

# Single Mode Soil Yielding in Masonry Vaulted Structures

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**Abstract**— The paper addresses the problem of soil yielding in masonry vaulted structures, setting up a procedure for theoretical treatment of barrel vaulted constructions and forecast of structural response induced by local sinking or rotation of piles. This is a primary issue in handling masonry constructions since collapse mechanisms may be activated, rapidly leading to crisis and failure of the entire structure.

Effects of foundation rotation or subsiding should not be neglected since they may actually deeply affect the correct behavior of ancient masonry structures, and, in particular, of their vaulted elements, and are often able to compromise their static operation. Single rotational or translational motion modes are considered in the paper, by developing the numerical investigation and showing results from implementation in ad-hoc built in codes of the theoretical set up.

**Keywords**—Masonry, Vaults, No Tension material, Subsiding, Geometric constraints, Theoretical treatment, Response forecast, Numerical investigation.

## I. INTRODUCTION

**M**ONUMENTAL or historical masonry constructions often experience during time differential subsiding of soil, which causes some local imposed displacements at their geometrical external constraint locations.

Due to the behavior and common failure modes of vaulted structures, the activation of imposed displacements does represent a concrete risk, since it is able to compromise the overall behavior of the structure, reducing its safety margins or, in some case, causing its crisis.

Analysis of masonry constructions, because of the complexity of the behavior of the basic material, which is usually assumed to be unable to suffer tensile stresses by adopting the No Tension (NT) mechanical hypothesis, is a still open problem [1]-[21]. Available tools, although still require further developments, may be successfully used for analyzing the construction under the current environmental and loading

conditions, allowing to lead, under certain assumptions, to reliable forecasts on its behavior.

Things get more and more complicated according to the complexity of the geometry when dealing with vaulted surfaces with complex shapes .

The primary interest of the subject is also related to possible applications for the forecasting and protection of monumental buildings, by current reinforcements techniques [22]-[27] or, also, by means of dynamic control techniques [28]-[35], especially useful in the absence of environmental forecasts and for vulnerability assessment [36]-[41].

Specializing of theoretical settlement for investigating the structural response under differential foundation drifts can be performed in order to individuate the possible crisis conditions according to the occurred disease, and, starting from the monitored crack distribution, to recognize the causes.

## II. FOUNDATION DIFFERENTIAL SUBSIDING IN MASONRY CONSTRUCTIONS

As well known soil subsiding may often occur under the foundation of structures, because of a number of causes, ranging from liquid infiltrations in the ground to changes in transmitted loads, to loss of homogeneity in the soil mass composition, to residual effects after the occurrence of exceptional events related to soil break up and rearrangement of soil volumes, and so on.

The localized subsiding of the soil volumes devoted to absorb the stresses transmitted through the foundation structures is particularly insidious, mainly due to the circumstance that differential displacements occur causing changes in the stress distribution in the bodies of the construction and activation of kinematic mechanisms.

Actually the case of uniformly distributed soil sinking, which would result in lower damages to the structure basically causing its rigid downward motion and a crack distribution

related to the detachment of the structures from the surrounding elements, is rather rare.

Foundation motion modes are usually due to subsiding of the foundation level of the construction caused by the achievement of the soil strength limits or by strains in the soil, which, in turn, may be related to many sources relevant to the direct transmission of loads by the upper construction, to load changes due to excavations realized in the surrounding areas, to dynamic solicitations or presence of water sources.

Usually not uniform foundation motion occurs inducing a crack dislocation mainly assembled along the contact lines between the structural parts subject to foundation subsiding or not, denoting the relevant detachment.

A number of factors determine the final appearance of the crack scenario, essentially depending on the position of subsiding, its typology and intensity, on the typology of foundation (continuous or not), on the structure, the dislocation of openings in the structure, the materials and the quality of their mechanical characteristics, the possibly present reinforcements or stiffer members, and so on.

In masonry constructions, where usually a long and various loading history has been experienced by the structure during its life-cycle and that are characterized by geometrical or mechanical discontinuities in the masonry bodies, not proper homogeneity of materials, previously cumulated damages that result in poor mechanical characteristics of materials, elements with reduced strength, and weak points vaulted monads, the development of fractures follows the minor resistance lines. In this case, the risks of crisis should not be underestimate. Effects of peripheral sinking are more dangerous of those related to central soil subsiding, especially in masonry structures, where the cooperation between the structural elements is of essential relevance, and is further reduced in the case of peripheral motion.

