Selection and Classification of Public Capital Projects using IF-Sets

Petr Hájek, and Vladimír Olej

Abstract—The paper presents basic notions of intuitionistic fuzzy sets (IF-sets) introduced by K.T. Atanassov. Further, we define a set of criteria for the selection and classification of public capital projects. The selection and classification process is realized by two approaches. Sanchez’s approach is based on the max-min-max composition of IF-relations, while Li’s approach consists in the optimization of IF-relations. The results show that IF-sets provide a good description of public capital projects by means of membership functions and non-membership functions since they enable processing of a great deal of uncertainty.

Keywords—IF-sets, IF-relations, max-min-max composition, selection, classification, optimization, public capital projects.

I. INTRODUCTION

The selection (and classification) of public capital projects (further only public projects) is considered a problem of multi-attribute decision-making which can be realized by various models. The selection (and classification) process of public projects resolves a great deal of uncertainty in the process of translating and mapping the information, especially in environmental and social domains. So far, fuzzy sets have been used for dealing this kind of uncertainty in many application areas. At this time, there are several generalizations of fuzzy set theory for various objectives. IF-sets theory represents one of the generalizations, the notion introduced by K.T. Atanassov [1]. IF-sets theory has been applied in different areas, for example optimization in an intuitionistic fuzzy environment [2], medical diagnosis [3], etc. The IF-sets are suitable for the selection (classification) of public projects as they provide a good description of object attributes by means of membership functions and non-membership functions. They also present a strong possibility to express uncertainty.

In public management, the integration of the investment decision to the organization’s strategic goals is critical to selecting (classifying) the successful capital projects [4]. Thus, an important element of the capital budgeting process is to adopt an investment approach in defining the rankings of the investment projects. Cost-benefit analysis is traditionally applied in the decision-making processes in public management. The major issue is that costs are easy to express while it is difficult to define benefits of capital projects in the public sector [4]. Therefore, multi-attribute decision-making methods have been used in public projects selection recently. Analytic hierarchy process (AHP) was used by [4] for capital projects selection in US Army. Further, in [5] AHP was used together with mixed integer programming in order to realize project selection in water and sewerage management. Social, political, and economic criteria were included.

The mixed integer programming was applied also by [6] where three types of interdependencies are involved: technical, resource, and benefit interdependencies. A multi-objective mixed integer linear programming was used by [7] for optimal project selection and scheduling that is especially geared toward public sector companies.

A multi-objective evaluation model was designed by [8] to support strategic urban planning. The three key objectives are optimized: the expected interest of actors for projects; the relevance of the projects with respect to the objectives of the strategic plan; and the resources required for implementing projects. A similar model is presented in [9] where scatter search approach is used for project portfolio selection.

Intuitionistic fuzzy AHP was designed by [10] aiming at the selection of environmental projects. This model makes it possible to handle both vagueness and ambiguity related uncertainties in the environmental decision-making process.

The paper presents the attributes design for the selection and classification of public projects. Next, the paper introduces basic notions of IF-sets, IF-relations, Sanchez’s approach [11], and an optimization method of IF-relations introduced by [12] (Li’s approach) for the selection and classification of public projects.

II. ATTRIBUTES DESIGN FOR PUBLIC PROJECTS SELECTION AND CLASSIFICATION

So far the financial criteria are stressed in the selection of public projects [13]. However, it is emphasized that public organizations should aim at the maximization of the realized outcomes. For example, Chang and Tuckman [14] point out the social efficiency. The main idea behind this concept is that the annual accounts cover both financial and non-financial information. Similarly, Chan [4] proposes the following attributes for public projects selection: health and safety issues, cash savings/payoff, assets maintenance, growth-related needs, and service enhancement.

Our design of the attributes for public projects selection and
classification is based on the principles of sustainable city and urban quality of life [15]. A sustainable city should be equipped with the following functions [16]:

- education system for gaining knowledge;
- equal opportunities;
- participation of citizens in decision-making;
- opportunities for economic development;
- ability to identify the needs of individual interest groups;
- responsibility for the environment;
- safety;
- sense of solidarity, etc.

