Robust Control of a Laboratory Circuit Thermal Plant

Libor Pekař and Petr Valenta

Abstract**—**At the Faculty of Applied Informatics, Tomas Bata University in Zlín, a laboratory circuit heating plant containing internal (state) delays was assembled. It provides unordinary step responses, which makes it difficult to be modeled, identified and controlled in general. This contribution aims controller design and its verification for the appliance by algebraic means in a robust sense. A simple negative control feedback is utilized. The ring of quasipolynomial meromorphic functions (R_{MS}), which has been recently revised and extended, is briefly presented and serves as a primary algebraic tool for controller design. A mathematical model of the plant derived in our previous works based on the anisochronic principle is introduced. A free (selectable) controller parameter is set such that requirements of robust stability and robust performance are satisfied. The final controller is verified by simulations in Matlab-Simulink. Because of the plant is controlled via a discrete-time program in a PC, a simple controller discretization based on delta models is proposed. A simple user-program interface has been programmed. Finally, the controller structure and setting is verified on a real process and compared with simulations. The obtained results show the applicability of the controller design methodology. In the future research, we intent to utilize other control system structures and/or to perform an optimization procedure for robust controller tuning.

Keywords**—**Algebraic control, Delta models, Discretization, Heating system, Robust performance, Robust stability, Time-delay system.

I. Introduction

MODELLING, identification and control of systems with delays remain a challenging task in system and control theory [1] – [5]. Time-delay systems perform unconventional time- and frequency-domain characteristics, which disqualifies the use of many traditional modeling tools due to their infinite-dimensional nature and, consequently, the obtained models can not be matched with most-used controller design methodologies and approaches.

Among many others, circuit heating and thermal plants and processes are typical representatives of systems with internal (state) delays mainly due to transmission latencies in pipelines [6] – [9].

The objective of this paper is to control a laboratory heating (thermal) plant assembled at the Faculty of Applied Informatics of Tomas Bata University in Zlín in order to test control algorithms for systems with dead time. The original description of the apparatus and its electronic circuits can be found in [10]. The laboratory appliance can be viewed as a small-scale model of a real-word system, e.g. cooling system in cars. Although the plant was originally intended to test control algorithms for input delays only, it contains internal delays as well as it has been shown in [9]. In the cited literature, a detailed mathematical model of the process via anisochronic modeling principle [11] was derived and presented.

Since algebraic control design for time-delay systems is one of the main authors' interests, the ring of quasipolynomial meromorphic functions (RMS) originally developed in [12] and revised and extended in [13] is used in this paper as a primary tool for controller structure derivation for the heating plant. Moreover, a meromorphic transfer function representation as a fraction of quasipolynomials, which is a natural result of the use of the Laplace transform to the process model, is not suitable in an endeavor to meet some control requirements, such as internal stability, reference tracking, controller properness etc. [14], [15]. Controller design in RMS employs the Bézout identity to obtain stable and proper controllers along with the Youla-Kučera parameterization for reference tracking and load disturbance rejection.

Generally, the obtained controller structure contains free (selectable) parameters which have to set suitably. There are many ways how to deal with this task, see e.g. [5], [16] – [18]. In this paper, the objective is to meet basic requirements on the control feedback robustness, namely, robust stability and robust performance.

A simple discretization concludes the controller design procedure. It is based on the well-known delta models [19] and it provides easy-to-handle-with calculation of a discrete-time version of the controller, in contrast to some other more sophisticated methods operating in the state-space domain, se e.g. [20] – [21] which are rather usable for spectrum calculation [22] then for acquiring of an easy-handling control law.

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The paper is organized as follows. The definition of the R_{MS} ring and a concise overview of the algebraic controller design procedure in the ring are presented in Section II. A brief description and a model of the laboratory appliance are introduced in Section III. In Section IV, the controller structure designed using R_{MS} for the plant is derived. The selection of a suitable controller parameter together with robustness tests is the matter of Section V. Section VI provides controller discretization. Finally, simulation and real-experimental results are given in Section VII.

II. R_{MS} Ring – Definition and Controller Design

The definition of the R_{MS} ring and a controller design procedure for the simple feedback control structure using the ring are objectives of this section.

A. R_{MS} *Ring Definition*

Originally, the ring was defined for retarded time-delay systems only [12]; however the original definition of R_{MS} has some drawbacks (e.g. it does not constitute a ring, hence, it requires inclusion of neutral quasipolynomials etc.). Therefore the following revisited definition has been proposed in [13].

Definition 1 (R_{MS} ring). An element $T(s)$ of R_{MS} ring can be expressed as

$$T(s)=\frac{y(s)}{x(s)}\in H_{\infty}(\mathbb{C}^{+})$$
$$y(s)=\tilde{y}(s)\exp(-\tau s) \tag{1}$$

where $\deg\tilde{y}(s)=l$, $\deg x(s)=n$, $l\le n$ and $\tau\ge 0$. Neutral $T(s)$ must be formally stable. The difference between these two types of quasipolynomials can be found e.g. in [4], [15], and the definition of formal stability was presented e.g. in [23]. If $T(s)$ includes distributed delays, all zeros of $x(s)$ in $\mathbb{C}^{+}$ must be those of $y(s)$, i.e. it has removable singularities. ■

The properness can be alternatively given as follows [24]: A term $T(s)$ is proper if and only if there exists a real positive number R such that

$$\sup_{\mathrm{Re}\,s>0,|s|\ge R}|T(s)|<\infty \tag{2}$$

B. *Controller design in* R_{MS}

Consider the simple negative feedback loop depicted in Fig. 1. where $W(s)$ is the Laplace transform of the reference signal, $D(s)$ stands for that of the load disturbance, $E(s)$ is transformed control error, $U_0(s)$ expresses the controller output (control action), $U(s)$ represents the manipulated input affected by a load disturbance, and $Y(s)$ is the plant output controlled signal in the Laplace transform. All the presented signals are assumed to be ratios of elements from R_{MS}.

