A Lyapunov Approach for a PI-Controller with Anti-windup in a Permanent Magnet Synchronous Motor using Chopper Control

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***Abstract*—This paper deals with a parameter set up of a PI-regulator to be applied in a system for permanent magnet three-phase synchronous motors to obtain a smooth tracking dynamics even though a chopper control structure is included in the drive. In particular, an anti-windup control structure is considered to avoid saturation and conditions on all controller parameters are found which guarantee stability. High performance application of permanent magnet synchronous motors (PMSM) is increasing. In particular, application in electrical vehicles is very much used. The technique uses a geometric decoupling procedure and a Lyapunov approach to perform a PWM control to be used as a chopper. Chopper control structures are very popular because they are very cheap and easy to be realised. Nevertheless, using a chopper control structure smooth tracking dynamics could be difficult to be obtained without increasing the switching frequency because of the discontinuity of the control signals. No smooth tracking dynamics lead to a not comfortable travel effect for the passengers of an electrical vehicle or, more in general, it could be difficult to generate an efficient motion planning if the tracking dynamics are not smooth. The paper presents a technique to minimise these undesired effects. The presented technique is generally applicable and could be used for other types of electrical motors, as well as for other dynamic systems with nonlinear model structures. Through simulations of a synchronous motor used in automotive applications, this paper verifies the effectiveness of the proposed method and discusses the limits of the results.**

***Index Terms*—PD regulators, velocity control, adaptive control**

I. INTRODUCTION

As the high field strength neodymium-iron-boron (NdFeB) magnets become commercially available with affordable prices. PMSMs are receiving increasing attention due to their high speed, high power density and high efficiency. These characteristics are very favorable in some special high performance applications, e.g. robotics, aerospace, and electric ship propulsion systems [1], [2]. Permanent magnet synchronous motors (PMSMs) as traction motors are common in electric or hybrid road vehicles. For rail vehicles, PMSMs as traction motors are not widely used yet. Although the traction PMSM can bring many advantages, just a few prototypes of vehicles were built and tested. The next two new prototypes of rail vehicles with traction PMSMs were presented on InnoTrans fair in Berlin 2008. They were Alstom AGV high speed train and Skoda Transportation low floor tram 15T ForCity.

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Advantages of PMSM are well known. The greatest advantage is a low volume of the PMSM in comparison with other types of motors. It makes a direct drive of wheels possible. On the other hand, the traction drive with PMSM has to meet special requirements typical for overhead line fed vehicles. The drives and specially their control should be robust to wide overhead line voltage tolerance (typically from -30% to $+20\%$), voltage surges and input filter oscillations. These aspects may cause problems during flux weakening operation. There are several reasons to use flux weakening operation of a traction drive. The typical reason is a constant power operation in a wide speed range and reaching nominal power during low speed (commonly 1/3 of maximum speed). In the case of common traction, motors like asynchronous or DC motors, it is possible to reach the constant power region using flux weakening. This is also possible for traction PMSM, however, a problem with high back electromagnetic force (EMF) rises. To achieve the desired system performance, advanced control systems are usually required to provide fast and accurate response, quick disturbance recovery, and parameter variations insensitivity [3]. In [4] it is shown how using a flux weakening control strategy for PMSM a prediction control structure improves the dynamic performance of traditional feedback control strategies in terms, for instance, of overshoot and rising time. To realize an effective prediction control, it is known that an accurate knowledge of the model and its parameters is necessary. In [5] an identification technique is shown to detect parameters such as R_s, L_{dq} and Φ or the PMSM is shown. In the existing applications chopper control structures are very popular because they are very cheap and easy to be realised. Nevertheless, using a chopper control structure smooth tracking dynamics could be difficult to be obtained without increasing the switching frequency because of the discontinuity of the control signals. No smooth tracking dynamics lead to a not comfortable travel effect for the passengers of the electrical vehicle. PI regulators are very often used in industrial applications because of their simple structure and in the last years advanced PI controllers have been developed to control nonlinear systems, [6], [7]. One of the most important problems concerning PI or PID controllers is tuning their parameters. In the scientific literature one of the most important approaches is the optimal tuning. An interesting contribution in this context is that presented in [8] in which the authors proposes as tuning techniques two sets of evolutionary strategies which are differential evolution and genetic algorithms. In [8] the optimal PID control parameters are applied for a high order system, system with time delay and non-minimum phase system. This paper extends the results proposed in [9] and deals with a parameter set up of a PI regulator to be applied in a system for a permanent magnet three-phase synchronous motor to obtain smooth tracking dynamics even though a chopper control structure is included in the drive. Moreover, the proposed PI controller uses an anti-windup scheme to avoid saturations. When saturation happens, the feedback loop is effectively broken and if a regulator with an integrator is used, the error will continue to be integrated. The value at the output of the regulator can become very large and often degrades the closed-loop performance in the form of large overshoot, long settling time and sometimes even instability. This is more evident if compared with the expected linear performance for the systems. The phenomenon described is called windup. The windup phenomenon has attracted interest in academic and in industrial community already at the end of the eighties. The problem became to have the first solution, an overview of the basic schemes is available in [10]. Other contributions like in [11] analyse the conditions in order to find invariant subspaces and in [12] a pre-action is employed to avoid windup. The paper is organized in the following way. In Section II a sketch of the model of the synchronous motor and its behaviour are given. Section III is devoted to derive a decoupling controller which will be used to calculate parameters K_p and K_i of the controller with Lyapunov approach. Section V shows simulation results using real data. The conclusions close the paper.

