Integration of FAHP and TOPSIS Methods for the Selection of Appropriate Multimedia Application for Learning and Teaching

Tomislav Volarić, Emil Brajković, and Tomo Sjekavica

Abstract—Criterias for the selection of the multimedia applications for learning are based on Bloom's digital taxonomy. Since there are more then one criteria for the selection of multimedia application for learning, multiple criteria decision making (MCDM) methods are needed to be used. In this paper FAHP and TOPSIS methods are proposed to be integrated for the selection of the best multimedia application for learning and teaching. First the FAHP method is used for determining the weights of each criteria and priority values of multimedia applications. Triangular fuzzy numbers are used in FAHP method for determining the benefits of one criteria to another. Then the TOPSIS method is used to determine the final ranking of the multimedia applications. The best multimedia application for teaching and learning would be the one that is farthest from the negative ideal solution and nearest to the positive ideal solution. The integration of FAHP and TOPSIS methods enables teacher to efficiently select a more suitable multimedia applications for learning.

Keywords—Bloom's digital taxonomy, FAHP, fuzzy logic, learning, MCDM, multimedia applications, TOPSIS.

I. Introduction

PROCESS of learning and teaching has considerable changed in last few decades. Along with the large expansion of ICT technologies, use of new modern tools and applications in education is required. Besides the classical learning, eLearing is becoming very popular and widely used way of learning in todays digital enviroment. Multimedia applications are taking there place in the classical education, as well in the eLearning. Using multimedia applications process of learning and teaching can be optimized and improved [1], [2]. Since there is an extremely large volume of various multimedia applications for learning, the need for the selection of the most appropriate multimedia application for learning is inevitable.

For the generation of learning objectives from elementary to higher education Bloom's taxonomy [3] is widely used. The taxonomy seperates forms of learning in three domains: cognitive, affective and psychomotor domain. Hiearchy of cognitive domain of learning is divided into six levels from knowledge to evaluation. The higher the level, the more complex and more useful it is. To adjust Bloom's taxonomy for requirements of 21st century students and teachers revised Bloom's taxonomy was published in 2001 [4]. The most important change in revised Bloom's taxonomy is that they have expanded cognitive domain of learning to include affective and psychomotor domains. After the revised Bloom's taxonomy Churches defined Bloom's digital taxonomy [5] in which he suplemented levels of taxonomy with new active verbs and included new opportunities for learning using advantages of new Web 2.0 technologies through the proposal of some specific digital tools and applications.

This paper discusses the selection of most suitable multimedia application for learning based on Bloom's digital taxonomy. As we are talking about more than one criteria for multimedia application selection, most suitable approach is using multiple criteria decision making (MCDM) methods. Different MCDM methods like AHP, FAHP, TOPSIS, ELECTRE, PROMETHEE, etc. can be used in the decision making process. Usually two or more MCDM methods are combined in order to improve the decision making proccess. In [6] the best strategy for non-formal ways of learning is selected using FAHP method and SWOT analysis. Three MCDM methods: AHP, Fuzzy PreRa and incomplete linguistic preference relations methods are used in [7] for selection of the multimedia authoring system. FAHP and TOPSIS methods can be used together in complex decision problems [8], [9]. Suitable multimedia applications for learning and teaching can be selected using FAHP and TOPSIS methods.

Analytic Hierarchy Process (AHP) method is one of the most famous and in recent years most used method for deciding, when the decision-making process or the choice of some of the available alternatives and their ranking is based

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on several attributes that have different importance and that are expressed using different scales. AHP method allows flexibility of the decision making process and helps decision makers to set priorities and make good decisions, taking into account both qualitative and quantitative aspects of the decisions [10]. AHP is based on the motto divide and conquer. Problems that require MCDM techniques are complex and, as result, it is advantageous to break them down and solve one 'sub-problem' at a time. This breakdown is done in two phases of the decision process during: the problem structuring and the alicitation of priorities through pairwise comparisons [11].

Fuzzy Analytic Hierarchy Process (FAHP) is an extension of AHP method that uses fuzzy logic, fuzzy sets and fuzzy numbers. It facilitates determining the ranking of certain criterias using fuzzy numbers instead of specific numerical values [12]. Understanding and managing with quantitative and qualitative data used in MCDM problems is much easier with FAHP method. In this approach triangular fuzzy numbers are used to determine the benefits of single criteria to another.

Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) method is used for the final ranking of multimedia applications on mulitple criteria, whose importance is determined using a FAHP method. Best alternative multimedia application is the one that is closest to the positive ideal solution and the farthest from the negative ideal solution by this method [13].

