Multiplicative and additive approach in analytic hierarchy process – case study of competitiveness evaluation of Czech and Slovak regions

J. Nevima, Z. Kiszová

***Abstract*—** The article is focused on alternative approach in regional competitiveness assessment in case of the Czech and Slovak Republic. The aim of the contribution is to find out position of NUTS 2 regions in closed programming period of 2000 – 2006 years using the analytic hierarchy process. The method of analytic hierarchy process is presented in two different approaches to pairwise comparisons. The first one is multiplicative what is original and connected with the method from its beginning. The second one is additive which is more natural and acceptable for many decision makers. Analytic hierarchy process is used to calculate weights of selected criteria (macroeconomic indicators). Mentioned macroregional indicators are chosen with regard to expert estimation and accessibility of relevant statistic data. Based on the application of the method we can gain detailed view on regional competitiveness of regions by way of quantitative characteristics which can lead to more precise definition of reached competitiveness of NUTS 2 regional units in the EU.

***Keywords*—** competitiveness, region, analytic hierarchy process, European Union.

I. Introduction

Competitiveness has become quite a common term used in many professional and non-specialized publications. Evaluation of the competitiveness issue is not less complicated. In the absence of mainstream views on the assessment of competitiveness, there is sample room for the presentation of individual approaches to its evaluation. In our paper we will examine the possibility of evaluation the competitiveness of the regions of selected Czech and Slovak regions at NUTS 2 level in terms of analytic hierarchy process. The level of NUTS 2 regions for evaluation of competitiveness seems to be legitimate especially because of the fact that European Commission accents the level of regional units from aims of economic and social cohesion view and realization of structural aid in the EU member states. When making concept of suitable evaluation tools of national and regional competitiveness it is necessary to suggest not only difficult but also simple methods which enable quick evaluation of competitiveness by accessible tools

Effectively analyzed competitiveness means to be based on a defined concept of competitiveness. For evaluation of regional competitiveness, we face the problem of the basic concept and definition of competitiveness due to absence of a consistent approach of its definition.

II. Concept of Regional Competitiveness

The concept of competitiveness has quickly spread into the regional level, but the notion of regional competitiveness is also contentious. In the global economy regions are increasingly becoming the drivers of the economy and generally one of the most striking features of regional economies is the presence of clusters [2], or geographic concentrations of linked industries [6]. The regional competitiveness is also affected by the regionalization of public policy because of the shifting of decision-making and coordination of activities at the regional level. To talk of regional competitiveness would seem to imply that regional economies are like firms [7], or nation-states, and are in competition with one another.

Decomposition of aggregate macroeconomic indicators is the most common used approach at the regional level, as well as comprehensive (mostly descriptive) analysis aimed at identifying the key factors of regional development, productivity [1], [15] and economic growth [13], [14].

Within governmental circles, interest has grown in the regional foundations of national competitiveness, and with developing new forms of regionally based policy interventions to help improve competitiveness of every region and major city, and hence the national economy as a whole. Regions play an increasingly important role in the economic development of states.

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A. Approaches to Competitiveness Evaluation

Creation of competitiveness evaluation system in terms of the EU is greatly complicated by heterogeneity of countries and regions and also by own approach to the original concept of competitiveness. Evaluation of competitiveness in terms of differences between countries and regions should be measured through complex of economic, social and environmental criteria that can identify imbalanced areas that cause main disparities. Comparing instruments for measuring and evaluation of competitiveness in terms of the EU is not a simply matter. Evaluation of regional competitiveness is determined by the chosen territorial region level, especially in terms of the European Union through the Nomenclature of Units for Territorial Statistics – in our paper we apply NUTS 2 level.

Another approach is presented by EU structural indicators evaluation. These indicators are used for the assessment and the attainment of the objectives of the Lisbon Strategy [19]. Specific approach is macro econometric modelling and creation of an econometric panel data model [3]. The approach based on application of specific economic coefficients of efficiency includes two methods of multi-criteria decision making. The first one is the classical Analytic Hierarchy Process (AHP) where relevance of criteria's significance is determined by the method of Ivanovic deviation. The second method - FVK is a multiplicative version of AHP [4]. Also DEA methodology was presented in case of Visegrad four regions. DEA evaluates the efficiency of regions with regard to their ability to transform inputs into outputs [8]. In other words - what results a region can achieve while spending a relatively small number of inputs (resources). This fact is vital for us to perceive the efficiency like a "mirror" of competitiveness [5], [17]. This aspect is also crucial in this paper, where we present AHP to gain more detailed view on competitiveness of regions by way of quantitative characteristics.

B. Database

Data base for evaluation of regional competitiveness in the NUTS 2 regions of the Czech and Slovak Republic countries is made up of regional data, which were taken from the database of the OECD iLibrary – section Statistics - OECD Regional statistics. Under regional data have been used time series of 7 indicators (in our case indicators mean "criteria"), annual basis, including: Gross domestic product (GDP), Gross fixed capital formation (GFCF), Gross expenditure on research and development (GERD), Net disposable income of households (NDI), Rate of employment (ER), Knowledge intensive services (KIS) and Patents created by regions – inventors (PAT). Comparability of data over time was ensured by using time series of the available indicators in purchasing power parity (PPS) per capita in euro currency. The data analysis cover reference period 2000 - 2006.

C. Description of Entrance Criteria for Evaluation of Competitiveness

GDP was chosen as it is one of the most important macroeconomic aggregate which is simultaneously suitable basic for competitiveness assessment of the country, but also for the regional level, where also NUTS 2 regions belong. It is obviously not always valid that with increasing level of GDP [13], [16], [18] (i.e. increasing efficiency of regions) also the rate of obtained competitiveness/competition advantage grows.