Similarly, urban quality of life comprises the attributes presented in [15]. Usually, sustainable development indicators are used to substitute these attributes. However, there are also problems which could appear while choosing and using the sustainable development indicators. One major difficulty lies in the subjectivity of the selection of the representative sustainable development indicators, and the evaluation of the results [17]. The selection of the sustainable development indicators is realized by experts with limited knowledge which is referred-to as dependence on a false model [18]. Other problems include lack of appropriate data which may result in missing vital information, and over aggregation of too many things resulting in unclear meaning, and therefore bad communication and analysis capability [18].

Therefore, we will use the attributes as they are presented in Table 1. Thus, the objective of decision-makers is to evaluate how the concrete public project contributes to the improvement of these attributes and, moreover, how important these attributes are for the particular city. In this evaluation process one has to deal with a great portion of uncertainty. We will use IF-sets for the selection and classification of public projects since they represent a strong possibility to express uncertainty in the decision-making process.

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### III. IF-SETS FOR DECISION-MAKING

The concept of IF-sets [1], [19], [20], [21], [22] is the generalization of the concept of fuzzy sets, the notion introduced by L.A. Zadeh [23]. The theory of IF-sets is well suited to deal with vagueness. Recently, the IF-sets have been used to classification models which can accommodate imprecise information [24]. E. Sanchez [11] adopted Zadeh’s max-min composition rule as an inference mechanism for IF-sets.

#### A. Basic Notions of IF-Sets

Let a set X be a non-empty fixed set. An IF-set A in X is an object having the form [1]

$$ A = \{ (x, \mu_A(x), \nu_A(x)) \mid x \in X \}, $$

where the function $\mu_A: X \to [0,1]$ defines the degree of membership function and the function $\nu_A: X \to [0,1]$ defines the degree of non-membership function, respectively, of the element $x \in X$ to the set A, which is a subset of X, and $A \subseteq X$, respectively; moreover for every $x \in X$, $0 \leq \mu_A(x) + \nu_A(x) \leq 1$, $\forall x \in X$ must hold.

The amount $\pi_\Delta(x) = 1 - (\mu_A(x) + \nu_A(x))$ is called the hesitation part, which may cater to either membership value or non-membership value, or both. For each IF-set in X, we will call $\pi_\Delta(x) = 1 - (\mu_A(x) + \nu_A(x))$ as the intuitionistic index of the element x in set A. It is a hesitancy degree of x to A. It is obvious that $0 \leq \pi_\Delta(x) \leq 1$ for each $x \in X$. The intuitionistic index $\pi_\Delta(x)$ is such that the larger $\pi_\Delta(x)$ the higher a hesitation margin of the decision maker.

If A and B are two IF-sets of the set X, then [1]

$$ A \cup B = \{ (x, \min(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x))) \mid x \in X \}, $$

$$ A \cap B = \{ (x, \max(\mu_A(x), \mu_B(x)), \min(\nu_A(x), \nu_B(x))) \mid x \in X \}, $$

$$ A \subseteq B \text{ iff } \forall x \in X, (\mu_A(x) \leq \mu_B(x)) \text{ and } (\nu_A(x) \geq \nu_B(x)), $$

$$ A = B \text{ iff } B \subseteq A, $$

$$ \bar{A} = \{ (x, \nu_A(x), \mu_A(x)) \mid x \in X \}. $$

Let X and Y be two sets. Then the IF-relation R from X to Y (will be denoted $R(X \to Y)$) is an IF-set of $(X \times Y)$ characterized by the membership function $\mu_R(x)$ and the non-membership function $\nu_R(x)$. If A is an IF-set of X, then the max-min composition [25] of the IF-relation $R(X \to Y)$ with A is an IF-set B of Y (denoted by $B = R \cdot A$) and is defined by the membership function [1]

$$ \mu_{R \cdot A}(y) = \bigvee_x [\mu_A(x) \land \mu_R(x,y)], $$

and the non-membership function

$$ \nu_{R \cdot A}(y) = \bigwedge_x [\nu_A(x) \lor \nu_R(x,y)], $$
∀y∈Y, where ∨=max, ∧=min.

Let Q(X→Y) and R(Y→Z) be two IF-relations. Then the max-min-max composition T=R⋅Q is the IF-relations from T(X→Z), defined by the membership function [1]

\[ µ_{R⋅Q}(x,z) = \bigvee_y [µ_Q(x,y) \land µ_R(y,z)] \]  

and the non-membership function

\[ ν_{R⋅Q}(x,z) = \bigwedge_y [ν_Q(x,y) \lor ν_R(y,z)] \]  

∀(x,z)∈(X×Y) and ∀y∈Y.

