Some Simulation Results of Heat Transfer through the Wall Model

Jana Mižáková, Ján Piteľ, Stella Hrehová

***Abstract*—**This paper deals with simulation results of model of heat transfer through the wall with different parameters of wall. Next, presented model has been implemented into model of heating system using of outdoor temperature compensation, in which have been also included others models: model of heating body and model of equitherm curves based on the temperature levels. Finally, results of simulation of whole heating system based on the outdoor temperature compensation are shown.

In detail, there are presented dynamics of heat transfer through the wall and there are described inverse Laplace transform of special functions, which occur in solving of partial differential equations system.

***Keywords*—**model, heat transfer through the wall, heating system using outdoor temperature compensation

I. Introduction

The goals of the sustainable development in the field of heat production and supply can only be achieved by automatic process control using digital controllers and others control devices, which perform advanced control algorithms base on the intelligent control methods. In a cases, where the classical verified model can not be obtained, it is possible to use the "black box" modeling, which in this case allows finding a mathematical dependence of outputs values on the input values, while error is minimal. Neural networks can be considered as universal approximators, which can approximate any function with any accuracy. [1, 2, 3, 4, 5]

The classical verified approach to the heating process control uses outdoor temperature compensation, which provides for the optimal temperature of supply water to heating bodies according to outdoor temperature. To achieve this requirement, it has been necessary to find a equilibrium between the supplied heat output and heat losses, i. e. to ensure optimum temperature of heating water. Usually, this is realized by such methods to the water temperature in the heating system was controlled by so-called equitherm curves. [6, 7] For that reason it has been necessary to design partial models of heating system, including model of heat transfer through the wall, model of heating body and model of equitherm curves. Sub-models, which have been implemented into the complex model, are the basis for heating control where different controller can be implemented and compared.

The model of heat transfer dynamics through the wall has been designed on the base of mathematical describing of the energy balance for the elementary layer of plane wall and specific problem of solving of partial differential equations system by Laplace transform has occurred.

The research work is supported by the Project of the Structural Funds of the EU, Operational Programme Research and Development, Measure 2.2 Transfer of knowledge and technology from research and development into practice, title of the project: Research and development of intelligent control systems for biomass based heat production and supply. ITMS code: 26220220030. And also is supported by VEGA - Grant agency of the Ministry of Education, Science, Research and Sport of the Slovak Republic, Title of project: „ Research of algorithms and methods for predictive control of biomass combustion processes“, no: 1/0881/2013

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II. The basis for model design

A. Heat transfer dynamic through the wall

For design of the model of heat transfer dynamics through the wall we have considered a plane wall, where the wall has been considered as continuum with continuously distributed thermal resistance and capacity.

We have chosen elementary layer with following parameters (Fig. 1):

- dy thickness of layer in the plane wall,
- d_w thickness of plane wall,
- y distance of layer from the heated surface.

Let's others variable have been used for mathematical description:

- θ_{w1} temperature of the heated wall surface,
- θ_{w2} temperature of the refrigerated wall surface,
- θ_w temperature of elementary layer,
- Φ_1 the heat flow supplied into heated wall surface,
- Φ_2 the heat flow taken away from refrigerated wall surface,
- Φ the heat flow inputs into unit surface of layer dy, and the heat flow $\Phi + d\Phi$ outputs from it.

Parameters of wall have been:

- c specific heat capacity,
- ρ volume weight,
- λ heat conductivity.

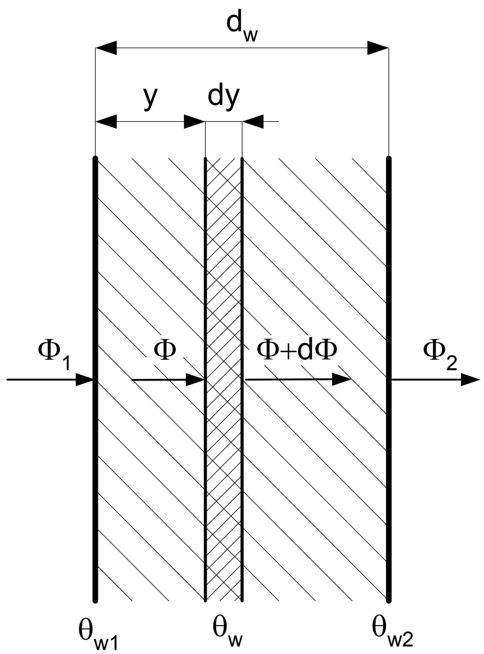

Fig. 1 Plane wall

B. Mathematical description of energy balance

According to [8] the heat energy does not originate either does not dissolve in considering elementary layer of the wall. Then difference of input heat and output heat in the layer has to be equal to the time variation of the energy in layer.