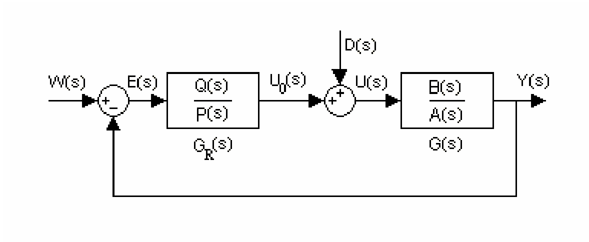

Fig. 1 Simple control negative feedback loop

External inputs, reference and load disturbance signals, respectively, have forms

$$W(s)=\frac{H_W(s)}{F_W(s)},\ D(s)=\frac{H_D(s)}{F_D(s)} \tag{3}$$

where $H_W(s),\ H_D(s),\ F_W(s),\ F_D(s)\in R_{MS}$.

The plant transfer function is depicted as

$$G(s)=\frac{B(s)}{A(s)} \tag{4}$$

where $A(s), B(s)\in R_{MS}$ are coprime, i.e. there does not exist a non-trivial (non-unit) common factor of both elements, see [13] for details.

The controller transfer function reads

$$G_R(s)=\frac{Q(s)}{P(s)}$$
$$P(s), Q(s)\in R_{MS} \tag{5}$$

An overview of basic steps in the controller design procedure is presented below (according to e.g. [25], [26]).

Feedback stabilization

Theorem 1 (Stabilization). Given a Bézout coprime pair $A(s), B(s)\in R_{MS}$ the closed-loop system is stable (in R_{MS} sense) if and only if there exists a coprime pair $P(s), Q(s)\in R_{MS}$ satisfying the Bézout identity

$$A(s)P(s)+B(s)Q(s)=1 \tag{6}$$

A particular stabilizing solution of (6), say $P_0(s), Q_0(s)$, can be further parameterized as

$$P(s)=P_0(s)\pm B(s)Z(s)\neq 0$$
$$Q(s)=Q_0(s)\mp A(s)Z(s) \tag{7}$$

where $Z(s)\in R_{MS}$. ■

A proof of Theorem 1 based on [12] is going to be found in [26]. Parameterization (7) is used to satisfy remaining control and performance requirements, such as reference tracking, disturbance rejection etc.

Remark 1. Any two elements $A(s), B(s) \in H_\infty(\mathbb{C}^+)$ form a Bézout (coprime) factorization if and only if

$$\inf_{\operatorname{Re} s \geq 0} \left(|A(s)| + |B(s)| \right) > 0 \tag{5}$$

see e.g. [23], [24]. Note that (5) does not hold for formally unstable systems. ∎

Reference tracking

The task is to find $Z(s) \in R_{MS}$ in (7) so that the reference signal is being tracked. The analysis on the scheme in Fig. 1 yields the following requirement.

Theorem 2 (Reference tracking). The reference signal $w(t) = L^{-1}\{W(s)\}$ is tracked if and only if $F_W(s)$ divides the product $A(s)P(s)$ in R_{MS}. ∎

A detailed analysis of Theorem 2 and a suggestion how to select the structure of $Z(s)$ can be found in [25], [26].

Load disturbance rejection

Analogously to the previous subsection, one can derive the following statement.

Theorem 3 (Load disturbance rejection). The load disturbance $d(t) = L^{-1}\{D(s)\}$ is asymptotically rejected if and only if $F_D(s)$ divides $B(s)P(s)$ in R_{MS}. ∎

Again, see details in [25], [26].

III. Circuit Thermal Plant Description and Model

Prior to controller design for the laboratory plant, its description and a proposed anisochronic model ought to be introduced.

The original appliance description was presented in [10] and a mathematical model was proposed in [9], the reader is hence referred therein for details.

A. Plant Description

A photo and a sketch of the appliance are depicted in Fig. 2.

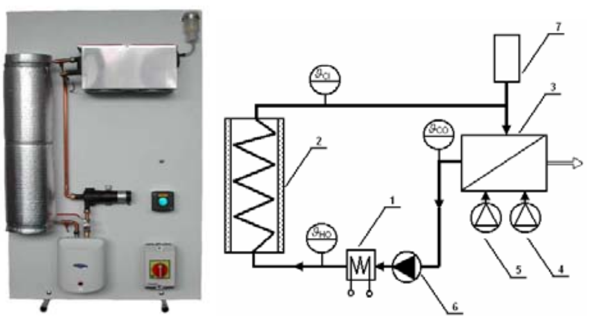

Fig. 2 Circuit heating plant

The heat transferring fluid (namely distilled water) is transported using a continuously controllable DC pump {6} into a flow heater {1} with maximum power $P_H(t)$ of 750 W. The temperature of a fluid at the heater output is measured by a platinum thermometer giving value of $\vartheta_{HO}(t)$. Warmed liquid then goes through a 15 meters long insulated coiled pipeline {2} which causes the significant delay in the system. The air-water heat exchanger (cooler) {3} with two cooling fans {4, 5} represents a heat-consuming appliance. The speed of the first fan can be continuously adjusted, whereas the second one is of on/off type. Input and output temperatures of the cooler are measured again by platinum thermometers giving $\vartheta_{CI}(t)$ and $\vartheta_{CO}(t)$, respectively. The expansion tank {7} compensates for the expansion effect of the water.

B. Mathematical (Anisochronic) Model

Since the modeling and identification of the laboratory plant was thoroughly derived and presented in [9], only basic ideas and necessary results are given here.

The methodology is based on the comprehension of all significant (lumped) delays and latencies in the model which is built in two steps: First, models of separate functional parts of the plant are found; second, the obtained sub-models are combined by means of their common physical quantities.

Models of the heater, coiled insulated pipelines and the cooler, respectively, are given by the well-known energy (heat) balance equations

$$\begin{aligned}
cM_H \frac{\mathrm{d}\vartheta_{HO}(t)}{\mathrm{d}t} &= P_H(t - 0.5\tau_H) + c\dot{m}(t)\left[\vartheta_{HI}(t-\tau_H) - \vartheta_{HO}(t)\right] \\
&\quad - K_H(t)\left[\frac{\vartheta_{HO}(t) + \vartheta_{HI}(t-\tau_H)}{2} - \vartheta_A\right] \\
cM_P \frac{\mathrm{d}\vartheta_{CI}(t)}{\mathrm{d}t} &= c\dot{m}(t)\left[\vartheta_{HO}(t-\tau_{HC}) - \vartheta_{CI}(t)\right] \\
&\quad - K_P\left[\frac{\vartheta_{CI}(t) + \vartheta_{HO}(t-\tau_{HC})}{2} - \vartheta_A\right] \\
cM_C \frac{\mathrm{d}\vartheta_{CO}(t)}{\mathrm{d}t} &= c\dot{m}(t)\left[\vartheta_{CI}(t-\tau_C) - \vartheta_{CO}(t)\right] \\
&\quad - K_C(t)\left[\frac{\vartheta_{CO}(t) + \vartheta_{CI}(t-\tau_C)}{2} - \vartheta_A\right]
\end{aligned} \tag{6}$$