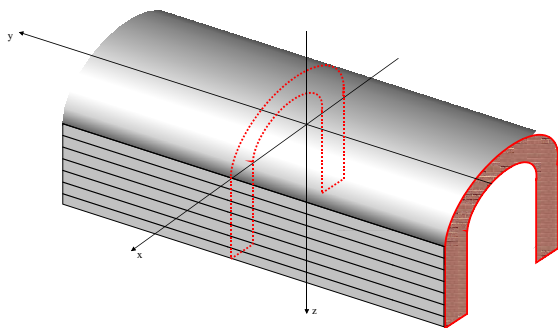


Fig.1: Barrel barrel vault with horizontal directrix.

### III. BARREL VAULTS MADE OF TUFF BRICKS

As regards to barrel vaults, first of all, one should consider that, since the vault geometrically derives by the translation along a directrix of a generating arch curve, in this case, the meridian lines coincide with the generatrix in their shapes; if one considers a rectilinear directrix, the vault parallels are

horizontal and rectilinear as well (Fig.1). The surface of the shell representing the mid-surface of the vault may be defined by the equation  $z = f(x)$ .

Because of the vault geometry, one has that

$$\begin{aligned} \theta = 0, \quad \tan \varphi &= \frac{\partial z}{\partial x}, \quad \tan \theta = \frac{\partial z}{\partial y} = 0 \\ ds_x &= \frac{dx}{\cos \varphi}, \quad ds_y = \frac{dy}{\cos \theta} = dy \\ dA &= ds_x ds_y = \frac{dx}{\cos \varphi} \frac{dy}{\cos \theta} = \frac{dx}{\cos \varphi} dy \end{aligned} \quad (1)$$

where  $ds_x$  and  $ds_y$  denote the length of the sides of the generic vault element ABCD of area  $dA$   $dx$  and  $dy$  the length of the corresponding sides on the element A'B'C'D' projected in the  $xy$ -plane, and  $\varphi$  and  $\theta$  denote the angles formed by the meridian sides AB and DC of the element with the  $x$ -axis and by the parallel sides AD and BC with the  $y$ -axis, respectively.

As concerns equilibrium, hypothesizing that the vault is in a membrane state of stress, a correspondence can be established between forces acting on the element ABCD (stresses  $N_x, N_y, N_{xy} = N_{yx}$  and applied load for unit area,  $p_x, p_y, p_z$ ) and projected forces acting on the associated element A'B'C'D' ( $\bar{N}_x, \bar{N}_y, \bar{N}_{xy} = \bar{N}_{yx}$  and  $\bar{p}_x, \bar{p}_y, \bar{p}_z$ ) in the  $xy$ -plane.

In absence of horizontal loads and if the vertical load is not dependent on "y", as it happens when the vault is subject to only vertical loads due to the self-weight (i.e.  $\bar{p}_z = \bar{p}_z(x) \geq 0$ ), and, additionally, assuming that the vault has an indefinite length in the direction  $y$ , equilibrium may be expressed in the form

$$\frac{\partial^2 \psi(y)}{\partial y^2} \frac{\partial^2 z}{\partial x^2} = -\bar{p}_z, \quad \frac{\partial^2 \psi(y)}{\partial y^2} = \bar{N}_x \quad (2)$$

which reduces the problem to the determination of stress function  $\psi(y)$ .

Assuming that the directrix curve of the vault is a circular arch (Fig.2) of radius  $R$ , with constant thickness "s" and unit weight  $\gamma$ , and imposing suitable constraint conditions, one yields the final solution

$$z(t) = -\gamma s \frac{R^2}{H} \left[ \arcsin(t) + \sqrt{1-t^2} + C \right] \quad (3)$$

with

$$C = -\left(1 + \frac{H z_0}{R^2 \gamma s}\right),$$

$$H = \gamma s \frac{R^2}{(z_1 - z_0)} \left[1 - t_1 \arcsin(t_1) - \sqrt{1 - t_1^2}\right] \quad (4)$$

where  $z_0$  and  $z_1$  are arbitrary ordinates, conditioned by the fact that  $z(t)$  should be contained in the interior of the profile of the vault.

After this result, it is possible to calculate the internal forces  $\bar{N}_x \leq 0, \bar{N}_y = \bar{N}_{xy} = 0$  and  $N_x \leq 0, N_y = N_{xy} = 0$

$$N_x = \frac{\bar{N}_x}{\cos \varphi} = \frac{H}{\cos \varphi} \quad (5)$$

It is also possible to realize that the equilibrium solution allows the structure to behave as a sequence of identical independent arches.