B. Sanchez’s Approach

Each project \( x_j \in X \) is assessed by attributes \( a_i \in A \), where \( A \) is an IF-set of attributes. Further, let \( R \) be an IF-relation, \( R(A→Ω) \). Then the max-min-max composition B of the IF-set A with the IF-relation \( R(A→Ω) \) denoted by \( B=A⋅R \) signifies the state of the project \( x_j \in X \) as an IF-set B of decision objective \( ω_k \in Ω \) with the membership function given in the following way

\[ µ_{B}(ω_k) = \bigvee_{a_i \in A} [µ_A(a_i) \land µ_B(a_i,ω_k)] \]  

and the non-membership function

\[ ν_{B}(ω_k) = \bigwedge_{a_i \in A} [ν_A(a_i) \lor ν_B(a_i,ω_k)] \]  

If the state of a given project \( x_j \in X \) is described in terms of the IF-set A of the attributes \( A=\{a_1,a_2,\ldots,a_m\} \), then the project \( x_j \in X \) is assumed to be assigned to decision objective \( ω_k \in Ω \) in terms of IF-set B of \( ω_k \) through an IF-relation of \( R(A→Ω) \).

Next, let be given \( n \) projects \( x_j \in X, j=1,2,\ldots,n \) and let \( R \) be an IF-relation \( R(A→Ω) \). Then an IF-relation Q can be constructed from the set of projects \( x_j \in X \) to the set of attributes \( A, Q(X→A) \). The composition \( T \) of IF-relations R and Q, \( T=R⋅Q \), describes the state of the project \( x_j \in X \) in terms of the decision objective \( ω_k \) as an IF-relation from \( X \) to \( Ω \), \( T(X→Ω) \) given by the membership function

\[ µ_T(x_j,ω_k) = \bigvee_{a_i \in A} [µ_Q(x_j,a_i) \land µ_R(a_i,ω_k)] \]  

and the non-membership function

\[ ν_T(x_j,ω_k) = \bigwedge_{a_i \in A} [ν_Q(x_j,a_i) \lor ν_R(a_i,ω_k)] \]  

∀(x_j,ω_k)∈(X×Ω) and ∀y∈Y.

The association index \( ψ_T \), which can be computed in this way

\[ ψ_T = µ_T(x_j,ω_k) = ν_T(x_j,ω_k) \times π_T(x_j,ω_k) \]  

assigns a single value of decision objective to projects \( x_j \). It emphasizes high values of the membership function \( µ_T(x_j,ω_k) \) (association) and reduces low values of the non-membership function \( ν_T(x_j,ω_k) \) (non-association).

C. Li’s Approach

Let there exists \( n \) decision-making alternatives (public projects) \( x_1,x_2,\ldots,x_n \) from which the most preferred one has to be selected. Each project is assessed by \( m \) attributes \( a_1,a_2,\ldots,a_m \). For the further considerations, let \( µ_{ij} (=µ_Q(x_i,a_i)) \) and \( ν_{ij} (=ν_Q(x_i,a_i)) \) be the membership and non-membership functions, respectively, of the project \( x_i \) with respect to the attribute \( a_i \), where \( 0 ≤ µ_{ij} + ν_{ij} ≤ 1 \) and \( π_{ij} = 1 - µ_{ij} - ν_{ij} \). Similarly, we can define the importance of the attribute \( a_i \) for the decision objective \( ω_k \). The final decision can be defined as a degree of acceptance of a project. Then \( µ_{ik} (=µ_B(a_i,ω_k)) \) and \( ν_{ik} (=ν_B(a_i,ω_k)) \) are the membership and non-membership functions, respectively, of the attribute \( a_i \) with respect to the decision objective \( ω_k \), where \( 0 ≤ µ_{ik} + ν_{ik} ≤ 1 \) and \( π_{ik} = 1 - µ_{ik} - ν_{ik} \).