with the following selected approximations and interpolations

$$\begin{aligned}
\vartheta_{HI}(t) &= \vartheta_{CO}(t - \tau_{CH}) \\
K_H(t) &= \frac{h_0 P_H^2(t) + h_1 \dot{m}^2(t) + h_2 P_H(t)\dot{m}(t) + h_3}{h_4 P_H(t) + h_5 \dot{m}(t)} \\
K_C(t) &= c_2 u_C^2(t - \tau_{KC}) + c_1 u_C(t - \tau_{KC}) + c_0 \\
\dot{m}(t) &= p_0\left[u_P(t) + p_1\right]^{p_2}
\end{aligned} \tag{7}$$

where the notation is the following

c [J kg^{-1} K^{-1}] – the specific water heat capacity
$\dot{m}(t)$ [kg s^{-1}] – the water mass flow rate
M_H [kg] – the overall water mass in the heater
M_C [kg] – the overall water mass in the cooler
M_P [kg] – the overall water mass in the pipeline

$\vartheta_{HI}(t)$ [°C] – heater input temperature
ϑ_A [°C] – ambient temperature
τ_H [s] – the delay of a water flow through the heater
τ_{HC} [s] – the delay of a water flow between the heater and the cooler
τ_C [s] – the delay of a water flow through the cooler
τ_{KC} [s] – the delay between a control signal to the cooling fan and the output temperature of the cooler
τ_{CH} [s] – the delay of a water flow between the cooler and the heater
$u_P(t)$ [V] – pump input voltage
$u_C(t)$ [V] – cooling fan input voltage
$K_H(t)$ [W K^{-1}] – the overall heater wastage energy heat transmission coefficient
$K_C(t)$ [W K^{-1}] – the overall cooler heat transmission coefficient
K_P [W K^{-1}] – the overall long pipeline heat transmission coefficient
h_0, h_1, h_2, h_3, h_4, h_5 – weighting coefficients for the estimation of the overall heat transmission coefficient of the heater
c_0 [W K^{-1}], c_1[W K^{-1} V^{-1}], c_2 – weighting coefficients for the estimation of the overall heat transmission coefficient of the cooler
p_0 [m^3 s^{-1}], p_1 [V], p_2 – weighting coefficients for the estimation of the mass flow rate of water

Although there are three continuous-time manipulated inputs, i.e. $P_H(t)$, $u_P(t)$, $u_C(t)$ and three measured outputs $\vartheta_{HO}(t), \vartheta_{CI}(t), \vartheta_{CO}(t)$, the intention is to control $\vartheta_{CO}(t)$ only by $P_H(t)$. For this relation, it was derived the following transfer function

$$G(s)=\frac{\vartheta_{CO}(s)}{P_H(s)}=\frac{[b_{0D}\exp(-\tau_0 s)+b_0]\exp(-\tau s)}{s^3+a_2 s^2+a_1 s+a_0+a_{0D}\exp(-\vartheta s)} \tag{8}$$

It was determined that for the operating point

$$\begin{aligned}&[u_p, u_c, P_H, \vartheta_{HO}, \vartheta_{CI}, \vartheta_{CO}, \vartheta_A]\\&=[5\,\text{V}, 3\,\text{V}, 300\,\text{W}, 44.1\,°\text{C}, 43.8\,°\text{C}, 36\,°\text{C}, 24\,°\text{C}]\end{aligned} \tag{10}$$

that the parameters in (8) are

$$\begin{aligned}&b_{0D}=2.334\cdot 10^{-6}, b_0=-2.146\cdot 10^{-7}, a_2=0.1767, a_1=0.009,\\&a_0=1.413\cdot 10^{-4}, a_{0D}=-7.624\cdot 10^{-5}, \tau_0=1.5, \tau=131, \vartheta=143\end{aligned} \tag{11}$$

IV. Controller Structure Design for the Plant

The task of this section is to design a controller structure using principles described in Section II. Hence, let the plant be described by the transfer function (8) and the external inputs be from the class of linearwise functions, i.e. (3) reads

$$W(s)=\frac{H_W(s)}{F_W(s)}=\frac{\frac{w_0}{m_w(s)}}{\frac{s}{m_w(s)}},\; D(s)=\frac{H_D(s)}{F_D(s)}=\frac{\frac{d_0}{m_d(s)}}{\frac{s}{m_d(s)}} \tag{12}$$

where $m_w(s)$ and $m_d(s)$ are arbitrary (quasi)polynomials of degree one with zeros in $\mathbb{C}^+$, say, $m_w(s)=m_d(s)=s+m_0$, $m_0>0$, for the simplicity, and w_0 and d_0 are real constants.

The plant transfer function can be factorized analogously as

$$G(s)=\frac{\frac{[b_{0D}\exp(-\tau_0 s)+b_0]\exp(-\tau s)}{m(s)}}{\frac{s^3+a_2 s^2+a_1 s+a_0+a_{0D}\exp(-\vartheta s)}{m(s)}}=\frac{B(s)}{A(s)} \tag{13}$$

where $m(s)$ is a stable (quasi)polynomial of degree three, for instance, $m(s)=(s+m_0)^3$, again for the simplicity.