Gross fixed capital formation (GFCF) due to international accounting is a basic part of gross capital (capital investments), in which is also the change of inventories and net acquisition of valuables included. According to ESA 95 methodology GFCF consists of the net assets acquisition minus decrease of fixed assets at residential producers during the time period plus certain increasing towards the value of non-produced assets originated as a consequence of production activity of producers or institutional units. It is estimated in purchase price including costs connected with installment and other costs on transfer of the ownership. Fixed assets are tangible or intangible/invisible assets produced as the output from production process and are used in production process repeatedly or continuously during the one-year period. It is an index of innovating competitiveness which enables to increase production on modern technical base.

Gross domestic expenditures on research and development (GERD) are sources for further economic growth increasing as stimulation of basic and applied research creates big multiplication effects with long-term efficiency and presumptions for long-term economic growth in economics. R&D is defined as creative work undertaken on a systematic basis in order to increase the stock of knowledge, including human knowledge, culture and society and the use of this stock of knowledge to devise new applications.

Net disposable income (NDI) is the result of current incomes [11] and expenditures, primary and secondary disposal of incomes. It explicitly excludes capital transfers, real profits and loss from possession and consequences of the events as disasters. In contrast to gross disposable income, it does not cover fixed capital consumption. Disposable income (gross or net) is the source of expenditures on final consumption cover and savings in the sectors: governmental institutions, households and non-profit institutions for households.

Next represented entrance criteria is rate of employment in age group 20 – 64 years (ER). From the economic relevance rate of employment is important in accordance to number of economic active people in above mentioned age group. Employed population consists of those persons who during the reference week did any work for pay or profit for at least one hour, or were not working but had jobs from which they were temporarily absent.

Knowledge intensive services (KIS) as % of total employment are among the fastest growing and dynamic sectors of the economy. Knowledge intensive services are characterized by high degrees of contact intensity and a high number of variants. Typical examples are professional business services like consulting, IT and marketing.

Knowledge-intensive services are supplied mainly to final consumers, as public services (e.g. health) or private professional ones (consumer financial advice [12] or computer repair).

Patents (PAT) are a key measure of innovation output, as patent indicators reflect the inventive performance of regions. Patent indicators can serve to measure the output of R&D, its productivity, structure and the development of a specific technology/industry. Among the few available indicators of technology output, patent indicators are probably the most frequently used. Patents are often interpreted as an output indicator; however, they could also be viewed as an input indicator, as patents are used as a source of information by subsequent inventors.

III. Analytic Hierarchy Process

A. *Multiplicative approach*

We use multicriteria decision-making method called analytic hierarchy process (AHP) to evaluate competitiveness of Czech regions. This method allows including both quantitative and qualitative criteria and is used to determine priorities. Using hierarchies and pairwise comparisons are important attributes of AHP.

Hierarchies allow dividing the problem of evaluation into hierarchical levels. Three-level hierarchy is classical example (Fig. 1). The goal of the problem is situated on the top level, the level of criteria follows. Criteria represent properties of elements on the lowest level, i.e. of alternatives. The principle of hierarchy ensures that an element located at a higher level influence elements on lower level, but not vice versa.

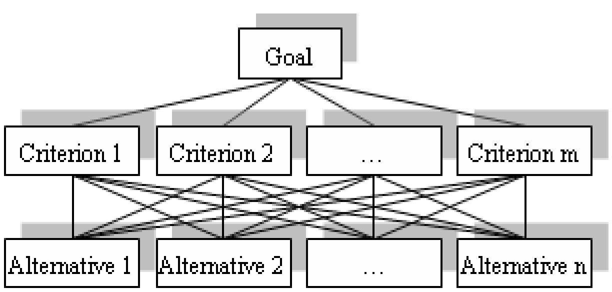

Fig. 1 Three-level hierarchic structure

The essence of pairwise comparison (which has multiplicative character in Saaty's concept) is mutual measure of all pairs of considered elements within the same hierarchical level with respect to the level immediately above. We compare criteria among themselves or alternatives with respect to given qualitative criterion. For numerical expression of intensity of relations between compared elements Saaty created nine-point scale [10], see Table 1.

Data obtained through pairwise comparisons are inserted into the pairwise comparison matrix A, its entries are signed generally aij. An (square) matrix is created, see Fig. 2.

Table 1 Saaty's fundamental scale

Intensity of importance	**Definition**
1	Equal importance
2	Weak
3	Moderate Importance
4	Moderate plus
5	Strong Importance
6	Strong plus
7	Very strong Importance
8	Very, very strong
9	Extreme importance

$$\begin{array}{c} \\ \text{element } x_1 \\ \text{element } x_2 \\ \vdots \\ \text{element } x_k \end{array} \begin{array}{c} \begin{array}{cccc} \text{element } x_1 & \text{element } x_2 & \dots & \text{element } x_k \end{array} \\ \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1k} \\ a_{21} & a_{22} & \cdots & a_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ a_{k1} & a_{k2} & \cdots & a_{kk} \end{bmatrix} \end{array}$$

Fig. 2 General multiplicative pairwise comparison matrix

Such a matrix is created whenever there is no absolute evaluation of the element with respect to an element from a higher level, i.e. when it is not possible to compare the elements in the given hierarchical level based on their values with respect to an element of the level immediately above. Entries of the pairwise comparison matrix represent estimation of weight ratio of two compared elements of the same hierarchic level (we have to determine these weights through numerical operations). If a_{ij} is an element of pairwise comparison matrix, $a_{ij} \in \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, w_i is wanted weight of the element x_i, w_j is wanted weight of the element x_j for all i and j, we can write:

$$a_{ij} = \frac{w_i}{w_j}, \; a_{ij} \in \{1,2,3,4,5,6,7,8,9\} \tag{1}$$

$$a_{ji} = \frac{1}{a_{ij}}, \; a_{ji} \in \{1,2,3,4,5,6,7,8,9\} \tag{2}$$

$$a_{ij} \cdot a_{ji} = 1 \qquad ('2)$$

Formula (2) corresponds to one of the pairwise comparison matrix characteristic – the reciprocity.