Let's c is specific heat capacity (specific heat) and ρ is volume weight of the wall material, then:

$$\Phi - \left(\Phi + d\Phi\right) = \frac{\partial}{\partial t}\left(c \cdot \rho \cdot \theta_w \cdot dy\right) \tag{1}$$

Considering that heat flow $d\Phi$ is:

$$d\Phi = \frac{\partial \Phi}{\partial y} dy \tag{2}$$

If specific heat capacity c and volume weight ρ of the used wall material are constant, then:

$$-\frac{\partial \Phi}{\partial y} = c \cdot \rho \cdot \frac{\partial \theta_w}{\partial t} \tag{3}$$

According to Fourier's law the heat flow is directly proportional to the temperature gradient

$$\Phi = -\lambda \frac{\partial \theta_w}{\partial y} \tag{4}$$

where λ is heat conductivity coefficient of the used wall material.

Partial differential equations (3) and (4) with relevant initial and border conditions completely describe non-stationary one-dimensional heat flow.

It is considerable for automatic control the dynamic dependence of the control deviation according to changes of variables which have effect on the deviation. For that reason, we have expressed dependent variables by their values and their increments as:

$$\begin{aligned} \Phi &= \Phi_0 + \Delta\Phi, \\ \theta_w &= \theta_{w0} + \Delta\theta_w \end{aligned} \tag{5}$$

By substitution (5) to (3) and (4) we have got:

$$-\frac{\partial \Phi_0}{\partial y} - \frac{\partial \Delta\Phi}{\partial y} = c \cdot \rho \cdot \left(\frac{\partial \theta_{w0}}{\partial t} + \frac{\partial \Delta\theta_w}{\partial t}\right) = c \cdot \rho \cdot \frac{\partial \Delta\theta_w}{\partial t} \tag{6}$$

$$\Phi_0 + \Delta\Phi = -\lambda \cdot \left(\frac{\partial \theta_{w0}}{\partial y} + \frac{\partial \Delta\theta_w}{\partial y}\right) \tag{7}$$

For initial steady-state is valid:

$$\frac{\partial \Phi_0}{\partial y} = 0 \tag{8}$$

$$\Phi_0 = -\lambda \frac{\partial \theta_{w0}}{\partial y} = -\lambda \frac{\theta_{w20} - \theta_{w10}}{d_w} = \lambda \frac{\theta_{w10} - \theta_{w20}}{d_w} \tag{9}$$

and then by substitution (8) to (6) and subtraction (9) from (7) we have got partial differential equations system of heat transfer dynamic through the wall:

$$-\frac{\partial \Delta\Phi}{\partial y} = c \cdot \rho \cdot \frac{\partial \Delta\theta_w}{\partial t} \tag{10}$$

$$\Delta\Phi = -\lambda \frac{\partial \Delta\theta_w}{\partial y} \tag{11}$$

To simplify computation, we have expressed each of dependences in non-dimensional form (relative changes of variables, i. e. changes compared with initial values of variables):

$$\begin{aligned} x_\Phi &= \frac{\Delta\Phi}{\Phi_0}, \\ x_{\theta w} &= \frac{\Delta\theta_w}{\theta_{w10} - \theta_{w20}} \end{aligned} \tag{12}$$

Then equations (10) and (11) are:

$$-\frac{\partial x_\Phi}{\partial y} = c \cdot \rho \cdot \frac{\left(\theta_{w10} - \theta_{w20}\right)}{\Phi_0} \cdot \frac{\partial x_{\theta w}}{\partial t} \tag{13}$$

$$x_\Phi = -\lambda \cdot \frac{\left(\theta_{w10} - \theta_{w20}\right)}{\Phi_0} \cdot \frac{\partial x_{\theta w}}{\partial y} \tag{14}$$

According (9) it have been possible to express:

$$\frac{\theta_{w10}-\theta_{w20}}{\Phi_0}=\frac{d_w}{\lambda} \tag{15}$$

and then

$$\frac{\partial x_{\Phi}}{\partial y}+c\cdot\rho\cdot\frac{d_w}{\lambda}\frac{\partial x_{\theta w}}{\partial t}=0 \tag{16}$$

$$x_{\Phi}+d_w\frac{\partial x_{\theta w}}{\partial y}=0 \tag{17}$$

C. Laplace transform

By the Laplace transform of partial differential equations (16) and (17) and by the other mathematical operations it has been possible to get a system of equations, which describes dependence of non-dimensional variables for heat flows Φ_1, Φ_2 and temperatures θ_{w1}, θ_{w2}. [9]

We have substituted thickness d_w and time T_s as thickness η and time τ due to exclusion of constants, we have used substitutions:

$$\eta=\frac{y}{d_w} \tag{18}$$

$$\tau=\frac{t}{T_s},\ T_s=\frac{d_w^2.\rho.c}{\lambda} \tag{19}$$

Let system of partial differential equations (16) and (17) is system of partial differential equations of variables thickness η, time τ:

$$\frac{\partial x_{\Phi}}{\partial\eta}+\frac{\partial x_{\theta w}}{\partial\tau}=0 \tag{20}$$

$$x_{\Phi}+\frac{\partial x_{\theta w}}{\partial\eta}=0 \tag{21}$$

Using Laplace transform defined with complex argument p as:

$$\overline{X}(\eta,p)=\int_0^{\infty}x(\eta,p).e^{-p\tau}\,d\tau \tag{22}$$

we have found image of function of real argument τ with initial condition $x_{\theta w}(\eta,0)=0$ (see substitution (19)) and we have got equations:

$$\frac{d\overline{X}_{\Phi}}{d\eta}+p.\overline{X}_{\theta w}(p)=0$$

$$\overline{X}_{\Phi}(p)+\frac{d\overline{X}_{\theta w}}{d\eta}=0\,.$$

Secondly we have used Laplace transform, which is defined as

$$\overline{\overline{X}}(q,p)=\int_0^{\infty}\overline{X}(\eta,p).e^{-q\eta}\,d\eta \tag{23}$$

of real argument η and complex argument q, initial condition are $\overline{X}_{\theta w}(0,p)=\overline{X}_{\theta w1}(p)$, $\overline{X}_{\Phi}(0,p)=\overline{X}_{\Phi 1}(p)$:

$$q.\overline{\overline{X}}_{\Phi}-\overline{X}_{\Phi 1}+p.\overline{\overline{X}}_{\theta w}=0$$

$$\overline{\overline{X}}_{\Phi}+p.\overline{\overline{X}}_{\theta w}-\overline{X}_{\theta w1}=0\,.$$

Laplace transform image of partial differential equations system is:

$$\overline{\overline{X}}_{\Phi}=\frac{q}{q^2-p}.\overline{X}_{\Phi 1}-\frac{p}{q^2-p}.\overline{X}_{\theta w1}$$

$$\overline{\overline{X}}_{\theta w}=-\frac{1}{q^2-p}.\overline{X}_{\Phi 1}+\frac{q}{q^2-p}.\overline{X}_{\theta w1}\,.$$

According to inverse Laplace transform defined by integral (23), i.e. complex argument q we have got following equations:

$$\overline{X}_{\Phi}=\cosh\sqrt{p}.\overline{X}_{\Phi 1}-\sqrt{p}.\sinh\sqrt{p}.\overline{X}_{\theta w1}$$

$$\overline{X}_{\theta w}=-\frac{\sinh\sqrt{p}}{\sqrt{p}}.\overline{X}_{\Phi 1}+\cosh\sqrt{p}.\overline{X}_{\theta w1}$$

Let denote $\overline{X}_{\Phi}=\overline{X}_{\Phi 2}$ and $\overline{X}_{\theta w}=\overline{X}_{\theta w2}$. And simultaneously due to the fact that the unknowns have been heat flow taken away from refrigerated wall surface Φ_2 and temperature of the heated wall surface θ_{w1}, we have expressed $\overline{X}_{\Phi 2}$ and $\overline{X}_{\theta w1}$. We have got:

$$\overline{X}_{\Phi 2}=\frac{1}{\cosh\sqrt{p}}.\overline{X}_{\Phi 1}-\sqrt{p}.tgh\sqrt{p}.\overline{X}_{\theta w2} \tag{24}$$

$$\overline{X}_{\theta w1}=-\frac{tgh\sqrt{p}}{\sqrt{p}}.\overline{X}_{\Phi 1}+\frac{1}{\cosh\sqrt{p}}\overline{X}_{\theta w2} \tag{25}$$

If we denote:

$$G_1(p)=\frac{1}{\cosh\sqrt{(p)}}, \tag{26}$$

$$G_2(p)=\sqrt{(p)}\cdot tgh\sqrt{(p)} \tag{27}$$

$$G_3(p)=\frac{tgh\sqrt{(p)}}{\sqrt{(p)}}. \tag{28}$$

Equations (24) and (25) are: [3, 5]:

$$\overline{X}_{\Phi 2}(p) = G_1(p)\cdot \overline{X}_{\Phi 1}(p) - G_2(p)\cdot \overline{X}_{\theta w2}(p) \tag{29}$$

$$\overline{X}_{\theta w1}(p) = G_3(p)\cdot \overline{X}_{\Phi 1}(p) - G_1(p)\cdot \overline{X}_{\theta w2}(p) \tag{30}$$

Next problem has been to find function which correspond to (29) and (30) using inverse Laplace transform with complex argument p. View of the fact, that transfers function (26) - (28) are not defined as images, we have had to use Taylor series:

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} ...,$$

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} ...,$$

$$tgh\, x = \frac{\sinh x}{\cosh x}$$

and we have got transfer functions:

$$G_1(p) = \frac{1}{\cosh\sqrt{p}} = \frac{1}{1 + \frac{\sqrt{p^2}}{2} + \frac{\sqrt{p^4}}{24}} = \frac{24}{24 + 12p + p^2}$$

$$G_2(p) = \sqrt{p}\frac{\sinh\sqrt{p}}{\cosh\sqrt{p}} = \sqrt{p}\frac{\left(\sqrt{p} + \frac{\sqrt{p}^3}{6}\right)}{1 + \frac{\sqrt{p^2}}{2} + \frac{\sqrt{p^4}}{24}} =$$

