j+1})\right) > T\left(\beta_{j+1}^*(s_{j+1}), \beta_{j+1}(s_{j+1})\right) \quad (9)$$

Clearly, (6) and (9) inconsistent, (2) do not hold.

(ii) Proof of not holding (3)

Assume that

$$\beta'_j(s_j) \neq \beta^*_j(s_j),\ \beta^*_{j+1}(s_{j+1}) = T\left(\beta'_j(s_j), B^*_{j+1}(s_{j+1}-s_j)\right) \quad (10)$$

$\beta^*_{j+1}(s_{j+1})$ is the optimal buffer allocation.

$$T\left(\beta^*_j(s_j), B^*_{j+1}(s_{j+1}-s_j), \beta_{j+1}(s_{j+1})\right) < T\left(\beta^*_{j+1}(s_{j+1}), \beta_{j+1}(s_{j+1})\right) \quad (11)$$

Using (10),

$$T\left(\beta^*_{j+1}(s_{j+1}), \overline{\beta}_{j+1}(s_{j+1})\right) = T\left(\beta'_j(s_j), B^*_{j+1}(s_{j+1}-s_j), \overline{\beta}_{j+1}(s_{j+1})\right) \quad (12)$$

$\beta^*_j(s_j)$ is the optimal allocation until L_j.

For $^\forall\beta_{j+1}(s_{j+1})$,

$$T\left(\beta'_j(s_j), B^*_{j+1}(s_{j+1}-s_j), \overline{\beta}_{j+1}(s_{j+1})\right) < T\left(\beta^*_j(s_j), B^*_{j+1}(s_{j+1}-s_j), \overline{\beta}_{j+1}(s_{j+1})\right) \quad (13)$$

Hence,

$$T\left(\beta^*_j(s_j), B^*_{j+1}(s_{j+1}-s_j), \overline{\beta}_{j+1}(s_{j+1})\right) > T\left(\beta^*_{j+1}(s_{j+1}), \overline{\beta}_{j+1}(s_{j+1})\right) \quad (14)$$

Clearly, (11) and (14) inconsistent, (3) do not hold.

(iii) Proof of not holding (4)

Assume that $B'_j(s_{j+1}-s_j) \neq B^*_{j+1}(s_{j+1}-s_j)$, $\beta'_j(s_j) \neq \beta^*_j(s_j)$, $\beta^*_{j+1}(s_{j+1}) = T\left(\beta'_j(s_j), B'_{j+1}(s_{j+1}-s_j)\right)$ (15)

$\beta^*_{j+1}(s_{j+1})$ is the optimal allocation.

$$T\left(\beta^*_j(s_j), B^*_{j+1}(s_{j+1}-s_j), \overline{\beta}_{j+1}(s_{j+1})\right) < T\left(\beta^*_{j+1}(s_{j+1}), \overline{\beta}_{j+1}(s_{j+1})\right) \quad (16)$$

Here, compare to $T\left(\beta^*_j(s_j), B^*_{j+1}(s_{j+1}-s_j), \overline{\beta}_{j+1}(s_{j+1})\right)$,

$T\left(\beta^*_j(s_j), B'_{j+1}(s_{j+1}-s_j), \overline{\beta}_{j+1}(s_{j+1})\right)$ and $T\left(\beta'_j(s_j), B'_{j+1}(s_{j+1}-s_j)\right)$

pattern of $T\left(\beta'_j(s_j), B'_{j+1}(s_{j+1}-s_j), \overline{\beta}_{j+1}(s_{j+1})\right)$. (17)

We will get the relationship of

$$T\left(\beta^*_j(s_j), B^*_{j+1}(s_{j+1}-s_j), \overline{\beta}_{j+1}(s_{j+1})\right) > T\left(\beta^*_j(s_j), B'_{j+1}(s_{j+1}-s_j), \overline{\beta}_{j+1}(s_{j+1})\right) > T\left(\beta'_j(s_j), B'_{j+1}(s_{j+1}-s_j), \overline{\beta}_{j+1}(s_{j+1})\right). \quad (18)$$