This paper is structured as follows: in the next section Bloom's taxonomy, revised Bloom's taxonomy and digital Bloom's taxonomy are shown and compared. Third section gives an overview on criterias of the multimedia applications for learning, while the fourth section presents fuzzy logic, fuzzy sets and fuzzy numbers. FAHP method that can be used for determining the weight criteria in MCDM problems is shown in fifth section. Then the TOPSIS method used for final ranking is described in sixth section. In seventh section, last one before conclusion empirical study for selecting appropriate multimedia application for learning and teaching is illustrated.

II. Bloom's Taxonomies

A. *Bloom's taxonomy*

According to the American psychologist Benjamin Samuel Bloom forms of learning can be divided into three categories [3]:

1. cognitive (knowledge),
2. affective (attitudes), and
3. psychomotor (skills).

Whitin the cognitive category Bloom differ six diferent hierachial levels of learning. These are, from the simplest level to the most complex cognitive domain level: i) knowledge, ii) comprehension, iii) application, iv) analysis, v) synthesis and vi) evaluation. Bloom's taxonomy is a useful tool that can help teachers in directing cognitive activities of students in all categories of thinking, especially those associated with higher mental operations.

B. *Revised Bloom's taxonomy*

Revised Bloom's taxonomy [4] was published after six year work of numeros team of experts among whom were Bloom's student Lorin Anderson and his associate David Krathwohl. Nouns that marked levels they replaced by verbs. Then they extended the synthesis to creation and changed the order of the two highest levels. Levels of cognitive domain of learning from the Lower Order Thinking Skills (LOTS) to the Higher Order Thinking Skills (HOTS) in revised Bloom's taxonomy are: i) remember, ii) understand, iii) apply, iv) analyze, v) evaluate and vi) create. Comparision of original Bloom's taxonomy and revised Bloom's taxonomy is shown in Fig. 1.

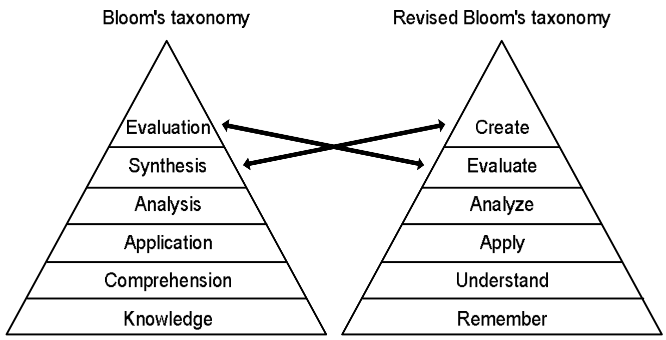

Fig. 1 Comparison of Bloom's and revised Bloom's taxonomy

Finally and most important, they expanded the cognitive domain of learning (knowledge) to include both affective (attitude) and pyschomotor domain of learning (skills). Their intention was to adjust Bloom's taxonomy for the 21st century teachers and students.

C. *Bloom's digital taxonomy*

Teacher and enthusiast Andrew Churches went one step further when he tried to accommodate taxonomy in the digital enviroment of 21st century, including additional learning opportunities which are provided with new Web 2.0 technologies [5].

Every level of Bloom's taxonomy he supplemented with new active verbs: i) remembering, ii) understanding, iii) applying, iv) analyzing, v) evaluating and iv) creating. He also proposed approach to specific digital tools for each level of taxonomy.

Fig. 2 shows various Web 2.0 tools and applications that can enable or enhance the process of learning and teaching for each level of cognitive domain of learning in Bloom's digital taxonomy that is proposed by Samantha Penney [14]. So for example, if the learning objective for students is to remember tools like Flickr and Delicious can be used. If goal for students is the creation, then tools like Prezi and Gimp would be suitable.

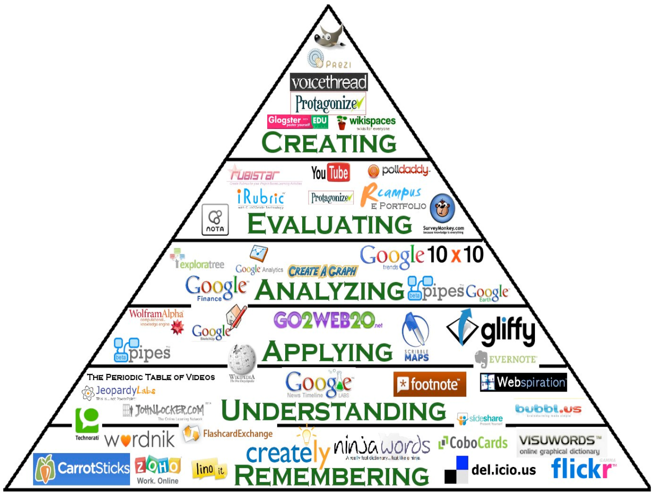

Fig. 2 Bloom's digital taxonomy pyramid by Samantha Penney

III. Criterias of the Multimedia Applications for Learning

Criterias of the multimedia applications for learning are relying on the Bloom's digital taxonomy [5]:

C_1 Remembering – Key terms are: recognising, listing, describing, identifying, retrieving, naming, locating and finding. Digital additions are: bullet pointing, highlighting, bookmarking or favouriting, social networking, social bookmarking and searching or „googling“.

C_2 Understanding – Key terms are: interpreting, summarising, inferring, paraphrasing, classifiying, comparing, explaining and exemplifying. Digital additions are: advanced and Boolean searching, blog journalling, categorising, tagging, commenting, annotating and subscribing.

C_3 Applying – Key terms are: carrying out, using, executing, implementing, showing and exhibiting. Digital additions are: running and operating, playing, uploading, sharing, hacking and editing.

C_4 Analysing – Key terms are: comparing, organising, deconstructing, attributing, outlining, finding, structuring and integrating. Digital additions are: smashing, linking, reverse-engineering and cracking.

C_5 Evaluating – Key terms are: checking, critiquing, hypothesising, experimenting, judging, testing, detecting and monitoring. Digital additions are: Blog/vlog commenting and reflecting, posting, moderating, collaborating and networking, testing and validating.

C_6 Creating – Key terms are: designing, constructing, planning, producing, inventing, devising and making. Digital additions are: programming, filming, animating, videocasting, podcasting, mixing, remixing, directing, producing and publishing.

Multimedia applications for learning are valued on the basis of the above mentioned criterias marked from C_1 to C_6. Weight of each criteria is determined with MCDM methods. Finally applications are ranked on the basis of all six criterias. The applications in the study are marked as APP1, APP2 and APP3.

IV. Fuzzy Sets and Fuzzy Numbers

Fuzzy logic is an extension of classical Boolean logic that is able to use the concept of partial truth. Standard Boolean logic supports only two values: 0 (false) and 1 (true), while fuzzy logic supports a range of values from a complete lie to the complete truth covering the whole range of values from 0 to 1 [15]. In classic set theory for each element is strictly determined if it belongs or does not belongs to a particular set. Fuzzy set is an extension of the classic set. With fuzzy sets one element may partially belong to the set. Fuzzy number is a generalization of real numbers. It is specified with interval of real numbers between 0 and 1. It is possible to use different fuzzy numbers according to the situation, but in practice trapezoidal and triangular fuzzy numbers are most used [16].

A. Triangular fuzzy number

Triangular fuzzy number is shown in Fig. 3.

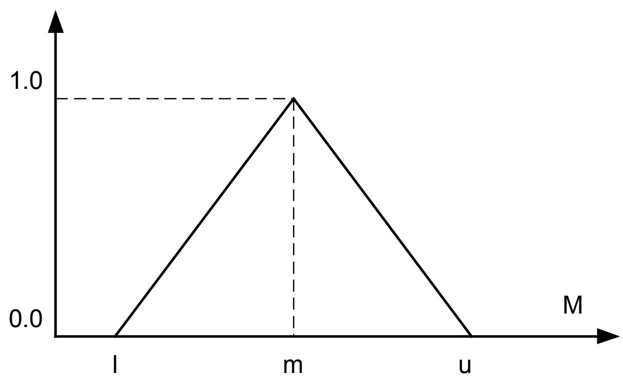

Fig. 3 A triangular fuzzy number

Triangular fuzzy number is defined by three real numbers, expressed as ordered triplet (l,m,u). The parameters l, m and u respectively show the lowest possible value, the most expected value and the maximum value that describes fuzzy event. If we define two positive triangular fuzzy numbers (l_1,m_1,u_1) and (l_2,m_2,u_2) then:

$$(l_1,m_1,u_1)+(l_2,m_2,u_2)=(l_1+l_2,m_1+m_2,u_1+u_2) \quad (1)$$

$$(l_1,m_1,u_1)\cdot(l_2,m_2,u_2)=(l_1\cdot l_2,m_1\cdot m_2,u_1\cdot u_2) \quad (2)$$

$$(l_1, m_1, u_1)^{-1} \approx (1/u_1, 1/m_1, 1/l_1) \tag{3}$$

For $x < l$ or for $x > u$ membership function $\mu(x/M)$ takes the value 0. For $l \leq x \leq m$ membership function takes the value $\frac{x-l}{m-l}$, while for $m \leq x \leq u$ function became $\frac{u-x}{u-m}$.