The primary aim is to stabilize the control feedback loop using (6). If $Q_0(s)=1$, the following particular stabilizing solution is obtained

$$P_0(s)=\frac{(s+m_0)^3-[b_{0D}\exp(-\tau_0 s)+b_0]\exp(-\tau s)}{s^3+a_2 s^2+a_1 s+a_0+a_{0D}\exp(-\vartheta s)} \tag{14}$$

For reference tracking and disturbance rejection, both conditions from Theorem 2 and Theorem 3 must be satisfied simultaneously, i.e. $F_W(s)\,|\,(A(s)P(s)) \wedge F_D(s)\,|\,(B(s)P(s))$. Equivalently, $P(s)$ must include at least one zero root which can be expressed by the condition

$$P(0)=0 \tag{15}$$

Thus, try to choose the following structure

$$Z(s)=\frac{(s+m_0)^3}{s^3+a_2 s^2+a_1 s+a_0+a_{0D}\exp(-\vartheta s)}Z_0 \tag{16}$$

where $Z_0\in\mathbb{R}$, to obtain $P(s)$ in an arbitrarily simple form. Condition (15) results in

$$Z_0=\frac{m_0^3}{b_{0D}+b_0}-1 \tag{17}$$

Finally, the controller structure is given by inserting (14), (16) and (17) into (6) as

$$G_R(s) = \frac{m_0^3\left[s^3 + a_2 s^2 + a_1 s + a_0 + a_{0D}\exp(-\vartheta s)\right]}{(b_{0D}+b_0)(s+m_0^3) - m_0^3\left([b_{0D}\exp(-\tau_0 s) + b_0]\exp(-\tau s)\right)} \tag{18}$$

The controller contains only one selectable (free) parameter m_0 and it has anisochronic structure including internal delays (in its dynamics); however, it is simply realizable by integrators and delay elements, see the Matlab-Simulink scheme as in Fig. 3.

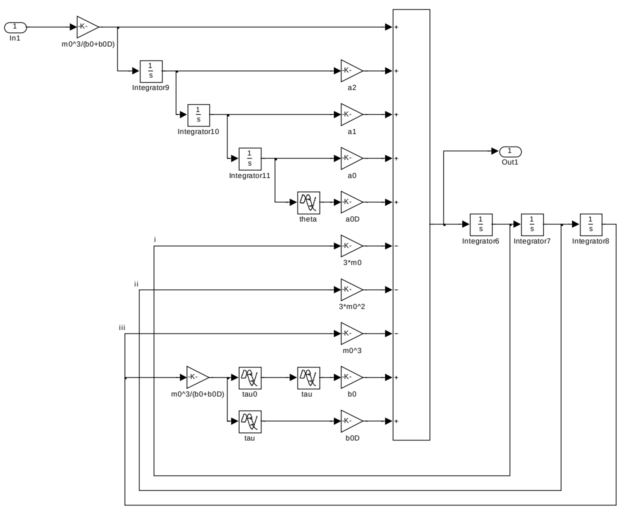

Fig. 3 Controller structure scheme

V. Robust Stability and Robust Performance Tests

Robust analysis constitutes a set of possible tools for controller quality and performance evaluation, particularly when an ideal plant mathematical model does not match the real system behavior ideally.

A. *Theory overview*

We pay attention to unstructured uncertainty, more precisely multiplicative disk uncertainty which enables to develop simple general analytics methods and results. Let $G_0(s)$ be the nominal plant transfer function and $G(s) = [1 + \Delta(s) W_M(s)] G_0(s)$ be a family of perturbed transfer functions. Here $W_M(s)$ is a fixed stable weight function expressing the uncertainty frequency distribution. Perturbation $\Delta(s)$ is a variable stable transfer function satisfying $\|\Delta(s)\|_\infty \le 1$. Moreover, $G(s)$ and $G_0(s)$ have the same number of unstable poles. It holds that

$$\left|\frac{G(\mathrm{j}\omega)}{G_0(\mathrm{j}\omega)} - 1\right| \le |W_M(\mathrm{j}\omega)|, \forall\omega \tag{19}$$

which means that $G(\mathrm{j}\omega)/G_0(\mathrm{j}\omega)$ lies in the disk with center 1 and radius $|W_M(\mathrm{j}\omega)|$. The weight function is selected so that it covers all systems from the family

$$\max_{G(s)}\left|\frac{G(\mathrm{j}\omega)}{G_0(\mathrm{j}\omega)} - 1\right| \le |W_M(\mathrm{j}\omega)|, \forall\omega \tag{20}$$

The closed-loop system is called robust stable if it is stable for the whole family of perturbed plant models. For multiplicative uncertainty, the feedback system as in Fig. 1 is robust stable if and only if

$$\|W_M(\mathrm{j}\omega) T_0(\mathrm{j}\omega)\|_\infty < 1 \tag{21}$$

where $T_0(s) = G_{WY}(s) = Y(s)/W(s)$ is the so-called (nominal) complementary sensitivity function, see e.g. [27].

The general notion of robust performance is that both, internal robust stability (21) and performance expressed by

$$\|W_P(\mathrm{j}\omega) S(\mathrm{j}\omega)\|_\infty < 1 \tag{22}$$

should hold for the whole family of perturbed plants where $S(s) = G_{WE}(s) = E(s)/W(s)$ stands for the sensitivity function and $|W_P(\mathrm{j}\omega)|$ is the sensitivity weighting function.

Conditions (21) and (22) results in the following summarized condition

$$\left\| |W_M(\mathrm{j}\omega) T_0(\mathrm{j}\omega)| + |W_P(\mathrm{j}\omega) S_0(\mathrm{j}\omega)| \right\|_\infty < 1 \tag{23}$$

where S_0 means the nominal sensitivity function.

B. *Controller Robust Analysis*

Now let us analyze the robustness of the designed controller (18) for various settings of m_0 by means of the previous subsection.

First, it is necessary to determine the family of plant transfer functions which is obtained by variations within the ranges of selected model parameters. We have selected three parameters the values of which are affected by measurements uncertainties or ambient conditions, namely, K_C, K_P and ϑ_A. Intervals for K_C and K_P have been chosen on the basis of two identification measurements [9], [28], ϑ_A has been selected according to room temperature variations during the year. Hence, the intervals are the following

$$K_P \in [0.1, 0.5], K_C \in [15, 22], \vartheta_A \in [16, 30]^\circ C \tag{24}$$

The set of Bode plots $|G(\mathrm{j}\omega)/G_0(\mathrm{j}\omega) - 1|$ for all eight combinations of margin values in (24) is depicted in Fig. 4. This set was covered by a plot expressing $|W_M(\mathrm{j}\omega)|$ given by the transfer function (25).

