Practical aspects regarding spare parts reliability evaluation within an integrated management system

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Abstract—Product quality means its degree or level of correspondence to the consumption needs, therefore reliability means, the use of products for their design parameters, their safe and continuous exploitation in well determined conditions during a given period of time. Due to the fact that a series of parts and subsets have a large proportion in the amount of expenses of a company, their operation monitoring is imposed in order to control their influence on the operational costs and therefore taking a series of decisions in order to increase their reliability. Monitoring their operational behavior has to be made during the entire life span of these spare parts, starting with their entering the company until taking them out of operation, the scope being that of determining their operational reliability. Therefore, in an Integrated Management System, it is imposed to implement not only the procedure of following the operation of spare parts during the defects notification period, procedure which only handles the means of solving the defects which may appear by the suppliers, but also a procedure for the evaluation of the reliability of parts supplied.

Keywords— biparametric Weibull distribution, Integrated Management System, Kolmogorov – Smirnov test, reliability.

I. THEORETIC CONSIDERATIONS

Operational reliability represents the reliability determined by the results regarding the operational behavior during a certain period of time, of a large number of products used by the beneficiary. That is to say, reliability is the quality of the product extended in time. The objectives of a reliability study are:

• Knowing the defect (establishing the causes as well as the means of repairing them);
• Defects’ physical analysis of defects;
• Quantitative and qualitative appreciation in time of the behavior of products and systems considering the influence of internal and external factors;
• Determination of mathematic models and determination and prognosis methods of the reliability based on trial operations and operational analyses of products and systems;
• Establishing the means of ensuring, maintaining and increasing the reliability during the design, creation, and operation phases of products;
• Establishing reliability data selection and processing.

II. SPARE PARTS RELIABILITY EVALUATION STEPS

An Integrated Management System, together with the procedure of following the operation of products during the warranty period also contains a series of procedures for the evaluation of the reliability of the supplied products. Therefore the steps to take for the evaluation of the reliability of spare parts are the following:

• Choosing the product, sample or lot which will be the basis of the determination of reliability;
• Establishing the data collection and information systematization system;
• Statistic analysis of malfunctions based on observation results;
• Identifying the malfunction repair procedure, determination of parameters of theoretic distribution of malfunctions and estimation of reliability parameters; testing the repair chosen procedure;
• Final result analysis, conclusions, measures, forecast.

III. EVALUATION OF SUPPLIED SPARE PARTS’ RELIABILITY

The reliability determination method based on acquired data processing by following the operational behavior of products, is characterized by minimum expenses concerning the acquisition and process of information, but require as well either a long period of time of supervision or the supervision of a large number of products during a short period of time. The success of such a reliability analysis depends mostly on the correctness of information regarding the behavior of products.

In what the volume of the sample is concerned, the larger it is the more real are the result of the analysis. A sample with a number of elements is considered insignificant for the balance of the reliability of complex products. If there aren’t enough exemplars of the specified product, data for a longer period of time are gathered for general statistics.

Data processing is meant to stabilize statistic information, obtained through a series of specific operations, which may be found in the following paragraphs.

A. Appointing the team of experts

The team will evaluate the reliability of the parts or of group of spare parts, according to specific responsibilities, time spent for the analysis, and the report presentation date regarding the
evaluation of the reliability of spare parts.

B. Delimitation

The volume of the sample is being established, i.e. population or statistic group to be observed.

Delimitation represents a division of the studied phenomenon, being also necessary either because of a large number of units or because the destructive characteristic of experiments. Therefore, a delimitation corresponding to a partition of reality is made, such as for the analysis of the phenomenon to be made on a reality level following the extension to the entire population.

C. Data acquisition

Continuous operation of spare parts is observed during the entire time of the analysis, being monitored.

D. Filling in the continuous operation chart of the product, with all the events related its operation

Therefore, the following will be noted for each breakdown in the operation of the machinery:

- Name of the defect,
- Date and time of the defect,
- Machineries starting time,
- Duration of the defect,
- Operation period between two events.