II. Model and behavior of a synchronous motor

To aid advanced controller design for PMSM, it is very important to obtain an appropriate model of the motor. A good model should not only be an accurate representation of system dynamics but also facilitate the application of existing control techniques. Among a variety of models presented in the literature, since the introduction of PMSM, the two axis dq-model obtained using Park's transformation is the most widely used in variable speed PMSM drive control applications, see [3] and [13]. The Park's dq-transformation is a coordinate transformation that converts the three-phase stationary variables into variables in a rotating coordinate system. In dq-transformation, the rotating coordinate is defined relative to a stationary reference angle as illustrated in Fig. 1. The dq-model is considered in this work.

$$\begin{bmatrix} u_d(t) \\ u_q(t) \\ u_0(t) \end{bmatrix} = \begin{bmatrix} \frac{2\sin(\omega_{el}t)}{3} & \frac{2\sin(\omega_{el}-2\pi/3)}{3} & \frac{2\sin(\omega_{el}+2\pi/3)}{3} \\ \frac{2\cos(\omega_{el}t)}{3} & \frac{2\cos(\omega_{el}-2\pi/3)}{3} & \frac{2\cos(\omega_{el}+2\pi/3)}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} u_a(t) \\ u_b(t) \\ u_c(t) \end{bmatrix}, \quad (1)$$

$$\begin{bmatrix} i_d(t) \\ i_q(t) \\ i_0(t) \end{bmatrix} = \begin{bmatrix} \frac{2\cos(\omega_{el}t)}{3} & \frac{2\cos(\omega_{el}-2\pi/3)}{3} & \frac{2\cos(\omega_{el}+2\pi/3)}{3} \\ \frac{-2\sin(\omega_{el}t)}{3} & \frac{-2\sin(\omega_{el}-2\pi/3)}{3} & \frac{-2\sin(\omega_{el}+2\pi/3)}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} i_a(t) \\ i_b(t) \\ i_c(t) \end{bmatrix}. \quad (2)$$

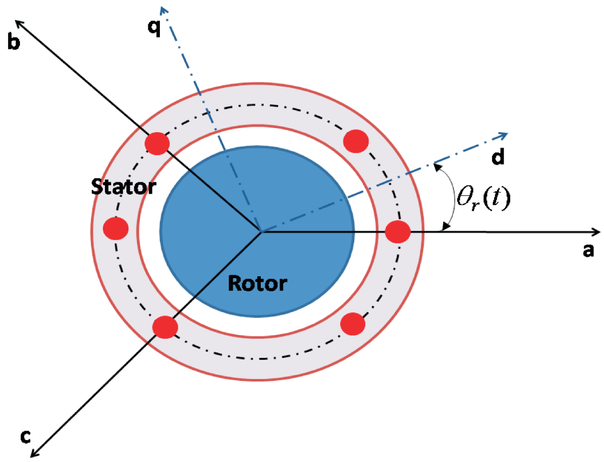

Fig. 1. Park's transformation for the motor

The dynamic model of the synchronous motor in d-q-coordinates can be represented as follows:

$$\begin{bmatrix} \frac{di_d(t)}{dt} \\ \frac{di_q(t)}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{R_s}{L_d} & \frac{L_q}{L_d}\omega_{el}(t) \\ -\frac{R_s}{L_q} & -\frac{L_d}{L_q}\omega_{el}(t) \end{bmatrix} \begin{bmatrix} i_d(t) \\ i_q(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{L_d} & 0 \\ 0 & \frac{1}{L_q} \end{bmatrix} \begin{bmatrix} u_d(t) \\ u_q(t) \end{bmatrix} - \begin{bmatrix} 0 \\ \frac{\Phi\omega_{el}(t)}{L_q} \end{bmatrix}, \quad (3)$$

and

$$M_m = \frac{3}{2}p\{\Phi i_q(t) + (L_d - L_q)i_d(t)i_q(t)\}. \quad (4)$$

In (3) and (4), $i_d(t)$, $i_q(t)$, $u_d(t)$ and $u_q(t)$ are the dq-components of the stator currents and voltages in synchronously rotating rotor reference frame; $\omega_{el}(t)$ is the rotor electrical angular speed; parameters R_s, L_d, L_q, Φ and p are the stator resistance, d-axis and q-axis inductance, the amplitude of the permanent magnet flux linkage, and p the number of couples of permanent magnets, respectively. At the end the motor torque is indicated with M_m. Considering an isotropic motor for that $L_d \simeq L_q = L_{dq}$, it follows:

$$\begin{bmatrix} \frac{di_d(t)}{dt} \\ \frac{di_q(t)}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{R_s}{L_{dq}} & \omega_{el}(t) \\ -\frac{R_s}{L_{dq}} & \omega_{el}(t) \end{bmatrix} \begin{bmatrix} i_d(t) \\ i_q(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{L_{dq}} & 0 \\ 0 & \frac{1}{L_{dq}} \end{bmatrix} \begin{bmatrix} u_d(t) \\ u_q(t) \end{bmatrix} - \begin{bmatrix} 0 \\ \frac{\Phi\omega_{el}(t)}{L_{dq}} \end{bmatrix}, \quad (5)$$

and

$$M_m = \frac{3}{2}p\Phi i_q(t) \quad (6)$$

with the following movement equation:

$$M_m - M_w = J\frac{d\omega_{mec}(t)}{dt}, \quad (7)$$

where $p\omega_{mech}(t) = \omega_{el}(t)$ and M_w is an unknown mechanical load.