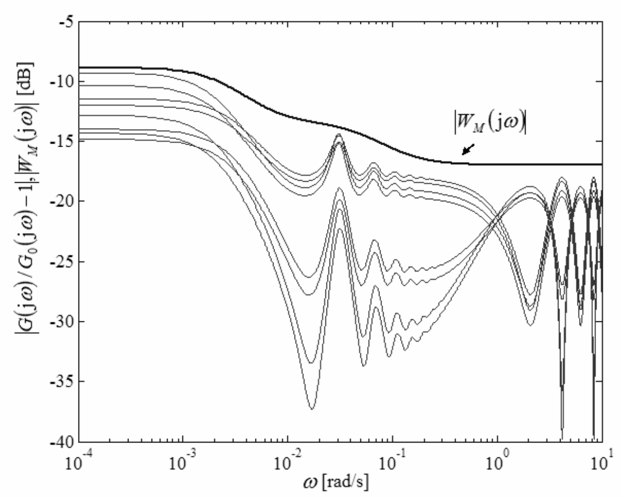

Fig. 4 Determination of $|W_M(j\omega)|$

$$W_M(s) = 0.36\frac{(200s+1)(10s+1)}{(340s+1)(15s+1)} = \frac{720s^2+75.6s+0.36}{5100s^2+355s+1} \quad (25)$$

Verification of the robust stability criterion for several settings of m_0 is displayed in Fig. 5.

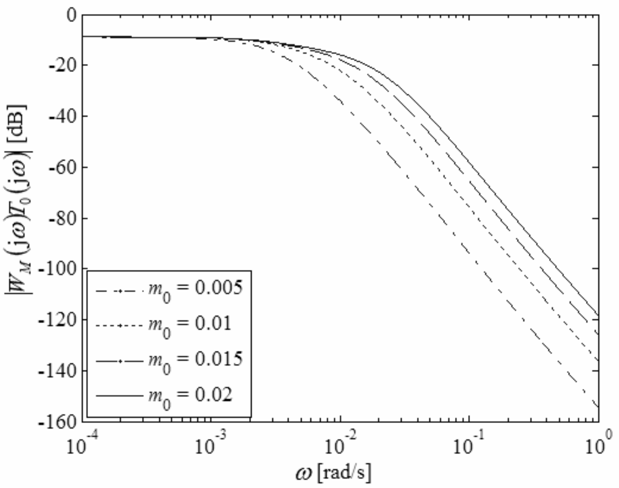

Fig. 5 Robust stability verification

The weighting function $|W_P(j\omega)|$ has been chosen so that the nominal performance condition

$$\|W_P(j\omega)S_0(j\omega)\|_\infty < 1 \quad (26)$$

holds for a selected range of m_0, as

$$1/W_P(s) = 900\frac{s(40s+1)}{(350s+1)(90s+1)} = \frac{36000s^2+900s}{31500s^2+440s+1} \quad (27)$$

see Fig. 6.

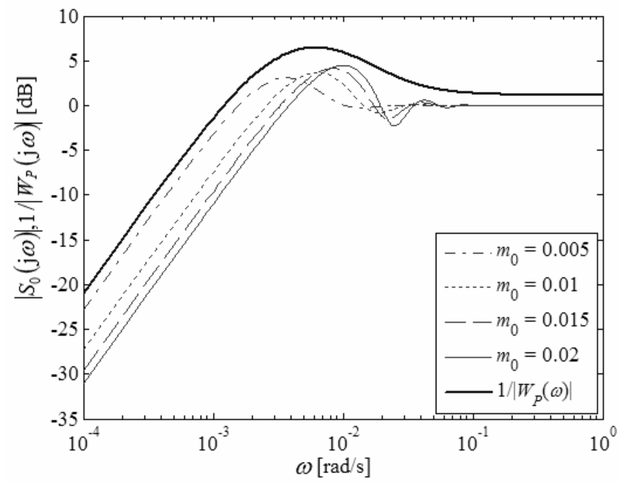

Fig. 6 Nominal performance – determination of $|W_P(j\omega)|$

Obviously, the decreasing of m_0 would lead to poor nominal performance at lower frequencies, whereas its increasing would cause the same effect at middle frequencies.

Finally, test the robust performance condition (23) with $W_M(s)$ and $W_P(s)$ given by (25) and (27), respectively, as it is depicted in Fig. 7.

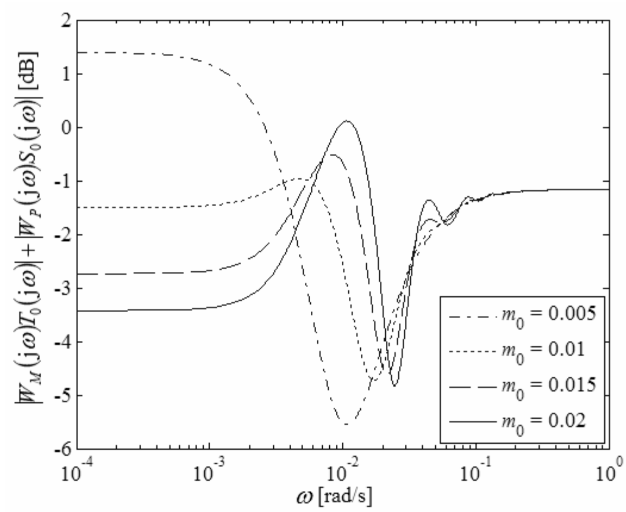

Fig. 7 Robust performance test

The results confirm the preceding idea since for $m_0 = 0.005$ and $m_0 = 0.02$ the feedback system has poor robust performance. Hence, we finally chose the range $m_0 \in [0.008, 0.012]$ for simulations and real experiments, see Section VII.

VI. Discrete-Time Controller

The computer connected with the laboratory appliance which serves for monitoring and control tasks utilizes CTRL-V3 unit working with discrete-time samples. Because of this it is inevitable to discretize the control algorithm. There were investigated a large number of discretization approaches, mainly for the spectrum estimation, e.g. [20], [21], [29]. A simple yet sufficient input-output method based on delta models [19] and linear delay interpolation [30] in this paper.