From an informational content point of view, this document comprises general data as well as data regarding the information which is gathered based on observation. It is really important to consider whether the products may or may not be repaired. For repairable spare parts the chart may have the following content:

a. General data referring to:
   - Name of the place where the monitored product is used,
   - Name of the product, The identification code of the product, Manufacturing year of the product, Starting operation date of the product, Name of the supplier.

b. Data comprising breakdown information:
   - Date and time of the breakdown: month, day, and hour of breakdown; The number of operational hours until the breakdown: the number of hours during operation or other parameters (km, number of cycles, etc.);
   - Cause of the defect: it will be specified if the defect is due to normal wear, breaking, corrosion, etc. If it's a visible effect the cause may be found right away. If it is the other way around, then we proceed with the total or partial dismantling of the product, mentioning the name of the malfunctioning element or elements;
   - The conditions in which the malfunction had appeared: it will be specified the factors leading to the malfunction such as overrun, wrong manipulation, etc.;
   - The means of remedying the malfunction: it will be specified the means of remedying the malfunction (replacement of parts with new or refurbished spare parts, adjustment operations, etc.);
   - The period of time for the remediation: the number of hours and minute will be specified for the remediation of the malfunction;
   - Out of operation period: is marked by the number of hours and minutes when the product was out of operation, since the appearance of the malfunction until the new operation;
   - Cost of repairs: the cost of materials and man hours;
   - Observations: technical revision or the category of repairs (current or capital) or recommendations for the improvement of operation in order to avoid the appearance of similar malfunctions will be specified;
   - Comprises the signature of the person filling in the chart.

For un-repairable spare parts, the chart may comprise the following:

a. General data:
   - The name of the place the product is monitored, The name of the product, The identification code of the product, The name of the supplier, The name or code of the contract.

b. Data comprising information concerning malfunctions:
   - Operation/installation date; Quantity, number of benchmarks put in operation; Measuring unit; Dismantling date; Number of operation hours until the malfunction;
   - Installation / dismantle confirmation document (minutes of meeting, observation, etc.); Date of the confirmation document; Reason for the replacement; Observations;
   - Comprises the signature of the person filling in the chart.

E. Data centralization

This operation consists in the acquisition of all data following to be processed. The operation is required when the steps B and C are realized distributive, i.e. in more experimental and observation places. The data centralization degree influences the compatibility of information to the model and the real situation.

F. Ordering

In order to obtain element referring to the statistic significance, experimental data are rearranged in an ascending manner. The result of such an operation is the acquisition of a statistic series.

G. Organization

The acquired statistic series obtained in step 3.6 is classified according to the following categories: (S1) – for which all the values are disjunctive values; (S2) – for which some of the values are repeated; (S3) – for which statistic data are grouped in intervals of disjunctive values. Statistic series are organized according to tables I, II and III.

Table I Data organization for series type (S1)

<table>
<thead>
<tr>
<th>No.</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>i₁</td>
</tr>
<tr>
<td>2</td>
<td>i₂</td>
</tr>
<tr>
<td>…</td>
<td>…</td>
</tr>
<tr>
<td>i</td>
<td>iᵢ</td>
</tr>
<tr>
<td>…</td>
<td>…</td>
</tr>
</tbody>
</table>
\[ t_1, t_2, \ldots, t_n \quad \forall i, k = 1, 2, \ldots, n; \quad i \neq k \]

\[ N = \text{sample volume} \quad \text{and the number of the values obtained by measurements.} \]

**Table II Data organization for series type (S2)**

<table>
<thead>
<tr>
<th>No.</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( t_1 )</td>
</tr>
<tr>
<td>2</td>
<td>( t_2 )</td>
</tr>
<tr>
<td>\ldots \ldots</td>
<td>\ldots</td>
</tr>
<tr>
<td>( i )</td>
<td>( t_i )</td>
</tr>
<tr>
<td>\ldots \ldots</td>
<td>\ldots</td>
</tr>
<tr>
<td>( t_v )</td>
<td>( t_v )</td>
</tr>
</tbody>
</table>

\[ t_1, t_2, \ldots, t_i, t_v \quad \forall i, k = 1, 2, \ldots, v; \quad i \neq k, v < N, \quad N = \text{sample volume.} \]

**Table III Data organization for series type (S3)**

<table>
<thead>
<tr>
<th>No.</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>([d_0, d_1])</td>
</tr>
<tr>
<td>2</td>
<td>([d_1, d_2])</td>
</tr>
<tr>
<td>\ldots \ldots</td>
<td>\ldots</td>
</tr>
<tr>
<td>( i )</td>
<td>([d_{i-1}, d_i])</td>
</tr>
<tr>
<td>\ldots \ldots</td>
<td>\ldots</td>
</tr>
<tr>
<td>( d_{m-1}, d_m )</td>
<td>( d_{m-1}, d_m )</td>
</tr>
</tbody>
</table>

