Portfolio selection in the BRICs stocks markets using Markov processes

F. Petronio, S. Ortobelli, L. Tamborini, and T. Lando

Abstract**—**In this paper, we examine the performance of classical portfolio strategies in the BRIC's stock markets using a Markov approximation of the portfolio returns. In particular, we try to evaluate whether these markets can be a valid investment for non satiable and risk averse investors. First, we examine the main statistical characteristics of the returns in each market. Secondly, we provide a methodology to approximate the portfolios sample paths when the returns follow a Markov process. Finally, we examine the profitability of the classic investment strategies in each of the four BRICs markets individually and in all markets jointly, under the assumption the returns are approximated by a non parametric Markov chain. In particular, we compare the ex-post sample paths of the wealth obtained optimizing a mean-variance performance with and without assuming the Markovian hypothesis.

Keywords**—**Markov processes, optimization problems, portfolio selection, financial markets, Sharpe performance.

I. Introduction

In the recent years, the BRIC's markets performance, i.e. Brazil, Russia, India and China (see O'Neill in [14]), have exceeded the Europe and the U.S. performance, both during the pre-2007 boom and during the crisis. These countries, in fact, on the one hand had suffered less the effects of the crisis and on the other hand, a much more rapid recovery, compared to advanced countries. Clearly, it is undeniable that emerging markets had a significant economic slowdown in recent years, due to less demand from its trading partners in Europe and North America. However, it is a less dramatic decline than what happened in Europe and the United States (see [13]). In fact, even during the global financial crisis, the default rate in developing countries has always been lower than that of the Eurozone and the United States (see [21]). Moreover, the BRICs GDP (in nominal terms) grows despite their economic dimension still rather small (see [13]). This is due in part to the recent appreciation of their currencies, in part because, during the crisis, the BRICs have continued to grow very quickly, while the advanced countries had growth rates far below their average. In this context, the IMF, in a Staff Discussion Notes (see Samar et al. in [18]) asserts that also the BRIC's currencies may have interesting developments despite their volatility (especially for Brazil and Russia where the value of their currencies is largely determined by the commodity price).

The strong growth registered by the BRIC markets in recent years and their surprising independence from the developed economies, especially during the financial crisis, have created many opportunities for the investors. In this paper, we examine the BRIC's stock financial markets and we evaluate their portfolio performance comparing the ex-post wealth obtained optimizing the Sharpe ratio with and without considering the returns Markovian behavior. In particular, we first analyze, from a statistical point of view, all the returns of the active stocks in the BRICs markets during the period (2001-2012). Secondly, we propose an ex-post comparison between two investment strategies, (based on two different non parametric distributional assumptions) assuming to invest in each of the four BRICs markets individually and in all markets jointly, to compare the level of wealth created in each of them. Thus, the first contribution of this paper consists in the portfolio and distributional analysis of BRIC's stock financial markets before and during the last global financial crisis (from 2001 till 2012). The second contribution of this paper is the evaluation of the financial impact of using non-parametric Markov processes in portfolio problems applied to BRICs markets. As a matter of fact, while Markov processes are probably the most used to approximate the return evolution in financial problems (see, among others, Cox et al. in [7], Rachev and Mittnik in [15], D'Amico et al. in [9]), only in the recent literature homogeneous Markov chains were used

This paper has been supported by the Italian funds ex MURST 60% 2014 and MIUR PRIN MISURA Project, 2013–2015. The research was also supported through the Czech Science Foundation (GACR) under project 13-13142S and SP2013/3, an SGS research project of VSB-TU Ostrava, and furthermore by the European Social Fund in the framework of CZ.1.07/2.3.00/20.0296 (second author) and CZ.1.07/2.3.00/30.0016 (first and fourth author).

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to forecast and approximate the future wealth distribution (see Angelelli and Ortobelli in [1] and Angelelli et al. in [2], [3] and the reference therein) and, never (by our knowledge) in the BRICs stock markets. Thus, in this context, we can evaluate the impact of approximating the Markovian behavior of the BRICs' stock returns with a proper non-parametric Markov chain.

The paper is organized as follows: in Section 2 we briefly examine the main features of BRICs financial markets and the empirical evidence of their stock returns. In Section 3 we describe the portfolio selection problem and we propose the Markovian approximation of the returns. In Section 4 we propose the ex-post empirical analysis of the BRICs markets. Conclusion are given in Section 5.

II. BRICS FINANCIAL MARKETS: MAIN FEATURES AND EMPIRICAL EVIDENCE

The Emerging Markets (EMs), and BRICs in particular, have many common characteristics and represent a good opportunity, often very profitable, of diversification for investors.