The decision maker can change his attributes weights \( a_i \) during the process of selection. Concretely, he can increase \( a_i \) by adding the value of the intuitionistic index \( π_{ik} \). Then the importance (weight) of the attributes lies in the interval \( [µ_{ik},\mu^u_{ik}] = [µ_{ik}, µ_{ik} + π_{ik}], \) where \( µ_{ik} \) are \( µ^o_{ik} \) the lowest and the highest values of the membership function \( µ_{ik} \), respectively. For each attribute \( a_i \) it holds that \( 0 ≤ µ^i_{ik} ≤ µ^u_{ik} ≤ 1 \). Since the weights \( µ_{ik} \) of the attributes \( a_i \) can change we have to define both the objective criterion and the limitations under which this criterion is satisfied. The objective criterion \( z \) is defined in the following way

\[ \max z = \frac{\sum_{j=1}^{n} \sum_{i=1}^{m} π_j µ_{ik}}{n} \]  

subject to \( 0 ≤ µ^i_{ik} ≤ µ_{ik} ≤ µ^u_{ik} ≤ 1 \) and \( \sum_{i=1}^{m} µ_{ik} = b \),

where \( b \) is set by the decision-maker in the interval \( [\sum µ_{ik},\sum µ^o_{ik}] \). The optimal solution \( µ^*_{ik} = (µ^*_{ik}, µ^u_{ik}, \ldots, µ^u_{ik})^T \) can be obtained by linear programming technique. The optimal comprehensive value of the project \( x_j \) can be computed as an interval \( [z^o_{j}, z^o_{j}] \), where

\[ z^o_j = \sum_{i=1}^{n} µ^o_{ik} µ^o_{ik} = \sum_{i=1}^{m} µ^o_{ik} µ^u_{ik} \]  

for each \( j=1,2,\ldots,n \). The final ranking order of the projects is
based on the index $\xi_j$ defined as follows

$$\xi_j = \frac{D(A_i^j, B)}{D(A_i^j, B) + D(A_i^j, G)}, \quad (15)$$

where $0 \leq \xi_j \leq 1$, $A_i^j$ is an IFC-set corresponding to the optimal value $z_j^*_{ij}$ of the project $x_i$, $G$ is an IFC-set corresponding to the evaluation of an ideal project $g$ for which $\mu(g) = 1$ and $\nu(g) = 0$, $B$ is an IFC-set corresponding to the evaluation of the negative ideal project $b$ for which $\mu(b) = 0$ and $\nu(b) = 1$, and $D(A_i^j, B)$ and $D(A_i^j, G)$ are distance measures between IFC-sets.

We will use the normalized Hamming distance between two IFC-sets $A$ and $B$ which is defined as follows

$$D(A, B) = \frac{1}{2n} \sum_{j=1}^{n} \left( |\mu_A(x_j) - \mu_B(x_j)| + |\nu_A(x_j) - \nu_B(x_j)| \right) + |\pi_A(x_j) - \pi_B(x_j)|, \quad (16)$$

It is proved that this distance is a metric [26]. In a similar manner, other distances can be defined such as Hamming, Euclidean and normalized Euclidean distance.

If $\xi_j = 0$ then the project $x_i$ is the negative ideal alternative $b$, while for $\xi_j = 0$ it represents the ideal alternative $g$. As a result, the higher $\xi_j$ shows up a better project $x_i$. The equation (15) can be rewritten as follows

$$\xi_j = \frac{z_{ij}^*}{1 + z_{ij}^* - z_{ij}^*}, \quad (17)$$

Based on the previous considerations we define the best project $x_{rj}$ as the project for which $\xi_j = \max(\xi_j | x_j)$. 

IV. MODELLING AND ANALYSIS OF THE RESULTS

A. Selection of Projects

Let $x_1, x_2$, and $x_3$ be three possible combinations (portfolios) of public projects considering a given budget limitation. Within this budget limitation only selected public projects can be realized. The concrete selection of these projects is dependent both on the specific needs of citizens and on the political direction and related authorities of the public authority.