**H. Frequencies and empirical distribution functions determination**

The Absolute frequency and, and the relative frequency, are determined. The empirical distribution function, \( \hat{F}(t) \), is given by the values of the relative frequencies smaller or equal to a value \( t \). Therefore, for the data organized in operation 3.5, it results:

a) for the series type (S1) the frequencies are not determined because these are equal to all the values which may appear,

\[ n_i = 1; \quad f_i = \frac{1}{N}; \quad \forall i = 1, 2, \ldots, N \]  \( \text{(1)} \)

The empirical distribution function is determined with the relations:

\[ \hat{F}(t) = \frac{i}{N+1}, \quad t_{i-1} \leq t < t_i, \quad i = 1, 2, \ldots, N, \]  \( \text{(2)} \)

Determined based on the progression;

\[ \text{for } N > 10 \]

\[ \hat{F}(t) = \frac{i - 0.3}{N + 0.4}, \quad t_{i-1} \leq t < t_i, \quad i = 1, 2, \ldots, N, \quad \text{(3)} \]

Results in considering the median;

b) For the statistic series type (S2), table IV is filled in for the determination of the frequencies.

**Table IV Frequencies for the statistic series type (S2)**

<table>
<thead>
<tr>
<th>No.</th>
<th>Values</th>
<th>Absolute frequency, ( n_i )</th>
<th>Relative frequency, ( f_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( t_1 )</td>
<td>( n_1 )</td>
<td>( f_i = n_i / N )</td>
</tr>
<tr>
<td>( \ldots \ldots \ldots \ldots )</td>
<td>( \ldots \ldots \ldots \ldots )</td>
<td>( \ldots \ldots \ldots \ldots )</td>
<td>( \ldots \ldots \ldots \ldots )</td>
</tr>
<tr>
<td>( i )</td>
<td>( t_i )</td>
<td>( n_i )</td>
<td>( f_i = n_i / N )</td>
</tr>
<tr>
<td>( \ldots \ldots \ldots \ldots )</td>
<td>( \ldots \ldots \ldots \ldots )</td>
<td>( \ldots \ldots \ldots \ldots )</td>
<td>( \ldots \ldots \ldots \ldots )</td>
</tr>
<tr>
<td>( v )</td>
<td>( t_v )</td>
<td>( n_v )</td>
<td>( f_v = n_v / N )</td>
</tr>
<tr>
<td>( \sum_{i=1}^{v} n_i = N )</td>
<td>( \sum_{i=1}^{v} t_i = 1 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The empirical distribution function is determined here with the following relation:

\[ \hat{F}(t) = \sum_{j=1}^{v} f_j, \quad \text{pentru } \quad t_{i-1} \leq t < t_i, \quad i = 1, 2, \ldots, v \]  \( \text{(4)} \)

c) For the statistic type (S3), the absolute frequency, \( n_i \), is determined for the value interval \([d_{i-1}, d_i]\) being given by the characteristic value number found in the specified interval. Conventionally, it is considered the values corresponding to the frequencies are given by:

\[ t_i = \frac{d_{i-1} + d_i}{2} \]  \( \text{(5)} \)

The empirical distribution function is determined with the relation:

\[ \hat{F}(t) = \sum_{j=1}^{i} f_j + \frac{t - d_{i-1}}{d_i - d_{i-1}} f_i, \quad \text{for } \quad d_{i-1} \leq t < d_i \]  \( \text{(6)} \)

where \( d_i - d_{i-1} \) represents the length of the interval, while \( i = 1, 2, \ldots, m \).

For \( d_0 < t \leq d_m \) (6) becomes:

\[ \hat{F}(t) = \sum_{j=1}^{i} f_j + \frac{f_i}{2}, \quad i = 1, 2, \ldots, m \]  \( \text{(7)} \)
The graphic representations of experimental data may show the type of the theoretic distribution followed by the random variable.

**I. Presenting the information obtained as appearance frequencies and as empirical distribution functions**

The graphic representations of experimental data may show the type of the theoretic distribution followed by the random variable.

**J. Determining the statistic indicator describing the behavior of the characteristic as random phenomenon**

The following will be determined:

- Average values showing the central tendency of the characteristic, respectively the simple and ponderous arithmetic progression;
- The median of the values series, \( M_e \);
- Module, \( M_o \);
- Dispersion, \( D \);
- Average square average deviation, \( s \);
- Variation coefficient \( CV \).