$$= \frac{4.\left(6p + p^2\right)}{24 + 12p + p^2}$$

$$G_3(p) = \frac{\sinh\sqrt{p}}{\cosh\sqrt{p}}\cdot\frac{1}{\sqrt{(p)}} = \frac{\sqrt{(p)}}{\sqrt{(p)}}\cdot\frac{1 + \frac{p}{6} + \frac{p^2}{120}}{1 + \frac{p}{2} + \frac{p^2}{24}} =$$

$$= \frac{120 + 20\cdot p + p^2}{\left(24 + 12\cdot p + p^2\right)\cdot 5}$$

Using substitution according (19) $p = T_s \cdot s$

$$G_1(p) = \frac{24}{24 + 12T_s s + T_s s^2} \tag{31}$$

$$G_2(p) = \frac{4.\left(6T_s s + T_s s^2\right)}{24 + 12T_s s + T_s s^2} \tag{32}$$

$$G_3(p) = \frac{120 + 20.T_s s + T_s s^2}{\left(24 + 12.T_s s + T_s s^2\right)5} \tag{33}$$

where $T_s = \frac{d_w^2 \cdot \rho \cdot c}{\lambda}$ is constant, which is depended on wall properties.

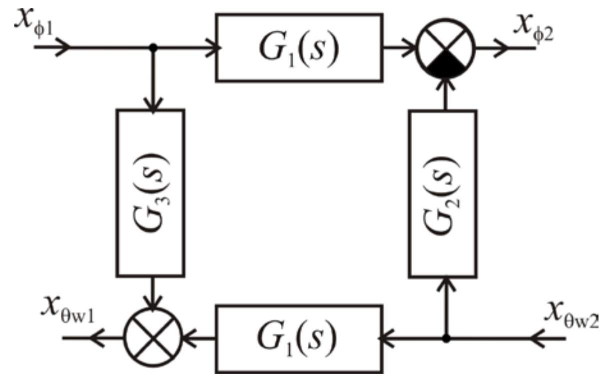

Fig. 2 Block diagram based on the (31) - (33)

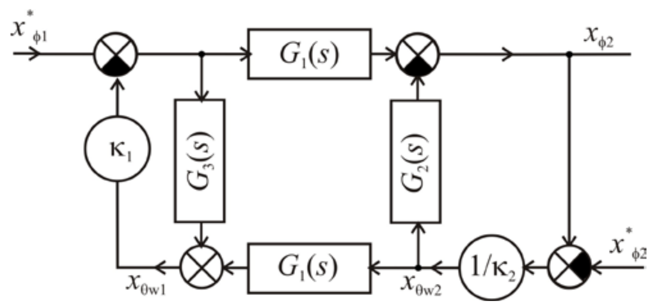

Fig. 3 Block diagram with $x^*_{\Phi 1}$, $x^*_{\Phi 2}$

III. Application of solution of partial differential equations for simulation model of wall

Relations expressed by system of partial differential equations (29) and (30) can be shows as a block diagram in fig. 2. [8, 10]:

Equations (16) - (17) or (20) - (21) still need to be supplemented by the equations of heat transfer on both sides of the wall surfaces.

In dimensionless form for internal surface area of the equation:

$$x_{\Phi 1} = x^*_{\Phi 1} - \kappa_1 x_{\theta w1} \tag{34}$$

where $x^*_{\Phi 1}$ includes external conditions on heat flow changes and $\kappa_1 = \frac{\alpha_1 . d_w}{\lambda}$. For external surface area is valid equation

$$x_{\Phi 2} = x^*_{\Phi 2} + \kappa_2 x_{\theta w2} \tag{35}$$

where $x^*_{\Phi 2}$ includes external conditions on heat flow changes and $\kappa_2 = \frac{\alpha_2 . d_w}{\lambda}$.

Block diagram in fig. 2 have been extended using (34), (35) to ensure impact of heat flow supplied into heated wall surface Φ_1 and Φ_2 and impact of external conditions. (Fig. 3).

Finally, simulation model of heat transfer dynamic through the wall based on the block diagrams in fig. 2 and fig. 3 in Matlab Simulink has been created. Problem of transfer functions (31), (32), (33) specifying has been to designed system (29) - (30) has been stable. If we have used more terms of Taylor series system was unstable. Transfer functions in the forms (31) - (33) have had negative roots and therefore system is stable. [3, 11, 12, 13, 14]

IV. Implementation Model into Model of Heating System Based on the Outdoor Temperature Compensation

In fig. 4 is presented simulation model of dynamic of heat transfer through the wall in Matlab.

Model has been implemented into model of heating system using of outdoor temperature compensation, in which others models: model of heating body and model of equitherm curves based on the temperature levels have been also included. Whole model is shown in fig. 5.

For creation first of them, hot-water heating body simulation model, analytic identification method based on mathematical description of a heat exchanger has been chosen. The heat body can be described as a heat transfer system between heating water and warmed-up air layer. Heating water circulates inside a radiator and deliver heat through surface layer of body. External side of radiator is surrounded by air layer, which is warmed-up and naturally flows up due to difference of specific heat weight. It was considered ideal mixing heating water in internal space of radiator and air in boundary layer of radiator. Input parameters have been temperature of refrigerated wall surface obtained from model of wall with some time delay and temperature of heating water obtain from model of equitherm curves. Output is temperature on the heating body surface. (Fig. 6)

Sub-model of heating body, mathematical description of model, differential equations, etc. were in detail mentioned in the works [15]. Simulations have not been published yet.