From this result, in the following one refers to the portal arch model, treated by the NT assumption, whose analytical problem implementation has been shown to give theoretical results in perfect agreement with experimental data, in order to forecast the structural response under assigned failure of geometrical environmental constraints due to soil sinking.

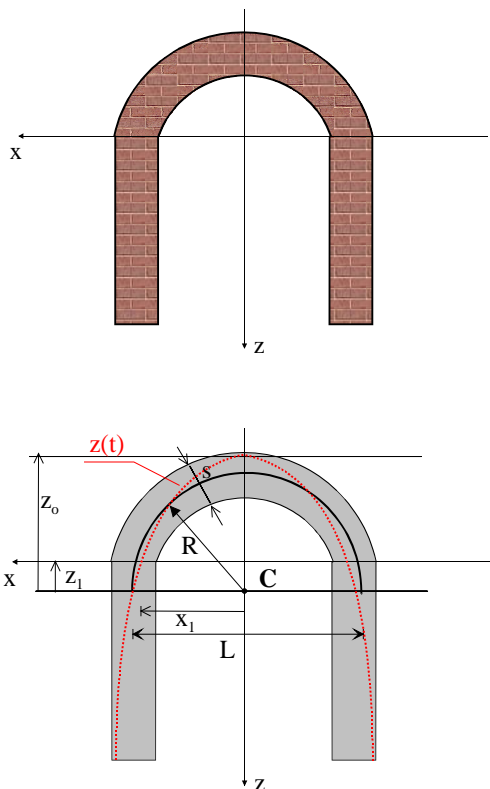


Fig.2: Cross section of a barrel vault with circular arch generatrix.

#### IV. NUMERICAL INVESTIGATION

##### A. Set Up of the Problem

Soil subsiding and rotation is a quite frequent cause of disease in masonry vaulted structures. Actually, due to the behavior and usual failure modes of masonry vaulted structures, which are mainly due to the activation of collapse mechanisms caused by the formation of a number of unilateral hinges properly placed on the structure, sinking or rotation of piles may quickly lead the structure to crisis, and, anyway, cause damage and subsequent degradation of its behavior.

According to what shown in the previous Sect.3, the analysis of masonry vaulted structure is quite complex, but, sometimes, for particular geometries, such as barrel vaults with indefinite length, it may be reduced to the analysis of a number of independent arches, allowing the theoretical treatment to be handled in a simpler way, because of the reduction to the plane geometry.

The problem is still complex as regards the analysis under active loads or partial failure of environmental geometric constraints, but additional simplifications are possible when addressing the problem of arch since mono-axial stress states may be assumed as regards to material admissibility, which considerably simplifies the problem.

Under the No-Tension hypothesis of the masonry material, set up and subsequent implementation in calculus codes of the relevant theoretical problem allows to give reliable estimates of local stresses, strains and displacements at any point of the structure.

The code is aimed at the numerical search of the constrained minimum of the complementary energy functional calculated with reference to the F.E.M. model of the structure, by assuming as primary variables the stresses, which are required to be satisfying equilibrium with applied loads and NT material admissibility.

In the following some results are shown from numerical investigation implementing the problem in the case of an elliptical masonry vaulted portal, subject or not to a single rotational or translational motion mode of the left pile caused by differential soil sinking under the structure, and resulting in some kinematic mechanism related to the occurred rotation or downward skid.

##### B. Numerical analysis of the portal arch in absence of foundation motion

In the following one first considers a masonry portal arch with span of 2.04 m, rise of 1.02 m and thickness of the vault of 0.3 m, subject to its own weight in the absence of soil sinking, and modeled according to the NT assumption.

In the set of Fig.3-9 some sketches are reported captured after the running of the calculus code, built on purpose for implementing the problem.

With reference to the portal arch subject uniquely to its self-weight, some stress solutions that are statically admissible do exist, which are represented by funicular lines included in the arch profile, which means that no collapse occurs under

applied loads.

This is clear from Fig.3 that reports one statically admissible stress field found (and captured in the picture) during the running of the calculus code.

The stress solution for the problem is represented through the corresponding funicular line in solution traced in Fig.9, where one may check that no mechanism is occurring under the applied loads since the resultants line is at any cross-section included in the portal arch profile.