V. FAHP METHOD

The AHP is based on the subdivision of the problem in a hierarchical form. The traditional AHP method is problematic in that it uses an exact value to express the decision-maker's opinion in a pair-wise comparison of alternatives. Chang [12] introduced a new approach for handling FAHP, with the use of triangular fuzzy numbers for pair-wise comparison scale of FAHP, and the use of the extent analysis method for the synthetic extent values of the pair-wise comparisons.

Let

$$X = \{x_1, x_2, ..., x_n\} \tag{4}$$

be an object set , and

$$U = \{u_1, u_2, ..., u_n\} \tag{5}$$

be a goal set.

According to the Chang's extent analysis method [12], each object is taken and extent analysis for each goal is performed respectively:

$$M_{gi}^1, M_{gi}^2, ..., M_{gi}^m, i = 1,2,..., n \tag{6}$$

where all the $M_{gi}^j (j = 1,2,..., m)$ are triangular fuzzy numbers. The value of fuzzy synthetic extent with respect to the i_{th} object is defined as:

$$S_i = \sum_{j=1}^{m} M_{gi}^j \otimes \left[\sum_{i=1}^{n} \sum_{j=1}^{m} M_{gi}^j \right]^{-1} \tag{7}$$

To obtain $\sum_{j=1}^{m} M_{gi}^j$, perform the fuzzy addition operation of m extent analysis values for a particular matrix such that:

$$\sum_{j=1}^{m} M_{gi}^j = \left(\sum_{j=1}^{m} l_j, \sum_{j=1}^{m} m_j, \sum_{j=1}^{m} u_j \right) \tag{8}$$

And to obtain $\left[\sum_{j=1}^{n} \sum_{j=1}^{m} M_{gi}^j \right]^{1}$, perform the fuzzy addition operation of $\sum_{j=1}^{m} M_{gi}^j, (J = 1,2,..., m)$ values such that:

$$\sum_{i}^{n} \sum_{j=1}^{m} M_{gi}^j = \left(\sum_{j=1}^{m} l_j, \sum_{j=1}^{m} m_j, \sum_{j=1}^{m} u_j \right) \tag{9}$$

As M_1 and M_2 are two triangular fuzzy numbers, the degree of possibility of $M_1 > M_2$ is defined as:

$$V(M_1 \geq M_2) = \underset{x \geq y}{SUP} \left[\min \left(\mu M_1(x), \mu M_2(y) \right) \right] \tag{10}$$

When a pair (x, y) exists such that $x \geq y$ and $\mu M_1(x) = \mu M_2(y)$, then we have $V(M_1 \geq M_2) = 1$.

Since M_1 and M_2 are convex fuzzy numbers we have that: $V(M_1 \geq M_2)$ is 1 if $m_1 \geq m_2$, 0 if $l_2 \geq u_1$ and $\frac{l_2 - u_1}{(m_1 - u_1) - (m_2 - l_2)}$, otherwise.

$$V(M_1 \geq M_2) = hgt(M_1 \cap M_2) = \mu M_1(d) \tag{11}$$

Where d is the ordinate of the highest intersection point D between $\mu(M_1)$ and $\mu(M_2)$ like shown on Fig. 4.

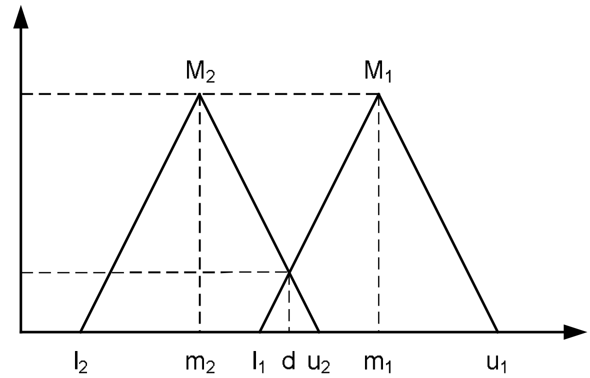

Fig. 4 The intersection between two triangular fuzzy numbers M_1 and M_2

The degree possibility for a convex fuzzy number to be greater than k convex fuzzy numbers $M_1(i = 1,2,..., k)$ can be defined by:

$$V(M \geq M_1, M_2, ..., M_k) = V[\ (M \geq M_1) \text{and} (M \geq M_2) \text{and} ... \text{and} (M \geq M_k)\ ] = \min V(M \geq M_i), i = 1,2,3,..., k \tag{12}$$

Assume that $d(A_i) = \min V(S_i \geq S_k), k = 1,2,..., n; k \neq i$, then the weight vector is given by:

$$W' = (d'(A_1), d'(A_2), ..., d'(A_n))^T \tag{13}$$

where A_i are n elements.