Consistency is characteristic of pairwise comparison matrix which expresses how much individual pairwise comparisons are mutually consistent. This characteristic can be expressed by the following formula illustrating transitivity of pairwise comparisons:

$$a_{ij} = a_{ik} \cdot a_{kj}, \; i, j, k = 1,2,\dots n. \tag{3}$$

We have to compute the eigenvector $w = (w_1, w_2, ..., w_n), \sum_{i=1}^{n} w_i = 1$ corresponding to the maximal eigenvalue $\lambda_{\max}$ of the pairwise comparison matrix A to determine result element priorities of the given matrix. Eigenvector w contents information about result priorities.

$$Aw = \lambda_{\max} w \quad (4)$$

Pairwise comparison matrix is square, nonnegative and irreducible. These characteristics ensure existence of maximal eigenvalue $\lambda_{\max}$ and corresponding positive eigenvector [9].

The Wielandt theorem is used to compute the eigenvector, where e is unit vector and c is constant.

$$cw = \lim_{k \to \infty} \frac{A^k e}{e^T A^k e} \quad (5)$$

It is possible to measure the consistency, respective inconsistency of multiplicative pairwise comparisons using multiplicative consistency index $I_{mc}(A)$ of pairwise comparison matrix A:

$$I_{mc}(A) = \frac{\lambda_{\max} - n}{n - 1} \quad (6)$$

In case of consistent pairwise comparison matrix $I_{mc}(A) = 0$.

As it follows from formula (6) the multiplicative consistency index $I_{mc}(A)$ depends on dimension of the matrix. Therefore the multiplicative consistency ratio $CR_{mc}(A)$ was implemented. It is defined as ratio of multiplicative consistency index $I_{mc}(A)$ and its mean value $R_{mc}(n)$ calculated for randomly generated reciprocal matrices satisfying characteristics of multiplicative pairwise comparison matrices. Values of $R_{mc}(n)$ are published e.g. in [10]. It is formulated as follows:

$$CR_{mc}(A) = \frac{I_{mc}(A)}{R_{mc}(n)}. \quad (7)$$

Generally the maximal acceptable value is 10 %.

B. Additive approach

Additive approach is an alternative to multiplicative pairwise comparisons. There is not used fundamental nine point scale in this approach. Total evaluation, i.e. 100 %, is divided between two compared elements. Resulting value gives how much of the total preferences the element x_i shares in comparison to element x_j. Results of these comparisons are inserted into the additive pairwise comparison matrix B with entries $0 \le b_{ij} \le 1$ for all i and j. The additive pairwise comparison matrix is square, its dimension n corresponds to number of elements of given hierarchical level (Fig. 3).

$$\begin{array}{c c} & \begin{array}{cccc} \text{element } x_1 & \text{element } x_2 & \cdots & \text{element } x_k \end{array} \\ \begin{array}{c} \text{element } x_1 \\ \text{element } x_2 \\ \vdots \\ \text{element } x_k \end{array} & \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1k} \\ b_{21} & b_{22} & \cdots & b_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ b_{k1} & b_{k2} & \cdots & b_{kk} \end{bmatrix} \end{array}$$

Fig. 3: additive pairwise comparison matrix B

Fundamental relation for elements of this matrix is expressed by following formula [21]:

$$b_{ij} + b_{ji} = 1,\ i, j = 1, 2, \ldots, n. \quad (8)$$

According to the formula (8) the matrix B is additive reciprocal, there are values $b_{ii} = 0.5$ on its diagonal.

Pairwise comparison consistency is ensured by transitivity of pairwise comparisons, which is expressed in the case of additive approach by formula [21]:

$$b_{ij} + b_{jk} + b_{ki} = 1.5 \text{ for all } i, j, k. \quad (9)$$

If elements b_{ij} of the matrix B are computed according to function $\varphi(a_{ij})$ [20]:

$$b_{ij} = \varphi(a_{ij}) = \frac{1}{2}\left(1 + \log_9 a_{ij}\right),\ a_{ij} \in \left[\frac{1}{9};9\right], \quad (10)$$

then it is possible to calculate elements a'_{ij} of multiplicative pairwise comparison matrix corresponding to elements b_{ij} of additive pariwise comparison matrix according to the function $\varphi^{-1}(b_{ij})$ [21]:

$$a'_{ij} = \varphi^{-1}(b_{ij}) = 9^{2b_{ij}-1},\ b_{ij} \in [0;1]. \quad (11)$$

This operation ensures transformation of the additive reciprocal matrix B into multiplicative reciprocal matrix A'. Priority vector of the matrix B is defined as weights determined on the base of transformed matrix A'. These weights are calculated according to (4) and (5).

The formula for computation of additive transitivity index $I_{at}(B)$ determines the additive transitivity of matrix B:

$$I_{at}(B) = I_{mc}(A'). \quad (12)$$

It is possible to measure the additive transitivity independent on the matrix dimension by additive transitivity ratio $CR_{at}(B)$:

$$CR_{at}(B) = \frac{I_{at}(B)}{R_{mc}(n)}. \quad (13)$$

Its acceptable value must not extend the threshold of 10 %.

Ramík suggests another way of additive pairwise comparison assessment [21].

If function $\phi(a_{ij})$ is defined as:

$$b_{ij} = \phi(a_{ij}) = \frac{a_{ij}}{1 + a_{ij}},\ a_{ij} > 0, \quad (14)$$

then it is possible to calculate elements a''_{ij} of multiplicative pairwise comparison matrix A'' analogical to given additive pariwise comparison matrix B according to the inverse function $\phi^{-1}(b_{ij})$:

$$a''_{ij} = \phi^{-1}(b_{ij}) = \frac{b_{ij}}{1 - b_{ij}},\ b_{ij} \in [0;1]. \quad (15)$$

The function $\phi^{-1}(b_{ij})$ transforms the additive reciprocal matrix B into the multiplicative reciprocal matrix A'' similarly as the function $\varphi^{-1}(b_{ij})$ does. The priority vector of matrix B is identical to the weight vector of matrix A'' and it is possible to calculate it by (4) and (5).