The second part of model of system based on then outdoor temperature compensation depends on the equithermic curves. The model of curves has contained table of 16 such curves, which have been characterized by their slope as their input parameters. If chosen slope is higher as heated space requires, permanent object overheating occurs. On the other hand, if chosen slope is lower than heated space requires, insufficient heating occurs. For that reason suitable curve has to be chosen for every heated space.

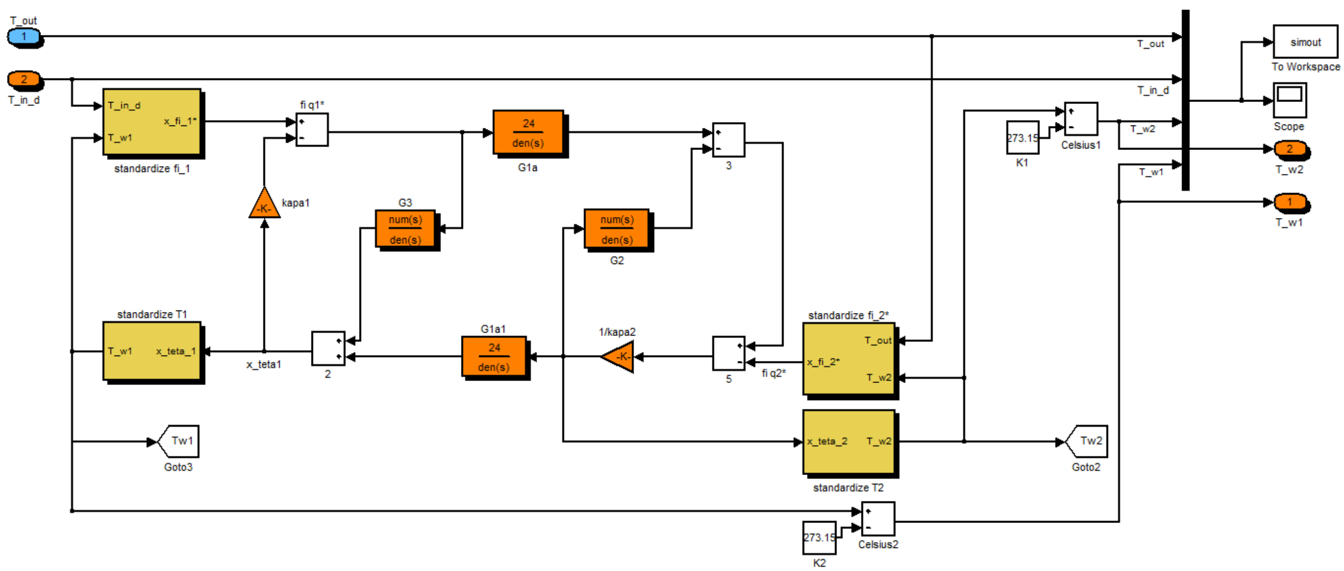

Fig. 4 Simulation model of heat transfer dynamic through the wall

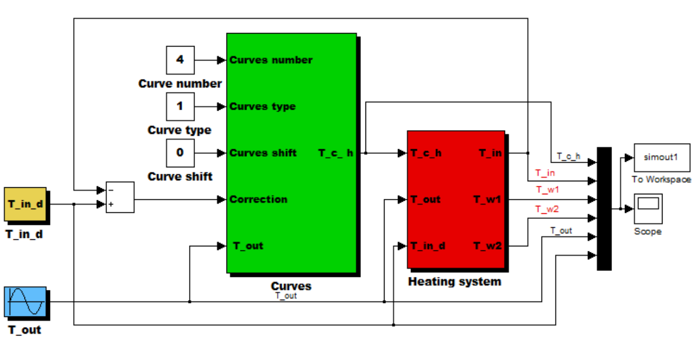

Fig. 5 Simulation model of whole heating system with model of equitherm curves

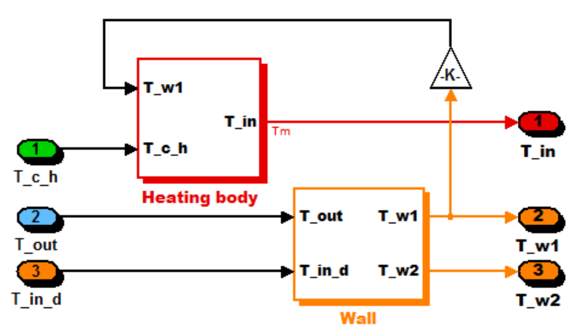

Fig. 6 Simulation model of heating system which consists from model of heating body and model of wall

Equithermic curve represents heat isolating attributes the heated object. Heating systems with higher thermal gradient have curves with higher heating water temperature and for curves with lower heating water temperature can be chosen for heating system with good isolation attributes.

