Mathematical modeling for the influence of initial stress on the reflection of plane waves at thermo-piezoelectric medium

Fatimah A. Alshaikh [1] and Abo-el-nour N. Abd-alla [2]

Abstract—This paper investigates the influence of the initial stress on reflection of plane waves at a perfect interface between an anisotropic thermo-piezoelectric medium and the vacuum in the context of Lord-Shulman theory. The equations of elastic waves, heat conduction equation, quasi-static electric field, and constitutive relationships for the thermo-piezoelectric medium are obtained.An analytical solution based on the wave theory is developed to obtain the reflection coefficients using the elastic and electric continuity conditions across the interface are satisfied simultaneously.The study shows that there exist independent wave modes satisfying the general Snell's law and propagating along the interface for the incident wave angle, namely quasi longitudinal (*QP*) wave, quasi shear vertical (*QSV*) wave and quasi thermal (*QT*) wave. The expressions for the reflection coefficients of quasi plane waves are investigated where three relations between the reflection coefficients are obtained.A particular model is chosen for the numerical computations of reflection coefficients. Effects of anisotropy and thermal relaxation time, initial stress as well as parameters of electric potential are observed on reflection coefficients. This study is relevant to signal processing, sound systems, wireless communications, surface acoustic wave (SAW) devices and military defense equipment.

Keywords—Thermo-piezoelectricity; Thermal relaxation time; Reflection coefficients; Hexagonal crystals; Cadmium Selenide; quasi-*P* waves; quasi-*SV* waves.

Nomenclature

$E_i = -\varphi_{,i}$ is the electric field,
u_i, φ, and T are the mechanical displacement, electric potential and absolute temperature,
D_i is the electric displacement,
$\sigma_{ij}, \sigma^o_{kj}$ are the stress and initial stress tensors,
ε_{ij} is the strain,
γ_{ij} is the thermal elastic coupling tensors
ρ is the density of the medium,
C_{ijkl} is the elastic parameters tensor,
e_{ijk} is the piezoelectric constants,
P_{ij} is the dielectric moduli,
d_i is the pyroelectric moduli,
t_o are thermal relaxation times,
K_{ij} is the heat conduction tensor,
T_o is the reference temperature.
C_E are the specific heat at constant strain,
δ_{ik} is the Kronecker delta

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I. INTRODUCTION

Thermo-piezoelectric ceramics and composites have been extensively used in many engineering applications such as sensors, actuators, intelligent structures, etc. The response of a thermo-piezoelastic material entails an interaction of mechanical, thermal and electrical properties. One application of thermo-piezoelectric materials is to detect the response of a structure by measuring the electric charge (sensing) or to reduce excessive responses by applying an additional electric charge or a thermal load (actuating). If sensing and actuating can be integrated smartly, a so-called intelligent structure can be designed. Piezoelectric materials are often used as resonators whose frequencies must be precisely controlled. Because of the coupling between the thermoelastic and pyroelectric effects, it is important to quantify the effect of heat dissipation on the propagation of a wave at low and high frequencies [36], [40], [49] and [52].

The theory of thermo-piezoelectricity was first proposed by Mindlin [29]. He also derived governing equations of a thermo-piezoelectric plate [30]. The physical laws for the thermo-piezoelectric materials have been explored by Nowacki [32] and [33]. Chandrasekharaiah [10], [11], [12] has generalized Mindlin's theory of thermo-piezoelectricity to account for the finite speed of propagation of thermal disturbances. Pal [35] studied surface waves in a thermo-piezoelectric medium of monoclinic symmetry. Abd-allaand Alsheikh [3] and [4]investigated the effect of the initial stresses on the reflection and transmission of plane quasi-vertical transverse or quasi longitudinal waves in piezoelectric materials.

Since 1970, composites consisting of piezoelectric and piezomagnetic materials have attracted interest because they possess a novel magneto-electric effect [49]. Many papers on experimental fabrication and theoretical predictions of the effective material constants for magneto-electric composites have been published [1], [2], [19], [34], [41], [46]. Furthermore, piezoelectric ceramics and composites have been extensively used in many engineering applications such as sensors, actuators and intelligent structures [36]. The mechanics of these so-called smart materials thus have attracted considerable academic attention [40], [44], [46].

Thermoelasticity deals with the dynamical systems whose interactions with surroundings include not only mechanical work and external work but also exchange of heat. Biot [9] explained thermoelasticity by deriving dilatation based on the thermodynamics of irreversible process and coupling it with elastic deformation. But the diffusion type heat equation used in this study predicted infinite speed for propagation of thermal signals. Lord and Shulman [21] defined the generalized theory of thermoelasticity in which a hyperbolic equation of heat conduction with a relaxation time ensured the finite speed for thermal signals. Using two relaxation times, Green and Lindsay [16] developed another generalized theory of thermoelasticity. Dhaliwal and Sherief [15] extended Lord and Shulman [21] generalization of thermoelasticity for anisotropic case. Chandrasekhariah [11] presented a review of work done in the theory of thermoelasticity. The thermoelasticity has wide applications in various fields such as earthquake engineering, soil dynamics, aeronautics, astronautics, nuclear reactors, high energy particle accelerator, etc. Thermoelasticity is also used in polymer coating and to evaluate the stress redistribution in ceramic matrix composites, for example see Mackin and Purcell [26].