As in the case of multiplicative pairwise comparison matrix there is investigated the additive consistency index $I_{ac}(B)$ of the additive reciprocal matrix, which is determined by:

$$I_{ac}(B) = I_{mc}(A''). \quad (16)$$

The additive consistency ratio $CR_{ac}(B)$ of additive pairwise comparison matrix, which value must not exceed the ten-percent threshold, is calculated as follows:

$$CR_{ac}(B) = \frac{I_{ac}(B)}{R_{mc}(n)} \tag{17}$$

Transformations of matrix B by (11) and (15) may give different weights of elements which are compared in this matrix.

C. Synthesis

Through formulas (4) and (5) are gained priority vectors of individual pairwise comparison matrices which determine weights of criteria with respect to the goal or weights of alternatives with respect to given criterion. The required result, i.e. weights of alternatives with respect to the goal, is obtained through synthesis of this information.

It is appropriate to normalize values of alternatives in the case of quantitative criteria. If value of j-th alternative with respect to maximizing criterion f_i is signed $h_j(f_i)$, then the weight of j-th alternative with respect to this criterion $v_j(f_i)$ is:

$$v_j(f_i) = \frac{h_j(f_i)}{\sum_{j=1}^{n} h_j(f_i)}, \; i = 1,2,\ldots,m. \tag{18}$$

If value of j-th alternative with respect to minimizing criterion f_i is signed $h_j(f_i)$, then the weight of j-th alternative with respect to this criterion $v_j(f_i)$ is:

$$v_j(f_i) = \frac{\frac{1}{h_j(f_i)}}{\sum_{j=1}^{n} \frac{1}{h_j(f_i)}}, \; i = 1,2,\ldots,m. \tag{19}$$

If weight of i-th criterion with respect to the goal is w_i and weight of j-th alternative with respect to criterion f_i is $v_j(f_i)$, the overall weight of j-th alternative with respect to the goal is:

$$H_j = \sum_{i=1}^{m} w_i v_j\left(f_i\right), \; j = 1, 2, \ldots, n. \tag{20}$$

On the basis of overall weights it is possible to rank evaluated alternatives from the best to the worst. The optimal alternative gains the highest weight.

IV. Application

In our case the goal is to assess competitiveness of Czech and Slovak regions. Alternatives are NUTS 2 Czech regions, i.e. Praha (CZ 01), Stredni Cechy (CZ 02), Jihozapad (CZ 03), Severozapad (CZ 04), Severovychod (CZ 05), Jihovychod (CZ 06), Stredni Morava (CZ 07) and Moravskoslezsko (CZ 08) and also Slovak regions, i.e. Bratislavsky kraj (SK 01), Zapadni Slovensko (SK 02), Stredni Slovensko (SK 03) and Vychodni Slovensko (SK 04) These alternatives are evaluated by following criteria: gross domestic product (GDP), net disposable income of households (NDI), gross fixed capital formation (GFCF), gross expenditure on research and development (GERD), knowledge intensive services (KIS), rate of employment (ER) and patents created by regions – inventors (PAT). All criteria are maximizing.

The pairwise comparison method is applied to determine weights of criteria with respect to the goal. The multiplicative pairwise comparison matrix A of criteria and the additive pairwise comparison matrix B are shown on Fig. 4 and Fig. 5. These matrices are based on expert estimation.

According to (11) and (15) are calculated transformed pairwise comparisons A' (see Fig. 6) and A'' (see Fig. 7) which are multiplicative reciprocal.

$$\begin{array}{rc}
 & \begin{array}{ccccccc} GDP & NDI & GFCF & GERD & KIS & ER & PAT \end{array} \\
\begin{array}{r} GDP \\ NDI \\ GFCF \\ A = GERD \\ KIS \\ ER \\ PAT \end{array} &
\begin{bmatrix}
1 & 3 & 5 & 7 & 8 & 8 & 9 \\
1/3 & 1 & 2 & 2 & 3 & 4 & 5 \\
1/5 & 1/2 & 1 & 3 & 4 & 5 & 5 \\
1/7 & 1/2 & 1/3 & 1 & 2 & 3 & 5 \\
1/8 & 1/3 & 1/4 & 1/2 & 1 & 3 & 4 \\
1/8 & 1/4 & 1/5 & 1/3 & 1/3 & 1 & 3 \\
1/9 & 1/5 & 1/5 & 1/5 & 1/4 & 1/3 & 1
\end{bmatrix}
\end{array}$$

Fig. 4: The multiplicative pairwise comparison matrix A

$$\begin{array}{rc}
 & \begin{array}{ccccccc} GDP & NDI & GFCF & GERD & KIS & ER & PAT \end{array} \\
\begin{array}{r} GDP \\ NDI \\ GFCF \\ B = GERD \\ KIS \\ ER \\ PAT \end{array} &
\begin{bmatrix}
0.5 & 0.7 & 0.75 & 0.8 & 0.85 & 0.9 & 0.95 \\
0.3 & 0.5 & 0.55 & 0.6 & 0.7 & 0.75 & 0.8 \\
0.25 & 0,45 & 0.5 & 0.65 & 0.7 & 0.8 & 0.8 \\
0.2 & 0.4 & 0.35 & 0.5 & 0.55 & 0.6 & 0.7 \\
0.15 & 0.3 & 0.3 & 0.45 & 0.5 & 0.6 & 0.65 \\
0.1 & 0.25 & 0.2 & 0.4 & 0.4 & 0.5 & 0.6 \\
0.05 & 0.2 & 0.2 & 0.3 & 0.35 & 0.4 & 0.5
\end{bmatrix}
\end{array}$$

