Dual Methods for Optimal Allocation of Total Network Resources

I.V. Konnov, A.Yu. Kashuba, E. Laitinen

Abstract—We consider a general problem of optimal allocation of a homogeneous resource (bandwidth) in a wireless communication network, which is decomposed into several zones (clusters). The network manager must satisfy different users requirements. However, they may vary essentially from time to time. This makes the fixed allocation rules inefficient and requires certain adjustment procedure for each selected time period. Besides, sometimes users requirements may exceed the local network capacity in some zones, hence the network manager can buy additional volumes of this resource. This approach leads to a constrained convex optimization problem. We discuss several ways to find a solution of this problem, which exploit its special features. We suggest the dual Lagrangian method to be applied to selected constraints. This in particular enables us to replace the initial problem with one-dimensional dual one. We consider the case of the affine cost (utility) functions, when each calculation of the value of the dual function requires solution of a special linear programming problem. We can also utilize the zonal resource decomposition approach, which leads to a sequence of one-dimensional optimization problems. The results of the numerical experiments confirm the preferences of the first method.

Keywords—Resource allocation, wireless networks, bandwidth, zonal network partition, dual Lagrange method, linear search, zonal resource decomposition, linear programming.

I. Introduction

The current development of telecommunication systems creates a number of new challenges of efficient management mechanisms involving various aspects. One of them is the efficient allocation of limited communication networks resources. In fact, despite the existence of powerful processing and transmission devices, increasing demand of different communication services and its variability in time, place, and quality, leads to serious congestion effects and inefficient utilization of significant network resources (e.g., bandwidth and batteries capacity), especially in wireless telecommunication networks. This situation forces one to replace the fixed allocation rules with more flexible mechanisms; see e.g. [1]–[4]. Naturally, treatment of these very complicated systems is often based on a proper decomposition/clustering approach, which can involve zonal, time, frequency and other decomposition procedures for nodes/units; see e.g. [5], [6], [7], [8], [9]. In [10], [11], several optimal resource allocation problems in telecommunication networks and proper decomposition based methods were suggested. They assumed that the network manager can satisfy all the varying users requirements. However, zonal resource amounts may be not sufficient in some time periods due to instable behavior of many users, hence the network manager can buy additional volumes of the resource. We note that such a strategy is rather typical for contemporary wireless communication networks, where WiFi or femtocell communication services are utilized in addition to the usual network resources; see e.g. [12]. This approach leads to a constrained convex optimization problem for some selected time period. We discuss several ways to find a solution of this problem, which exploit its special features. We suggest the dual Lagrangian method to be applied to selected constraints. The utilization of the dual decomposition in this problem was also considered in [13]. It was based on an explicit volume resource allocation procedure with a a sequence of one-dimensional optimization problems and gave a multi-level iterative procedure.

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In this paper, we discuss several possible approaches to the zonal resource allocation problem and give some other way to enhance the performance of the solution method. It consists in utilization of the Lagrangian multipliers only for the total resource bound, which yields an one-dimensional dual optimization problem. We consider the case of the affine cost (utility) functions, when each calculation of the value of the dual function requires solution of a special linear programming problem. The results of the numerical experiments confirms the preferences of the new method over the previous ones.

II. Problem description

Let us consider a network with nodes (attributed to users), which is divided into n zones (clusters) within some fixed time period. For the k-th zone ($k = 1, \dots, n$), I_k denotes the index set of nodes (currently) located in this zone, b_k is the maximal fixed resource value. We suppose that users can move but that all the assignments of users to zones are fixed within this time period. The network manager satisfies users resource requirements in the k-th zone by allocation of the own (inner) resource value $x_k \in [0, b_k]$ and also by taking the external resource value $z_k \in [0, c_k]$. Clearly, these values require proper maintenance expenses $f_k(x_k)$ and side payments $h_k(z_k)$ for each $k = 1, \dots, n$. We suppose also that there exists the upper bound B for the total amount of the inner resource of the network. Next, if the i-th user receives the resource amount y_i with the upper bound a_i, then he/she pays the charge $\varphi_i(y_i)$. The problem of the network manager is to find an optimal allocation of the resource among the zones and can be written as follows:

$$\max \to \sum_{k=1}^{n} \left[\sum_{i \in I_k} \varphi_i(y_i) - f_k(x_k) - h_k(z_k) \right] \qquad (1)$$

subject to

$$\sum_{i \in I_k} y_i = x_k + z_k, \ k = 1, \dots, n; \qquad (2)$$

$$0 \leq y_i \leq a_i, \ i \in I_k, \ 0 \leq x_k \leq b_k, \ 0 \leq z_k \leq c_k, \ k = 1, \dots, n; \qquad (3)$$

$$\sum_{k=1}^{n} x_k \leq B. \qquad (4)$$

In what follows we shall also utilize the assumptions that all the functions $\varphi_i(y_i)$, $f_k(x_k)$, and $h_k(z_k)$ are affine, i.e.

$$\begin{array}{rcl} \varphi_i(y_i) & = & \alpha_i' y_i + \alpha_i'', \ \alpha_i' > 0, \ i \in I_k, \ k = 1, \dots, n, \\ f_k(x_k) & = & \beta_k' x_k + \beta_k'', \ \beta_k' > 0, \ k = 1, \dots, n, \\ h_k(z_k) & = & \gamma_k' z_k + \gamma_k'', \ \gamma_k' > 0, \ k = 1, \dots, n. \end{array} \qquad (5)$$

However, the basic exposition of the problem will be given for the general case where the functions $-\varphi_i(y_i)$, $f_k(x_k)$, and $h_k(z_k)$ are convex.