The topics of anisotropic elastic solids, generalized theories of thermoelasticity, piezoelectricity and the coupling between the thermoelastic and pyroelectric effects can be found in many papers and monographs. It is impossible to embrace all the related literature on this matter; some examples are: Achenbach [7], Auld [8], Nayfeh [31], Parton, and Kudryavtsev [36], Royer, and Dieulesaint [40], Wu [49], Yang [50-51], andYe [52]. In addition to that, a study on the total internal reflection of nano-acoustic waves (NAWs) at the air/GaN interface presented by Hsieh et al. [20]. Keith and Crampin [22] derived a formulation for calculating the energy division among waves generated by a plane wave incident to a boundary between generally anisotropic media. A comprehensive account is presented for *P, SV* and *SH* waves incident to an isotropic half-space on an orthorhombic olivine half-space. Kumar et al. [24] studied wave propagation at the boundary surface for elastic and initially stressed visco-thermoelastic diffusion in media with voids. Furthermore, Song et al. [45] used coupled generalized thermoelastic equations with thermal relaxation time and plasma theory to study the reflection problem at the surface of a semi-infinite semiconducting medium during a photothermal process.

Some studies are relevant to the topics under consideration. Quiligotti et al. [39] inspect a particular generalized continua constituted by porous deformable solids infused with an inviscid compressible fluid.Placidi et al. [37] also study the propagation of bulk transversal and longitudinal waves and the influence of pre-stresses, deriving evolution equations through a variational approach. dell'Isola et al. [13] starting from an extended Hamilton-Rayleigh principle, define a general set of boundary conditions at fluid-permeable interfaces between dissimilar fluid-filled porous matrices. Madeo and Gavrilyuk [27] study propagation of linear waves impinging at a pure-fluid/porous-medium interface and deduce novel expressions for the reflection and transmission coefficients.dell'Isola et al. [14] study plane waves and their reflection and transmission at plane displacement discontinuity surfaces, for a specific class of second gradient continua.Madeo et al. [28] develop for deformable porous media the aforementioned model by dell'Isola et al. [14] and describe the effects of confined fluid streams on compression wave propagation and on reflection and transmission at an interface due to a porous matrix discontinuity. In second gradient porous media,Placidi et al. [38] explore reflection and transmission of compression and shear waves at structured interfaces between second-gradient continua.

Latterly, the subject of reflection plane waves at the interface between a semi-infinite pyroelectric medium and a vacuum are studied by many authors. Distinguished among them are: Guo [18] derived the complete set of uncoupled thermo-electromagnetic wave equations for piezoelectric solids, and a new phenomenon of thermo-electromagnetic waves with low propagation speeds was obtained. Kumar and Rupender [23] discussed the reflection of plane waves at the free surface of an electro-microstretch generalized thermoelastic solid. Sharma et al. [42] discussed the reflection of quasi-longitudinal, quasi-transverse, and thermal waves from a stress-free, thermally insulated or isothermal open-circuit boundary of a transversely isotropic, piezo-thermoelastic half-space under the influence of thermal relaxation. Singh [43], [44] contributed significantly on this subject. For instance, he discussed the reflection coefficients of various reflectedwaves for plane wave propagation in generalized thermoelasticity theory for piezoelectric materials. In addition, he investigated wave propagation in a pre-stressed, piezoelectric half-space. Tomar and Khurana [47] presented thereflection phenomenon of planeelasticwaves from a stress-free plane boundary of an electro-microelastic solid half-space. They obtained the amplitude ratios and the energy ratios of various reflectedwaves when an elasticwave is incident obliquely at the stress-free plane boundary of an electro-microelastic solid half-space. Abd-alla et al. [5] discussed the reflection phenomena for the propagation of plane, vertical and transverse waves at an interface of a semi-infinite piezoelectric elastic medium under the influence of initial stresses. Abd-alla et al. [6] investigated the reflection and the refraction of waves in nano-smart materials, or anisotropic thermo-piezoelectric materials. Gevorgyan and Rafayelyan [17] considered reflection of light from half-space of anisotropic metamaterial at arbitrary direction of optical axis in the plane of light incidence. Kumar and Kaur [25] investigated the problem of reflection and refraction of waves at the interface of an elastic solid and microstretchthermoelastic solid with micro-temperatures. Vashishth and Sukhija [48] formulated a mathematical model for the propagation of harmonic plane waves at the boundary of an anisotropic piezo-thermoelastic medium and solved it for slowness surfaces. Yuan et al. [53] analyzed the reflection and the refraction of plane waves at a perfect interface between two anisotropic piezoelectric media.