The diagrams of rotations (Fig.4), curvatures (Fig.5), bending moments (Fig.6), compressive stresses (Fig.7), deformed configuration (Fig.8) and crack distribution (Fig.9) give an exhaustive and complete representation of results coming out from the developed numerical analysis.

As one can observe from Fig.8 and Fig.9, some deformation of the portal occurs with the formation of some cracks that can be considered as typical of the phenomenological behaviour of masonry structures, that, in order to absorb stresses, develop a fracture strain field that superposes to the elastic strain field ensuring compatibility with the displacement field, according to the NT model.

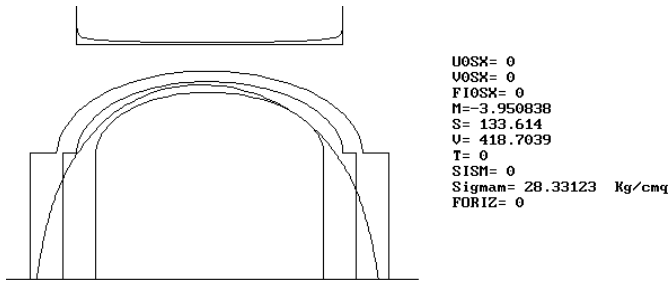


Fig.3. Sketch from calculus code.

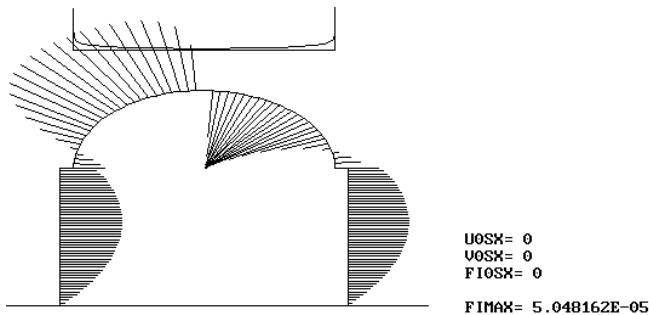


Fig.4. Diagram of rotations reported on the mid-line of the NT model.

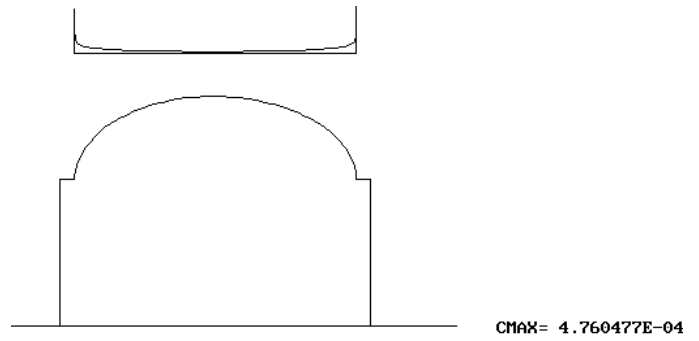


Fig.5. Diagram of curvatures reported on the mid-line of the NT model.

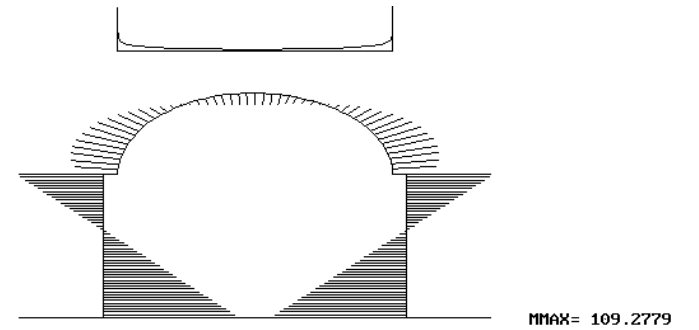


Fig.6. Diagram of bending moments reported on the mid-line of the NT model.

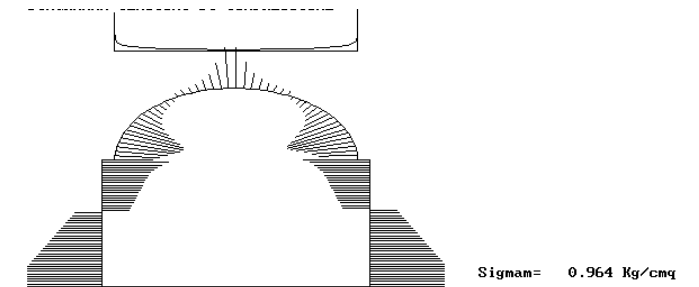


Fig.7. Diagram of compressive stresses reported on the mid-line of the NT model.