**K. Determining the theoretic distribution function corresponding to the empirical function established in at \( H \)**

Extrapolation is mainly used in order to determine the theoretic distribution function, which may determine the appearance of an error. It is therefore determined the corresponding theoretic model which will be the base while estimating the studied characteristics.

Passing on to the empirical model, obtained by the appearance or truncate the reality, to generalizing results is sustained by Glivenco-Cantelli theory, according to which in case of a big enough volume population, the event that the deviation of the empirical distribution function \( \hat{F}(t) \) from the theoretic function \( F(t) \) be zero represent the only event, i.e.

\[
P \left( \lim_{n \to \infty} \sup_{t \in \mathbb{R}} |\hat{F}(t) - F(t)| \right) = 1 \tag{8}
\]

where \( R \) represents the array of real numbers.

In order to adopt the theoretic model of the distribution there is the need to apply the accordance tests between the empirical and theoretic distributions.

**L. Determining the parameters of the theoretic functions and reliability indicators**

It is determined, based on adequate methods, parameters defining theoretic distributions which describe the studied phenomenon. Based on these parameters, the reliability indicators will be determined.

Speaking of a statistic analysis of random values, respectively the frequency of appearance of malfunctions during the operation of the machineries, choosing a distribution law was necessary.

In the case of machineries and installations with an increased degree of complexity, in order to determine the reliability of their components, the bi-parametric Weibull distribution law of the malfunction frequency, using the least square method.

This distribution model uses three parameters:

- The form parameter \( \beta \);
- The distribution parameter \( \lambda \);
- The scale parameter \( \eta \).

The form of the reliability function is:

\[
R(t) = e^{-\left(\frac{t}{\eta}\right)^\beta} \tag{9}
\]

In order to apply the least square method, the input data group and form a series type (S3), and in order to establish the length of an interval the Sturges relation is used,

\[
Ad = \frac{d_{\text{max}} - d_{\text{min}}}{1 + 3.322 \log N} \tag{10}
\]

where: \( d_{\text{min}} \) is the minimum value of the statistic series; \( d_{\text{max}} \) - is the maximum value of the statistic series.

**IV. DETERMINING THE EMPIRIC DISTRIBUTION PARAMETERS**

The study was made considering the teeth of bucket wheel excavators in Berbești, Roșia, Pesteana and Oltet quarries belonging to the National Company of brown coal Oltenia. The following simplifying hypotheses were considered:

- The teeth were installed on the same type of excavator in the same quarry;
- It has been considered that the teeth were kept under the same force independent on the nature of rock they sank into;
- The wear does not depend on the nature of the rock they operate in, considering the war due to the abrasive phenomenon is equivalent to the wear due to coal cutting phenomenon;
- The entire set is replaced at once.

### Table V Frequencies for the statistic series (S3)

<table>
<thead>
<tr>
<th>No</th>
<th>Value Interval</th>
<th>Conventional Value</th>
<th>Absolute Freq.</th>
<th>Relative Frequency, ( f_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>([d_1, d_2])</td>
<td>( t_1 = \frac{d_2 + d_1}{2} )</td>
<td>( n_1 )</td>
<td>( f_1 = \frac{n_1}{N} )</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>i</td>
<td>([d_{i-1}, d_i])</td>
<td>( t_i = \frac{d_i + d_{i-1}}{2} )</td>
<td>( n_i )</td>
<td>( f_i = \frac{n_i}{N} )</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>m</td>
<td>([d_{m-1}, d_m])</td>
<td>( t_m = \frac{d_m + d_{m-1}}{2} )</td>
<td>( n_m )</td>
<td>( f_m = \frac{n_m}{N} )</td>
</tr>
<tr>
<td></td>
<td>( \sum_{i=1}^{m} n_i = N )</td>
<td>( \sum_{i=1}^{m} f_i = 1 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The estimation of reliability parameters was made considering that until they fall off the set of teeth follow the Weibull distribution law, using the least square method.

The results of the analysis of the empiric distribution function and the necessary measures for the determination of the parameters of the biparametric Weibull distribution are presented in Table VI.