Fig. 5: The additive pairwise comparison matrix B

$$\begin{array}{rc}
 & \begin{array}{ccccccc} GDP & NDI & GFCF & GERD & KIS & ER & PAT \end{array} \\
\begin{array}{r} GDP \\ NDI \\ GFCF \\ A' = GERD \\ KIS \\ ER \\ PAT \end{array} &
\begin{bmatrix}
1 & 2.41 & 3.00 & 3.74 & 4.66 & 5.80 & 7.22 \\
0.42 & 1 & 1.25 & 1.55 & 2.41 & 3.00 & 3.74 \\
0.33 & 0,80 & 1 & 1.93 & 2.41 & 3.74 & 3.74 \\
0.27 & 0.64 & 0.52 & 1 & 1.25 & 1.55 & 2.41 \\
0.21 & 0.42 & 0.42 & 0.80 & 1 & 1.55 & 1.93 \\
0.17 & 0.33 & 0.27 & 0.64 & 0.64 & 1 & 1.55 \\
0.14 & 0.27 & 0.27 & 0.42 & 0.52 & 0.64 & 1
\end{bmatrix}
\end{array}$$

Fig. 6: Transformed pairwise comparison matrix A'

$$\begin{array}{rc}
 & \begin{array}{ccccccc} GDP & NDI & GFCF & GERD & KIS & ER & PAT \end{array} \\
\begin{array}{r} GDP \\ NDI \\ GFCF \\ A'' = GERD \\ KIS \\ ER \\ PAT \end{array} &
\begin{bmatrix}
1 & 2.33 & 3 & 4 & 5.67 & 9 & 19 \\
0.43 & 1 & 1.22 & 1.50 & 2.33 & 3 & 4 \\
0.33 & 0,82 & 1 & 1.86 & 2.33 & 4 & 4 \\
0.25 & 0.67 & 0.54 & 1 & 1.22 & 1.50 & 2.33 \\
0.18 & 0.43 & 0.43 & 0.82 & 1 & 1.50 & 1.86 \\
0.11 & 0.33 & 0.25 & 0.67 & 0.67 & 1 & 1.50 \\
0.05 & 0.25 & 0.25 & 0.43 & 0.54 & 0.67 & 1
\end{bmatrix}
\end{array}$$

Fig. 7: Transformed pairwise comparison matrix A''

Maximal eigenvalue of multiplicative pairwise comparison matrix A is calculated: $\lambda_{\max}(A) =$ 7.510. The corresponding priority vector of criteria (in order GDP, NDI, GFCF, GERD, KIS, ER, PAT) is w_A = (0.456; 0.171; 0.157; 0.088; 0.064; 0.039; 0.025). Matrix A is considered as multiplicative consistent with the multiplicatively consistency ratio $CR_{mc}(A) = 6.3\,\%$.

Maximal eigenvalue of transformed pairwise comparison matrix A' is $\lambda_{\max}(A') =$ 7.050 and corresponding priority vector of criteria (in the same order as above) is $w_{A'}$ = (0.381; 0.173; 0.169; 0.098; 0.079; 0.058; 0.043). According to (13) additive transitivity ratio is calculated: $CR_{at}(B) = 0.6\,\%$, therefore matrix B is considered as additively transitive.

Maximal eigenvalue of transformed pairwise comparison matrix A'' is obtained: $\lambda_{\max}(A'') =$ 7.070. Corresponding priority vector of criteria (in the same order as above) is $w_{A''} =$ (0.443; 0.159; 0.155; 0.089; 0.070; 0.049; 0.035). Because the additive consistency ratio $CR_{ac}(B)$ is of value 0.9 %, matrix B is considered as additively consistent.

The final order of regions is presented in Table 2-Table 4. Consequently, our approach presented here could be considered as a suitable alternative for the evaluation of regional competitiveness not only in the Czech and Slovak Republic.

Table 2: Rank of regions in years 2000 – 2006 based on multiplicative pairwise comparisons

Region / Year	*2000*	*2001*	*2002*	*2003*	*2004*	*2005*	*2006*
Praha	1	1	1	1	1	1	1
Stredni Cechy	3	3	3	3	3	3	3
Jihozapad	5	4	5	5	5	5	4
Severozapad	9	9	9	7	9	10	10
Severovychod	6	6	6	6	6	6	7
Jihovychod	4	5	4	4	4	4	5
Stredni Morava	7	7	7	8	7	8	8
Moravskoslezsko	8	8	8	9	8	7	6
Bratislavsky kraj	2	2	2	2	2	2	2
Zapadne Slovensko	10	10	10	10	10	9	9
Stredne Slovensko	11	11	11	11	11	11	11
Vychodne Slovensko	12	12	12	12	12	12	12

Table 3: Rank of regions in years 2000 – 2006 based on additive pairwise comparisons transformed by function $\varphi^{-1}(b_{ij})$

Region / Year	*2000*	*2001*	*2002*	*2003*	*2004*	*2005*	*2006*
Praha	1	1	1	1	1	1	1
Stredni Cechy	3	3	3	3	3	3	3
Jihozapad	6	4	6	6	6	5	5
Severozapad	9	9	9	7	9	10	10
Severovychod	5	6	5	5	5	6	7
Jihovychod	4	5	4	4	4	4	4
Stredni Morava	7	7	7	9	7	8	8
Moravskoslezsko	8	8	8	8	8	7	6
Bratislavsky kraj	2	2	2	2	2	2	2
Zapadne Slovensko	10	10	10	10	10	9	9
Stredne Slovensko	11	11	11	11	11	11	11
Vychodne Slovensko	12	12	12	12	12	12	12

Table 4: Rank of regions in years 2000 – 2006 based on additive pairwise comparisons transformed by function $\phi^{-1}(b_{ij})$

Region / Year	*2000*	*2001*	*2002*	*2003*	*2004*	*2005*	*2006*
Praha	1	1	1	1	1	1	1
Stredni Cechy	3	3	3	3	3	3	3
Jihozapad	6	4	6	6	6	5	5
Severozapad	9	9	9	7	9	10	10
Severovychod	5	6	5	5	5	6	7
Jihovychod	4	5	4	4	4	4	4
Stredni Morava	7	7	7	9	8	8	8
Moravskoslezsko	8	8	8	8	7	7	6
Bratislavsky kraj	2	2	2	2	2	2	2
Zapadne Slovensko	10	10	10	10	10	9	9
Stredne Slovensko	11	11	11	11	11	11	11
Vychodne Slovensko	12	12	12	12	12	12	12