Further, let $a_1, a_2, \ldots, a_{10}$ be attributes defined in Table 1. Domain experts have been asked to assign the membership $\mu_{ij}$ and non-membership $\nu_{ij}$ functions of the portfolios of projects $x_1, x_2$, and $x_3$ to each of the attributes $a_1, a_2, \ldots, a_{10}$. The decision-making authority (city council) has been asked to assign weights (membership $\mu_k$ and non-membership $\nu_k$ functions) to attributes $a_1, a_2, \ldots, a_{10}$. A great deal of uncertainty is associated with both of the steps. The less certain the domain experts (decision-makers) in their assessments the higher the intuitionistic index $\pi_j$. As a result the membership $\mu_j$ and non-membership $\nu_j$ functions are defined in Appendix A. Similarly, the membership $\mu_k$ and non-membership $\nu_k$ functions are proposed in Appendix B.

First, we use the Sanchez’s approach introduced above. This approach consists of the following steps:

$$\mu_{ij}(x_1, x_2, x_3) = \bigvee \{\mu_{ij}(x_1, x_2) \land \mu_{ij}(x_1, x_3)\} = \sum_{a_1 \in A} \min\{\mu_{ij}^{a_1}(x_1, x_2), \mu_{ij}^{a_1}(x_1, x_3)\},$$

$$\nu_{ij}(x_1, x_2, x_3) = \bigwedge \{\nu_{ij}(x_1, x_2) \lor \nu_{ij}(x_1, x_3)\} = \sum_{a_1 \in A} \max\{\nu_{ij}^{a_1}(x_1, x_2), \nu_{ij}^{a_1}(x_1, x_3)\}.$$
Then, the index $\xi_2$ for the projects are as follows

$$\xi_1 = 2.1423, \xi_2 = 2.2035, \xi_3 = 2.0954.$$  

The best portfolio of projects is $x_2$. The objective of the project selection process lies in the finding of optimal ranking order of the alternatives. This ranking is different for the Sanchez’s ($x_1 > x_2 > x_3$) compared to the Li’s approach ($x_2 > x_1 > x_3$). The results of the Sanchez’s approach are affected by the highest degrees of membership $\mu_{ij}$ and $\mu_{ik}$ for the same attribute $a_i$. Thus, one important attribute $a_i$ has a dominant effect on the results. Moreover, the weights $\mu_{ik}$ of the attributes may not be altered during the process of decision-making.

In the Li’s approach, the optimum values of the attributes’ weights $\mu_{ik}$ are altered based on the intuitionistic indices $\pi_{ij}$, $i=1,2,\ldots,m$, $j=1,2,\ldots,n$. The results of Li’s approach are affected by the degrees of membership $\mu_{ik}$ of all the attributes $a_i$, $i=1,2,\ldots,m$, which are weighted by the degrees of membership $\mu_{ij}$ for lower values $z^{(i)}$ and the degrees of non-membership $\nu_{ij}$ (for upper values $z^{(i)}$). Then, the degrees of membership $\mu_{ij}$ have main impact on the resulting criterion $\xi_{ij}$.

### B. Classification of Projects

Contrary to previous example, the objective of the classification of public projects is to assign a class $o_k$ to the given public projects, where $o_1$ stands for the projects supporting economy, $o_2$ for social projects, and $o_3$ for environmental projects. In practice, the classification of the public projects into the mentioned classes is ambiguous. Most of the public projects have an effect on all three pillars of sustainable development, i.e. economic, social, and environmental. Additionally, it is not easy to quantify the amount of this effect precisely. Therefore, IF-sets represent a suitable tool for the classification of the public projects.

Let $x_1$, $x_2$, and $x_3$ are three public projects. The objective is to provide an estimation of the projects’ effects on the three areas of sustainable development: $o_1$ (economic), $o_2$ (social), and $o_3$ (environmental).

Further, let attributes $a_{11},a_{12},\ldots,a_{10}$ defined in Table 1. Again, the domain experts have been asked to assign the membership $\mu_{ij}$ and non-membership $\nu_{ij}$ functions of the projects $x_1$, $x_2$, and $x_3$ to each of the attributes $a_{11},a_{12},\ldots,a_{10}$. Moreover, they assigned the membership $\mu_{ik}$ and non-membership $\nu_{ik}$ functions of the attributes $a_{11},a_{12},\ldots,a_{10}$ to the classes $o_1$, $o_2$, and $o_3$, i.e. how each attribute contributes to economic, social, and environmental area. Again, the intuitionistic index $\pi_{ij}$ ($\pi_{ik}$) represents the level of uncertainty in the evaluation performed by the domain experts.