The distribution parameter formulas for the biparametric Weibull formula are the following:

\[ \beta = \frac{n \left( \sum_{i=1}^{n} \ln \frac{1}{R(t_i)} \right) - \left( \sum_{i=1}^{n} \ln t_i \right)^2}{\left( \sum_{i=1}^{n} \ln t_i \right)^2} \] (11)

The distribution parameter is:

\[ \hat{i} = \exp \left[ \frac{\left( \sum_{i=1}^{n} \ln \frac{1}{R(t_i)} \right) \left( \sum_{i=1}^{n} \ln t_i \right)^2 - \left( \sum_{i=1}^{n} \ln t_i \right)^3}{n \left( \sum_{i=1}^{n} \ln t_i \right)^2} \right] \] (12)

The estimated scale parameter is:

\[ \hat{\eta} = \frac{1}{\hat{i}^{\beta}} \] (13)

Introducing the values of the parameters \( \hat{\beta} = 1.513614904 \), \( \hat{i} = 0.000123 \), \( \hat{\eta} = 382.50 \) hours.

The necessary determination formulas for the application of the Kolmogorov-Smirnov test, for tooth bucket 1400M II, are as follows: consider a period of time \( t \) as the probability between the experimental function and the theoretic one. It is appreciated that for \( P(k) > 0.05 \) the accordance is corresponding.

Table VI Necessary measures for the application of Kolmogorov-Smirnov test, for tooth bucket 1400M II, Berbești Quarry

| i  | \( \bar{d}_i \) | \( F(\bar{d}_i) \) | \( \hat{F}(\bar{d}_i) \) | \( |F(\bar{d}_i) - \hat{F}(\bar{d}_i)| \) |
|----|-------------|----------------|----------------|----------------|
| 1  | 50          | 0.04493        | 0.050          | 0.005070       |
| 2  | 150         | 0.215311       | 0.200          | 0.015311       |
| 3  | 250         | 0.408651       | 0.400          | 0.008651       |
| 4  | 350         | 0.582821       | 0.550          | 0.032821       |
| 5  | 450         | 0.721654       | 0.683          | 0.038320       |
| 6  | 550         | 0.823209       | 0.750          | 0.073209       |
| 7  | 650         | 0.892611       | 0.967          | 0.074060       |

The value \( d = 0.07406 \) results from the analysis. Relation (15) gives the following result:

\[ k = 0.07406 \cdot \sqrt{30} = 0.4056 \] (16)

\( P(k) \) measure is determined from the verification tables of the Kolmogorov-Smirnov test, variant II:

\[ P(k) = 0.9972 >> 0.05 \] (17)

And therefore the proposed accordance between the empiric and theoretic function is true.

V. SPARE PARTS SUPPLY USING RELIABILITY THEORIES

A. Spare parts supply model

In its simplest form, the problem of spare parts stock may be expressed as follows: consider a period of time \( t = 0 \), the first spare element comes into operation; it breaks down after a period of time \( t = \tau_1 \) and it is replaced by a second spare element. This breaks down as well after a period of time \( t = \tau_1 + \tau_2 \) and is replaced by a third element and so on. The determination of the number \( n \) of spare parts to be reserved comes into question, such as maintenance needs during the time of operation \( T \) be expressed by the probability \( \gamma \), (e.g. \( \gamma = 0.9 \)).

It is supposed that \( \tau_1, \tau_2, ... \) are independent random values, having a distribution function \( F(t) \). Therefore, the smallest value of \( n \) needs to be determined, for which

\[ \Pr \left\{ \tau_1 + \tau_2 + ... + \tau_n > T \right\} = \gamma \] (18)

It may be shown that, if the number of spare parts from the
following relation is determined

\[ n = \frac{T}{MTBF}, \]  

(19)

Where MTBF is the mathematic expectancy of a flawless operation time of the spare element, maintenance needs will be covered for 50% of the cases. Thus it is necessary to reserve more spare parts than \( T / \text{MTBF} \), because maintenance needs should be covered by the required safety degree.

The probability of the left side of equation (18) may be expressed by the element \( (n-1) \) of the distribution function \( F(x) \). The solution may be analytically given only for the normal and exponential gamma distributions. For example, using the exponential distribution \( F(x) = 1 - \exp(-\lambda t) \) the Poisson distribution is obtained.

\[ \text{Pr}\{\tau_1 + \tau_2 + \ldots + \tau_n > T\} = \sum_{i=0}^{\infty} \frac{\lambda T}{i!} e^{-\lambda T} \]  

(20)

The Weibull biparametric distribution \( F(x) = 1 - \exp(-\lambda x^\gamma) \) is mostly used for current applications. If \( n \) is higher, then an analytical solution may be found,

\[ n = \frac{d}{2} + \sqrt{\left(\frac{d}{2}\right)^2 + \left(\frac{T}{MTBF}\right)^2} \]  

(21)

The value of \( d \) may be taken from Table VII depending on the variation coefficient \( \nu \) of the Weibull distribution.