In these markets operate both local and foreign investors. In both cases, however, the institutional investors are more frequent than retail (see Thompson, in [19]) and in most cases, their operation is limited to the traditional investment operations and does not include the creation of innovative financial products.

The number of European investors operating in the Emerging markets is increasing; however, there is a significant shortage of investors who regularly invest in these markets. Most of them, in fact, so-called cross-over investors, also operates in other markets. This characteristic determines an "on-off" access of Emerging Markets in the international market resulting in the assets prices volatility (see Donald and Garry in [11]).

Moreover, another problem that affects emerging markets is known as "shortage phenomenon", i.e. the lack of securities issuance (in both equity and bond market especially) (see De la Torre and Schmukler in [10] and Thompson in [19]). The shortage assets gives some distortion effects, like the so called "buy-and-hold investment strategy" that makes illiquid these markets.

On the other hand, the Information Technologies development, deregulation and globalization with consequently consolidation process (in the banking sector, mutual funds and stock exchanges), have made these markets highly attractive.

Currently, concerning the BRICs countries, the main Stock Exchanges are: the BM&FBovespa created in Brazil in 2008 from merge of São Paulo Stock Exchange (Bovespa) and the Brazilian Mercantile and Futures Exchange (BM&F); the National Stock Exchange (NSE) in India; the Russian Trading System (RTS), that currently is one of the largest stock markets in the regulated markets of Europe; finally, in China, the Shanghai and Shenzhen Stock Exchanges are often jointly considered, while Hong Kong Stock Exchange is a separate entity.

A. *Empirical evidences*

For purposes of our study, we first analyze 2690 stocks that were traded in the BRICs markets from January 2001 till January 2012. We take the assets from DataStream

The fact that log returns present a distribution with heavier tail than distributions with finite variance is documented in several empirical research works (see, among others, Rachev and Mittnik in [15] and Biglova et al. in [4]). Central theories in finance and important empirical studies assume that asset returns follow a normal distribution. The justification of this assumption is often cast in terms of its asymptotic approximation. However, this can be only a partial justification because the Central Limit Theorem for normalized sums of independent and identically distributed (i.i.d.) random variables determines the domain of attraction of an α stable distribution, i.e., any return r_i is asymptotically approximated as a stable Paretian distribution $r_i = S_\alpha(\sigma, \beta, \mu)$, where $\alpha \in (0,2]$ is the index of stability, σ is the scale parameter, μ is the location parameter and β is the skewness parameter. In particular, in Table 1 we report the average statistics of the log returns of each country: mean, standard deviation, Fisher skewness and kurtosis, percentage of assets for which is rejected the normality hypothesis with Jarque-Bera tests, the maximum likelihood stable Paretian parameters and the 5%, 25%, 50%, 75%, 95% quantiles.

It is not surprising that when we consider tests for normality such as the Jarque-Bera tests (with a 95% confidence level) the null hypothesis of normality for the daily log returns is rejected for more the 90% of stocks in each country (see J-B 95% in Table 1).

	BRAZIL	**RUSSIA**	**INDIA**	**CHINA**
Mean	0.00084	0.00111	0.00226	0.00108
St. Dev.	0.03314	0.03405	0.04162	0.03117
Skewnes s	0.47034	0.45113	0.52363	0.31489
Kurtosis	49.1727	35.4753	8.20238	8.61869
J-B 95%	0.94082	0.92757	0.92865	0.90122
Alpha	1.21571	1.29066	1.47768	1.44482
Beta	-0.00988	0.04752	0.25209	0.07572
Sigma	0.00474	0.00845	0.01769	0.01438
delta	-2.2E-05	0.00073	0.00387	0.00065
Perc 5%	-0.0315	-0.0382	-0.0575	-0.047
Perc 25%	-0.00446	-0.0079	-0.0166	-0.0136
Perc 50%	1.5E-05	6.9E-05	4.2E-05	0.00032
Perc 75%	0.00478	0.0086	0.0183	0.01459
Perc 95%	0.03677	0.0447	0.07131	0.05143

Table 1: Statistics on the ex-ante returns

Moreover, this result is also confirmed by the very high average of kurtosis and by the average of the maximum likelihood estimates of the α stable Paretian parameters. In particular, the very low indexes of stability α of the stock returns (less than 1.3 for the Brazil and Russia and less than 1.5 for India and China) suggest that most of the returns are leptokurtic. In addition, we observe that while 50% average quantile is around zero, the absolute value of the 5% and 25% average quantiles are respectively lower than the 95% and 75%, average quantiles. Thus, the returns of all BRICs markets generally present a positive asymmetry that is also confirmed by the positive skewness and by the stable skewness parameter β. This preliminary analysis implicitly suggests to use a more flexible distributional model to describe the portfolio return evolution. For this reason in the following section we propose to approximate the Markovian behavior of returns using a proper approximating Markov chain that accounts the possible evolution of the future wealth.