Source: Own elaboration and calculations, 2012

APPENDIX

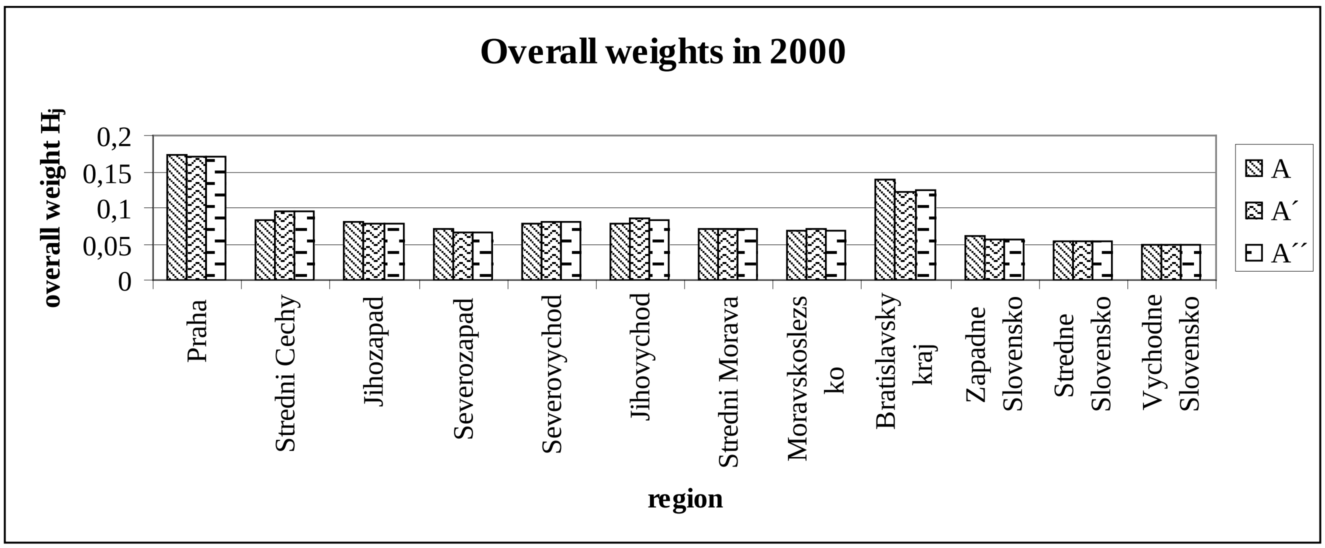

Fig. 8: Overall weights of regions in 2000 based on multiplicative and transformed additive pairwise comparisons

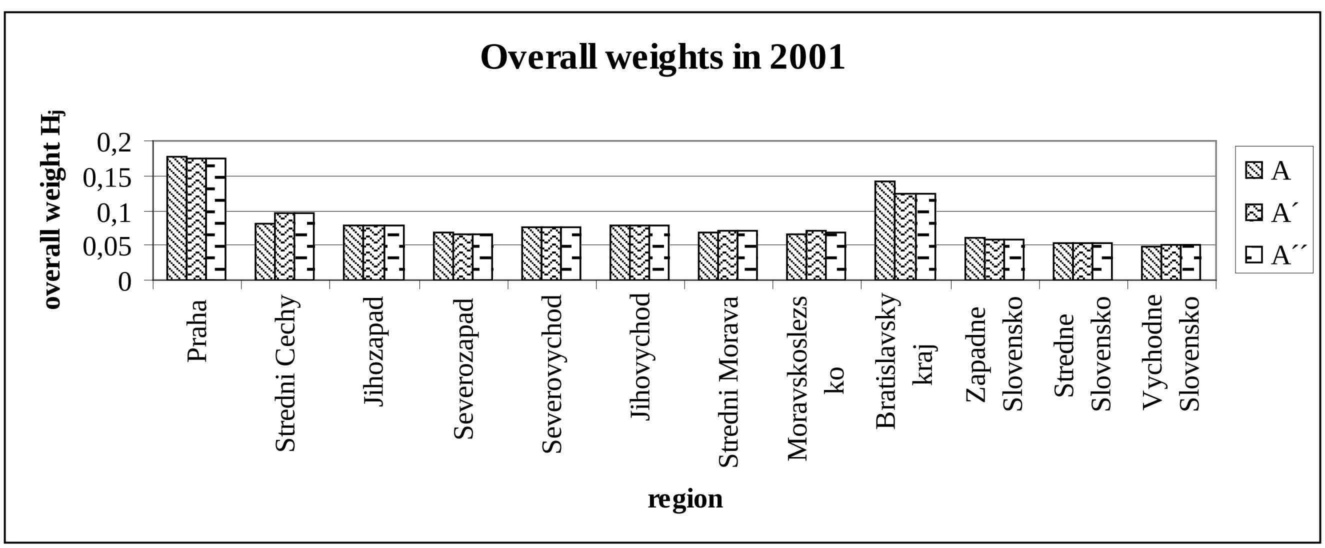

Fig. 9: Overall weights of regions in 2001 based on multiplicative and transformed additive pairwise comparisons