III. STRUCTURE OF THE DECOUPLING CONTROLLER

To achieve a decoupled structure of the system described in equation (5) a matrix $\mathbf{F}$ is to be calculated such that:

$$(\mathbf{A}+\mathbf{BF})\mathcal{V} \subseteq \mathcal{V}, \quad (8)$$

where $\mathbf{u} = \mathbf{Fx}$ is a state feedback with $\mathbf{u} = [u_d, u_q]^T$ and $\mathbf{x} = [i_d, i_q]^T$,

$$\mathbf{A} = \begin{bmatrix} -\frac{R_s}{L_{dq}} & \omega_{el}(t) \\ -\frac{R_s}{L_{dq}} & \omega_{el}(t) \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \frac{1}{L_{dq}} & 0 \\ 0 & \frac{1}{L_{dq}} \end{bmatrix}, \quad (9)$$

and $\mathcal{V} = im([1,0]^T)$ is a controlled invariant subspace. More explicitly it follows:

$$\mathbf{F} = \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix}, \text{and} \begin{bmatrix} u_d(t) \\ u_q(t) \end{bmatrix} = \mathbf{F} \begin{bmatrix} i_d(t) \\ i_q(t) \end{bmatrix},$$

then, according to [14], the decoupling of the dynamics is obtained considering the following relationship:

$$\text{im}\left(\begin{bmatrix} -\frac{R_s}{L_{dq}} & \omega_{el}(t) \\ -\frac{R_s}{L_{dq}} & \omega_{el}(t) \end{bmatrix}\right) + \text{im}\left(\begin{bmatrix} \frac{1}{L_{dq}} & 0 \\ 0 & \frac{1}{L_{dq}} \end{bmatrix} \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) \subseteq \text{im} \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad (10)$$

where parameters F_{11}, F_{12}, F_{21}, and F_{22} are to be calculated in order to guarantee condition (10) and a suitable dynamics for sake of estimation, as it will be explained in the next. Condition (10) is guaranteed if:

$$F_{21} = R_s. \quad (11)$$

After decoupling the first equations of the system represented in (5) becomes as follows:

$$\frac{di_q(t)}{dt} = \omega_{el}(t) i_q(t) + \frac{u_q(t)}{L_{dq}}. \quad (12)$$

IV. A LYAPUNOV APPROACH TO SET THE PI-CONTROLLER PARAMETERS

Considering the following PI-controller:

$$u_c(t) = K_p(\omega_{mec_d} - \omega_{mec}(t)) + K_i \int_0^t (\omega_{mec_d} - \omega_{mec}(\tau)) d\tau + A_w(t), \quad (13)$$

with $A_w(t)$ is the anti-windup signal with

$$A_w(t) = K_b\Big(u_c^*(t) - K_p(\omega_{mec_d} - \omega_{mec}(t)) - K_i \int_0^t (\omega_{mec_d} - \omega_{mec}(\tau)) d\tau\Big), \quad (14)$$

where

$$u_c^*(t) = \begin{cases} u_c(t) & if \;\; u_c(t) < Sat_{out} \\ Sat_{out} & if \;\; u_c(t) \geq Sat_{out}, \end{cases} \quad (15)$$

where Sat_{out} is the constat of the saturating level of the output of the controller which corresponds to the saturation level of the compressor output, K_b is a constant to be set.

A. PI-Controller parameters conditions in case of no saturation

If ω_{mec_d} is a constant, it follows that:

$$\frac{\partial u_c(t)}{\partial t} = -K_p \frac{\partial \omega_{mec}(t)}{\partial t} + K_i(\omega_{mec_d} - \omega_{mec}(t)), \quad (16)$$

equation (12) can be written in the following way:

$$i_q(t) = \frac{1}{\omega_{el}(t)}\Big(-\frac{di_q(t)}{dt} + \frac{u_q(t)}{L_{dq}}\Big). \quad (17)$$

Combining equations (6) and (17), then the following expression is obtained:

$$M_m = \frac{3}{2} p \Phi \frac{1}{\omega_{el}(t)}\Big(-\frac{di_q(t)}{dt} + \frac{u_q(t)}{L_{dq}}\Big). \quad (18)$$

Considering equation (18) and equation (7), then the following relation is obtained:

$$\frac{3}{2} p \Phi \frac{1}{\omega_{el}(t)}\Big(-\frac{di_q(t)}{dt} + \frac{u_q(t)}{L_{dq}}\Big) - M_w = J\frac{d\omega_{mec}(t)}{dt}. \quad (19)$$