As a result the membership $\mu_{ij}$ and non-membership $\nu_{ij}$ functions are defined in the same manner as in the selection process (Appendix A). Similarly, the membership $\mu_{ik}$ and non-membership $\nu_{ik}$ functions are proposed in Appendix D.

### Economic area

The Sanchez’s approach:

$$\mu_{T}(x_o,o_k) = \bigvee_{a_i \in A} [\mu_{T}(x_o,a_i) \land \mu_{T}(a_i,o_k)]$$

$$\begin{align*}
\mu_{T}(x_o,o_k) &= \max_{a_i \in A} \left\{ \mu_{T}(x_o,a_i) \land \mu_{T}(a_i,o_k) \right\} \\
&= \max \{ a_{11} = 0.35, a_{12} = 0.70, a_{13} = 0.15, a_{14} = 0.10, a_{15} = 0.25, a_{16} = 0.80, a_{17} = 0.35, a_{18} = 0.30, a_{19} = 0.60, a_{20} = 0.10 \}
\end{align*}$$

The composite matrix $T$ and the association index $\psi_{T}$ are as follows:

$$T = \{ (x_1, \mu_{T}(x_1), \nu_{T}(x_1)) | x_j \in X \} = \{ (x_1, 0.80, 0.10), (x_2, 0.75, 0.15), (x_3, 0.60, 0.15) \},$$

$$\psi_{T} = \{ (x_1, \psi_{T}(x_1)) | x_j \in X \} = \{ (x_1, 0.79), (x_2, 0.74), (x_3, 0.56) \}.$$

According to these results, the best project in economic area is $x_1$.

In the Li’s approach the intervals for the weights of attributes $[\mu_{ik}, \mu_{ik}']$ are defined as presented in Appendix E. Then, the objective criterion $z$ is defined in the following way

$$\max \{ z = (0.50\mu_{1k} + 0.80\mu_{2k} + 0.85\mu_{3k} + 0.70\mu_{4k} + 0.35\mu_{5k} + 0.30\mu_{6k} + 0.65\mu_{7k} + 0.40\mu_{8k} + 0.35\mu_{9k} + 0.50\mu_{10k}) / 3 \},$$

subject to $0 \leq \mu_{ik} \leq \mu_{ik}' \leq 1$ and $\sum_{i=1}^{m} \mu_{ik} = b = 5.0$,

where $b$ is set by the decision-maker in the interval $[\sum \mu_{1k}, \sum \mu_{1k}'] = [4.55, 5.50]$.

The optimal solution $\mu_{o} = (\mu_{o1}, \mu_{o2}, \ldots, \mu_{o10})^T$ is obtained by linear programming technique as follows

$$\mu_o = (0.95, 0.85, 0.20, 0.15, 0.30, 0.09, 0.55, 0.30, 0.60, 0.20)^T.$$

The optimal comprehensive value $z=2.5963$ can be expressed also for individual projects $x_j, j=1,2,3$, in the form of interval $[z_{ol}, z_{ol}']$ so that $[z_{ol} = 2.7800], [z_{ol}' = 3.6675], [z_{ol} = 2.9225], [z_{ol}' = 3.7913], [z_{ol} = 2.6750], [z_{ol}' = 3.5150]$. Then, the index $\xi$ for the projects are as follows

$$\xi_1 = 1.9430, \xi_2 = 2.0287, \xi_3 = 1.9103.$$

The best portfolio of projects is $x_2$. The objective of the project selection process lies in the finding of optimal ranking order of the alternatives. This ranking is different for the Sanchez’s ($x_1 > x_2 > x_3$) compared to the Li’s approach ($x_2 > x_1 > x_3$).
\( x_1 > x_2 \).