Moreover,

$$T\left(\beta^*_j(s_j), B'_{j+1}(s_{j+1}-s_j), \overline{\beta}_{j+1}(s_{j+1})\right) < T\left(\beta^*_j(s_j), B^*_{j+1}(s_{j+1}-s_j), \overline{\beta}_{j+1}(s_{j+1})\right) \quad (19)$$

Hence,

$$T\left(\beta^*_j(s_j), B^*_{j+1}(s_{j+1}-s_j), \overline{\beta}_{j+1}(s_{j+1})\right) > T\left(\beta^*_j(s_j), \overline{\beta}^*_{j+1}(s_{j+1}-s_j)\right). \quad (20)$$

Clearly, (16) and (20) inconsistent, (4) do not hold.

The conjecture implies that the optimal allocation until L_{j+1} is obtained by adding the optimal allocation in L_{j+1} to the optimal allocation until L_j.

This leads to the following dynamic programming algorithm.

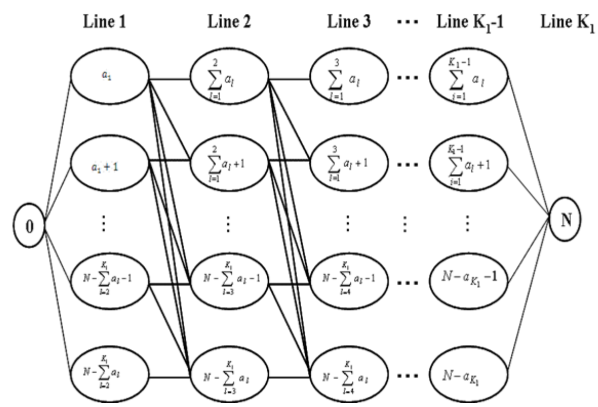

Fig 6. Dynamic Programming

4. Dynamic Programming

Based on discussion in this section, here we propose a dynamic programming algorithm to get optimal throughput of the system.

Notations :

0 :origin

a_j :Buffer-size in L_j $(j = 1,2,3,\cdots,K_1)$

N :Total buffer-size in buffer allocation until L_{K_1} $(j = 1,2,3,\cdots,K_1)$

S_j: $\{s_j | s_j = \sum_{l=1}^{K_1} a_l, \sum_{l=1}^{K_1} a_l + 1, \cdots, N - \sum_{l=1}^{K_1} a_{l+1}\}$ $(j = 1,2,3,\cdots,K_1)$, $(l = 1,2,3,\cdots,K_1)$

Fig 6 denote the dynamic programming. Using the previous section, we must derive the max throughput added by buffer allocation between L_1 and L_j.We consider the possible buffer size in L_1. For the possible buffer size in L_1, we can derive buffer allocation. The case of N=14 shows that the optimal buffer size in L_1 is 5.

As well as, L_2, L_3 and L_4 adapt the process of L_1. The thick arrows indicate the optimal route to get the optimal solution.

Holding buffer allocation in L_1, we consider the optimal buffer allocation in L_2. In this case, the best buffer allocation of L_1 is β_1(1,1,2,1), the best buffer allocation of L_2 is β_2(1,1,2,1,1,2). As well as, β_3 and β_4 is (1,1,2,1,1,2,1,2,2),(1,1,2,1,1,2,1,2,2,1)