In order to set up parameters K_p and K_I of the controller, the following Lyapunov function is chosen:

$$V_{\mathcal{L}}(\omega_{mec}(t)) = \frac{1}{2}\big(\omega_{mec_d}(t) - \omega_{mec}(t)\big)^2. \quad (20)$$

Considering the derivative of (20), then:

$$\frac{\partial V_{\mathcal{L}}(\omega_{mec}(t))}{\partial t} = -(\omega_{mec_d}(t) - \omega_{mec}(t))\frac{\partial \omega_{mec}(t)}{\partial t}. \quad (21)$$

From equation(19) it follows that:

$$\frac{\partial V_{\mathcal{L}}(\omega_{mec}(t))}{\partial t} = -\frac{\big(\omega_{mec_d}(t) - \omega_{mec}(t)\big)}{J} \times \Big(\frac{3}{2} p\Phi \frac{1}{\omega_{el}(t)}\Big(-\frac{di_q(t)}{dt} + \frac{u_q(t)}{L_{dq}}\Big) - M_w\Big). \quad (22)$$

B. Monotonic increasing dynamics

Considering the expression in (16), equation (22) becomes as follows:

$$\frac{\partial V_{\mathcal{L}}(\omega_{mec}(t))}{\partial t} = -\frac{\big(\frac{K_p}{K_i}\frac{\partial \omega_{mec}(t)}{\partial t} + \frac{\partial u_c(t)}{\partial t}\frac{1}{K_i}\big)}{J} \times \Big(\frac{3}{2} p\Phi \frac{1}{\omega_{el}(t)}\Big(-\frac{di_q(t)}{dt} + \frac{u_q(t)}{L_{dq}}\Big) - M_w\Big) \leq 0. \quad (23)$$

If

$$\frac{\partial \omega_{mec}(t)}{\partial t} \geq 0, \quad (24)$$

and

$$\frac{\partial u_c(t)}{\partial t} \geq 0, \quad (25)$$

condition (23) is guaranteed $\forall\ K_p > 0$ and $\forall\ K_i > 0$.

Remark 1: Condition (24) should be guaranteed with a suitable choice of the parameters of the controller. This condition states monotonic dynamics and thus dynamics of the motor without oscillations for the monotonic increasing dynamics. In automotive field, this condition is an ideal one for optimality of the electrical driver consumption and comfort of the passengers. □

To show under which conditions the following relation

$$\frac{\partial\omega_{mec}(t)}{\partial t} \leq 0 \tag{26}$$

holds, let consider PI controller defined by equation (16) which can be rewritten in the following way:

$$\frac{\partial u_c(t)}{\partial t} + K_i(\omega_{mec_d} - \omega_{mec}(t)) = K_p\frac{\partial\omega_{mec}(t)}{\partial t} \geq 0. \tag{27}$$

Choosing K_p and K_i big enough, it is possible to consider

$$\frac{\partial u_c(t)}{\partial t} << K_i(\omega_{mec_d} - \omega_{mec}(t)) - K_p\frac{\partial\omega_{mec}(t)}{\partial t}. \tag{28}$$

The following condition must hold:

$$K_i(\omega_{mec_d} - \omega_{mec}(t)) - K_p\frac{\partial\omega_{mec}(t)}{\partial t} \geq 0, \tag{29}$$

which is equivalent to prove that:

$$K_p\frac{\partial\omega_{mec}(t)}{\partial t} + K_i\omega_{mec}(t) \geq K_i\omega_{mec_d}. \tag{30}$$

Considering the solution of the following differential equation, then:

$$K_p\frac{\partial\omega_{mec}(t)}{\partial t} + K_i\omega_{mec}(t) = 0, \tag{31}$$

then it must be:

$$\omega_{mec}(t) = \omega_{mec}(0)e^{-\frac{K_i}{K_p}t} \geq K_i\omega_{mec_d}, \tag{32}$$

then the following final general condition is obtained:

$$\boxed{K_i \leq \frac{\omega_{mec}(0)}{\omega_{mec_d}}K_p.} \tag{33}$$

Concerning assumption (25):

$$\frac{\partial u_c(t)}{\partial t} \geq 0, \tag{34}$$

let us consider equation (16) in which ω_{mec_d} is a constant, then:

$$\frac{\partial u_c(t)}{\partial t} - K_i - \omega_{mec}(t) = K_p\frac{\partial\omega_{mec}(t)}{\partial t} \geq 0. \tag{35}$$

Considering that it should be guaranteed $\frac{\partial u_c(t)}{\partial t} \geq 0$, then:

$$-K_p\frac{\partial\omega_{mec}(t)}{\partial t} + K_i(\omega_{mec_d} - \omega_{mec}(t)) \geq 0,$$

which it is equivalent to proof that:

$$K_p\frac{\partial\omega_{mec}(t)}{\partial t} + K_i\omega_{mec}(t) \leq K_i\omega_{mec_d}, \tag{36}$$