In heating objects, temperature changes are realized by a curve shift to plus or minus values. Usually in a practice, temperature is reduced by a shift to minus values during economy mode [3].

Input parameters have been external temperature, internal temperature, curve type (1 or 2, because heating temperatures are arranged into two tables, depended on isolation attributes), curve number (can be 1-8) and curve shift (from -20°C till 20°C). Output is heating water temperature. Sub-model of equitherm curves has not been in detail published yet.

V. Discussion and Results

Next, we can present some results of simulations. For the simulation model of wall have been used the following real parameter values, which have corresponded to parameters measured on typical wall of building: wall thickness $d_w = 0{,}52$ (m), where wall has consisted of internal plaster with thickness 0,015 (m), external plaster with thickness 0,015 (m), internal isolation layer 0,15 (m) and external isolation layer 0,05 (m) and finally thickness of brick layer 0,29 (m). Volume of wall material $\rho = 1\,400$ (kg/m^3) and specific heat capacity $c = 840$ (J/(kg.K)). Thermal conductivity is $\lambda = 0{,}15$ (W/(m.K)) and it has also consisted of thermal conductivities of individual wall layers: $\lambda_1 = 0{,}7$, $\lambda_2 = 0{,}032$, $\lambda_3 = 0{,}52$, $\lambda_4 = 0.04$, $\lambda_5 = 0.87$, (W/(m.K)) and has been calculated as

$$\lambda = \frac{1}{R}$$

where R is thermal resistance obtained by form:

$$\lambda = \frac{d_{w1}}{\lambda_1} + \frac{d_{w2}}{\lambda_2} + \frac{d_{w3}}{\lambda_3} + \frac{d_{w4}}{\lambda_4} + \frac{d_{w5}}{\lambda_5}.$$

Outdoor temperature has been simulated as sine wave with frequency 2π/86400 (rad/s) and amplitude 5 (°C). Internal requested constant temperature has been 20 (°C). Fig. 7 captures 2 days, i. e. 172800 (sec). Behavior of temperatures has corresponded to simulated sine wave, but internal wall layer has been refrigerated by about 3 degrees of Celsius.

In Fig. 8 is presented similar situation, but wall has consisted of wood layer without isolation layers. Parameters are thickness of wall $d_w = 0{,}5$ (m), volume of wall material $\rho = 600$ (kg/m^3), specific heat capacity $c = 1\,450$ (J/(kg.K)) and finally thermal conductivity of whole wall is $\lambda = 0{,}84$ (W/(m.K)).

As we can see in this figure, heat losses appeared. Refrigerated wall surface has had higher temperature due to higher heat transfer through the wall. And on the other hand, heated wall surface has had lower temperature as requested internal temperature is.