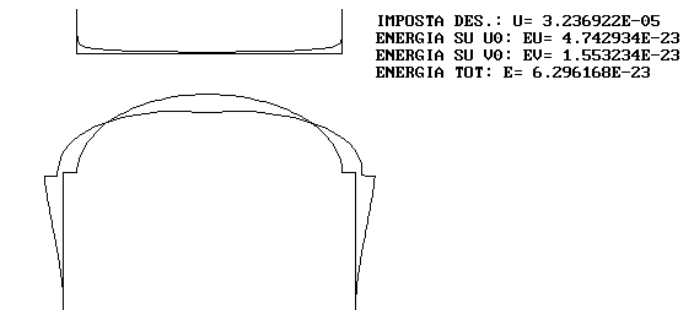


Fig.8. Deformed configuration referred to the mid-line of the NT model.

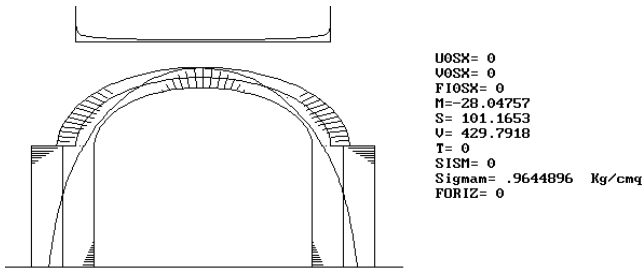


Fig.9. Funicular line.

C. Numerical analysis of the portal arch under foundation rotational mode

Rotational foundation motion may be induced by differential drifts of the foundation level, or by an horizontal thrust, causing detachments between different parts of the structure or within the single structural element.

The crack distribution follows a path that depends on a number of factors and also on the degree of cooperation between the construction components.

Vaulted structural members, because of their own nature and operation mode, are more sensitive to the release of some of the original environmental kinematic constraints of the structure, which may lead to the loss of mutual contrast among voussoirs or macro-elements composed by sets of voussoirs, activating failure mechanisms.

Foundation rotational motions are usually characterized, accompanied and preceded by the occurrence of a number of phenomena such as detachments, out of plane rotations, drifts and differential rotations.

In the following some results are shown from numerical investigation implementing the problem in the case of the NT portal arch referred in the previous Sect.3.2, subject to its self-weight and a soil rotational motion at the foot of its left pile of  $\phi_0 = 0.03^\circ$ .

In Fig.10-16 some sketches are reported, captured after the running of the calculus code, built on purpose for implementing the problem under the assigned imposed counter-clockwise rotation  $\phi_0$  of the left pile.

By analysing the solution funicular line traced in Fig.16, one may observe that it touches the extrados and intrados profiles of the arch at 3 cross-sections, at the extrados keystone and at the intrados impostos of the arch, thus allowing, as clear from the diagram of curvatures represented in Fig.12, the activation of three unilateral hinges at the corresponding locations. At those locations some unilateral pivotal points are formed allowing relative rotations between the macro-elements into which the arcade is disarticulated.

The stress resultants at those points, in a ideal model, are then allowed to pass uniquely through those contact points between the masonry elements, thus assuming compressive stresses a theoretically infinite intensity due to the restricted (punctual) area of contact between elements. This assertion is confirmed, for the specific case, by the diagram of compressive stresses in Fig.14.

Therefore, mechanisms involving the arcade failure are

possible as clear from Fig.15, where the deformed configuration of the portal arch under the assigned rotational subsiding of the left pillar is shown, amplified by a magnification coefficient for improving the readability of the mechanism.

One can observe that, due to the induced rotation of the left pile, the failure mechanism of the arcade involves the formation of the above described hinges, and the crack distribution represented in Fig.16, with cracks interesting the intrados of the arcade in the area surrounding the keystone, and the extrados areas close to the impostos.

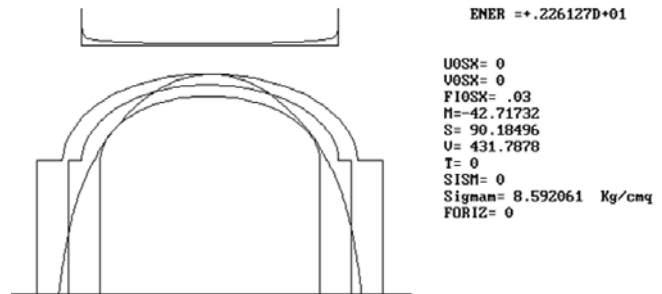


Fig.10. Sketch from calculus code.