In this paper, reflection of the plane waves from stress-free thermally insulated surface of a transversely isotropic thermo-piezoelectric half-space is studied. Reflection coefficients of various reflected waves are obtained and studied numerically for a particular model. It was noticed that the amplitude ratios of the reflection coefficients of plane quasi-longitudinal waves (*QP*-waves) depend upon the angle of incidence, the parameters of the electrical potential, the material constants of the medium, the thermal parameters, the thermal relaxation times of the medium as well as the initial stress. The reflection coefficients were computed for the thermo-piezoelectric material Cadmium Selenide (CdSe) using the Gaussian elimination procedure. The graphs of the results show the significant effects of the thermal and electrical potentials and the anisotropy of the materials on the various reflection coefficients.

II. GOVERNING EQUATIONS

Following Lord and Shulman [21] and Sharma and Walia[42], the constitutive relations and hexagonal thermopiezoelectric equations under initial stress and relaxation time effect for two dimensions motion are given by:
Equation of motion and Gauss's equation

$$\left.\begin{aligned}\sigma_{ij,j} + \left(u_{ik}\sigma_{kj}^{o}\right)_{,j} &= \rho\ddot{u}_i,\\ D_{i,i} &= 0\end{aligned}\right\}(1)$$

Heat conduction equation

$$K_{ij}T_{,ij} = T_o\left[\gamma_{ij}\left(\dot{u}_{i,j} + t_o\delta_{ik}\ddot{u}_{i,j}\right) - d_{ij}\left(\dot{\varphi}_{,i} + t_o\delta_{ik}\ddot{\varphi}_{,j}\right)\right] + \rho C_E\left(\dot{T} + \delta_{ik}\ddot{T}\right)(2)$$

The strain-displacement relation and the electric field according to the quasi-static approximation have the forms as:

$$\left.\begin{aligned}\varepsilon_{ij} &= \tfrac{1}{2}\left(u_{i,j} + u_{j,i}\right),\\ E_i &= -\varphi_{,i}\end{aligned}\right\} i,j = 1,2,3.(3)$$

Stress-strain-temperature and electric field relations

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl} - e_{kij}E_k - \gamma_{ij}T \qquad (4)$$

The relation between the electric displacement, strain, electric field, and temperature

$$D_i = e_{ikl}\varepsilon_{kl} + P_{ik}E_k + d_iT \qquad (5)$$

III. SOLUTION OF THE PROBLEM FOR INCIDENT *QP*-WAVE

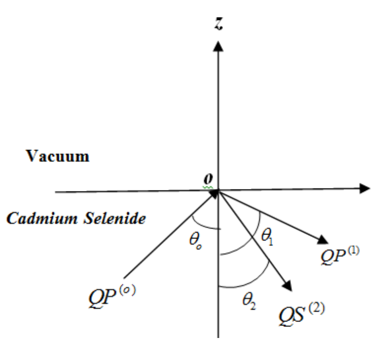

Figure 1

We consider a thermopiezoelectric plane wave *QP*propagating under initial stress through the medium, which we identify as the region $z \le 0$ and falling at the plane z = 0, with its direction of propagation making an angle θ_o with the normal to the surface. Corresponding to incident wave, we get the waves in mediumas reflected *QP*-, *QS*-, φ- and T- waves. The complete geometry of the problem is shown in Figure 1. Let the wave motion in this medium be characterized by the displacement $\vec{u}(u,0,w)$, the temperature T vector, and the electric potential functionφ, all these quantities being dependent only on the variablesx, z, t. We assume solution of the form (Achenbach [7]):

$$(u^{(o)}, w^{(o)}, \varphi^{(o)}, T^{(o)}) = (A_o\sin\theta_o, A_o\cos\theta_o, B_o, C_o)\exp[\vartheta_o]$$
$$(u^{(1)}, w^{(1)}, \varphi^{(1)}, T^{(1)}) = (A_1\sin\theta_1, -A_1\cos\theta_1, B_1, C_1)\exp[\vartheta_1]$$
$$(u^{(2)}, w^{(2)}, \varphi^{(2)}, T^{(2)}) = (A_2\cos\theta_2, A_2\sin\theta_2, B_2, C_2)\exp[\vartheta_2] \qquad (6)$$

where

$$\vartheta_0 = ik_o(x\sin\theta_o + z\cos\theta_o - C_{Lo}t),$$
$$\vartheta_1 = ik_1(x\sin\theta_1 - z\cos\theta_1 - C_{L1}t),$$
$$\vartheta_2 = ik_2(x\sin\theta_2 - z\cos\theta_2 - C_{T2}t),$$

Also, here $C_{L0} = \omega/k_0, C_{L1} = \omega/k_1, C_{T2} = \omega/k_2$are the velocity of incident *QP*, reflected *QP*, reflected *QS* wave.