III. Portfolio selection model with Markov processes

In this section, we propose a distributional analysis of the time evolution of the portolio wealth when the portfolios dynamic is described by a homogeneous Markov chain. Throughout the paper we consider n risky assets with gross returns[1] $z_{t+1} = [z_{1,t+1}, \dots, z_{n,t+1}]'$. If we denote by $x = [x_1, \dots, x_n]'$ the vector of the positions taken in the n risky assets, then the portfolio return during the period $[t, t+1]$ is given by

$$z_{(x),t+1} = x' z_{t+1} = \sum_{i=1}^{n} x_i z_{i,t+1}. \qquad (1)$$

A. The Markov hypothesis

Let the interval $(\min_k z_{(x),k}; \max_k z_{(x),k})$ be the range of the portfolio gross returns, where $z_{(x),k}$ is the k-th past observation of the portfolio $z_{(x)}$. The states are denoted by N gross return $z_{(x)}^{(i)}$ where $i \in \{1,2,...,N\}$. Without loss of generality we assume that $z_{(x)}^{(i)} > z_{(x)}^{(i+1)}$ for $i = 1,..., N-1$. The initial wealth W_0 is given and equal to 1, the wealth at time $t = 1,..., k$; W_t is a random variable with a number of possible values increasing exponentially with time t. In order to keep the complexity of the computation reasonable, we first divide the portfolio support $(\min_k z_{(x),k}; \max_k z_{(x),k})$ in N intervals $(a_{(x),i}; a_{(x),i-1})$ where $a_{(x),i}$ is decreasing with index i and it is given by:

$$a_{(x),i} = \left(\frac{\min_k z_{(x),k}}{\max_k z_{(x),k}} \right)^{i/N} \cdot \max_k z_{(x),k}, \quad i = 0,1,\dots,N.$$

Then, we compute the return associated to each state as the geometric average of the extremes of the interval $(a_{(x),i}; a_{(x),i-1})$, that is, for $i = 1,2,..., N$,

$$z_{(x)}^{(i)} = z_{(x)}^{(1)} u^{1-i} \text{ where } u = \left(\frac{\max z_{(x),k}}{\min z_{(x),k}} \right)^{1/N}$$

Thus, the final wealth W_t does not depend on the specific path followed by the process and we denote such property of a Markov chain as *recombining effect*. Thanks to the recombining effect of the Markov chain on the wealth W, the possible values after k steps of W_k are $1 + k(N-1)$, and the whole set of possible values of the random variables W_t ($t = 1,..., k$) can be stored in a matrix with k columns and $1 + k(N-1)$ rows resulting in $k + k^2(N-1) = O(Nk^2)$ memory space requirement. The $(N-1)k+1$ values of the wealth $W_k = [w^{(l,k)}]_{1 \le l \le (N-1)k+1}$ after k periods can be computed by the formula:

$$w^{(l,k)} = \left(z^{(1)}\right)^k \cdot u^{(1,l)}, l = 1, \dots, (N-1)k+1 \qquad (2)$$

thus, the l-th node at time k of the wealth-tree corresponds to wealth $w^{(l,k)}$. In this analysis we only consider homogeneous Markov chains, so transition matrix does not depend on time and it can be simply denoted by P. The entries $p_{i,j}$ of matrix P are estimated using the maximum likelihood estimates $\widehat{p}_{i,j} = \frac{\pi_{ij}(K)}{\pi_i(K)}$ where $\pi_{ij}(K)$ is the number of observations (out of K observations) that transit from the i-th state to the j-th state and $\pi_i(K)$ is the number of observations (out of K observations) in the i-th state. We refer to D'Amico in [8] for the statistical properties of these estimators.

The procedure to compute the distribution function of the future wealth is strictly connected to the recombining feature of the wealth-tree. Under these assumptions Iaquinta and Ortobelli in [12], have shown how to compute the unconditional and conditional (conditional on the initial state s_0, i.e. $z^{(s_0)}$) probability of each node of the future wealth W_t for any time t.