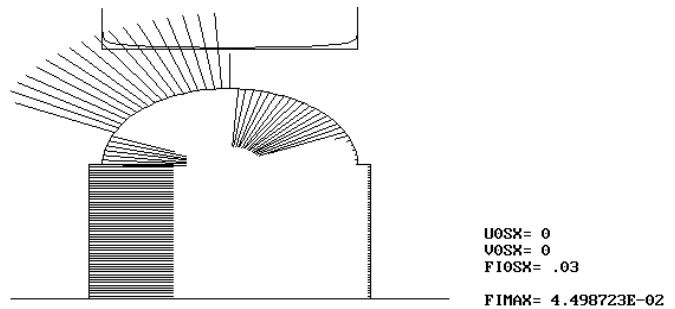


Fig.11. Diagram of rotations reported on the mid-line of the NT model.

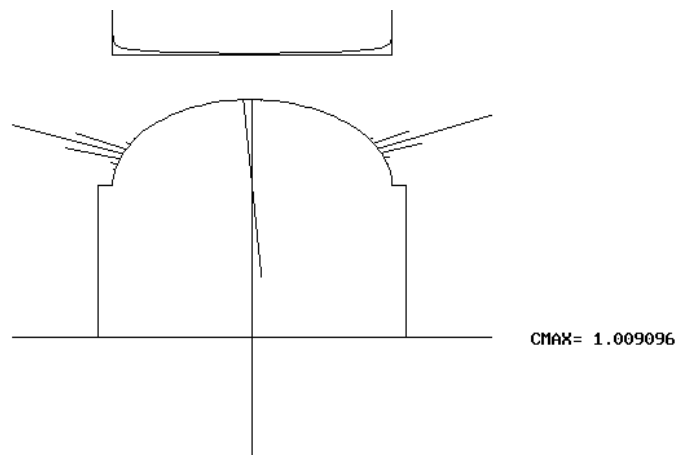


Fig.12. Diagram of curvatures reported on the mid-line of the NT model.

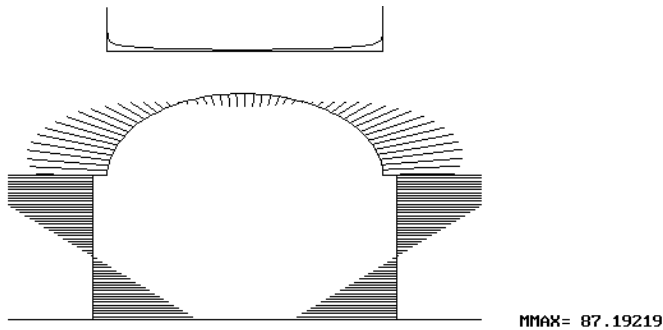


Fig.13. Diagram of bending moments reported on the mid-line of the NT model.

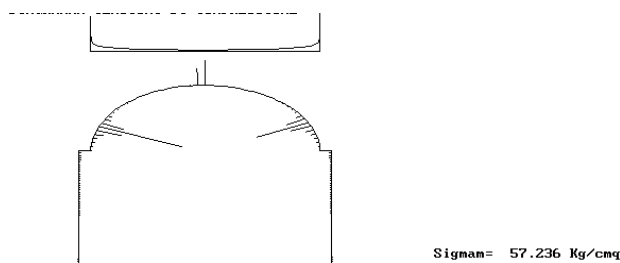


Fig.14. Diagram of compressive stresses reported on the mid-line of the NT model.

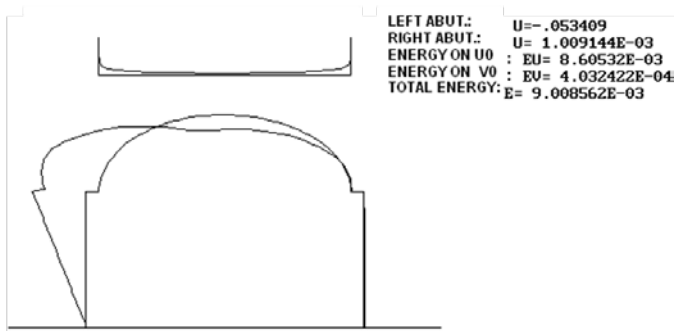


Fig.15. Deformed configuration referred to the mid-line of the NT model.