IV. THE BOUNDARY CONDITIONS

The free mechanical boundary conditions

$$\sigma_{zx}^{(O)} + \sigma_{zx}^{(1)} + \sigma_{zx}^{(2)} = 0 \quad (7)$$
$$\sigma_{zz}^{(O)} + \sigma_{zz}^{(1)} + \sigma_{zz}^{(2)} = 0 \quad (8)$$

The electrical condition

$$\varphi^{(O)} + \varphi^{(1)} + \varphi^{(2)} = 0 \quad (9)$$

The thermal condition

$$T^{(O)} + T^{(1)} + T^{(2)} = 0 \quad (10)$$

Using equations (3-4), and (6) of hexagonal (6 mm) crystals into equations (7)-(10), we obtain the following set of equations:

$$ik_o[C_{44}\sin 2\theta_o A_o + e_{15}\sin\theta_o B_o]\exp[\vartheta_0] - ik_1[C_{44}\sin 2\theta_1 A_1 - e_{15}\sin\theta_1 B_1]\exp[\vartheta_1] - ik_2[C_{44}\cos 2\theta_2 A_2 - e_{15}\sin\theta_2 B_2]\exp[\vartheta_2] = 0(11)$$

$$ik_o[(C_{13}\sin^2\theta_o + C_{33}\cos^2\theta_o)A_o + e_{33}\cos\theta_o B_o - (\gamma_3/ik_o)C_o]\exp[\vartheta_0] - ik_1[(C_{13}\sin^2\theta_1 + C_{33}\cos^2\theta_1)A_1 - e_{33}\cos\theta_1 B_1 - (\gamma_3/ik_1)C_1]\exp[\vartheta_1] + ik_2[(C_{13} - C_{33})sin\theta_2\cos\theta_2 A_2 - e_{33}\cos\theta_2 B_2 - (\gamma_3/ik_2)C_2]\exp[\vartheta_2] = 0 \qquad (12)$$

$$B_o exp[\vartheta_o] + B_1\, exp[\vartheta_1] + B_2\, exp[\vartheta_2] = 0(13)$$
$$C_o exp[\vartheta_o] + C_1\, exp[\vartheta_1] + C_2\, exp[\vartheta_2] = 0(14)$$

Equations (11-14) must be valid for all values of $t$and x, hence

$$\left.\begin{aligned}\vartheta_o &= \vartheta_1 = \vartheta_2,\\ k_o\sin\theta_o &= k_1\sin\theta_1 = k_2\sin\theta_2,\\ k_oC_{Lo} &= k_1C_{L1} = k_2C_{T2} = \omega\end{aligned}\right\} \quad (15)$$

From the above relations, we get

$$\left.\begin{aligned}&k_o = k_1, \theta_o = \theta_1, C_{Lo} = C_{L1},\\ &k_2/k_o = C_{Lo}/C_{T2} = \tau,\\ &\sin\theta_2 = \sin\theta_o/\tau\end{aligned}\right\}(16)$$

Furthermore, we should now use the equations (6) when z=0 of the media, i.e., using equations (1-2) which will give us additional relations between amplitudes

$$\left.\begin{aligned}\chi_oA_o + R_oB_o + \mu_oC_o &= 0,\\ \chi_1A_1 + R_1B_1 + \mu_1C_1 &= 0,\\ \chi_2A_2 + R_2B_2 + \mu_2C_2 &= 0\end{aligned}\right\}(17)$$

$$\left.\begin{aligned}L_oA_o + G_oB_o + S_oC_o &= 0,\\ L_1A_1 + G_1B_1 + S_1C_1 &= 0,\\ L_2A_2 + G_2B_2 + S_2C_2 &= 0\end{aligned}\right\}(18)$$

$$\left.\begin{aligned}E_oA_o + D_oB_o + F_oC_o &= 0,\\ E_1A_1 + D_1B_1 + F_1C_1 &= 0,\\ E_2A_2 + D_2B_2 + F_2C_2 &= 0\end{aligned}\right\}(19)$$

where

$$\chi_o = -\sin\theta_o[\rho C_{Lo}^2 - (C_{11} + \sigma_{xx}^o)\sin^2\theta_o - (C_{13} + 2C_{44} + \sigma_{zz}^o)\cos^2\theta_o]$$
$$R_o = (e_{13} + e_{15})\sin\theta_o\cos\theta_o, \quad \mu_o = -i\gamma_1\sin\theta_o/k_o$$
$$\chi_1 = -\chi_o, \qquad R_1 = R_o, \qquad \mu_1 = -\mu_o,$$
$$\chi_2 = \cos\theta_2[\rho C_{T2}^2 - (C_{11} + \sigma_{xx}^o)\sin^2\theta_2 - (C_{44} + \sigma_{zz}^o)\cos^2\theta_2 + (C_{13} + C_{44})\sin^2\theta_2]$$
$$R_2 = (e_{13} + e_{15})\sin\theta_2\cos\theta_2, \quad \mu_2 = -i\gamma_1\sin\theta_2/k_2,$$
$$L_o = -[(e_{13} + 2e_{15})\sin^2\theta_o\cos\theta_o + e_{33}\cos^3\theta_o],$$