The idea rests on the introduction of variable γ associated with the delta operator δ defined as

$$\gamma = \frac{z-1}{\alpha T_s z + (1-\alpha) T_s} \tag{28}$$

where z is the variable from the z-transform, $\alpha \in [0,1]$ represents a weighting parameter and T_s means a sampling period. The choice of α enables to obtain different first order models, such as forward ($\alpha = 0$), backward ($\alpha = 1$) or Tustin ($\alpha = 0.5$) one. The substitution $s \to \gamma$ in the transfer function system model results in a discrete-time model in z associated with the shifting operator *q*. However, this substitution is applied to *s*-powers expressing derivatives only, whereas delay exponential terms are subjected to a natural transformation

$$\exp(-\eta s) X(s) \to x(t-\eta) \tag{29}$$

followed by (linear) interpolation

$$x(t-\eta_i) \approx (1-\alpha_i) x(t-\tau_{d,i}) + \alpha_i x(t-\tau_{d+1,i}) \tag{30}$$

where $d_i = \lfloor \eta_i / T_s \rfloor$, $\tau_{d,i} = d_i T_s$, $\tau_{d+1,i} = (d_i + 1) T_s$, $\tau_{d,i} \le \eta_i \le \tau_{d+1,i}$ and a weighting coefficient $\alpha_i = (\eta_i - \tau_{d,i}) / T_s \in [0,1]$. Then, finally

$$x(t-kT_s) \to z^{-k} X(z) \tag{31}$$

The Tustin (trapezoidal) method was utilized in this paper with $T_s = 1$. The final discrete rule for controller (18) reads

$$G_{RD}(z) = \frac{Q_D(z)}{P_D(z)} \tag{32}$$

where

$$\begin{aligned} Q_D(z) &= q_0 + q_1 z^{-1} + q_2 z^{-2} + q_3 z^{-3} + q_{143} z^{-143} + q_{144} z^{-144} \\ &\quad + q_{145} z^{-145} + q_{146} z^{-146} \\ P_D(z) &= p_0 + p_1 z^{-1} + p_2 z^{-2} + p_3 z^{-3} + p_{131} z^{-131} + p_{132} z^{-132} \\ &\quad + p_{133} z^{-133} + p_{134} z^{-134} + p_{135} z^{-135} + q_{136} z^{-136} \end{aligned} \tag{33}$$

$$\begin{aligned} &q_0 = m_0^3 (8 + a_0 + 2a_1 + 4a_2),\, q_1 = m_0^3 (-24 + 3a_0 + 2a_1 - 4a_2), \\ &q_2 = m_0^3 (24 + 3a_0 - 2a_1 - 4a_2),\, q_3 = m_0^3 (-8 + a_0 - 2a_1 + 4a_2), \\ &q_{143} = q_{146} = m_0^3 a_{0D},\, q_{144} = q_{145} = 3 m_0^3 a_{0D} \\ &p_0 = (b_0 + b_{0D})(m_0^3 + 6m_0^2 + 12m_0 + 8), \\ &p_1 = (b_0 + b_{0D})(3m_0^3 + 6m_0^2 - 12m_0 - 24), \\ &p_2 = (b_0 + b_{0D})(3m_0^3 - 6m_0^2 - 12m_0 + 24), \\ &p_3 = (b_0 + b_{0D})(m_0^3 - 6m_0^2 + 12m_0 - 8), \\ &p_{131} = -b_0 m_0^3,\, p_{132} = m_0^3(-3b_0 - 0.5b_{0D}),\, p_{133} = m_0^3(-3b_0 - 2b_{0D}), \\ &p_{134} = m_0^3(-b_0 - 3b_{0D}),\, p_{135} = -2b_{0D} m_0^3,\, p_{136} = -0.5 b_{0D} m_0^3 \end{aligned} \tag{34}$$

VII. Controller Verification

In this closing section, simulation as well as real-experiment benchmarks and test are presented. Manipulated input and measured output simulation responses for continuous-time controller (18) as differences from the operating point, $\Delta P(t)$ and $\Delta \vartheta_{CO}(t)$, respectively, for three different values of m_0 within the selected range $m_0 \in [0.008, 0.012]$ are displayed in Fig. 8 and Fig. 9, respectively where step load disturbance $d_P(t) = -10$ enters at $t = 2000$.

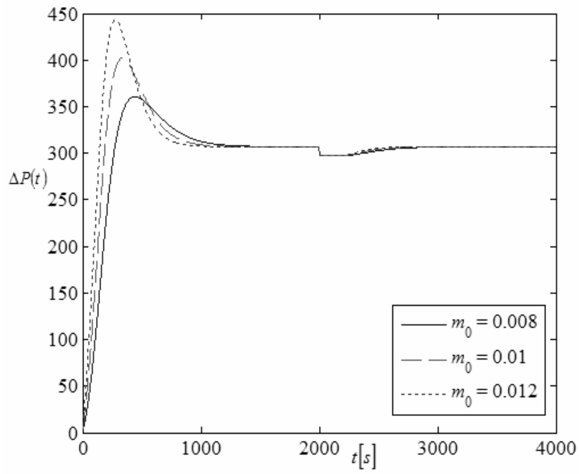

Fig. 8 Continuous-time control response – *u*(*t*)

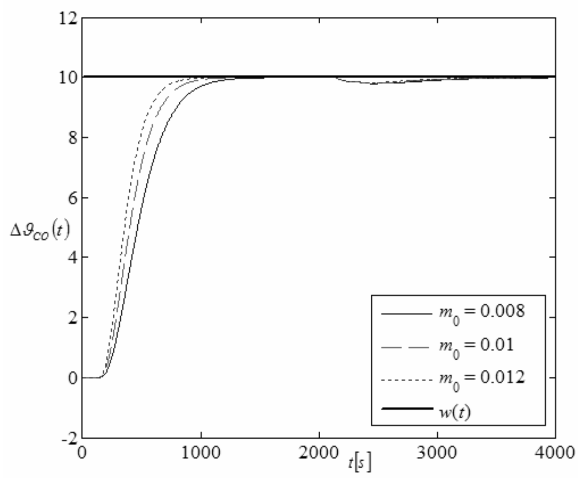

Fig. 9 Continuous-time control response – *y*(*t*)

The value $m_0 = 0.012$ was selected for real experiments since the control responses are satisfactory enough and the control input does not reach saturation point (note that the maximum feasible value of the heat power is $\Delta P_{MAX}(t) = 450\,\mathrm{W}$).

Let us compare continuous and discrete-time responses, as can bee seen in Figs. 10, 11 now. Obviously, both curves are nearly identical.