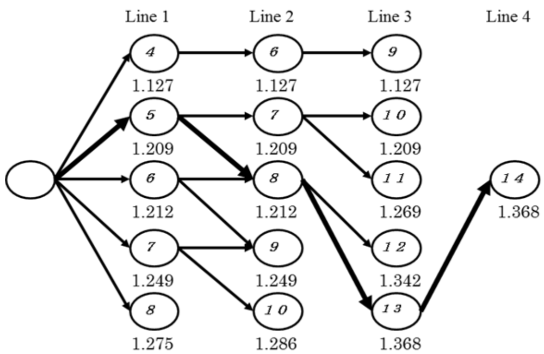

Fig 7.Throughput of N=14

Table 3.Buffer Allocation and Optimal Allocation in N=14

Line	Buffer allocation	Optimal allocation
1	$B_1(4)$=(1,1,1,1),$B_1(5)$=(1,1,2,1) $B_1(6)$=(1,2,2,1), $B_1(7)$=(1,2,2,2) $B_1(8)$=(1,2,3,2)	$B_1(5)$=(1,1,2,1)
2	$B_2(6)$=(1,1,1,1,1,1) $B_2(7)$=(1,1,2,1,1,1) $B_2(8)$=(1,2,2,1,1,1)or(1,1,2,1,1,2) $B_2(9)$=(1,2,2,2,1,1)or(1,2,2,1,1,2) $B_2(10)$=(1,2,2,2,1,2)	$B_2(8)$=(1,1,2,1,1,2)
3	$B_3(9)$=(1,1,1,1,1,1,1,1,1) $B_3(10)$=(1,1,2,1,1,1,1,1,1) $B_3(11)$=(1,1,2,1,1,1,1,1,2) $B_3(12)$=(1,1,2,1,1,2,1,1,2) **$B_3(13)$=(1,1,2,1,1,2,1,2,2)**	**$B_3(13)$=(1,1,2,1,1,2,1,2,2)**
4	**$B_4(14)$=(1,1,2,1,1,2,1,2,2,1)**	**$B_4(14)$=(1,1,2,1,1,2,1,2,2,1)**

respectively(See Table 3).

Remark. In L_j , s_1 signify the range of $a_1$by $\sum_{l=2}^{K_1} a_l$. For example, for $N = 14$, buffer allocation in L_1 represent buffer 5,buffer 8(1),buffer 10(1) and buffer 11 as buffer size in each L_j are all 1.In this case, minimized size in L_1denote $a_1 = 4$. Remaining buffer size $\sum_{l=2}^{4} a_l = 10$ remove L_1 regard with L_{K_1}. We assume that buffer in $L_2, L_3, \cdots L_{K_1}$ enter in L_{K_1}.

Now, maximum s_1 with feasible buffer size in L_1given $a_l = 8$ because this case can intend subtract $\sum_{l=2}^{4} a_l$ from N. Therefore, the numbers of feasible buffer allocation in L_2, L_3 and L_4 shown by $\sum_{l=2}^{4} a_l = 6$. Recall that remaining the number of buffer did not include L_1. In other words, $\sum_{l=2}^{4} a_l =$ 6 mean that minimum buffer size denoted by feasible buffer allocation from L_2 until final line.

For the reasons mentioned above, suppose buffer size in $L_1, L_2, \cdots L_{K_1}$located all 1, maximum buffer size get $s_1 = 8$ using$N - \sum_{l=2}^{4} a_l$. Therefore, the range allocation size of L_1will find between $s_1 = 4$ and $s_1 = 8$.

Next, using the buffer allocation size (s_1), we note buffer allocation by enumeration with simulation result (i.e. maximum T $\left(\boldsymbol{B_j}(n_j)|\overline{\boldsymbol{B}}_j(n_j)\right)$ and buffer allocation) and hold the buffer allocation state to derive the optimal buffer allocation in L_{K_1}. As well as L_1,L_2 consists of buffer 2 and buffer 6. Notice that minimum feasible buffer size in L_2 is not $a_2 = 2$ but mixing $a_1 = 4$ and $a_2 = 2$. $a_1 = 4$, $a_2 = 2$ signify minimum $\sum_{l=1}^{2} a_l$ (min $s_2$is 6) and then max $s_2$is given by remaining the number of buffer in L_3, L_4did not include L_2(i.e. max s_2 is 10). L_3, L_4 can derive the same as $L_1$and L_2method.

The algorithm for optimal buffer-size allocation:

Step 0) Set j=1.