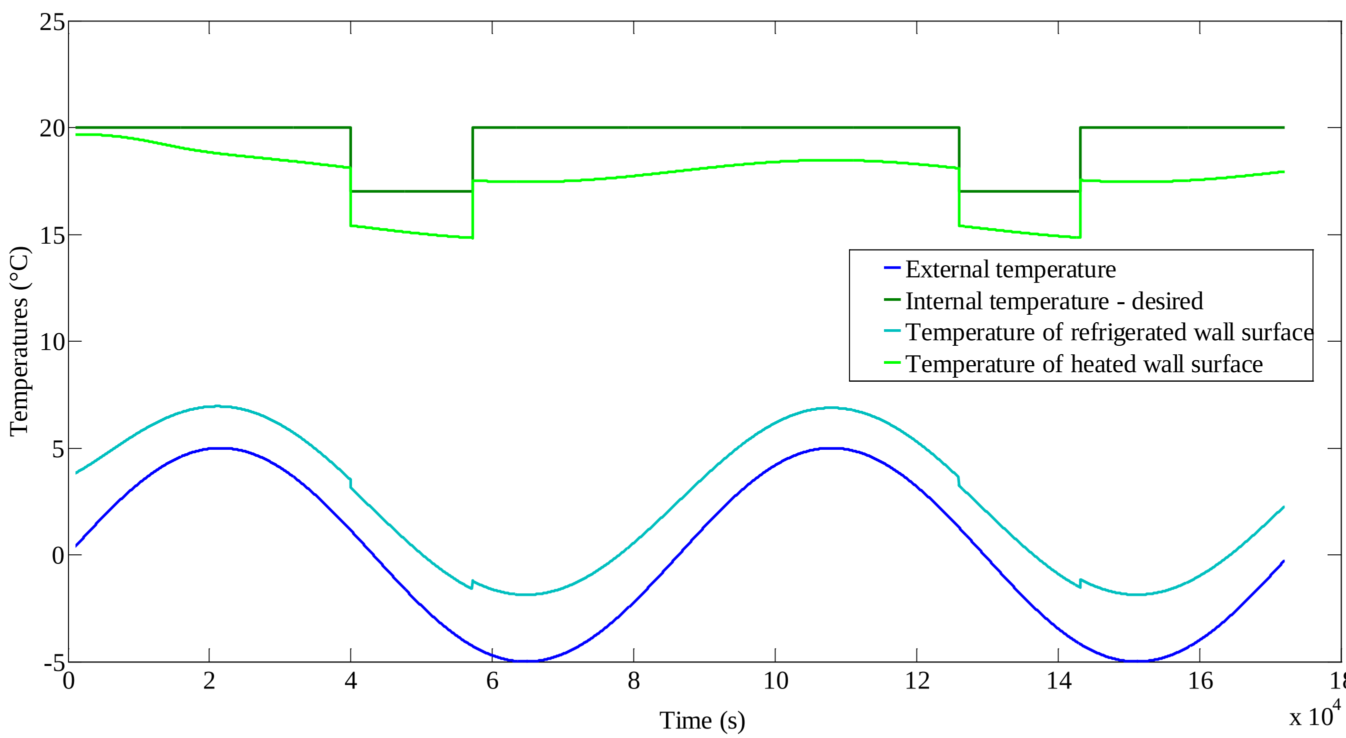

Fig. 7 Simulation results: internal temperature θ_{w10}, external temperature θ_{w20}, temperature of the heated wall surface θ_{w1}, temperature of the refrigerated wall surface θ_{w2} with input conditions parameters c, ρ, d_w.

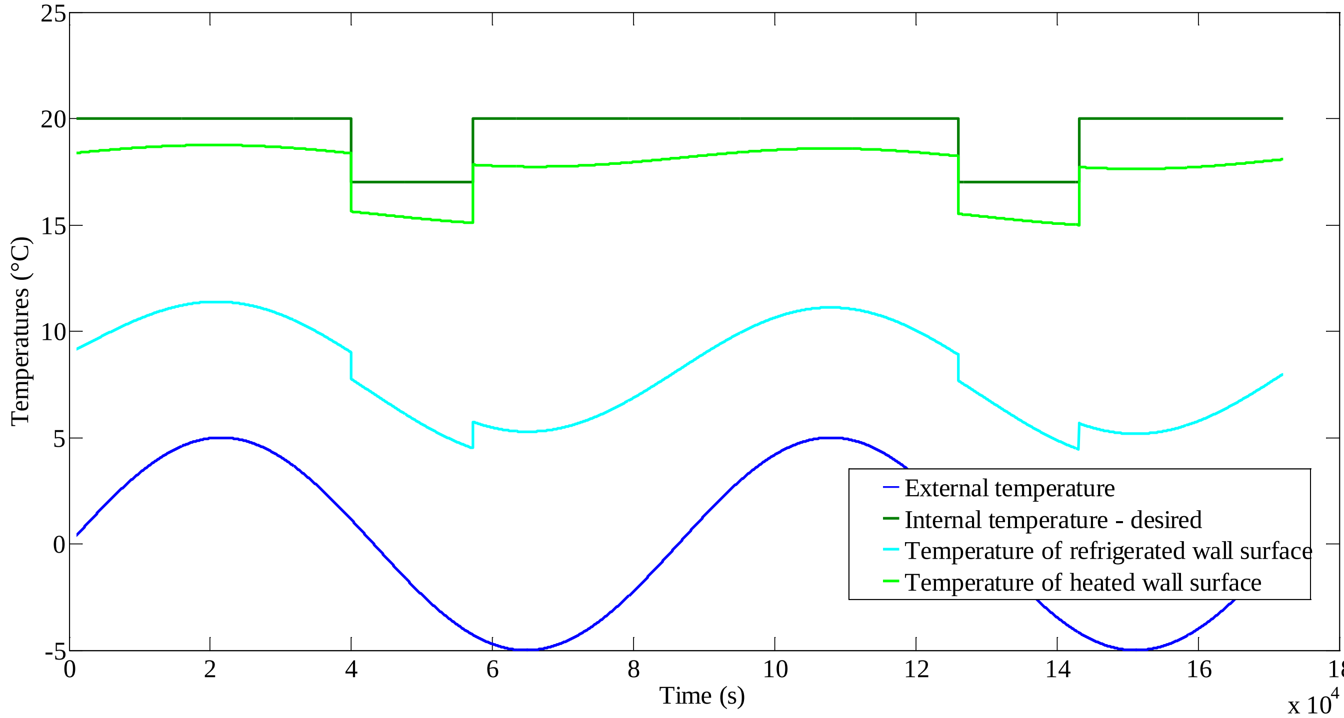

Fig. 8 Simulation results: internal temperature θ_{w10}, external temperature θ_{w20}, temperature of the heated wall surface θ_{w1}, temperature of the refrigerated wall surface θ_{w2} with input conditions parameters c, ρ, d_w.