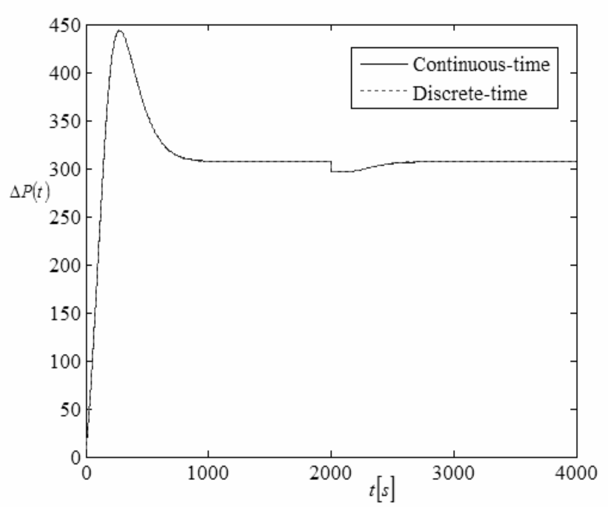

Fig. 10 Continuous-time vs. discrete-time response – $u(t)$

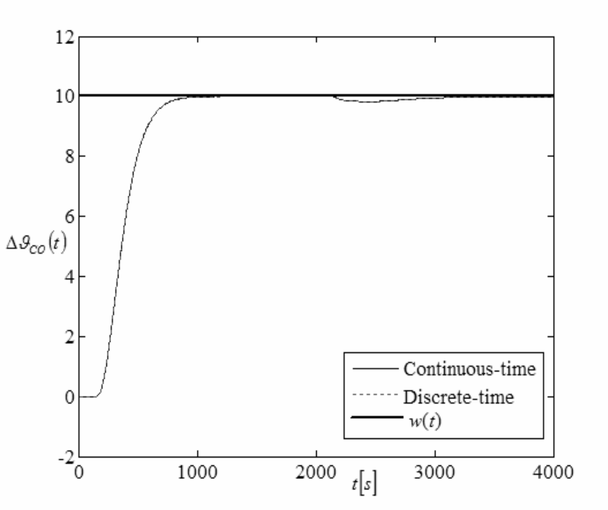

Fig. 11 Continuous-time vs. discrete-time response – $y(t)$

The discrete-time form of the controller law (32)-(34) is used to test real control performance on the laboratory appliance (without the impact of the load disturbance) and compared with simulations, see Figs. 12 and 13.

Finally, the ability of the controller to asymptotically reject a simple "stepwise" disturbance is displayed in Fig. 14 where the cooler input voltage was abruptly changed from $u_c = 3\,\mathrm{V}$ to $u_c = 9\,\mathrm{V}$.

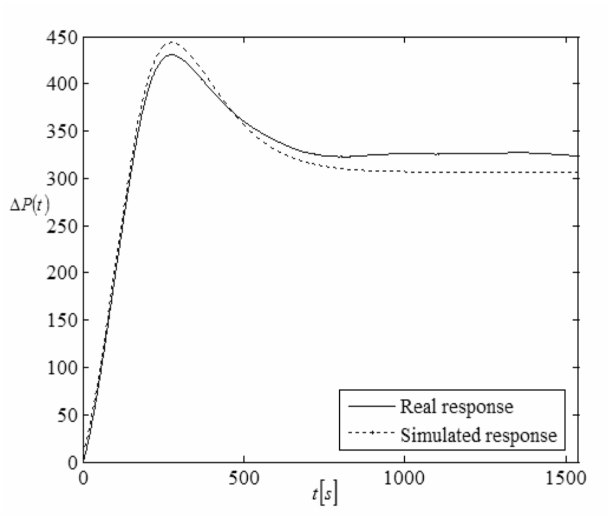

Fig. 12 Real vs. simulated response – $u(t)$

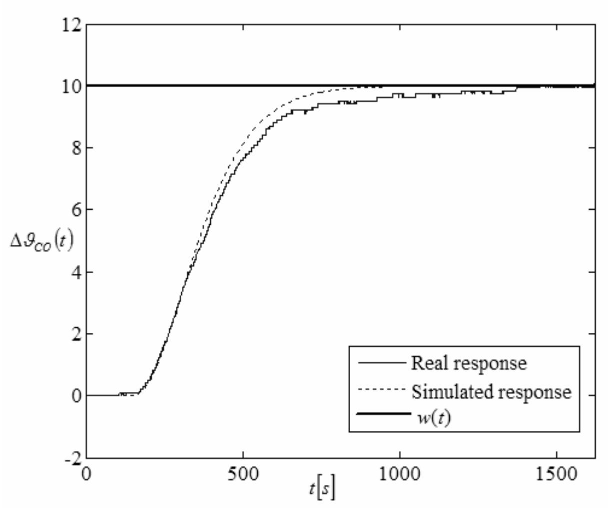

Fig. 13 Real vs. simulated response – $y(t)$

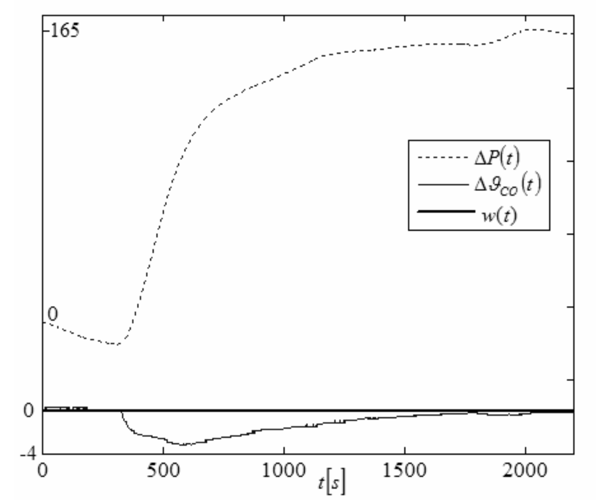

Fig. 14 – Step disturbance rejection verification

VIII. Conclusion

Algebraic robust controller design for a laboratory thermal plant in the R_{MS} ring has been the objective of the propounded paper.

A revised definition of the R_{MS} ring together with general algebraic controller design via the solution of the Bézout identity and the Youlla-Kučera parameterization have been presented first. A simple feedback loop has been utilized. The laboratory appliance appearance together with its model of a relation between one manipulated input and a measured output have been introduced in the second part of this contribution. presented first. Then, the particular controller derivation for the laboratory plant ensuring asymptotical reference tracking and (stepwise) load disturbance rejection has been given to the reader. Robust stability and robust performance tests have followed; as a result, a suitable range of a controller parameter has been chosen. A sketch of a simple discretization procedure based on delta models has been presented as well. Finally, simulation verification of both controller rules, i.e. continuous-time and discrete-time ones, together with their comparison with real laboratory control responses have been brought out. The results confirm the usability and applicability of the presented robust controller design approach.

The methodology is usable also for neutral time delay systems and those with distributed delays; however, in these cases a stabilizing controller may not be found.