B. The portfolio problem

The classic static portfolio selection problem when no short sales are allowed, can be represented as the maximization of a functional $f:(\Omega,\Im,P)\mapsto\mathbb{R}$ applied to the random portfolio of

[1] Generally, we assume the standard definition of gross return between time t and time t+1 of asset i, as $z_{i,t+1} = \frac{S_{i,t+1} + d_{i,[t,t+1]}}{S_{i,t}}$, where $S_{i,t}$ is the price of the i-th asset at time t and $d_{i,[t,t+1]}$ is the total amount of cash dividends paid by the asset between t and t+1. We distinguish the definition of gross return from the definition of return, i.e., $z_{i,t} - 1$ (or the alternative definition of log returns $r_{i,t} = \log z_{i,t}$).

gross returns $z_{(x),k+1}$ subject to the portfolio weights belonging to the simplex $S=\left\{x\in\Re^n \mid \sum_{i=1}^n x_i=1; x_i\geq 0\right\}$, i.e., $\max_{x\in S} f(z_{(x)})$. Typically, the functional f(.) is a performance measure or an utility functional. In both cases the functional f(.) should be isotonic with a particular ordering of preference $\succcurlyeq$, that is, if X is preferred to Y (X$\succcurlyeq$Y), then f(X)≥f(Y). The choice of the functional f(.) plays a crucial role in the portfolio strategy. Isotonic utility functionals with non satiable and risk averse preferences have been used in many financial applications (see Angelelli et al. in [2] and [3]). In these cases we have f(*X*)=E(v(*X*)) where *v* is an increasing and concave utility function. However, as suggested in behavioural finance, while all investors prefer more to less they could be neither risk averse nor risk lover (see Ortobelli et al. in [15] and Rachev et al. in [17]). For this reason it makes sense to consider functionals that are monotone, even though they are not consistent with an uncertainty/aggressive order (see, among others, Rachev et al. in [17]). We call *OA performance (utility) functional* any functional computed under the assumption that the gross return of each portfolio follows a Markov chain with N states. In this paper we will use and describe only some OA functionals that consider the forecasted wealth at time T. That is, investors have to periodically (every T periods) compute the portfolio $x\in S$ solution of the problem:

$$\max_{x\in S} f\left(W_T(x)\right) \tag{1}$$

The vector of weights x solution of the problem (3) represents the percentage of wealth that should be invested in each asset during the period [0,T]. Since the value of the assets change during the period [0,T], then even an OA portfolio strategy generally implies that the wealth must be recalibrated T times during the period [0,T] in order to maintain constant the percentages of the wealth invested in each asset. If T is very large and we do not recalibrate the portfolio periodically (the period should be the same used in the valuation) these percentages invested in the assets could be completely different at the end of investor's temporal horizon. This point has not been explicitly addressed in Angelelli and Ortobelli's analysis (see [1]) even if could have a very big impact in portfolio choices.

In portfolio literature more than one hundred static reward-risk performance measures have been proposed (see Cogneau and Hübner in [5] and [6]). Here, we list the Sharpe static strategy and the analogous OA performance functional isotonic with choices of non satiable investors that will be object of the following empirical analysis. For the OA portfolio strategy we assume that investors have temporal horizon equal to T.

Sharpe ratio (*SR*) The classic version of the Sharpe ratio (see Sharpe in [16]) values the expected excess return for unity of risk (standard deviation) without considering the time evolution of the portfolio wealth. Thus maximizing the classic version of the Sharpe ratio we implicitly assume that the future market trend, at least in the short term, will follow the recent past. Essentially, the Sharpe ratio characterizes how well the return of an asset compensates the investor for the risk taken. The Sharpe ratio is calculated by subtracting a benchmark gross return (often the risk-free gross return) from the portfolio gross return and dividing the result by the standard deviation of the portfolio excess return. Formally:

$$\mathrm{SR}(x'z)=\frac{E(x'z-r_b)}{\sigma_{x'z-r_b}} \tag{4}$$

where r_b is a benchmark return and $\sigma_{x'z-r_b}$ is the standard deviation of the portfolio excess return. When the benchmark r_b is the risk free rate, the Sharpe ratio is isotonic with non-satiable risk averse preferences. In our next analysis we assume that the riskless (or the benchmark asset) is not present (i.e., $r_b=1$).

OA-Sharpe ratio (*OASR*). With the OA-Sharpe ratio we value the expected excess final wealth for unity of risk, i.e.,

$$\mathrm{OASR}(W_T(x))=\frac{E(W_T(x)-W_T(r_b))}{\sigma_{W_T(x)-W_T(r_b)}} \tag{5}$$

where $W_T(r_b)$ is the final wealth at time T we obtain investing in the benchmark r_b. In the Markovian framework we should consider the bivariate evolution of the vector $(W_T(x)-W_T(r_b))$ to value the standard deviation $\sigma_{W_T(x)-W_T(r_b)}$ of $W_T(x)-W_T(r_b)$. However, in the following analyses we assume that the riskless asset (or the benchmark asset) is not allowed, thus, the OA-Sharpe Ratio is simply given by $\frac{E(W_T(x)-1)}{\sigma_{W_T(x)}}$.