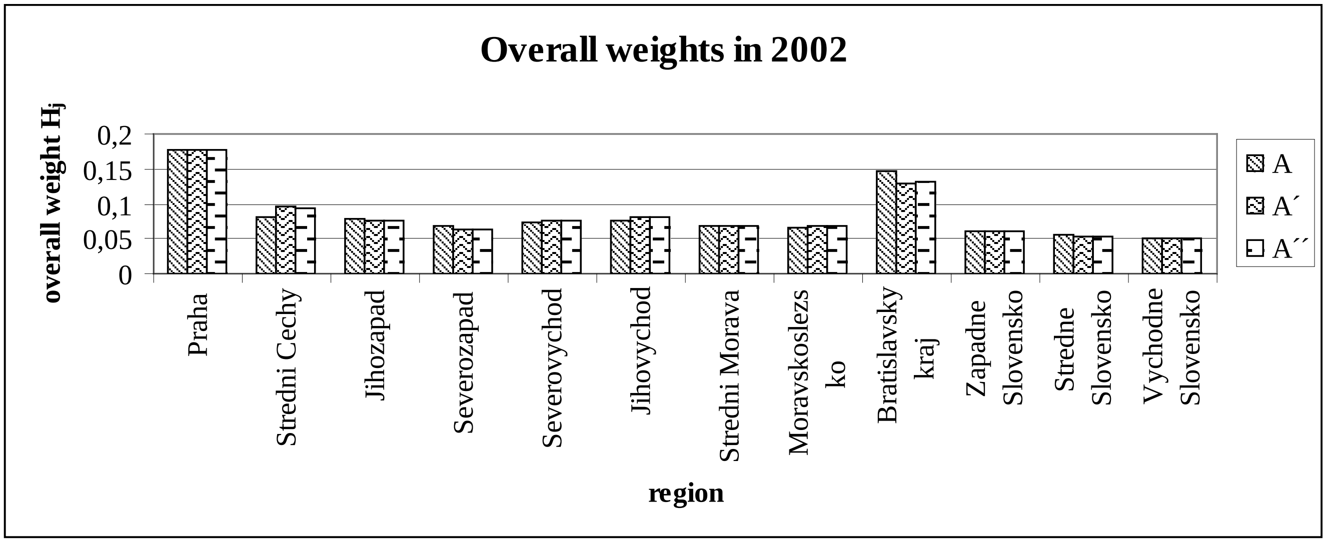

Fig. 10: Overall weights of regions in 2002 based on multiplicative and transformed additive pairwise comparisons

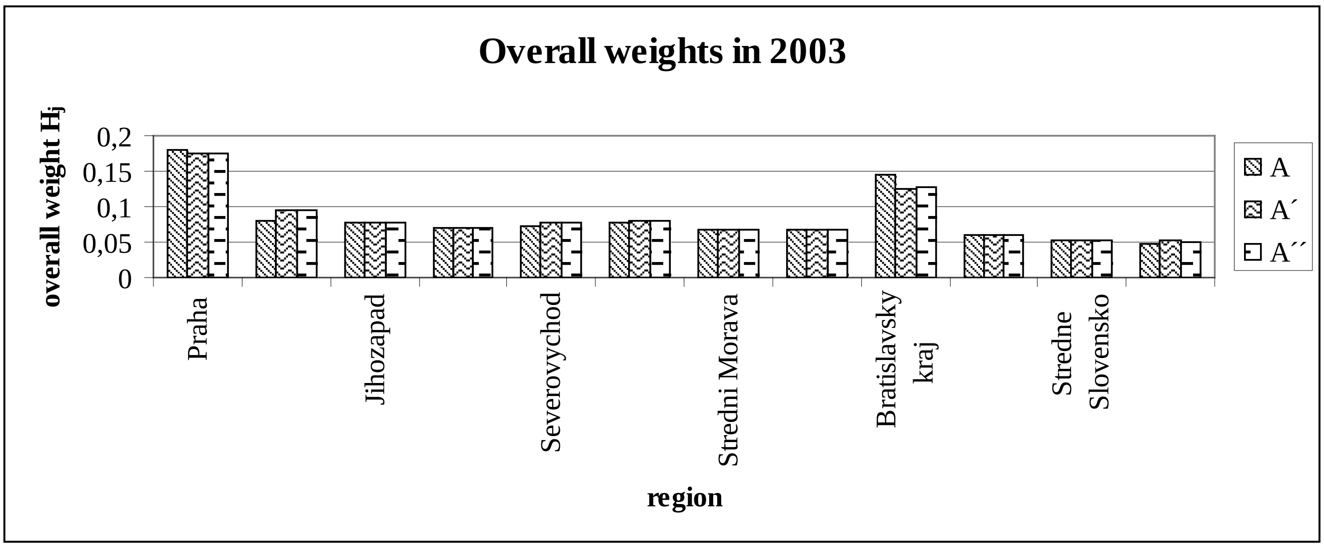

Fig. 11: Overall weights of regions in 2003 based on multiplicative and transformed additive pairwise comparisons

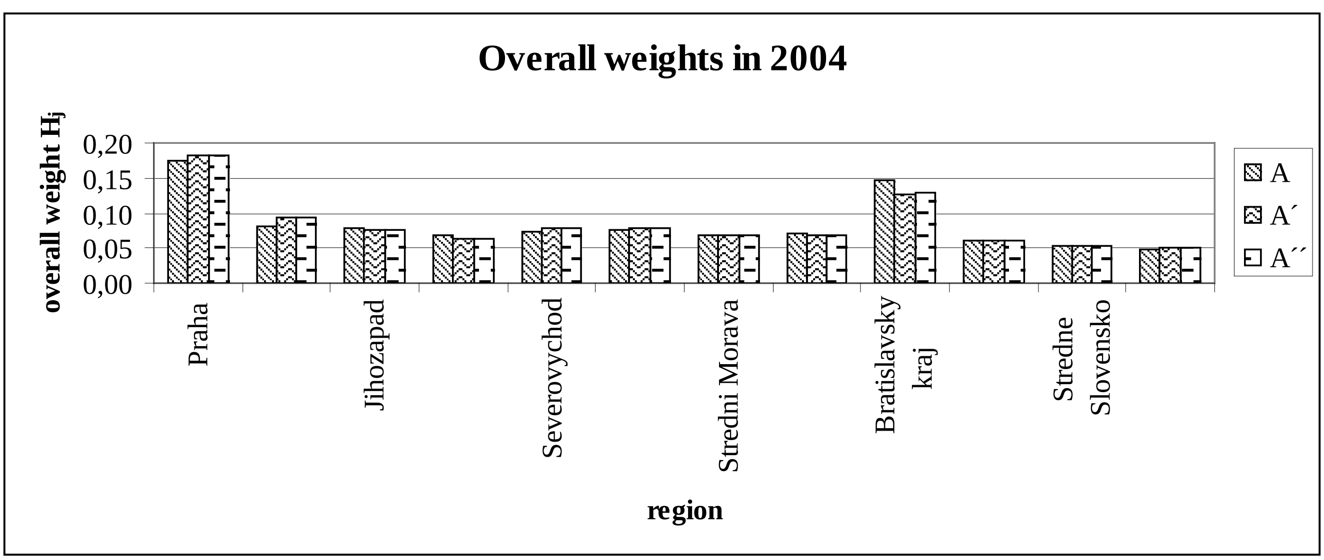

Fig. 12: Overall weights of regions in 2004 based on multiplicative and transformed additive pairwise comparisons