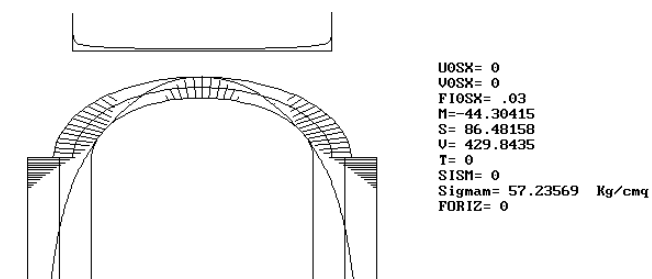


Fig.16. Funicular line.

#### D. Numerical analysis of the portal arch under translational subsiding

Downward translational motion of the structure or of its parts, or part of a single structural element may be caused by

breakup and fluidification of the soil due to liquid infiltrations, excessive compressibility of the foundation soil, execution of dig works in the building proximities, compressive crushing of the foundation masonry structures, increase in the transmitted loads and so on.

Also in this case the danger potential of this type of disease is rather high for masonry structures in general, and, in particular, for their vaulted structures.

In the following some results are shown relevant to the case of translational soil sinking causing lowering at the bottom of the left pillar of the portal arch with intensity  $v_0 = 0.08$  m.

In Fig.17-23 some sketches are reported, captured after the running of the calculus code, built on purpose for implementing the problem under the assigned imposed downward skid  $v_0$  of the left pile.

By analyzing the funicular line traced in Fig.23, one may observe that a different and asymmetrical dislocation of the hinges occur, since the stress resultants line touches the extrados and intrados profiles of the arch at the 3 cross-sections approximately corresponding to the intrados right impost of the arcade, to the extrados left rein of the arcade and to the extrados foot section of the left pillar.

As clear from the diagram of curvatures represented in Fig.19, the activation of three unilateral hinges occur at the corresponding locations, where some unilateral pivotal points are formed allowing relative rotations between the macro-elements into which the arcade is disarticulated.

This assertion is confirmed, for the specific case, by the diagram of compressive stresses in Fig.21.

Therefore, the mechanism involving the arcade and left pillar is possible shown in Fig.22, where the deformed configuration of the portal arch under the assigned translational subsiding of the left pillar is captured, amplified by a magnification coefficient for improving the readability of the mechanism.

The crack scenario is represented in Fig.23, with cracks interesting the extrados of the arcade in the area surrounding its right impost, and the intrados areas close to the arcade right rein and to the foot of the right pillar.

## V. CONCLUSION

The paper focuses on the possibility of forecasting the behavior of barrel masonry vaults under soil differential subsiding causing imposed rotations and skid at the foot of pillars. This is a phenomenon frequently occurring also in multi-span masonry constructions such as historical bridges, which may be recognized a common source of disease for masonry structures, in general, and for, vaulted constructions, in particular. The possibility of producing reliable forecasts allows to check safety margins on existing structures which have already experienced such disease and to give some estimate of response variables intensities throughout the structure.

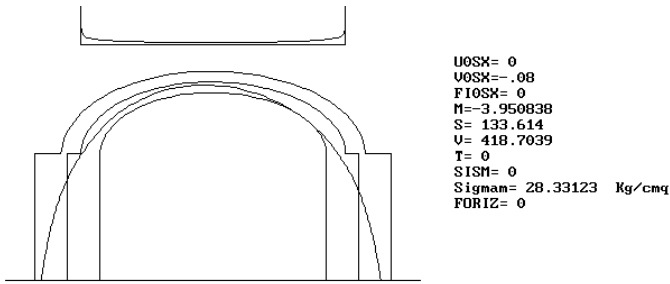


Fig.17. Sketch from calculus code.

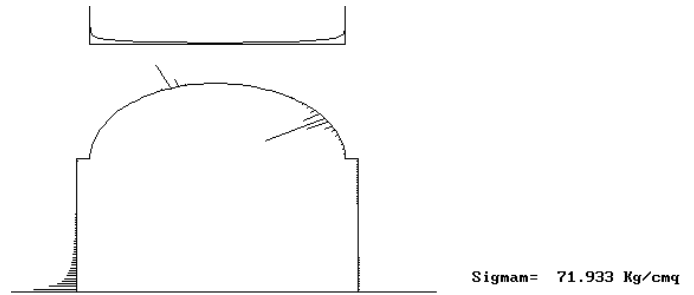


Fig.21. Diagram of compressive stresses reported on the mid-line of the NT model.