Considering the solution of

$$K_p\frac{\partial\omega_{mec}(t)}{\partial t} + K_i\omega_{mec}(t) = 0, \tag{37}$$

then it must be:

$$\frac{\partial\omega_{mec}(t)}{\partial t} = \omega_{mec}(0)e^{-\frac{K_i}{K_p}t} \leq \frac{\omega_{mec_d}}{K_p}K_i. \tag{38}$$

From equation (69) a boundary condition on K_i is obtained:

$$\boxed{K_i \geq \frac{\omega_{mec}(0)}{\omega_{mec_d}}K_p.} \tag{39}$$

Combining condition (33) with (71), the final sufficient condition on parameters K_p and K_i is obtained:

$$\boxed{K_i = \frac{\omega_{mec}(0)}{\omega_{mec_d}}K_p.} \tag{40}$$

C. Anti-windup and PI-controller parameters conditions in case of saturation and increasing dynamics

Let us consider equation (16) in which ω_{mec_d} is a constant, then:

$$\frac{\partial u_c(t)}{\partial t} = [1 - K_b]\Big(-K_p\frac{\omega_{mec}(t)}{\partial t} + K_i(\omega_{mec_d} - \omega_{mec}(t))\Big). \tag{41}$$

Considering that it should be guaranteed $\frac{\partial u_c(t)}{\partial t} \geq 0$, then:

$$\boxed{0 \leq K_b \leq 1,} \tag{42}$$

and

$$K_p\frac{\partial\omega_{mec}(t)}{\partial t} - K_i(\omega_{mec_d} - \omega_{mec}(t)) \leq 0,$$

which it is equivalent to proof that:

$$K_bK_p\frac{\partial\omega_{mec}(t)}{\partial t} + K_bK_i\omega_{mec}(t) \geq K_bK_i\omega_{mec_d}. \tag{43}$$

Considering the solution of

$$K_p\frac{\partial\omega_{mec}(t)}{\partial t} + K_i\omega_{mec}(t) = 0, \tag{44}$$

then it must be:

$$\omega_{mec}(t) = \omega_{mec_i}e^{-\frac{K_i}{K_p}t} \geq \frac{\omega_{mec_d}}{K_p}K_i, \tag{45}$$

where $\omega_{mec_i} = \omega_{mec}(0)$. From equation (69) a boundary condition on K_i is obtained:

$$\boxed{K_i \leq \frac{\omega_{mec}(0)}{\omega_{mec_d}}K_p.} \tag{46}$$

D. Monotonic decreasing dynamics

Considering the expression in (16), equation (22) becomes as follows:

$$\frac{\partial V_{\mathcal{L}}(\omega_{mec}(t))}{\partial t} = -\frac{\left(\frac{K_p}{K_i}\frac{\partial\omega_{mec}(t)}{\partial t} + \frac{\partial u_c(t)}{\partial t}\frac{1}{K_i}\right)}{J}\times \Big(\frac{3}{2}p\Phi\frac{1}{\omega_{el}(t)}\Big(-\frac{di_q(t)}{dt} + \frac{u_q(t)}{L_{dq}}\Big) - M_w\Big) \leq 0. \tag{47}$$

If

$$\frac{\partial\omega_{mec}(t)}{\partial t} \leq 0, \tag{48}$$

and

$$\frac{\partial u_c(t)}{\partial t} \leq 0, \tag{49}$$

condition (47) is guaranteed $\forall\ K_p > 0$ and $\forall\ K_i > 0$.

Remark 2: Condition (48) should be guaranteed with a suitable choice of the parameters of the controller. This condition states monotonic dynamics and thus dynamics of the motor without oscillations also for the monotonic decreasing dinamics. In automotive field, this condition is an ideal one for optimality of the electrical driver consumption and comfort of the passengers. □

To show under which conditions the following relation

$$\frac{\partial \omega_{mec}(t)}{\partial t} \leq 0 \tag{50}$$

holds, let consider PI controller defined by equation (16) which can be rewritten in the following way:

$$\frac{\partial u_c(t)}{\partial t} + K_i(\omega_{mec_d} - \omega_{mec}(t)) = K_p \frac{\partial \omega_{mec}(t)}{\partial t} \leq 0. \tag{51}$$

Choosing K_p and K_i big enough, it is possible to consider

$$\frac{\partial u_c(t)}{\partial t} << K_i(\omega_{mec_d} - \omega_{mec}(t)) - K_p \frac{\partial \omega_{mec}(t)}{\partial t}. \tag{52}$$

The following condition must hold:

$$K_i(\omega_{mec_d} - \omega_{mec}(t)) - K_p \frac{\partial \omega_{mec}(t)}{\partial t} \leq 0, \tag{53}$$

which is equivalent to prove that:

$$K_p \frac{\partial \omega_{mec}(t)}{\partial t} + K_i \omega_{mec}(t) \geq K_i \omega_{mec_d}. \tag{54}$$