**Social area**

The Sanchez’s approach:

\[
\mu_T(x_0, a_0) = \bigvee_{a_1 \in A} [\mu_Q(x_0, a_1) \land \mu_R(a_0, a_0)] = \max_{a_1 \in A} \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}\}
\]

\[
x_1 = 0.35, 0.70, 0.50, 0.30, 0.25, 0.70, 0.35, 0.60, 0.80, 0.30, 0.80, 0.80
\]

\[
x_2 = 0.30, 0.70, 0.55, 0.25, 0.30, 0.70, 0.50, 0.75, 0.80, 0.60, 0.80, 0.80
\]

\[
x_3 = 0.25, 0.60, 0.60, 0.40, 0.25, 0.60, 0.75, 0.60, 0.70, 0.60, 0.75
\]

The optimal comprehensive value \( z = 4.3224 \) can be expressed also for individual projects \( x_j \), \( j = 1, 2, 3 \), in the form of interval \([z^{a_1}_{a_1}, z^{a_1}_{a_1}]\) so that \( z^{a_1}_{a_1} = 3.9250, z^{a_1}_{a_1} = 5.5325 \), \( z^{a_2}_{a_2} = 4.3950, z^{a_2}_{a_2} = 5.8324 \), and \( z^{a_3}_{a_3} = 4.2800, z^{a_3}_{a_3} = 5.7375 \). Then, the index \( \xi_j \) for the projects are as follows

\[
\xi_1 = 2.0494, \xi_2 = 2.3929, \xi_3 = 2.3347
\]

The best portfolio of projects is \( x_2 \). The objective of the project selection process lies in the finding of optimal ranking order of the alternatives. This ranking is again different for the Sanchez’s ((\( x_1 = x_2 > x_3 \)) compared to the Li’s approach (\( x_2 > x_3 > x_1 \)).

**Environmental area**

The Sanchez’s approach:

\[
\mu_T(x_0, a_0) = \bigvee_{a_1 \in A} [\mu_Q(x_0, a_1) \land \mu_R(a_0, a_0)] = \max_{a_1 \in A} \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}\}
\]

\[
x_1 = 0.35, 0.25, 0.30, 0.25, 0.05, 0.10, 0.10, 0.15, 0.30, 0.50, 0.50
\]

\[
x_2 = 0.30, 0.25, 0.55, 0.25, 0.30, 0.05, 0.10, 0.15, 0.60, 0.60, 0.60
\]

\[
x_3 = 0.25, 0.25, 0.60, 0.40, 0.25, 0.05, 0.10, 0.15, 0.60, 0.60, 0.60
\]

The optimal comprehensive value \( z = 3.1228 \) can be expressed also for individual projects \( x_j \), \( j = 1, 2, 3 \), in the form of interval \([z^{a_1}_{a_1}, z^{a_1}_{a_1}]\) so that \( z^{a_1}_{a_1} = 2.0475, z^{a_1}_{a_1} = 4.755, z^{a_1}_{a_1} = 5.5755 \), \( z^{a_2}_{a_2} = 2.3700, z^{a_2}_{a_2} = 3.4278 \), and \( z^{a_3}_{a_3} = 2.4525, z^{a_3}_{a_3} = 3.6275 \). Then, the index \( \xi_j \) for the projects are as follows
ξ_1 = 1.5542, ξ_2 = 1.6658, ξ_3 = 1.6678.

The best portfolio of projects in environmental area is x_3. The objective of the project selection process lies in the finding of optimal ranking order of the alternatives. This ranking is again different for the Sanchez’ s (x_2 = x_3 > x_1) compared to the Li’s approach (x_1 > x_2 > x_3).

According to the Sanchez’s approach, the portfolios of projects can be classified as follows:

x_1 is economic project with μ = 0.80 (and is not with ν = 0.10),

x_1 is social project with μ = 0.80 (and is not with ν = 0.10),

x_1 is environmental project with μ = 0.50 (and is not with ν = 0.20),

x_2 is economic project with μ = 0.75 (and is not with ν = 0.15),

x_2 is social project with μ = 0.80 (and is not with ν = 0.10),

x_2 is environmental project with μ = 0.60 (and is not with ν = 0.15),

x_3 is economic project with μ = 0.60 (and is not with ν = 0.15),

x_3 is social project with μ = 0.75 (and is not with ν = 0.10),

x_3 is environmental project with μ = 0.60 (and is not with ν = 0.15).

V. CONCLUSION

The IF-s sets theory has been applied in different areas, for example in classification and prediction [27], [28], [29], [30], [31], [32]. IF-s sets are, for example, also suitable for the selection of public projects since they provide a good description of projects’ attributes by means of membership functions and non-membership functions. They also present a strong possibility to express uncertainty.