<table>
<thead>
<tr>
<th>( \gamma )</th>
<th>0.99</th>
<th>0.95</th>
<th>0.75</th>
<th>0.5</th>
<th>0.25</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \nu )</td>
<td>2.326</td>
<td>1.645</td>
<td>1.282</td>
<td>0.674</td>
<td>0.674</td>
</tr>
</tbody>
</table>

B. Centralised spare parts supply

All the spare parts are found in one storage area therefore if reference is made to a certain spare part, the purpose of the supply may be mentioned following an algorithm similar to the previous one.

Therefore, for the moment \( t = 0 \) the first spare part begins its operation; it breaks down at \( t = \tau_1 \) and it is replaced by a second spare part; it breaks down as well at \( t = \tau_1 + \tau_2 \) and is replaced by a third spare part and so on. The determination of the number \( n \) of spare parts to be reserved comes into question, such as maintenance needs during a total operation time \( T \), with a given probability \( \gamma = 0.9 \).

The smallest value of \( n \) needs to be determined for which the inequality is satisfied:

\[ \text{Pr}\{\tau_1 + \tau_2 + \ldots + \tau_n > T\} \geq \gamma \]  

(22)

It is usually supposed that \( \tau_i \) has random, independent and positive values, having the same distribution function \( F(t) \) and the same probability density \( f(t) \). Its average value is marked down with \( M[\tau] = M[\tau] = \text{MTBF} \).

The determination of the number \( n \) of spare parts uses (19). Still, this only covers the needs with a 50% probability. It should, therefore, be determined more than \( T / \text{MTBF} \) spare parts, in order to obtain a higher probability rate than 0.5.

In (22), the probability of the left member may be expressed with the following relation

\[ \text{Pr}\{\tau_1 + \tau_2 + \ldots + \tau_n > T\} = 1 - F_n(T) \]  

(23)

where:

\[ F_n(T) = F(T) \]  

and \( F_n(T) = \int_0^T f(x) dx \)  

(24)

Considering an exponential distribution, the most frequently met case in reliability practice,

\[ \text{Pr}\{\tau_1 + \tau_2 + \ldots + \tau_n > T\} = \frac{\lambda T}{n!} e^{-\lambda T} \]  

(25)

The application needs to be numerically solved for the Weibull distribution. Therefore, for high values of \( n \) it may be written:

\[ \lim_{n \to \infty} \text{Pr}\left\{ \frac{1}{n} \sum_{i=1}^n (\tau_i - M[\tau]) > x \right\} = \frac{1}{\sqrt{2\pi}} \int e^{-x^2} dy \]  

(26)

In this relation \( \text{var}[\tau] = D \), represents the variance or random variable dispersion for the considered distribution.

Applying the transformation \( \sqrt{n \text{var}[\tau]} + n M[\tau] = T \) the following is obtained

\[ \text{Pr}\left\{ \frac{1}{n} \sum_{i=1}^n (\tau_i - M[\tau]) > x \right\} = \frac{1}{\sqrt{2\pi}} \int e^{-\frac{x^2}{2}} dy \]  

(27)

For the given values \( \gamma \), \( T \), \( M[\tau] \) and \( \text{var}[\tau] \), \( n \) is determined using the tables regarding the standard normal distribution.

Thus, with \( (T - n M[\tau]) / \sqrt{n \text{var}[\tau]} = -d \) it may be written,

\[ n = \frac{d}{2} + \sqrt{\left(\frac{d}{2}\right)^2 + \left(\frac{T}{M[\tau]}\right)^2} \]  

(28)
where \( \nu \) represents the variation coefficient of the distribution, determined with following relation

\[
\nu = \frac{\sqrt{\text{var}[r]}}{M[r]} = \frac{\sqrt{D}}{M[r]} = \frac{\sigma}{M[r]}
\]  

(30)

where \( \sigma \) represents the average standard deviation of random variables.

For common values of the confidence level, \( \gamma \), the value of \( d \) is presented in table VIII bellow.

<table>
<thead>
<tr>
<th>( \gamma )</th>
<th>0.99</th>
<th>0.95</th>
<th>0.9</th>
<th>0.75</th>
<th>0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( d )</td>
<td>2.33</td>
<td>1.64</td>
<td>1.28</td>
<td>0.67</td>
<td>0</td>
</tr>
</tbody>
</table>

Table VIII The values of \( d \) for each confidence level \( \gamma \)

Relations (27), (28), (29) are valid for any distribution function \( F(t) \).