Considering the solution of the following differential equation, then:

$$K_p \frac{\partial \omega_{mec}(t)}{\partial t} + K_i \omega_{mec}(t) = 0, \tag{55}$$

then it must be:

$$\omega_{mec}(t) = \omega_{mec}(0) e^{-\frac{K_i}{K_p} t} \geq K_i \omega_{mec_d}, \tag{56}$$

then the following final general condition is obtained:

$$\boxed{K_i \geq \frac{\omega_{mec}(0)}{\omega_{mec_d}} K_p.} \tag{57}$$

Concerning assumption (25):

$$\frac{\partial u_c(t)}{\partial t} \leq 0, \tag{58}$$

let us consider equation (16) in which ω_{mec_d} is a constant, then:

$$\frac{\partial u_c(t)}{\partial t} - K_i - \omega_{mec}(t) = K_p \frac{\partial \omega_{mec}(t)}{\partial t} \leq 0. \tag{59}$$

Considering that it should be guaranteed $\frac{\partial u_c(t)}{\partial t} \geq 0$, then:

$$-K_p \frac{\partial \omega_{mec}(t)}{\partial t} + K_i(\omega_{mec_d} - \omega_{mec}(t)) \geq 0,$$

which is equivalent to proof that:

$$K_p \frac{\partial \omega_{mec}(t)}{\partial t} + K_i \omega_{mec}(t) \geq K_i \omega_{mec_d}, \tag{60}$$

Considering the solution of

$$K_p \frac{\partial \omega_{mec}(t)}{\partial t} + K_i \omega_{mec}(t) = 0, \tag{61}$$

then it must be:

$$\frac{\partial \omega_{mec}(t)}{\partial t} = \omega_{mec}(0) e^{-\frac{K_i}{K_p} t} \leq \frac{\omega_{mec_d}}{K_p} K_i. \tag{62}$$

From equation (69) a boundary condition on K_i is obtained:

$$\boxed{K_i \leq \frac{\omega_{mec}(0)}{\omega_{mec_d}} K_p.} \tag{63}$$

Combining condition (57) with (63), the final sufficient condition on parameters K_p and K_i is obtained:

$$\boxed{K_i = \frac{\omega_{mec}(0)}{\omega_{mec_d}} K_p.} \tag{64}$$

E. Anti-windup and PI-controller parameters conditions in case of saturation and decreasing dynamics

Let us consider equation (16) in which p_d is a constant, then:

$$\frac{\partial u_c(t)}{\partial t} = [1 - K_b] \Big(-K_p \frac{\omega_{mec}(t)}{\partial t} + K_i(\omega_{mec_d} - \omega_{mec}(t)) \Big). \tag{65}$$

Considering that it should be guaranteed $\frac{\partial u_c(t)}{\partial t} \leq 0$, then

$$\boxed{0 \leq K_b \leq 1,} \tag{66}$$

and

$$-K_p \frac{\partial \omega_{mec}(t)}{\partial t} + K_i(\omega_{mec_d} - \omega_{mec}(t)) \leq 0,$$

which it is equivalent to proof that:

$$K_p \frac{\partial \omega_{mec}(t)}{\partial t} + K_i \omega_{mec}(t) \geq K_i \omega_{mec_d}. \tag{67}$$

Considering the solution of

$$K_p \frac{\partial \omega_{mec}(t)}{\partial t} + K_i \omega_{mec}(t) = 0, \tag{68}$$

then it must be

$$\omega_{mec}(t) = \omega_{mec_i} e^{-\frac{K_i}{K_p} t} \leq \frac{\omega_{mec_d}}{K_p} K_i, \tag{69}$$

where $\omega_{mec_i} = \omega_{mec}(0)$. From equation (69) a boundary condition on K_i is obtained:

$$\boxed{K_i \geq \frac{\omega_{mec}(0)}{\omega_{mec_d}} K_p.} \tag{70}$$

To conclude, the intersection of all these conditions is the two following constraints:

$$K_i = \frac{\omega_{mec}(0)}{\omega_{mec_d}} K_p, \tag{71}$$

and

$$0 \leq K_b \leq 1. \tag{72}$$

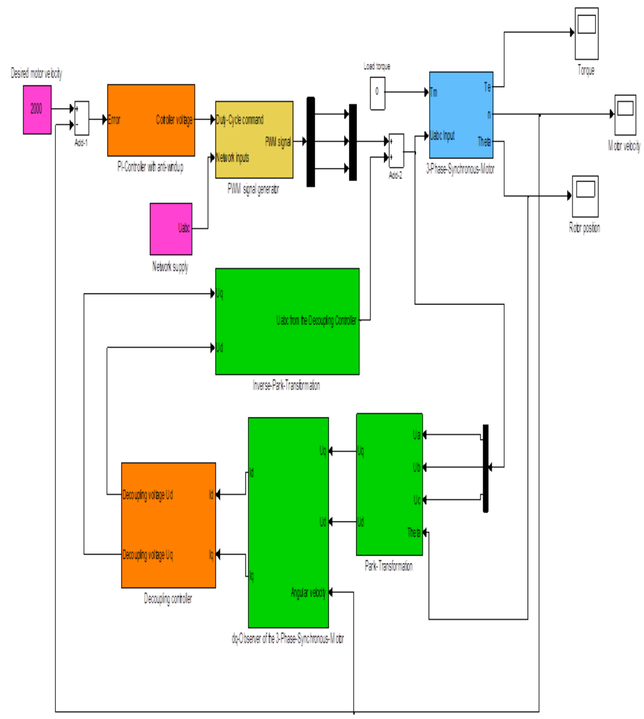

Fig. 2. Simulink structure of the whole control system

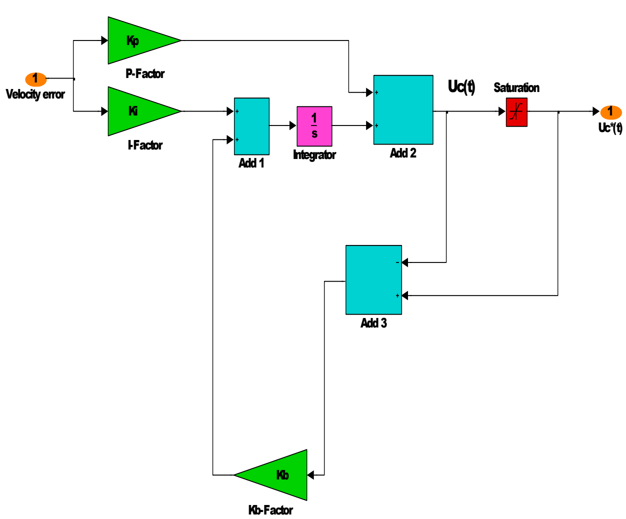

Fig. 3. Simulink structure of PI controller including anti-windup scheme

V. Simulation results

Simulations have been carried out using a special stand with a 58 kW traction PMSM. The stand consists of PMSM, tram wheel and a continuous rail. The PMSM is a prototype for low floor trams. PMSM parameters: nominal power 58 kW, nominal torque 852 Nm, nominal phase current 122 A and

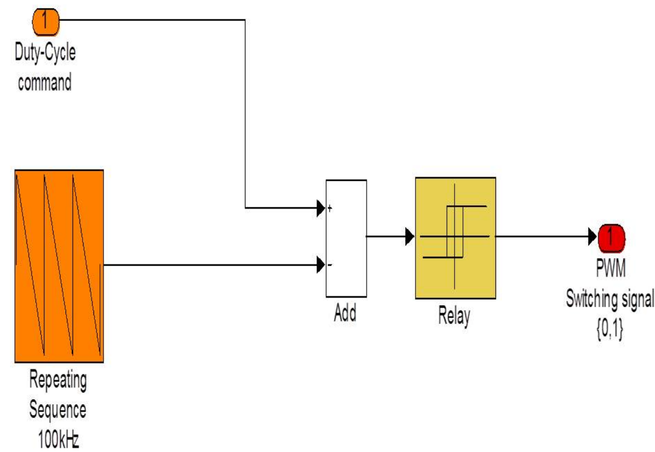

Fig. 4. PWM-Simulink-Block

number of poles 44. Model parameters: $R = 0.08723$ Ohm, $L_{dq} = Ld = Lq = 0.8$ mH, $\Phi = 0.167$ Wb. Surface mounted NdBFe magnets are used in PMSM. Advantage of these magnets is inductance up to 1.2 T, but theirs disadvantage is corrosion. The PMSM was designed to meet B curve requirements. In Fig. 2 the complete control scheme is shown. In this Simulink block diagram the transformed dq-observer is indicated together with Park's and inverse Park's transformation. PWM frequency equals 100 kHz and the structure of the simulink PWM block is shown in Fig. 4. Figure 6 shows the obtained and desired motor velocity profiles. Figure 3 shows PI controller with the anti-windup proposed scheme. Figure 7 shows the obtained and desired motor acceleration profiles. From these two results it is possible to remark that the effect of the chopper control is visible which does not allow the tracking to be precise. In particular, according to the theoretical condition $\frac{\partial \omega_{mec}(t)}{\partial t}$ the result should not present oscillation. Because of the realisation of the controller using a chopper which consists of discontinuous signals this is structurally not possible. Figure 8 shows PWM signal sequence with the maximal chopper switching frequency equals 2.5 kHz. Fig. 9 shows the chopper effect on the input of the motor.

VI. Conclusions and future work

This paper deals with a PI-controller for a three-phase synchronous motor. The technique uses a decoupling procedure. A Lyapunov approach is used to calculate parameters K_p and K_i to obtain soft velocity variation. Moreover, an anti-windup control structure is considered to avoid saturation and conditions on all controller parameters are found which guarantee stability. The resulting control structure is generally applicable for other dynamic systems with similar nonlinear model structure. Through simulations of a synchronous motor used in automotive applications, this paper verifies the effectiveness of the proposed method, the found theoretical and

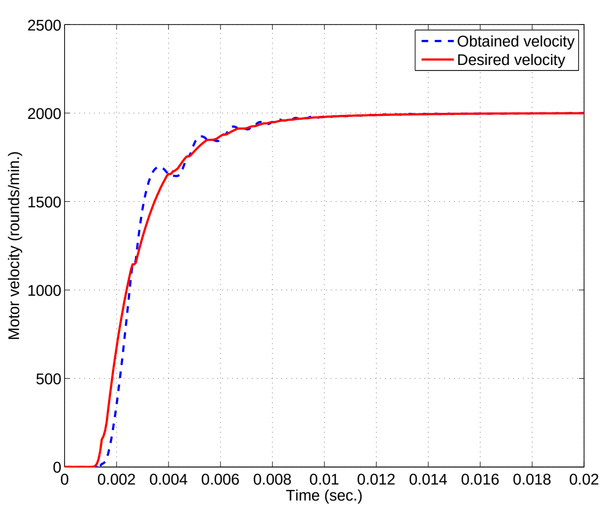

Fig. 5. Profile of the obtained and desired motor velocity using a maximal switching chopper frequency equals 2.5 kHz in case of positive desired velocity