Via normalization, the normalized weight vectors are

$$W = (d(A_1), d(A_2), ..., d(A_n))^T \tag{14}$$

where W is a non-fuzzy number.

FAHP method is used to determine the weight criteria for decision-making process. In the FAHP procedure, the pair-wise comparisons in the judgement matrix are fuzzy numbers.

VI. TOPSIS METHOD

The TOPSIS method requires only a minimal number of inputs from the user and its output is easy to understand. The only subjective parameters are the weights associated with criteria. The fundamental idea of TOPSIS method is that the best solution is the one which has the shortest distance to the ideal solution and the furthest distance from the antiideal solution [11].

TOPSIS method was firstly proposed by Hwang and Yoon [13]. According to this technique, the best alternative would be the one that is nearest to the ideal positive solution and farthest from the ideal negative solution. The positive ideal solution is a solution that maximizes the benefit criteria and minimizes the cost criteria, whereas the negative ideal solution maximizes the cost criteria and minimizes the benefit criteria. The method is calculated as follows [17].

Establish a decision matrix for the ranking. The structure of the matrix can be expressed as follows:

$$D = \begin{array}{c} \\ A_1 \\ A_2 \\ \dots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} F_1 & F_2 & \dots & F_n \end{array} \\ \begin{bmatrix} c_{11} & c_{12} & \dots & c_{1n} \\ c_{21} & c_{22} & \dots & c_{2n} \\ \dots & \dots & \dots & \dots \\ c_{m1} & c_{m2} & \dots & c_{mn} \end{bmatrix} \end{array} \tag{15}$$

where A_i denotes the alternatives i, $i = 1,2,..., m$. F_j represents j^{th} criteria, related to i^{th} alternative; and c_{ij} is a crisp value indicating the performance rating of each alternative A_i with respect to each criteria c_{ij}.

Calculate the normalized decision matrix. The normalized value r_{ij} is calculated as:

$$r_{ij} = \frac{w_{ij}}{\sqrt{\sum_{j=1}^{J} w_{ij}^2}} \tag{16}$$

where $j = 1,2,..., J; i = 1,2,..., n$.

The weighted normalized decision matrix is calculated by multiplying the normalized decision matrix by its associated weights. The weighted normalized value v_{ij} is calculated as:

$$v_{ij} = w_{ij} * r_{ij}, j = 1,2,..., J, i = 1,2,..., n \tag{17}$$

where w_j .represents the weight of the j^{th} criteria.

Positive ideal solution (PIS) and negative ideal solution (NIS) are calculated as follows:

$$A^* = \{v_1^*, v_2^*, ..., v_n^*\}, \text{ maximum values} \tag{18}$$

$$A^- = \{v_1^-, v_2^-, ..., v_n^-\}, \text{ minimum values} \tag{19}$$

Calculate the separation measures, using the m-dimensional Euclidean distance [18]. The distance of each alternative from PIS and NIS are calculated:

$$d_i^* = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_j^*)^2}, j = 1,2,..., J \tag{20}$$

$$d_i^- = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_j^-)^2}, j = 1,2,..., J \tag{21}$$

Calculate the relative closeness to the ideal solution and rank the alternatives in descending order. The closeness coefficient of each alternative is calculated:

$$CC_i = \frac{d_i^-}{d_i^* + d_i^-}, i = 1,2,..., J \tag{22}$$

where the index value of CC_i lies between 0 and 1. The larger the index value, the better performance of the alternatives.

By comparing CC_i values, the ranking of alternatives is determined.

VII. EMPIRICAL STUDY

A numerical example is illustrated and trial data is used for selecting the best multimedia application for learning. Assume that three multimedia applications: APP1, APP2, APP3 are evaluated under a fuzzy environment. Fig. 5 shows the all main criteria in hierarchic view. To create pairwise comparison matrix, linguistic scale [11] is used which is given in Table I.