Step1-1) Calculate $T\left(\boldsymbol{B}_j(n_j), \overline{\boldsymbol{B}}_j(n_j)\right)$ for all $\boldsymbol{B_j}(n_j)$'s to derive

Step1-2) Set $\boldsymbol{\beta}_j^*(n_j) = \boldsymbol{B}_j^*(n_j)$ for each feasible n_j

Step2-1) For each feasible s_j, keep $\boldsymbol{\beta}_j^*(s_j)$ unchanged and calculate $T\left(\boldsymbol{B}_{j+1}(s_{j+1} - s_j), \overline{\boldsymbol{B}}_{j+1}(s_{j+1} - s_j)\right)$ for all $\boldsymbol{B}_{j+1}(s_{j+1} - s_j)$'s to derive $\boldsymbol{B}_{j+1}^*(s_{j+1} - s_j)$ for each feasible s_{j+1}.

Step2-2) For each feasible s_{j+1}, select optimal $\boldsymbol{\beta}_{j+1}^*(s_{j+1}) = T\left(\boldsymbol{\beta}_j^*(s_j), \boldsymbol{B}_{j+1}^*(s_{j+1} - s_j)\right)$ from all feasible combina tions of s_j and $s_{j+1} - s_j$.

Step 3) If j=K_1, $\boldsymbol{\beta}_{j+1}^*(s_{j+1})$ is the optimal buffer-size allocation vector. Otherwise set j=j+1 and go back to Step 2-1).

5. Validation

In this section, we calculate Percentage Reduction (PR) in all allocation type with total buffer size. It defines Total Allocation (TA) and SA with search for allocation.

We have

$$PR = 1 - \frac{SA}{TA}$$

For N=14, we search for 210 allocation types. Buffer allocation type was increased by the number of total buffer size. Hence, increasing buffer allocation increased the magnitude of PR and allocation type. Consequently, independent from the magnitude of buffer size, an algorithm applied for holding the buffer allocation.

Table 4. Search Result

Total buffer size	Total buffer allocation type	Search for buffer allocation	PR
12	55	27	50.9%
13	220	83	62.2%
14	715	210	70.6%
15	2002	459	77.1%

6. Conclusion

In this paper, we discussed optimal buffer-size allocation problem in assembly-like queueing systems to propose a dynamic programming algorithm that maximizes throughput of the system. Different from previous studies on heuristic policies, we proposed a systematic algorithm.

However, future research is necessary to find theoretical support for the observation in Section 3.

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Masayoshi HASAMA is a part time teacher in Fukuoka Institute of Technology(FIT)He recieved Ph D degree in Engineering from FIT, Japan in 2007.

His research interest are in areas of Supply Chain Management and Operations Research.His articles have appeared in journals such as Trans. of the Japan Society for Production Management,Japan Industrial Management Association and proceedings of major conferences such as APIEMS,ISORA and WSEAS.

Takao ITO is a professor of the Department of Business Administration at Ube National College of Technology, Japan. He is serving concurrently as professorof Harbin Institute of Technology (HIT) in Weihai, China.

He received his Doctor degree of Economics from Kyoto University and PhD degree of Engineering from Miyazaki University, Japan. His current research interests include automaton theory, quantitative analysis of supply chain relationships using graph theory, theoretical analysis of the relationship between IT and corporate strategy, and engineering analysis of organizational structures using complex system theory. He was one of the winners of the Best Paper Award in the International Symposium on Artificial Life and Robotics (AROB) in January 2006.

He has published in several refereed journals and proceedings, particularly in the area of corporate management, IT strategy, and computer science. Dr. Ito is a member of the international advisory board of Scientific Journals International and the editorial review board of International Journal of Business Data Communications and Networking, International Journal of Data Mining, Modeling and Management, and International Journal of Information and Communication Technologies for the Advanced Enterprise.

He sits in the International Program Committee of Artificial Life and Robotics (AROB). And he also is a member of the Board of Directors of Asian Symbiosis Association, Japan.

Seigo MATSUNO is an associate professor in the Department of Business Administration at Ube National College of Technology, Japan. He received his PhD degree in Economics from Kyushu University, Japan in 2004.

His current research interests are in the areas of information systems (IS) outsourcing, IT management and strategy, and business-to-business e-commerce. He has published several research papers and books in that area.

His articles have appeared in journals such as Trans. of the Japan Society for Production Management and proceedings of major conferences such as WSEAS and AROB.