IV. An ex-Post empirical analysis

In this section, we compare the ex-post wealth obtained optimizing the Sharpe ratio and OA Sharpe ratio in each of the four BRICs markets individually and in all markets jointly. In particular, for each of the BRIC countries we take into account all stocks active during the period 1/1/2001 to 1/4/2012[2] and for all stocks we consider the prices in USD. Thus, we have created 5 clusters: Brazil, Russia, India, China and BRIC.

In the empirical comparison, we optimize each performance measure monthly (every 20 trading days) using one year of daily historical observations (250 trading days) to compute the performance measures we have to optimize. Thus, at any optimization time, every 20 trading days, we use a moving window of 250 trading days which are used in the optimization process. So, once we obtain the optimal solution of the optimization problem at the beginning of the monthly investment period, we have to recalibrate daily the wealth in order to maintain constant the optimal portfolio composition. This means that the investor, day by day, sells those securities whose price has increased and buys those securities whose

[2] In particular, within Chinese market we also consider the Hong Kong market.

price is lowered; so that the actual allocation reflects the optimal portfolio calculated at beginning of period, not only the first day, but every day of the investment period. Thus, the principal parameter settings used in the optimization process are:

a) the investors have a temporal horizon of $T = 20$ trading days;
b) the global ex-post investment period is 2622 trading days and, as already pointed out, the composition of the portfolio will be optimized every 20 days, for a total of 132 optimizations;
c) Markov chains have $N = 9$ states, so that the final wealth W_{20} presents 161 nodes in the Markov tree;
d) the initial wealth W_0 is equal to 1 at the date 18-Dec-2001;
e) two significant constraints were introduced in the optimization function: a) the maximum share of the portfolio that can be invested in a single title is 20%; b) short sales are not allowed, in other words, the percentage weight of each security in the portfolio can't be negative (i.e.: $x_i \in [0,0.2]$). The first constraint aims to achieve a well-diversified portfolio and not overly concentrated; while, the second constraints excludes the possibility of short selling, since the securities sale not owned directly is a technique not easily implementable by a private investor.

In addition, we introduce a preliminary "liquidity filter" in the optimization process, since BRICs markets present a low level of liquidity.

Therefore, we perform this empirical analysis first to examine the ex-post wealth of classic portfolio strategies (Sharpe ratio) in the BRICs markets and second to evaluate the impact of the return Markovian approximation in portfolio problems (OA Sharpe ratio) applied to Emerging Markets.

For each strategy, we have to compute the optimal portfolio composition 132 times and at the *k*-th optimization ($k = (0,1,2,\ldots,132)$), four main steps are performed to compute the ex-post final wealth:

Step 1 *liquidity filter*: To overcome the lack of liquidity, we introduce a filter that, for each recalibration, allows to exclude from the securities basket, those stocks whose volumes are below in average to 1 daily contract (minimal requirement) and whose price remain constant for more than 15% of the last year. This liquidity filter is repeated at each recalibration process and takes into account only the 250 observations used in the optimization step. Therefore, a stock discarded at a given optimization time could be kept in the subsequent optimization time.

Step 2 - *pre-selection*: Despite the liquidity filter, the number of assets in each cluster remained relatively large. Thus, as suggested by Angelelli et al in [2] and [3], we reduce the complexity of the problem using in the optimization problem only the 150 assets with the greatest performance (Sharpe Ratio or OA Sharpe ratio).

Step 3 - *optimization*: We implement an optimization function on 150 pre-selected assets. The optimization aim to identify which of these assets, and what percentage, should compose the portfolio that maximizes the performance measure. Therefore, we determine the market portfolio $x_M^{(k)}$ that maximizes the performance ratio $\rho(W(x))$ (formulas (4) or (5)) associated to the strategy (or Sharpe or OA Sharpe) i.e. the "ideal" solution of the following optimization problem:

$$\begin{aligned}&\max_{x^{(k)}} \rho(W(x^{(k)})\\&s.t.\\&\textstyle\sum_{i=1}^{n} x_i^{(k)} = 1,\\&x_i^{(k)} \le 0.2;\, x_i^{(k)} \ge 0\\&i = 1,\ldots,n\end{aligned} \tag{6}$$

Angelelli and Ortobelli in [1] have observed that the complexity of the portfolio problem is much higher in view of a Markovian evolution of the wealth process. In order to overcome this limit we use the Angelelli and Ortobelli's heuristic algorithm that could be applied to any complex portfolio selection problem that admits more local optima.