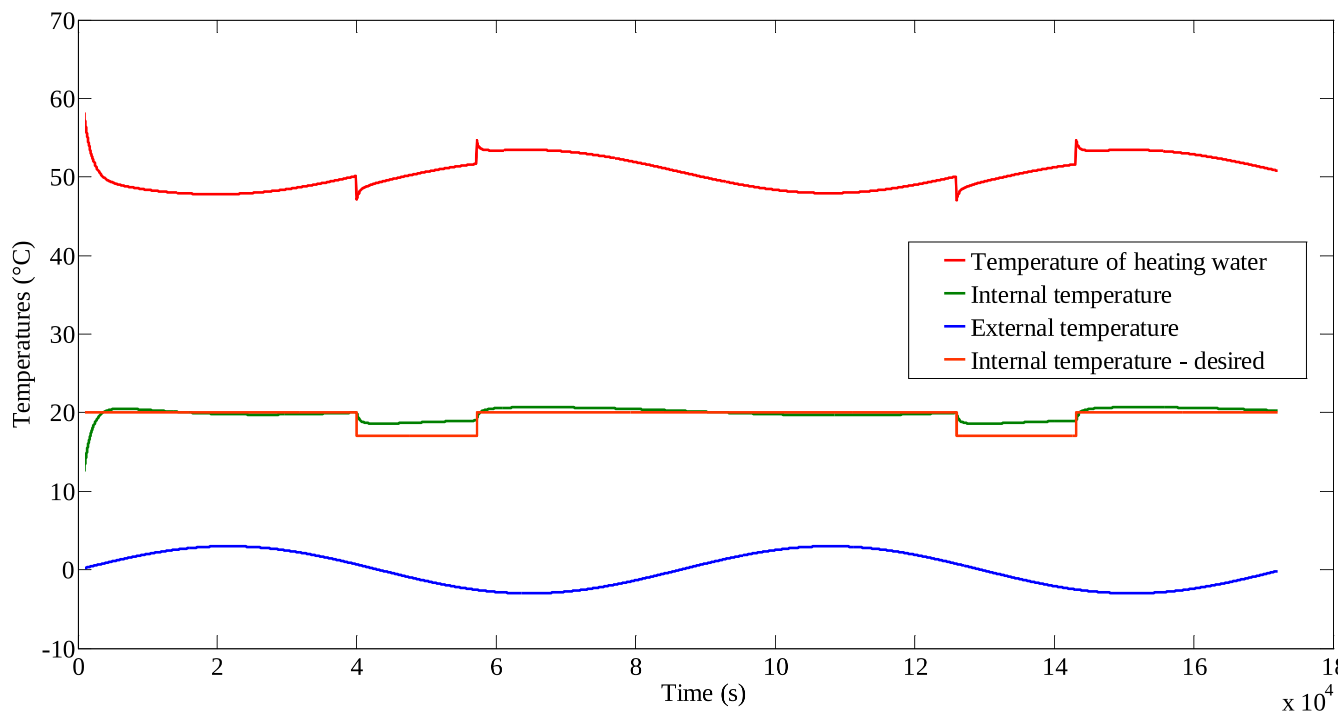

Fig. 9 Simulation results: internal temperature θ_{w10}, external temperature θ_{w20}, internal requested temperature θ_{10}, temperature of the heating water with input conditions parameters c, ρ, d_w.

When model of wall was implemented into model based on the outdoor temperature compensation, requested temperature contained intervals of economy mode. In our countries is a common using economy mode, which starts from 22.00 p.m. to 3.30 p.m. and requested temperature is lower (e.g. 17 (°C)). Usually, the aim of economy mode is saving energy and ensuring of temperature comfort during nights. In our model, such situation has been simulated from 39 600 (sec) till 55 800 (sec). Situation in fig. 7 and 8. can be compared to fig. 5 and 6 in work [16], where requested temperature was constant.

The last compared results are in fig. 9: temperature of heating water, requested internal temperature and simulated internal temperature and external temperature. Parameters of wall have been the same as in fig. 7. To internal temperature has been maintained at requested temperature, when external outdoor temperature decreases, heating water temperature must increase, e.g. from 30 000 (sec) till 100 000 (sec). Economy mode has also been respected. Parameters of equitherm curves have been: number of curve have been 4 (-), type of curve have been 1 (-) and shift have been 0 (°C). Simulation of external temperature has been shown in fig. 7.

VI. Conclusion

To summarize, the first part of paper deals with solving of partial differential equations system, where problem of solution inverse Laplace transform appears, because transfers function (26) - (28) are not defined as images. To solve this problem we have had to find Taylor series of image functions ($\sin x$, $\cos x$ and $\mathrm{tg}\, x$) as a result of first inverse Laplace transform.

These results of Laplace transform have been used in second part of the paper, where have been used as transfer functions for simulation model of heat transfer through the wall. Next problem has been to find such number of terms of Taylor series to the system has been stable. Basic model in Fig. 3 has had to be extended by, for example, standardization of input temperature variables or unit conversion from Celsius to Kelvin scale, etc. I would like to mention, that in our countries is more common using Celsius scale, for that reason results are in degrees of Celsius but model uses Kelvin scale.

The designed model mentioned above has been created by Matlab Simulink, has been described and obtained simulation results have been presented.

Finally, the model of wall has been implemented into designed model of heating system based on the outdoor temperature compensation containing model of heat transfer dynamic through the wall together with model of heating body and with model of equitherm curves. The model has been tested.

In the future work control will be simulations in Matlab compared to real heating system.

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