Considering the Weibull biparametric distribution, the average or mathematic expectancy of the random variable is expressed using the following relation

\[
M[r] = \eta \Gamma\left(1 + \frac{1}{\beta}\right)
\]  

(31)

where:

- \( \eta \) represents the real scale parameter of the distribution;
- \( \beta \) – is the form parameter of the distribution;
- \( \Gamma(p) = \int_0^{\infty} x^{p-1} e^{-x} \, dx \), width \( p > 0 \), represents the Gamma function or Euler integral of the second type.

Using the same parameters, the dispersion and respectively the standard average deviation of the distribution are determined with the following relation

\[
D = \sigma^2 = \eta^2 \left\{ \Gamma\left(1 + \frac{2}{\beta}\right) - \left[ \Gamma\left(1 + \frac{1}{\beta}\right) \right]^2 \right\}
\]  

(32)

Therefore the variation coefficient \( \nu \) may be expressed using the following relation

\[
\nu = \frac{\sqrt{D}}{M[r]} = \frac{\sqrt{\Gamma\left(1 + \frac{2}{\beta}\right) - \left[ \Gamma\left(1 + \frac{1}{\beta}\right) \right]^2}}{\Gamma\left(1 + \frac{1}{\beta} + 1\right)}
\]  

(33)

And it depends only on the form parameter \( \beta \). Considering the centralised logistic support, when all the spare parts are found in a single warehouse, the number of spare parts is determined using the following relation

\[
n = \frac{d \nu}{2} + \sqrt{\left(\frac{d \nu}{2}\right)^2 + \left(\frac{T}{M[r]} \right)^2}, \text{ piece,}
\]

where moreover, \( N \) represents the number beneficiaries supplied from the same warehouse.

It has been considered, for the above mentioned cases, that the parts cannot be repaired. The case of repairable spare parts is covered using the aleatory process theory.

One of the frequently met models is the first repairable passive redundancy of \( n \) or \( k \) of \( n \), considering that the spare part is used for the \( k \) time in the respective unit. Considering as well that the supply of the parts is made through a centralised manner using the supply base, determining the supply needs according to the models of the reliability theory leads to the decrease of the number of supplied spare parts and therefore the decrease of acquisition expenses.

C. Decentralised spare parts supply

For beneficiaries using a larger number of identical or resembling in operation, the decentralised supply of spare parts is often organised. Such a solution is justified if we consider its reaction time, safety and simplicity. The main disadvantage is that globally there is the need of a larger number of spare parts.

This explains why an optimal distribution is intended between the centralised logistic support on one hand and the decentralised one on the other hand. Depending on the application, the optimisation scope function may be different.

For example:

a) for a given capital, the number of local spare parts supply warehouses to be determined such as the average or total time of inactivity in the interval \( (0,1] \) be minimum;

b) for a given value of average inactivity time of for a given value of the total inactivity time in the interval \( (0,1] \) of the system, the number of local supply warehouses should be determined such as the necessary capital be minimum.

D. Results obtained during the determination of spare parts stock dimensions based on their reliability

Using the values of the parameters specific to the Weibull biparametric distribution determined in chapter IV based on the theoretic considerations presented in chapters V/A and V/B the need of spare parts has been determined for an individual supply for each unit respectively by the use of a centralised warehouse for several units.

It is mentioned that for the unit quantifying the need of spare parts is the complete set of teeth installed completely on the bucket of an excavator.

The determination was made for each type of tooth, considering several quarries when the parameters of the distribution have close average values or for each quarry when the values of the parameters for each tooth are sensitively different, resulting therefore in the following:

a) 1400 MII type teeth installed on the EsRe 1400 30/7 type excavators
The operation of such a tooth is characterised by the Weibull distribution having as form parameters $\beta=1.505$ and real scale ones $\eta=360.95$ hours.

Using relations (21) and (23), as well as their values, figure 1 represents the required sets of teeth for the insurance of a certain number of operation hours considering the use of 1400 MII type teeth.

The $n_1(T)$ curve in this figure represents the need of sets of teeth for the insurance of the number of operation hours if the supply is made individually for each excavator. The $n_2(T)$, respectively $n_3(T)$ curves represent the same thing considering the supply is made in a centralised manner from a central ware house, first of all for four excavators and second of all for ten excavators.

Finally, considering a centralised supply, translates to the decrease of the stock with 60, 72, and respectively 120 sets of teeth, resulting in a 25% and respectively 30% economy.