$G_o = P_{11}\sin^2\theta_o + P_{33}\cos^2\theta_o$, $S_o = id_3\cos\theta_o/k_o$,
$L_1 = -L_o$, $G_1 = G_o$, $S_1 = -S_o$,
$L_2 = [(e_{13} + e_{15} - e_{33})\sin\theta_2\cos^2\theta_2 - e_{15}\sin^3\theta_2]$
$G_2 = P_{11}\sin^2\theta_2 + P_{33}\cos^2\theta_2$, $S_2 = -id_3\cos\theta_2/k_2$
$E_o = T_o(1 - ik_o t_o \delta C_{Lo})(\gamma_1\sin^2\theta_o + \gamma_3\cos^2\theta_o)$,
$D_o = -T_o d_3(1 - ik_o t_o \delta C_{Lo})\cos\theta_o$,
$$F_o = [(K_1\sin^2\theta_o + K_3\cos^2\theta_o)/C_{Lo}] - [\rho i C_E(1 - ik_o t_o C_{Lo})/k_o]$$
$E_1 = -E_o$, $D_1 = D_o$, $F_1 = -F_o$,
$E_2 = -T_o(1 - ik_2 t_o \delta C_{T2})(\gamma_1 - \gamma_3)\sin\theta_2\cos\theta_2$,
$D_2 = -T_o d_3(1 - ik_2 t_o \delta C_{T2})\cos\theta_2$,
$$F_2 = [\rho i C_E(1 - ik_2 t_o C_{T2})/k_2] - [(K_1\sin^2\theta_2 + K_3\cos^2\theta_2)/C_{T2}]$$
Solving equations (17-19), we can determine the reflection coefficients as:
$A_1/A_o = (\eta_1 + \eta_2)/(\eta_1 - \eta_2)$, (20)
$A_2/A_o = 2/(\eta_2 - \eta_1)$(21)
$B_1/B_o = -A_1/A_o$ (22)
$B_2/B_o = (A_2/A_o - 1)$ (23)
$C_1/C_o = A_1/A_o$ (24)
$C_2/C_o = -(1 + A_1/A_o)$(25)
Where
$\eta_1 = Y_o Y_1/Y_2$, $\eta_2 = Y_3 Y_4/Y_5$
$Y_o = \tau(F_o R_o - D_o R_o)/(F_2 R_2 - D_2 R_2)$
$$Y_1 = (C_{13} - C_{33})(D_2\mu_2 - F_2 R_2)\sin\theta_2\cos\theta_2 + e_{33}(E_2\mu_2 - F_2\chi_2)\cos\theta_2 + \gamma_3(D_2\chi_2 - E_2 R_2)/ik_2$$
$$Y_2 = (C_{13}\sin^2\theta_2 + C_{33}\cos^2\theta_o)(D_o\mu_o - F_o R_o) - e_{33}(E_o\mu_o - F_o\chi_o)\cos\theta_o + \gamma_3(D_o\chi_o - E_o R_o)/ik_o$$
$Y_3 = \tau(D_o\mu_o - F_o R_o)/(D_2\mu_2 - F_2 R_2)$
$Y_4 = [C_{44}(D_2\mu_2 - F_2 R_2)\cos 2\theta_2 + e_{15}(E_2\mu_2 - F_2\chi_2)\sin\theta_2]$
$Y_5 = [C_{44}(D_o\mu_o - F_o R_o)\sin 2\theta_o - e_{15}(E_o\mu_o - F_o\chi_o)\sin\theta_o]$
We obtain the amplitude ratios from the equations (20-25).

V. APPLICATION TO PARTICULAR MODEL

The reflection coefficients of reflected QP, QS, T and φ waves depend upon angle of incidence, angle of reflection, various elastic and thermopiezoelectric parameters, initial stress and relaxation times. The effect of these parameters on the reflection coefficients may be analyzed for particular model of the medium. For the purpose of numerical computations, the following physical constants of Cadmium Selenide (CdSe) for lower medium are considered [42]