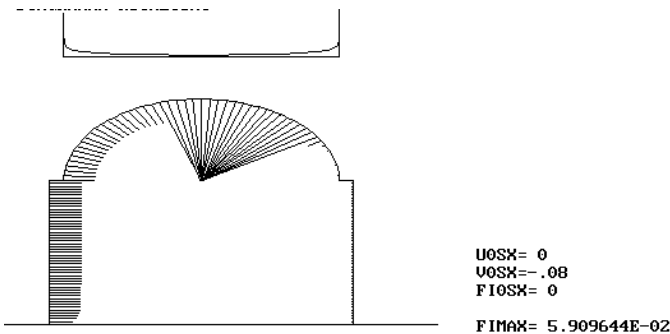


Fig.18. Diagram of rotations reported on the mid-line of the NT model.

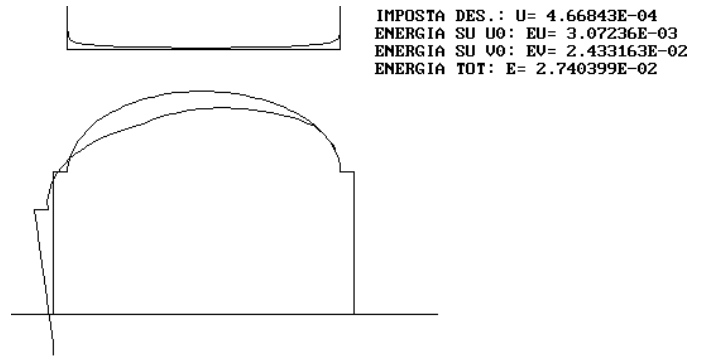


Fig.22. Deformed configuration referred to the mid-line of the NT model.

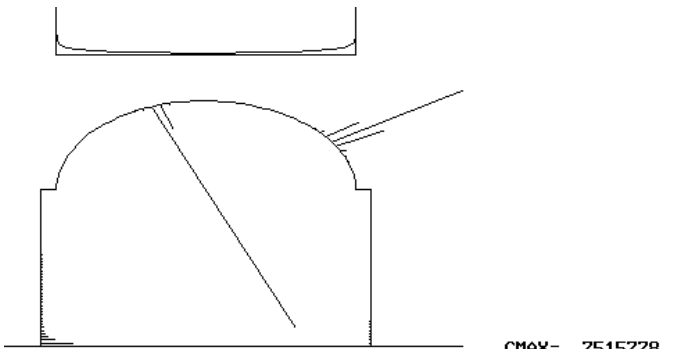


Fig.19. Diagram of curvatures reported on the mid-line of the NT model.

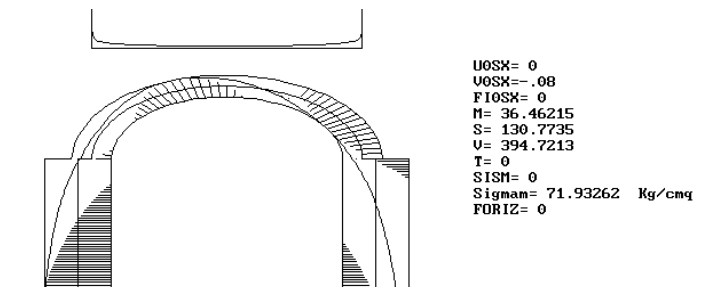


Fig.23. Funicular line.

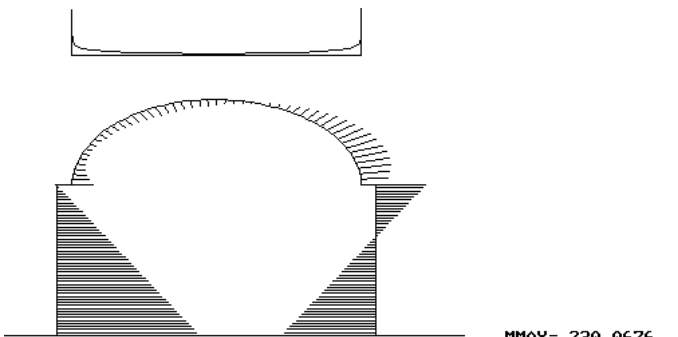


Fig.20. Diagram of bending moments reported on the mid-line of the NT model.

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