Step 4 - *recalibration*: We recalibrate daily the portfolio maintaining the percentages invested in each asset equal to those of the market portfolio $x^{(k)}$ during the period $[t_k, t_{k+1}]$ (where $t_{k+1} = t_k + T$). Thus, the ex-post final wealth is given by:

$$W_{t_{k+1}} = W_{t_k}\left(\prod_{i=1}^{T} (x_M^{(k)}{}')z_{(t_k+i)}^{(ex\ post)}\right) \tag{7}$$

where $z_{(t_k+i)}^{(ex\ post)}$ is the vector of observed daily gross returns between $t_k + i - 1$ and $t_k + i$.

Steps 1, 2, 3 and 4 are repeated for all performance ratios until some observations are available.

The results of this empirical analysis are reported in Table 2 and Figures 1,2,3,4,5,6,7.

	Mean	St dev	Sharpe	Skewness	Kurtosis	Final Wealth
SR Brazil	0.0013	0.012	0.105	1.81	20.94	24.52
OASR Brazil	0.0018	0.014	0.128	1.98	21.04	91.08
SR Russia	0.0013	0.018	0.071	17.13	547.35	21.00
OASR Russia	0.0013	0.020	0.066	14.00	396.34	21.14
SR India	0.0015	0.014	0.105	-0.46	3.59	35.90
OASR India	0.0018	0.022	0.084	17.57	605.56	70.92
SR China	0.0007	0.016	0.043	1.15	45.20	4.39
OASR China	0.0008	0.015	0.052	0.53	8.84	6.05
SR BRICs	0.0015	0.017	0.085	6.57	116.90	31.95
OASR BRICs	0.0018	0.013	0.137	2.75	68.73	85.33

Table 2: Statistics on the ex-post optimal returns

Table 2 reports some statistics (mean, standard deviation, Sharpe ratio, skewness, kurtosis and the final wealth at time 1/4/2012) of the optimal ex-post returns obtained with the two portfolio strategies (Sharpe, OA Sharpe).

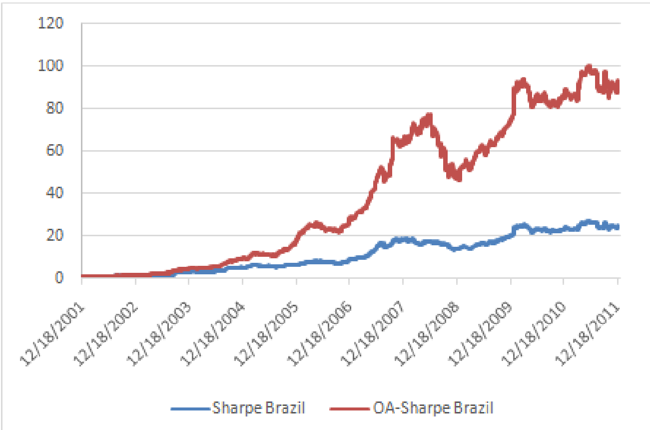

Fig. 1 Sharpe and OA Sharpe ex post wealth obtained using the Brazil cluster

In Figure 1 we report the ex-post wealth obtained optimizing the Sharpe and OA Sharpe strategies with Brazil cluster. In particular, we observe a growing trend of the wealth that involves the entire period of our analysis, and that becomes more intense in recent years. However, there is a significant divergence in the results obtained with two investment strategies, starting from 2003 and amplified over time, that clearly outline the superiority of the OA-Sharpe strategy. As a matter of fact, this last strategy allows to reach a final wealth equal to 91 times the initial one and offers an average annual rate of return of 57 % (see also Fig. 7 and Table 2). The Sharpe strategy, instead, leads to a final wealth equal to 24.5 times the initial one, allowing to achieve an average annual rate of return of 37.7%.

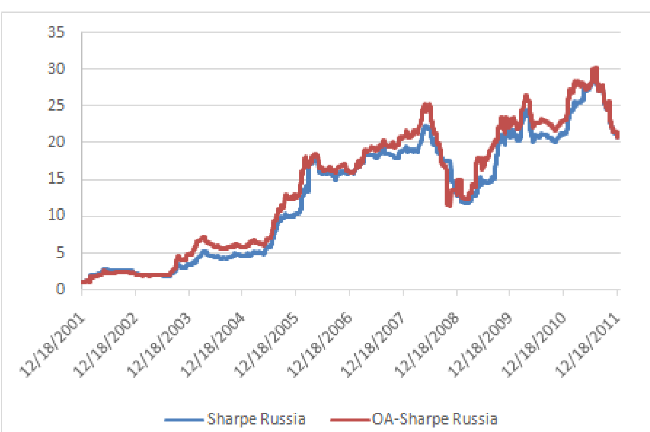

Fig. 2 Sharpe and OA Sharpe ex post wealth obtained using the Russia cluster

With regard to the Russian markets, we observe a general growing trend during the decade analyzed. However, there is a significant decline during the second half of 2008 and the first months of 2009. As we can see in Figure 2, other two falls occur during the first part of 2010 and the second part of 2011.