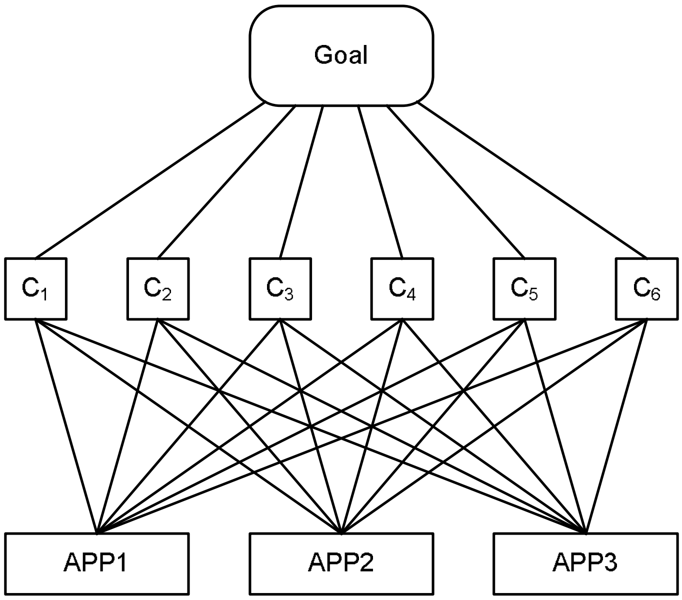

Fig. 5 Hierarchy of multimedia application selecting problem

TABLE I.
THE LINGUISTIC SCALE AND CORRESPONDING TRIANGULARY FUZZY NUMBERS

Linguistic scale	Explanation	TFN	Inverse TFN
Equal importance	Two activities contribute equally to the objective	(1, 1, 1)	(1, 1, 1)
Moderate importance	Experience and judgement slightly favor one activity over another	(0.33, 0.5, 1)	(1, 2, 3)
Strong importance	Experience and judgement strongly favor one activity over another	(0.75, 1, 1.25)	(0.8, 1, 1.33)
Very strong importance	An activity is favored very strongly over another, its dominance	(1, 2, 3)	(1/3, 1/2, 1)
Demonstrated importance	The evidence favoring one activity over another is highest possible order of affirmation	(1.33, 2, 4)	(1/4, 1/2, 3/4)

In our study we are ranking three multimedia applications by FAHP and TOPSIS methods. In step 1 with the help of improved AHP by fuzzy set theory, the procedure is as follows: first we should make the hierarchy structure. Proposed tree is shown in Tables II and III.

TABLE II.
EVALUATION MATRIX

	C_1	C_2	C_3	C_4	C_5	C_6
C_1	1	2	1	1	2	1
C_2	0.5	1	0.5	0.75	0.75	0.5
C_3	1	2	1	2	2	1
C_4	1	1.33	0.5	1	0.5	0.5
C_5	0.5	1.33	0.5	2	1	2
C_6	1	2	1	2	0.5	1

TABLE III.
FUZZY EVALUATION MATRIX

	C_1	C_2	C_3	C_4	C_5	C_6
C_1	(1, 1, 1)	(1, 2, 3)	(0.75, 1, 1.25)	(0.75, 1, 1.25)	(1, 2, 3)	(0.75, 1, 1.25)
C_2	(0.33, 0.5, 1)	(1, 1, 1)	(0.25, 0.5, 0.75)	(0.5, 0.75, 1)	(0.75, 1, 1.25)	(0.33, 0.5, 1)
C_3	(0.8, 1, 1.33)	(1.33, 2, 4)	(1, 1, 1)	(1, 2, 3)	(1, 2, 3)	(0.75, 1, 1.25)
C_4	(0.8, 1, 1.33)	(1, 1.33, 2)	(0.33, 0.5, 1)	(1, 1, 1)	(0.33, 0.5, 1)	(0.25, 0.5, 0.75)
C_5	(0.33, 0.5, 1)	(0.8, 1, 1.33)	(0.33, 0.5, 1)	(1, 2, 3)	(1, 1, 1)	(1, 2, 3)

C_6	(0.8, 1, 1.33)	(1, 2, 3)	(0.8, 1, 1.33)	(1.33, 2, 4)	(0.33, 0.5, 1)	(1, 1, 1)

In step 2 below results are obtained and have been brought in Tables IV and V.