For the future research, we plant to utilize and perform more sophisticated (optimal) controller tuning ideas and/or to use other (more complex) control system structures (with more degrees of freedom etc.).

References

[1] H. Górecki, S. Fuksa, P. Grabowski, and A Korytowski, *Analysis and Synthesis of Time Delay Systems*. New York: John Wiley & Sons, 1989.

[2] J. K. Hale and S. M. Verduyn Lunel, “Introduction to functional differential equations,” *Applied Mathematical Sciences*, vol. 99. New York : Springer-Verlag, 1993.

[3] S.-I. Niculescu, “Delay effects on stability,” in *Lecture Notes in Control and Information Sciences*, vol. 269. Berlin: Springer-Verlag, 2001.

[4] J.-P. Richard, “Time-delay systems: an overview of some recent advances and open problems,” *Automatica*, vol. 39, no. 10, pp. 1667-1694, 2003.

[5] W. Michiels, T. Vyhlídal, and P. Zítek, “Control design for time-delay systems based on quasi-direct pole placement,” *J. Process Control*, vol. 20, no. 3, pp. 337-343, 2010.

[6] P. Zítek and A. Víteček, *Control Design of Time-Delay and Nonlinear Subsystems*. Prague: ČVUT Publishing, 1999. (in Czech)

[7] L. Garbai and L. Barna, “Modelling of non-steady-state conditions in a gas boiler heated room,” in *Proc. 3rd IASME/WSEAS Int. Conf. on Heat Transfer, Thermal Engineering and Environment,* Corfu, Greece, 2006, pp. 6-11.

[8] L. Hacia and K. Domke, “Integral modeling and simulation in some thermal problems,” in *Proc. 5th IASME/WSEAS Int. Conf. on Heat Transfer, Thermal Engineering and Environment,* Athens, Greece, 2007, pp. 42-47.

[9] L. Pekař, R. Prokop, and P. Dostálek, “Circuit heating plant model with internal delays,” *WSEAS Trans. Systems*, vol. 8, no. 9, pp. 1093-1104, 2009.

[10] P. Dostálek, J. Dolinay, and V. Vašek, “Design and implementation of portable data acquisition unit in process control and supervision applications,” *WSEAS Trans. Systems and Control*, vol.3, no.9, 2008, pp. 779-788.

[11] P. Zítek, “Anisochronic modelling and stability criterion of hereditary systems,” *Problems of Control and Information Theory*, vol. 15, no. 6, pp. 413-423, 1986.

[12] P. Zítek and V. Kučera, “Algebraic design of anisochronic controllers for time delay systems,” *Int. J. Control*, vol. 76, no. 16, pp. 905-921, 2003.

[13] L. Pekař, “A ring for description and control of time-delay systems,” special issue on *Time Delay Systems: Modelling, Identification, Stability, Control and Applications* of *WSEAS Trans. Systems*, 15 p., 2012, accepted.

[14] J. J. Loiseau and D. Brethe, “An effective algorithm for finite spectrum assignment of single-input systems with delays,” *Mathematics and Computers in Simulation*, vol. 43, no. 3-4, pp. 339-348, 1998.

[15] J. J. Loiseau, “Algebraic tools for the control and stabilization of time-delay systems,” *Annual Reviews in Control*, Vol.24, pp. 135-149, 2000.

[16] L. Pekař and R. Prokop, “Optimal pole placement for LTI-TDS via some advanced iterative algorithms – part I: theory,” in *Recent Researches in Circuits and Systems, Proc. of the 16th WSEAS Int. Conf. on Systems*, Kos, Greece, 2012, pp. 196-201.

[17] L. Pekař, “On the optimal pole assignment for time-delay systems”, *Int. J. Math. and Computers in Simulation*, submitted for publication.

[18] P. Zítek, V. Kučera, and T. Vyhlídal, „Meromorphic observer-based pole assignment in time delay systems,“ *Kybernetika*, vol. 44, no. 5, pp. 633-648, 2008.

[19] R. H. Middleton and G. C. Goodwin, *Digital Control and Estimation: A Unified Approach*, Detroit: Prentice Hall, 1990.

[20] D. Breda, S. Maset, and R. Vermiglio, “Pseudospectral differencing methods for characteristic roots of delay differential equations,” *SIAM J. Scientific Computing*, vol.27, no.2, pp. 482-495, 2005.

[21] D. Breda, “Solution operator approximation for characteristic roots of delay differential equations,” *Applied Numerical Mathematics*, vol.56, no.3-4, pp. 305-317, 2006.

[22] T. Vyhlidal, *Analysis and Synthesis of Time Delay System Spectrum*, Ph.D. Thesis, Faculty of Mechanical Engineering, Czech Technical University in Prague, 2003.

[23] J. J. Loiseau, M. Cardelli and X. Dusser, “Neutral-type time-delay systems that are not formally stable are not BIBO stabilizable,” *IMA J. Mathematical Control and Information*, vol.19, no.1-2, pp. 217-227, 2002.

[24] J. R. Partington and C. Bonnet, “H_∞ and BIBO stabilization of delay systems of neutral type,” *Systems & Control Letters*, vol.52, no.3-4, pp. 283-288, 2004.

[25] L. Pekař and R. Prokop, “Implementation of a new quasi-optimal controller tuning algorithm for time-delay systems,” in *MATLAB for Engineers – Applications in Control, Electrical Engineering*, IT and Robotics, 1st ed., K. Perůtka, Ed. Rijeka, Croatia: InTech, 2011, pp 3-26.

[26] L. Pekař, *Control of Time Delay Systems – An Algebraic Approach*, Ph.D. Thesis, Faculty of Applied Informatics, Tomas Bata University in Zlín, 2012. To be published.

[27] J. C. Doyle, B. A. Francis, and A. R. Tannenbaum, *Feedback Control Theory*, New York: McMillan, 1992.

[28] M. Švehlíková, *Heating Plant Mathematical Model Verification*, Bachelors Thesis, Faculty of Applied Informatics, Tomas Bata University in Zlín, 2009.

[29] M. Hofreiter, “Discretization of continuous linear anisochronic models,” *Studies in Informatics and Control*, vol.12, no.1, pp. 69-76, 2003.

[30] T. Vyhlidal and P. Zítek, “Discrete approximation of a time delay system and delta model spectrum,” in *Proc. of the 16th IFAC World Congress*, Prague, 2005, [CD-ROM].