In this cluster the trend is very similar, considering both strategies (Sharpe and OA Sharpe). In fact, we obtain a final wealth equal to 21 times the one invested at time t0, using both Sharpe and OA-Sharpe maximization. Obviously also the average annual rate of return is quite similar (35.6% and 35.7% respectively).

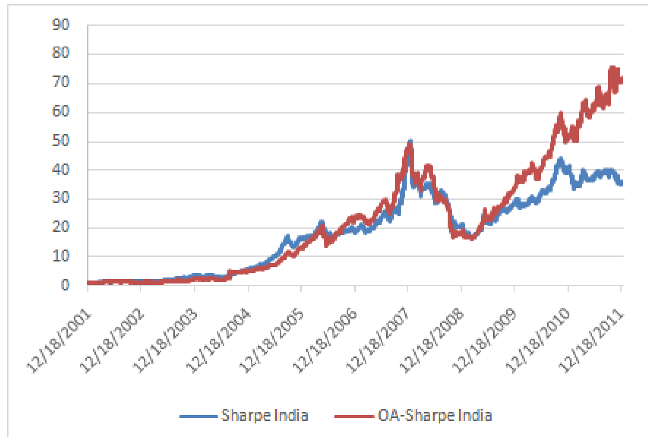

Fig. 3 Sharpe and OA Sharpe ex post wealth obtained using the India cluster

Also in the Indian markets we obtain a remarkable performances. As we can see in Figure 3, the OA-Sharpe strategy performs better after the half part of 2009; before this date the two strategies offer similar results: there is an increasing trend up to December 2007 and a decreasing phase during the subprime crisis from January 2008 to February 2009. Specifically, in the Indian cluster, the AO-Sharpe strategy allowing to reach a final wealth of 70.9 times the initial investment and offers an average annual rate of return of 53.1%. With Sharpe strategy, however, the investor get a final wealth of 35.9 times the initial wealth with an average annual rate of return of 43.1% (see also Fig. 7).

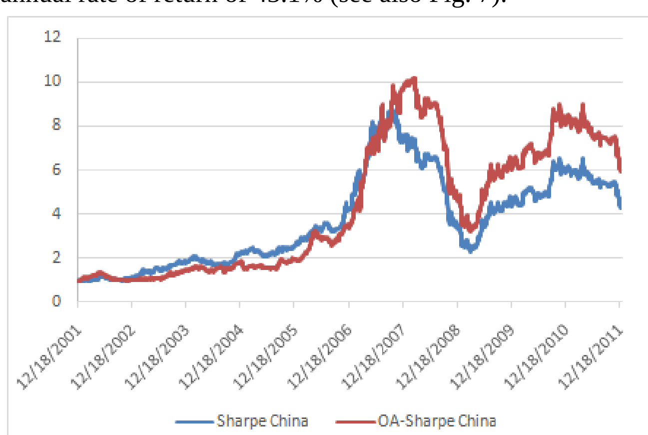

Fig. 4 Sharpe and OA Sharpe ex post wealth obtained using the China cluster

However, Table 2 remarks that there is a strong difference in the kurtosis of the two optimal strategies. Thus, even if the Sharpe strategy does not perform very well during the crisis, it is less risky and it presents an higher ex-post Sharpe ratio.

Figure 4 examines the ex-post wealth obtained investing in the China cluster. As for the others, we notice a remarkable growth till 2007 with both strategies. Then, there is a decline with a minimum touched at the beginning of 2009 followed by a recovery phase until the end of 2010, and a new slight decline till the examined period.

The ex-post wealth obtained with the two strategies applied to the China cluster are quite similar: the OA-Sharpe allows to obtain a final wealth equal to 6 times the invested at the initial period and offers an average annual rate of return of 19.7%; otherwise, with Sharpe strategy the investor gains a wealth of 4.4 times the initial one, with an average annual rate of return of 15.9%.

Concerning the BRICs cluster, Figure 5 confirms the trend observed in all the other single clusters with a growing trend before the crisis (2001-2007) a decreasing trend during the sub-prime crisis (2008- February 2009) and a slightly increasing trend during the credit risk crisis (March 2009-January 2012). In particular, we observe that both investment strategies present similar ex-post wealth until 2004, then; the OA-Sharpe trend is slightly lower than the Sharpe one until July 2007; while there is a high improvement of OA-Sharpe during the credit risk crisis. Investing in the jointly BRICs markets with the OA-Sharpe strategy, the investor reaches a wealth equal to 85.3 times the initial one, obtaining an average annual rate of return of 56%. Instead the wealth obtained with Sharpe strategy is only 31.9 times the initial one with an average annual rate of return of 41.4%.