$$C_{11} = 7.41\times10^{10}\ \mathrm{Nm^{-2}}, C_{12} = 4.52\times10^{10}\ \mathrm{Nm^{-2}},$$
$$C_{13} = 3.93\times10^{10}\ \mathrm{Nm^{-2}}, C_{33} = 8.36\times10^{10}\ \mathrm{Nm^{-2}},$$
$$C_{44} = 1.32\times10^{10}\ \mathrm{Nm^{-2}}, \rho = 5504\ \mathrm{Kgm^{-3}},$$
$$e_{13} = -0.160\ \mathrm{Cm^{-2}}, e_{33} = 0.347\ \mathrm{Cm^{-2}},$$
$$e_{15} = -0.138\ \mathrm{Cm^{-2}}, T_o = 298\ \mathrm{K},$$
$$\gamma_1 = 0.621\times10^6\mathrm{NK^{-1}m^{-2}}, \gamma_3 = 0.551\times10^6\mathrm{NK^{-1}m^{-2}},$$
$$d_3 = -2.94\times10^{-6}\mathrm{CK^{-1}m^{-2}}, K_1 = K_3 = 9\ \mathrm{Wm^{-1}K^{-1}}$$
$$\epsilon_{11} = 8.26\times10^{-11}\mathrm{C^2N^{-1}m^{-2}},$$
$$\epsilon_{33} = 9.03\times10^{-11}\mathrm{C^2N^{-1}m^{-2}}$$
$$C_E = 260\ \mathrm{J\,Kg^{-1}K^{-1}}, \omega = 2.14\times10^{13}\mathrm{s^{-1}},$$
$$t_o = 10^{-12} = 1\ \text{pico} - \text{sec}.$$

The variations of phase velocities computed from
$$c_{Lo} = c_{L1} = \sqrt{C_{44} + C_{11}\sin^2\alpha + C_{33}\cos^2\alpha + v_1}\Big/\sqrt{2\rho},$$
$$c_{T2} = \sqrt{C_{44} + C_{11}\sin^2\alpha + C_{33}\cos^2\alpha - v_1}\Big/\sqrt{2\rho},$$
where $v_1 = \sqrt{v_{11} + v_{12}}$,
$v_{11} = [(C_{11} - C_{44})\sin^2\alpha + (C_{44} - C_{33})\cos^2\alpha]^2$,
$v_{12} = (C_{13} + C_{44})^2\sin^2 2\alpha$

Numerical computations are restricted to incident QP wave only. For the incidence of QP wave, the reflection coefficients of QP, QS and T waves are computed for Lord and Shulman (L-S) theory with the angle of incidence after using the above physical constants. For L-S theory, the reflection coefficients are computed for the range $0 \le \theta_o \le 90^o$ of angle of incidence, and plotted in figures 2-9 which have the following observations:

Figures2,3,4and 5 represent the relation between the imaginary part of reflection coefficient A_1/A_o, A_2/A_o, B_2/B_o and C_2/C_o as function of incident angle θ_o for various value of relaxation time for (L-S) model. While **Figures** 6,7,8 and 9show the effect of initial stress on the imaginary part of reflection coefficient A_1/A_o, A_2/A_o, B_2/B_o and C_2/C_o as function of incident angle θ_o for (L-S) model.

The real parts of those reflection coefficients have not any influence by the change of the thermal relaxation time in the model of (Lord-Shulman). While the real parts of those reflection coefficients have some affected by the change of the initial stress (we did not give the figures and the details for these influence due to a shortcut the contribution and the details will be given in the coming search).

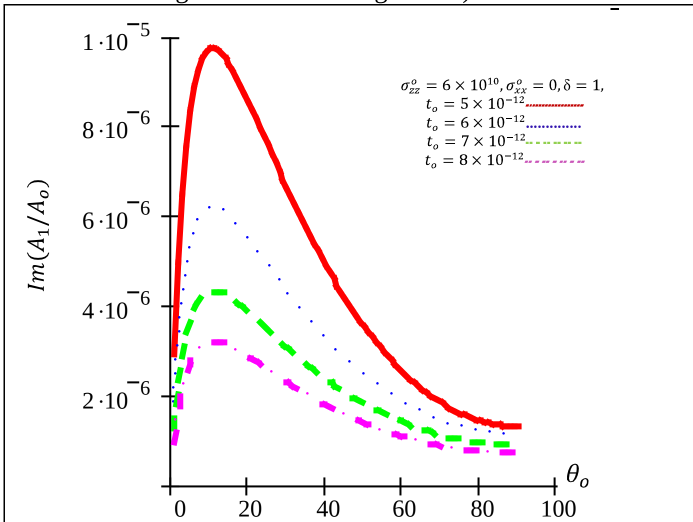

Fig. 2 Imaginary part of reflection coefficientA_1/A_o,as a function ofincidence angle $\theta_o$for various values of the relaxation times

θ_o

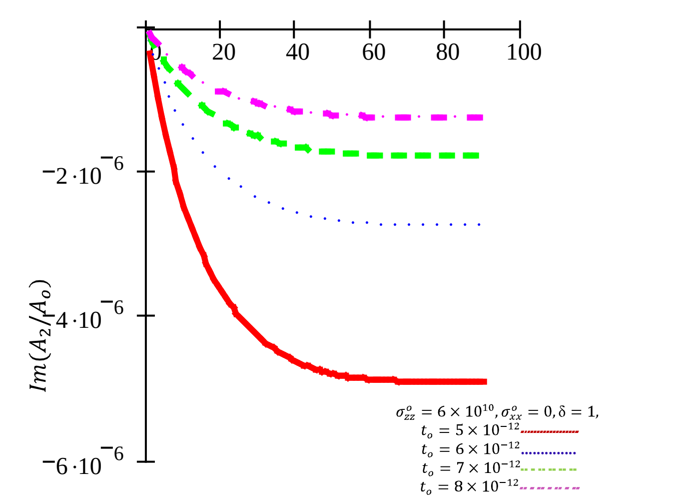

Fig. 3 Imaginary part of reflection coefficient A_2/A_oas a function of incidence angle θ_o for various values of the relaxation times.