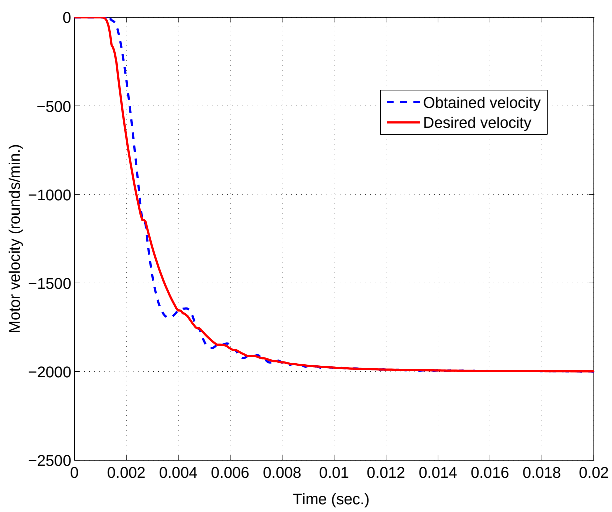

Fig. 6. Profile of the obtained and desired motor velocity using a maximal switching chopper frequency equals 2.5 kHz in case of negative desired velocity

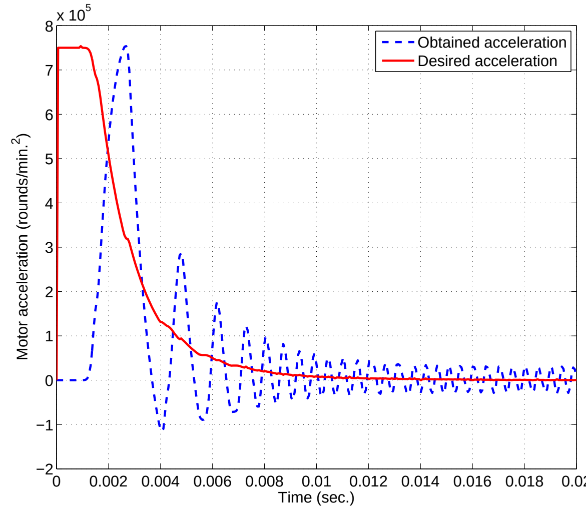

Fig. 7. Profile of the obtained and desired motor acceleration using a maximal switching chopper frequency equals 2.5 kHz in case of positive desired velocity

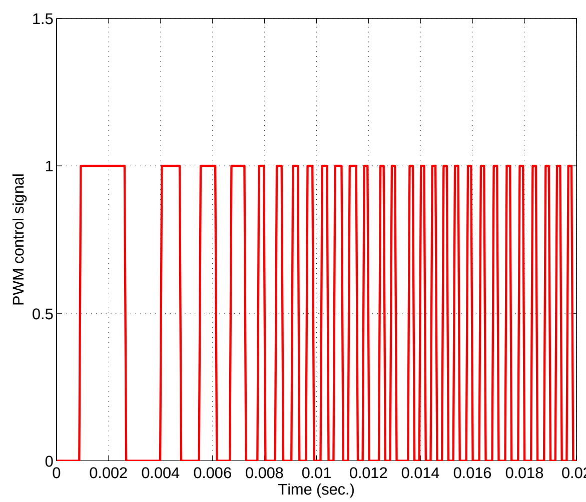

Fig. 8. PWM signal used as a chopper with a maximal switching frequency equals 2.5 kHz in case of positive desired velocity

simulation results. Future work includes the calculation of the optimal value of parameter K_p and K_b.

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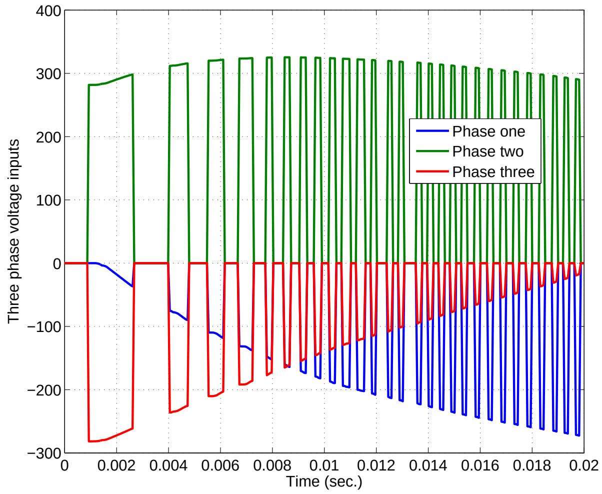

Fig. 9. Three-phase control signals after the chopper controller using a PWM singnal with a maximal switching frequency equals 2.5 kHz in case of positive desired velocity

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