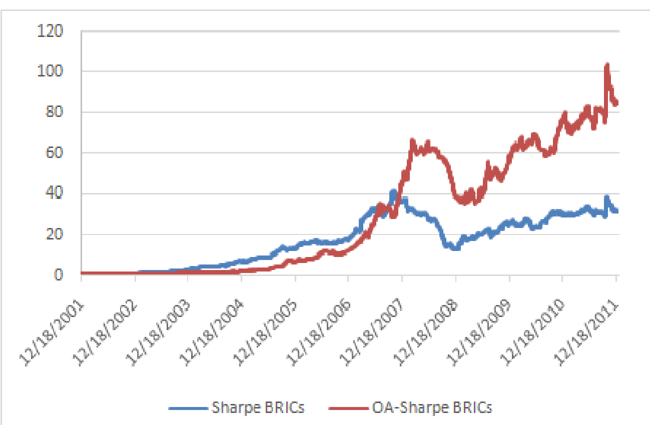

Fig. 5 Sharpe and OA Sharpe ex post wealth obtained using the BRICs cluster

Figure 6 shows the geographical composition of the optimal portfolio invested (in average) in the BRICs cluster using the OA-Sharpe strategy. We get very similar percentage composition (but obviously with different stocks), when we consider the average portfolio composition obtained optimizing the classic Sharpe ratio in the BRICs cluster.

In particular, we observe that the weight of the Chinese stocks is predominant; the Indian assets play a secondary role however still significant; while the Brazilian assets and especially the Russians assets have a marginal weight.

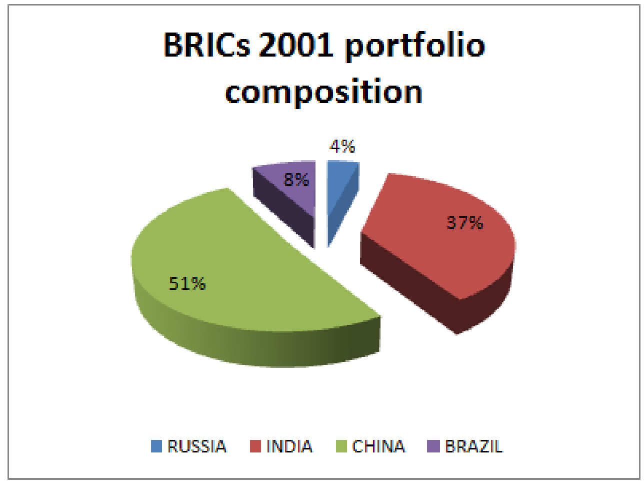

Fig. 6 Average portfolio composition in BRICs cluster

In Figure 7 we draw up a list of different clusters, based on the average annual returns obtained using the two different strategies. We can clearly observe that the OA-Sharpe strategy performs better in all clusters considered in terms of ex-post wealth. This is more evident especially for Brazil, India and BRICs jointly considered.

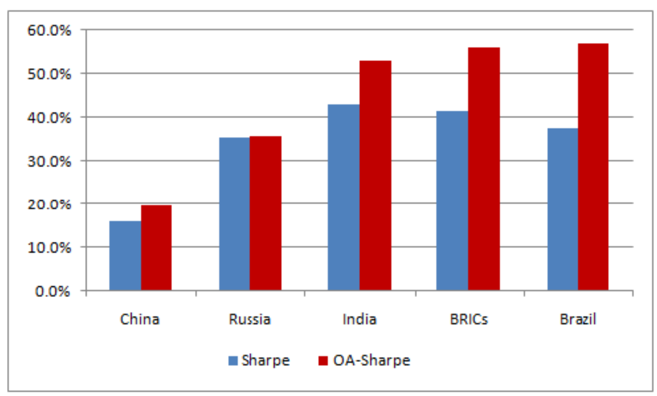

Fig. 7 Average annual returns comparison

V. Concluding Remarks

The analysis proposed in this paper essentially confirms the growing trends of financial BRIC markets (Brazil, Russia, India, China) by the point of view of non satiable risk averse investors that optimize their Sharpe ratio. Moreover we also observe a strong impact of the Markovian hypothesis in the portfolio selection. Thus the proposed analysis emphasize the importance to consider the time evolution of the portfolio wealth by using a proper approximating Markov chain.

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