TABLE IV.
THE SUMS OF HORIZONTAL AND VERTICAL DIRECTIONS

Criteria	Row Sums	Column Sums
C_1	(5.25, 8, 10.75)	(4.06, 5, 6.99)
C_2	(3.16, 4.25, 6)	(6.13, 9.33, 14.33)
C_3	(5.88, 9, 13.58)	(3.46, 4.5, 6.33)
C_4	(3.71, 4.83, 7.08)	(5.58, 8.75, 13.25)
C_5	(4.46, 7, 10.33)	(4.41, 7, 10.25)
C_6	(5.26, 7.5, 11.66)	(4.08, 6, 8.25)
Sum of row or column sums		(27.72, 40.58, 59.4)

After forming the fuzzy pair-wise comparison matrix, weights of all criteria are determined by the help of FAHP. According to the FAHP method, firstly synthesis values must be calculated. From Table V, synthesis values respect to main goal are calculated like Eq. (7):

$$SC_1 = (5.25, 8, 10.75) \otimes (27.72, 40.58, 59.4)^{-1} = \\ = (0.8838, 0.1971, 0.3878)$$

$$SC_2 = (3.16, 4.25, 6) \otimes (27.72, 40.58, 59.4)^{-1} = \\ = (0.0531, 0.1047, 0.2164)$$

$$SC_3 = (5.88, 9, 13.58) \otimes (27.72, 40.58, 59.4)^{-1} = \\ = (0.0989, 0.2217, 0.4898)$$

$$SC_4 = (3.71, 4.83, 7.08) \otimes (27.72, 40.58, 59.4)^{-1} = \\ = (0.0624, 0.1190, 0.2554)$$

$$SC_5 = (4.46, 7, 10.33) \otimes (27.72, 40.58, 59.4)^{-1} = \\ = (0.0750, 0.1724, 0.3726)$$

$$SC_6 = (5.26, 7.5, 11.66) \otimes (27.72, 40.58, 59.4)^{-1} = \\ = (0.0885, 0.1848, 0.4206)$$

TABLE V.
THE FUZZY SYNTHETIC EXTENT OF EACH CRITERIA

Criteria	SC_i
C_1	(0.8838, 0.1971, 0.3878)
C_2	(0.0531, 0.1047, 0.2164)
C_3	(0.0989, 0.2217, 0.4898)
C_4	(0.0624, 0.1190, 0.2554)
C_5	(0.0750, 0.1724, 0.3726)
C_6	(0.0885, 0.1848, 0.4206)

These fuzzy values are compared by using Eq. (11) and following values are obtained:

$$V(SC_1 \geq SC_2) = 0.9238, V(SC_1 \geq SC_3) = 1, \\ V(SC_1 \geq SC_4) = 1, V(SC_1 \geq SC_5) = 1, V(SC_1 \geq SC_6) = 1$$

$$V(SC_2 \geq SC_1) = 0.5808, V(SC_2 \geq SC_3) = 0.5008, \\ V(SC_2 \geq SC_4) = 0.9150, V(SC_2 \geq SC_5) = 0.6759, \\ V(SC_2 \geq SC_6) = 0.6149$$

$$V(SC_3 \geq SC_1) = 1, V(SC_3 \geq SC_2) = 1, \\ V(SC_3 \geq SC_4) = 1, V(SC_3 \geq SC_5) = 1, V(SC_3 \geq SC_6) = 1$$

$$V(SC_4 \geq SC_1) = 0.6813, V(SC_4 \geq SC_2) = 1, \\ V(SC_4 \geq SC_3) = 0.6035, V(SC_4 \geq SC_5) = 0.7712, \\ V(SC_4 \geq SC_6) = 0.7172$$

$$V(SC_5 \geq SC_1) = 0.9202, V(SC_5 \geq SC_2) = 1, \\ V(SC_5 \geq SC_3) = 0.8473, V(SC_5 \geq SC_4) = 1, \\ V(SC_5 \geq SC_6) = 0.9584$$

$$V(SC_6 \geq SC_1) = 0.9642, V(SC_6 \geq SC_2) = 1, \\ V(SC_6 \geq SC_3) = 0.8969, V(SC_6 \geq SC_4) = 1, \\ V(SC_6 \geq SC_5) = 1$$

Then priority weights are calculated by using Eq. (13).

$$d'(C_1) = \min(1, 0.9238, 1, 1, 1) = 0.9238$$

$$d'(C_2) = \min(0.5808, 0.5008, 0.9150, 0.6759, 0.6149) = 0.5008$$

$$d'(C_3) = \min(1, 1, 1, 1, 1) = 1$$

$$d'(C_4) = \min(0.6913, 1, 0.6035, 0.7712, 0.7172) = 0.6035$$

$$d'(C_5) = \min(0.9202, 1, 0.8473, 1, 0.9584) = 0.8473$$

$$d'(C_6) = \min(0.9942, 1.0.8969, 1, 1) = 0.8969$$

Calculated priority weights form vector:

$$W' = (0.9238, 0.5008, 1, 0.6035, 0.8473, 0.8969)$$

After the normalization of these values priority weights respect to main goal are calculated using Eq. (14) as:

$$W = (0.1935, 0.1049, 0.2095, 0.1264, 0.1775, 0.1879)$$

Decision matrix for the ranking is then established.