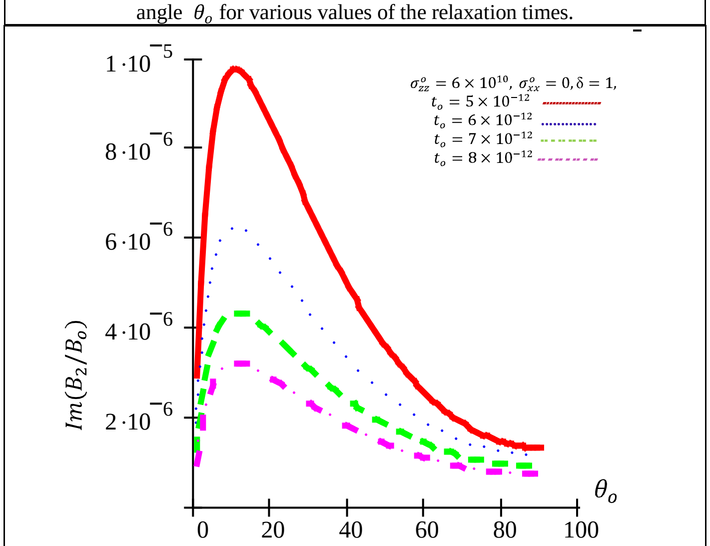

Fig. 4 Imaginary part of reflection coefficient(B_2/B_o)as a function of incidence angle$\theta_o$for various values of the relaxation times.

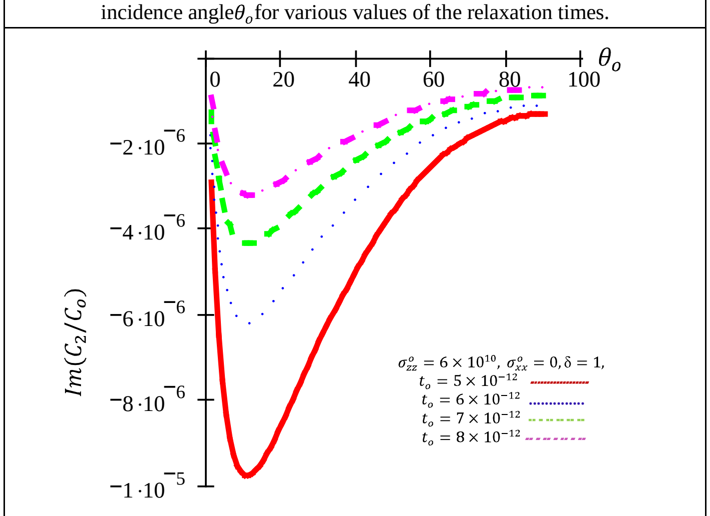

Fig. 5 Imaginary part of reflection coefficient(C_2/C_o)as a function of incidence angle $\theta_o$for various values of the relaxation times.

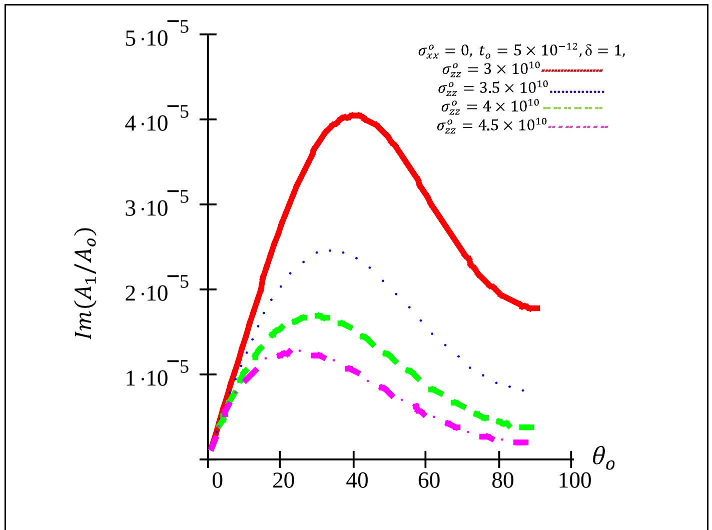

Fig. 6 Imaginary part of reflection coefficient (A_1/A_o) as a function of incidence angle $\theta_o$under effect of different values of the initial stress