In order to ensure 400 hours of operation for the 1400 MII type teeth, according to figure 1, two sets of teeth are required, considering the individual supply of an excavator, 5.5 sets of teeth for four excavators and respectively 13 sets of teeth for ten excavators. It results that if for the first case only 2 sets of spare teeth are necessary for each excavator, then for the other cases, 1.375 and respectively 1.3 sets of teeth are necessary for each excavator, resulting as well in the decrease of the stock with 75, respectively 84 teeth, determined for a set formed of 120 teeth, resulting in an economy of 31.25% respectively.

b) 1400 var. 3GX type teeth installed on the EsRc 1400 30/7 type excavator

The operation of this type of tooth is characterised by the Weibull distribution with the form parameter $\beta=1.787$ and the real scale one $\eta=322.435$ hours.

Figure 2 represents the need of spare teeth for the insurance of a certain number of operation hours using the 1400 var. 3GX type teeth.

Sensitively close values result from the presented diagram, interpreted the same as for the 1400 MII type teeth, resulting as well from the determination of the reliability indicators, resulting in a relatively poor quality of the teeth manufactured by METABET Piteşti, leading to the need of increasing the spare parts stock.

Therefore, in order to ensure 400 hours of operation for the 1400 var. 3GX type teeth, according to figure 2, two sets of teeth are required for individual supply of a single excavator, six sets of teeth for four excavators and 14 sets of teeth for ten excavators.

Finally, considering a centralised supply, translates to the decrease of the stock with 60, 72, and respectively 120 sets of teeth, resulting in a 25% and respectively 30% economy.

c) 1300 var. A0M type teeth installed on the EsRc 1400 30/7 type excavator

Figure 3 presents the necessary sets of spare teeth for the insurance of a certain number of operation hours using the 1300 var. A0M type teeth installed on the EsRc 1400 30/7 type excavator.

d) 1300 var. A0M type teeth installed on the EsRc 470 type excavator

Figure 4 presents the required sets of spare teeth in order to ensure a certain number of operation hours using the 1300 var. A0M type teeth installed on the EsRc 470 type excavator. In order to ensure 400 hours of operation of the 1300 var. A0M de 400 teeth, according to Figure 6.4 there is the need of 1.4 sets of teeth for an individual supply of a single excavator, 3.7 sets of teeth for four excavators and respectively 8.4 sets of teeth for 10 excavators. Finally, considering a centralised supply translates in the decrease of the stock with 30 and respectively 36 sets of spare teeth, determined for a set of 64 teeth, resulting therefore in an economy of 33% respectively.
Based on the data obtained during operation, using specific methods in the theory of reliability, the empiric parameters of an appropriate distribution have been determined, and the correctitude of the distribution was validated through a reliability model.

The determination of the reliability of products and applying it within the supply of results is laborious and difficult if there is a lack of observation in their operation and of an informational system for data processing.

The reliability forecast also allows the evaluation of maintenance tasks in case of repairs and the number of spare parts which should be provisioned in order to facing random defects of a machine park. On the other hand, during the same previsions, the same components recording progressive wear and need to be systematically replaced the statement of a maintenance schedule.

The reliability studies bring an important support for the maintenance services in all the problems related to the determination of the maintenance politics. The scope of this optimization may be either the minimization of maintenance costs (personnel and spare parts stock), or the maximization of the availability of the products, for given costs.

Based on the data obtained during operation, using specific methods in the theory of reliability, the empiric parameters of the bivariate Weibull distribution have been determined, and characterizes the operational behavior of analyses teeth and the correetude of the distribution was validated through accordance for the considered cases.

VI. CONCLUSIONS

- The determination of the reliability of products and applying it within the supply of results is laborious and difficult if there is a lack of observation in their operation and of an informational system for data processing.
- The reliability forecast also allows the evaluation of maintenance tasks in case of repairs and the number of spare parts which should be provisioned in order to faced random defects of a machine park. On the other hand, during the same previsions, the same components recording progressive wear and need to be systematically replaced the statement of a maintenance schedule.
- The reliability studies bring an important support for the maintenance services in all the problems related to the determination of the maintenance politics. The scope of this optimization may be either the minimization of maintenance costs (personnel and spare parts stock), or the maximization of the availability of the products, for given costs.
- Based on the data obtained during operation, using specific methods in the theory of reliability, the empiric parameters of an appropriate distribution have been determined, and the correctitude of the distribution was validated through accordance for the considered cases.

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