TABLE VI.
DECISION MATRIX

	C_1	C_2	C_3	C_4	C_5	C_6
APP1	2	3	4	2	3	2
APP2	5	2	3	2	3	4
APP3	4	2	5	3	2	4

Normalized decision matrix is calculated like in Table VII.

TABLE VII.
NORMALIZED DECISION MATRIX

	C_1	C_2	C_3	C_4	C_5	C_6
APP1	0.298	0.727	0.565	0.485	0.639	0.333
APP2	0.745	0.485	0.424	0.485	0.639	0.666
APP3	0.596	0.485	0.707	0.727	0.426	0.666

Weighted normalization matrix in Table VIII is formed by multiplying each value with their weights.

TABLE VIII.
WEIGHTED NORMALIZATION MATRIX

	C_1	C_2	C_3	C_4	C_5	C_6
APP1	0.057	0.076	0.118	0.061	0.113	0.062
APP2	0.144	0.050	0.088	0.061	0.113	0.125
APP3	0.115	0.050	0.148	0.092	0.075	0.125

In step 3 positive and negative ideal solutions are determined by talking the maximum and minimum values for each criteria using Eq. (18) and Eq. (19):

$$A^* = (0.144, 0.076, 0.148, 0.092, 0.113, 0.125)$$

$$A^- = (0.057, 0.050, 0.088, 0.061, 0.075, 0.062)$$

Then the distance of each alternative from PIS and NIS with to respect to each criteria is calculated with the help of Eq. (20) and Eq. (21).

$$d_1^* = \sqrt{\begin{aligned}&(0.057-0.144)^2+(0.076-0.076)^2+(0.118-0.148)^2+\\&(0.061-0.092)^2+(0.113-0.113)^2+(0.062-0.125)^2\end{aligned}}$$

$$d_1^- = \sqrt{\begin{aligned}&(0.057-0.057)^2+(0.076-0.050)^2+(0.118-0.088)^2+\\&(0.061-0.061)^2+(0.113-0.075)^2+(0.062-0.062)^2\end{aligned}}$$

$$d_2^* = \sqrt{\begin{aligned}&(0.144-0.144)^2+(0.050-0.076)^2+(0.088-0.148)^2+\\&(0.061-0.092)^2+(0.113-0.113)^2+(0.125-0.125)^2\end{aligned}}$$

$$d_2^- = \sqrt{\begin{aligned}&(0.144-0.057)^2+(0.050-0.050)^2+(0.0888-0.088)^2+\\&(0.061-0.061)^2+(0.113-0.075)^2+(0.125-0.062)^2\end{aligned}}$$

$$d_3^* = \sqrt{\begin{aligned}&(0.115-0.144)^2+(0.050-0.076)^2+(0.148-0.148)^2+\\&(0.092-0.061)^2+(0.075-0.075)^2+(0.125-0.062)^2\end{aligned}}$$

$$d_3^- = \sqrt{\begin{aligned}&(0.115-0.057)^2+(0.050-0.050)^2+(0.148-0.088)^2+\\&(0.092-0.061)^2+(0.075-0.075)^2+(0.125-0.062)^2\end{aligned}}$$

At the end closeness coefficient of each multimedia application is calculated by using Eq. (22).

TABLE IX.
RANKING OF THE MULTIMEDIA APPLICATIONS

	d_i^*	d_i^-	CC_i	Rank
APP1	0.115	0.0543	0.321	3
APP2	0.071	0.113	0.613	2
APP3	0.053	0.108	0.667	1

Considering the Table IX, prefered multimedia application for learning is APP3 for decision maker’s preference in this empirical study. Different rankings can be obtained by using different decision maker’s preference values.

VIII. Conclusion

The proposed approach in this paper is based on FAHP and TOPSIS methods. We have shown how FAHP method is first used to determine the weight criteria for decision-making using triangular fuzzy numbers. Then with TOPSIS method order of multimedia applications for learning was defined. Criterias for the selection of multimedia applications for learning and teaching were defined according to the Bloom's digital taxonomy. Empirical study for the selection of multimedia applications for learning was shown and discussed.

Our ongoing research is directed towards the development of a fuzzy decision making model for the selection of a suitable multimedia application for learning using subjective judgments of decision makers. In future studies other multi-criteria methods like fuzzy PROMETHEE and ELECTRE can be used to improve process of selecting multimedia applications for learning and teaching.

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