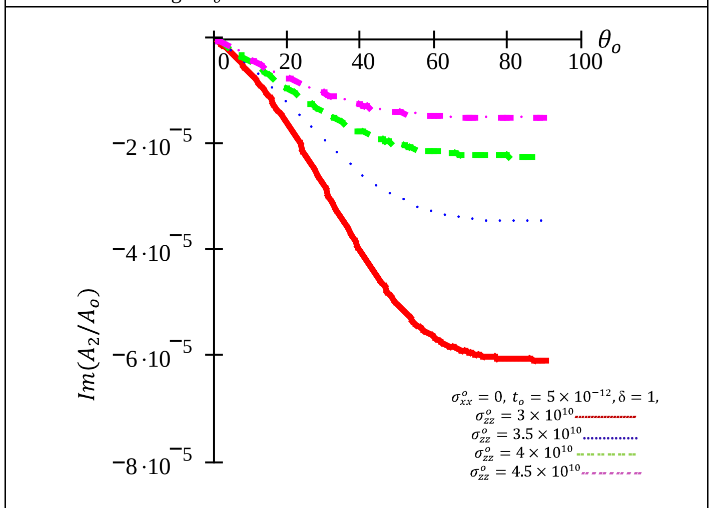

Fig. 7 Imaginary part of reflection coefficient A_2/A_o as a function of incidence angle $\theta_o$under effect of different values of the initial stress.

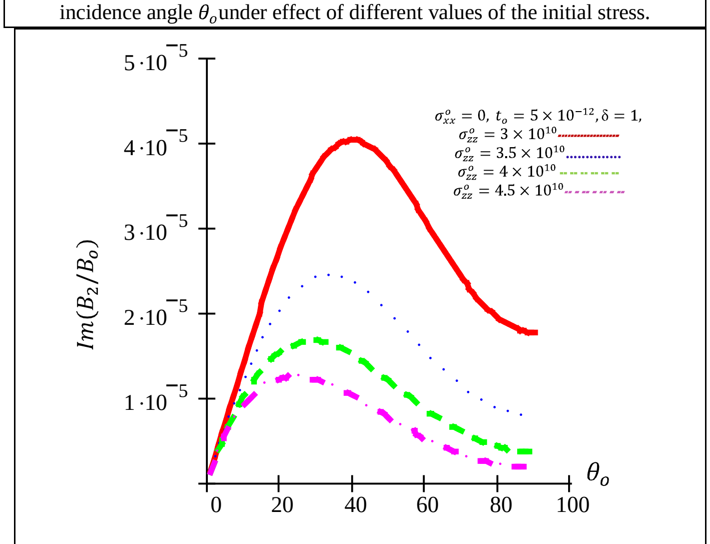

Fig. 8. Imaginary part of reflection coefficient (B_2/B_o) as a function of incidence angle $\theta_o$under effect of different values of the initial stress

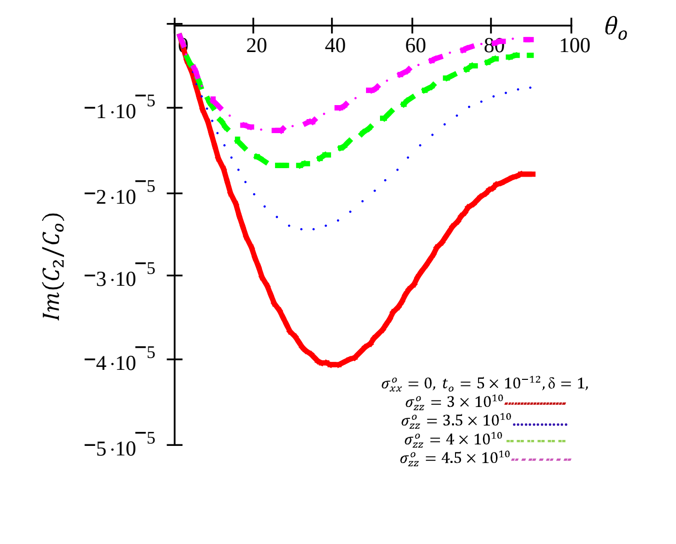

Fig. 9. Imaginary part of reflection coefficient (C_2/C_o) as a function of incidence angle θ_o under effect of different values of the initial stress.

VI. Conclusion

Reflection of the plane waves is studied under initial stress of hexagonal thermopiezoelectric solid half-space. The reflection coefficients which dependon the angle of incidence, angle of reflection, various elastic and thermopiezoelectric parameters are computed for a particular model. From numerical computations, it is noticed that the reflection coefficients of various quasi-plane waves are affected significantly due to the presence of relaxation time and anisotropy in the medium. In particular, the *T* wave is most affected due to the presence of relaxation time and anisotropy. However, the reflection coefficients of *T* wave are much less than that of *QP* at each angle of incidence.

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