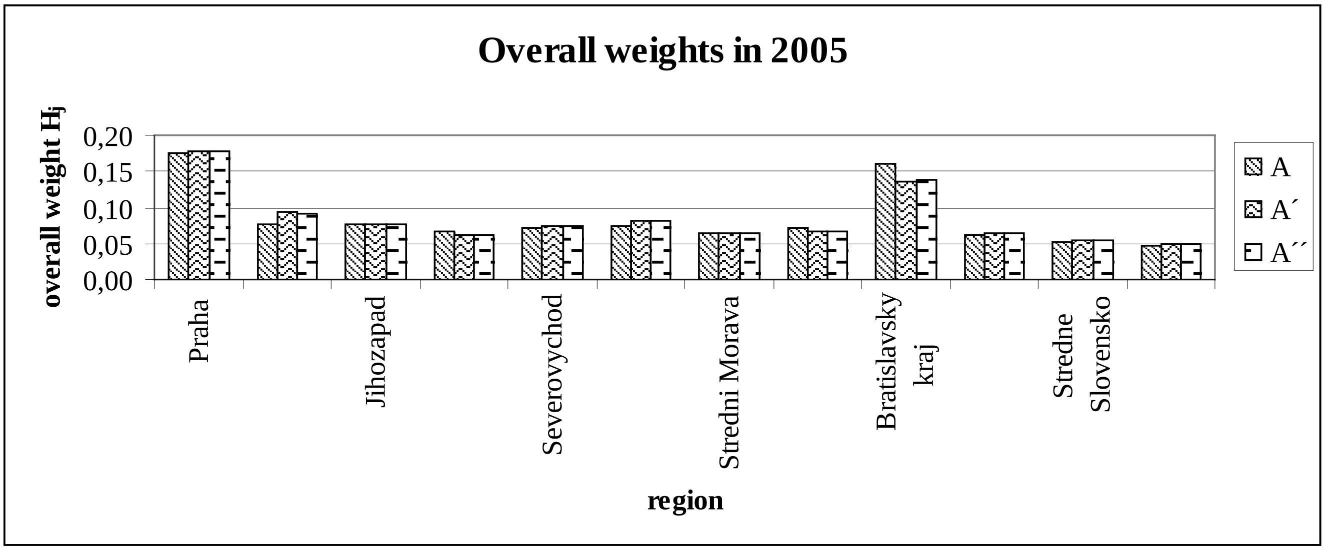

Fig. 13: Overall weights of regions in 2005 based on multiplicative and transformed additive pairwise comparisons

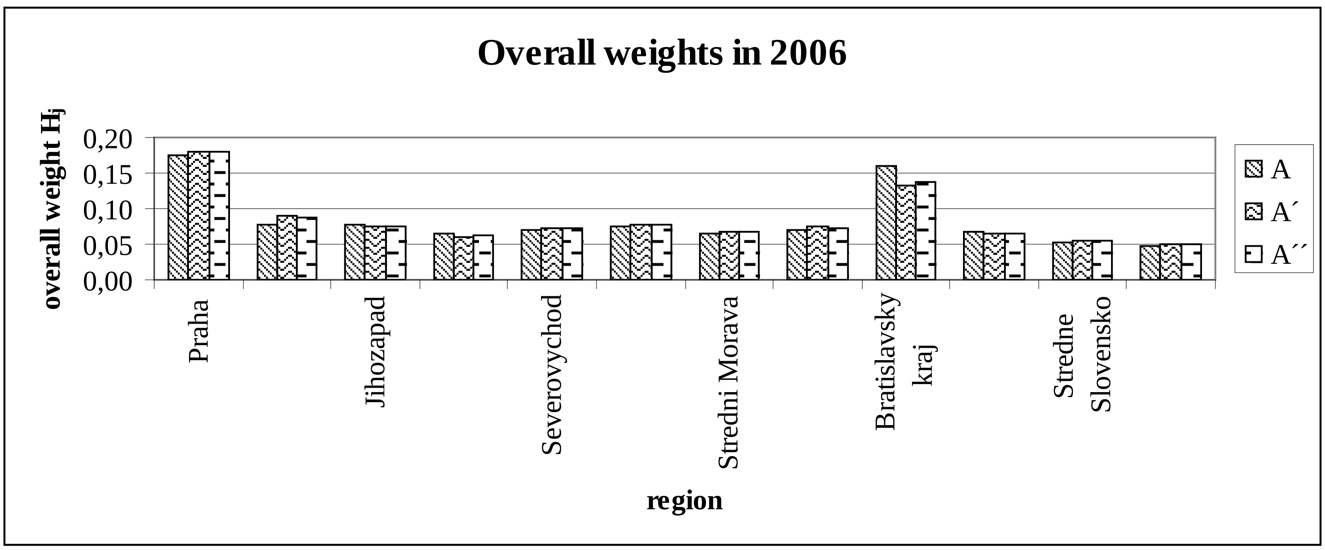

Fig. 14: Overall weights of regions in 2006 based on multiplicative and transformed additive pairwise comparisons
Source: Own elaboration and calculations, 2012

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