Therefore, the decision support systems based on IF-s sets are designed in this paper since they allow processing uncertainty and the expert knowledge. Based on IF-s sets, the paper presents the selection (and classification) process of public projects by using the Sanchez’s and Li’s approach. The results show that the Li’s approach works more effective than the Sanchez’s approach since it provides stronger possibility to accommodate imprecise information during the decision-making process and, at the same time, all the attributes have impact on the decision objective. Domain experts assign attribute values to projects, while decision-making authority assigns weights to attributes in the selection process. Thus, both groups are involved in the selection of public projects. In the classification of public projects, both groups of weights are assigned by domain experts.

The introduction of association index ψ_T (and index ξ) makes it possible to point out the rankings of the projects for the final decision.

The experiments were carried out in Matlab 7.1 in MS Windows XP operation system.

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APPENDIXES

Appendix A: Membership functions $\mu_{ij}$ and non-membership functions $\nu_{ij}$ for both the selection and the classification process

$$
\begin{align*}
    x_1 & : (0.35,0.50) (0.70,0.05) (0.50,0.20) (0.30,0.65) (0.25,0.60) (0.80,0.10) (0.35,0.35) (0.60,0.25) (0.80,0.05) (0.30,0.55) \\
    x_2 & : (0.30,0.50) (0.70,0.05) (0.55,0.15) (0.25,0.50) (0.30,0.60) (0.75,0.15) (0.50,0.30) (0.75,0.10) (0.85,0.05) (0.60,0.25) \\
    x_3 & : (0.25,0.60) (0.60,0.10) (0.60,0.15) (0.40,0.20) (0.25,0.65) (0.60,0.30) (0.75,0.10) (0.60,0.30) (0.70,0.20) (0.60,0.20)
\end{align*}
$$

Appendix B: Membership functions $\mu_{ik}$ and non-membership functions $\nu_{ik}$ for the selection process

$$
\begin{align*}
    o_1 & : (0.60,0.20) (0.50,0.30) (0.40,0.25) (0.85,0.05) (0.40,0.40) (0.80,0.05) (0.35,0.30) (0.75,0.10) (0.70,0.10) (0.40,0.35) \\
    o_2 & : (0.50,0.30) (0.70,0.05) (0.40,0.75) (0.85,0.95) (0.40,0.60) (0.80,0.95) (0.35,0.70) (0.75,0.90) (0.70,0.90) (0.40,0.65)
\end{align*}
$$

Appendix C: Intervals $[\mu_{ik}, \mu_{ik}]$ of the weights of attributes for the selection process

$$
\begin{align*}
    o_1 & : (0.90,0.05) (0.75,0.15) (0.15,0.80) (0.10,0.85) (0.30,0.60) (0.90,0.05) (0.45,0.45) (0.30,0.55) (0.60,0.25) (0.10,0.75) \\
    o_2 & : (0.50,0.30) (0.75,0.10) (0.75,0.10) (0.60,0.25) (0.75,0.10) (0.70,0.20) (0.85,0.05) (0.85,0.05) (0.80,0.10) (0.60,0.30) \\
    o_3 & : (0.65,0.20) (0.25,0.60) (0.80,0.10) (0.90,0.00) (0.85,0.05) (0.05,0.90) (0.10,0.80) (0.10,0.85) (0.15,0.75) (0.90,0.00)
\end{align*}
$$

Appendix D: Membership function $\mu_{ik}$ and non-membership $\nu_{ik}$ for the classification process

$$
\begin{align*}
    o_1 & : (0.90,0.95) (0.75,0.85) (0.15,0.20) (0.10,0.15) (0.30,0.40) (0.90,0.95) (0.45,0.45) (0.30,0.45) (0.60,0.75) (0.10,0.25) \\
    o_2 & : (0.50,0.70) (0.75,0.90) (0.75,0.90) (0.60,0.75) (0.75,0.90) (0.70,0.80) (0.85,0.95) (0.85,0.95) (0.80,0.90) (0.60,0.70) \\
    o_3 & : (0.65,0.80) (0.25,0.40) (0.80,0.90) (0.90,1.00) (0.85,0.95) (0.05,0.10) (0.10,0.20) (0.10,0.15) (0.15,0.25) (0.90,1.00)
\end{align*}
$$

Appendix E: Intervals $[\mu_{ik}, \mu_{ik}]$ of the weights of attributes for the classification process

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