III. Solution methods

Although problem (1)–(4) seems rather simple, but its dimensionality can be rather large, so that the streamlined application of the well-known iterative methods, say, projection or Newton type ones (see e.g. [14]), may cause significant computational expenses. This is the case even for the affine functions $\varphi_i(y_i)$, $f_k(x_k)$, and $h_k(z_k)$, i.e. when (1)–(4) is a linear programming problem. In fact, a decomposition approach, which exploits particular features of this problem (see e.g. [15], [16]), will be more suitable here. However, the creation of an efficient decomposition method is also not a trivial task. In fact, problem (1)–(4) has $n+1$ functional constraints and many box type ones. For instance, utilization of the Lagrangian function with respect to all the functional constraints leads to a non-smooth dual convex optimization problem in $n+1$ dual variables, whose solution may cause certain algorithmic difficulties. For this reason, we first describe a hierarchical approach from [13], which yields a sequence of one-dimensional problems.

For each k, we denote by $\mu_k(u_k)$ the optimal value of the parametric zonal optimization problem:

$$\max \to \sum_{i \in I_k} \varphi_i(y_i) \qquad (6)$$

subject to

$$\sum_{i \in I_k} y_i \leq u_k, \ 0 \leq y_i \leq a_i, \ i \in I_k; \qquad (7)$$

where u_k is the total amount of the resource for the k-th zone, which should be chosen by the network manager. We set $\mu_k(u_k) = -\infty$ if the constraints in (7) are inconsistent. Hence, $\mu_k(u_k)$ gives the total profit from consumers of zone k if the total resource value for this zone equals u_k. This function is concave if all φ_i are so.

Then the manager problem (1)–(4) can be equivalently rewritten as follows:

$$\max \to \sum_{k=1}^{n} [\mu_k(u_k) - f_k(x_k) - h_k(z_k)] \qquad (8)$$

subject to

$$x_k + z_k = u_k, \ 0 \leq x_k \leq b_k, \ 0 \leq z_k \leq c_k, \ k = 1, \dots, n; \qquad (9)$$

$$\sum_{k=1}^{n} x_k \leq B. \qquad (10)$$

Let us take the Lagrange function with respect to constraint (10):

$$\begin{array}{rcl} M(x, u, z, \lambda) & = & \displaystyle\sum_{k=1}^{n} [\mu_k(u_k) - f_k(x_k) - h_k(z_k)] \\ & - & \lambda \left(\displaystyle\sum_{k=1}^{n} x_k - B \right) \end{array}$$

and set

$$V = V_1 \times \ldots \times V_n,$$
$$V_k = \{(x_k, z_k, u_k) \mid x_k + z_k = u_k, \ 0 \leq x_k \leq b_k, 0 \leq z_k \leq c_k, \} \quad \text{for } k = 1, \dots, n.$$

Then we can replace (8)–(10) with its one-dimensional dual:

$$\min_{\lambda \geq 0} \to \psi(\lambda), \qquad (11)$$

where

$$\begin{array}{l} \psi(\lambda) = \displaystyle\max_{(x,z,u) \in V} L(x, u, z, \lambda) \\ = \lambda B + \displaystyle\max_{(x,z,u) \in V} \sum_{k=1}^{n} [\mu_k(u_k) - (f_k(x_k) + \lambda x_k) - h_k(z_k)] \\ = \lambda B + \displaystyle\sum_{k=1}^{n} \max_{(x_k, z_k, u_k) \in V_k} [\mu_k(u_k) - (f_k(x_k) + \lambda x_k) - h_k(z_k)]. \end{array}$$

Let us take the k-th inner problem above:

$$\max_{(x_k, z_k, u_k) \in V_k} \to [\mu_k(u_k) - (f_k(x_k) + \lambda x_k) - h_k(z_k)],$$

for each $k = 1, \dots, n$. We can again apply the dual approach. First we write the particular Lagrange function with respect to the constraint $u_k = x_k + z_k$:

$$\begin{array}{l} M_k(u_k, x_k, z_k, \eta_k) \\ = \mu_k(u_k) - (f_k(x_k) + \lambda x_k) - h_k(z_k) + \eta_k(u_k - x_k - z_k) \\ = (\mu_k(u_k) + \eta_k u_k) - (f_k(x_k) + x_k(\lambda + \eta_k)) \\ -(h_k(z_k) + \eta_k z_k) \end{array}$$

and then define the dual problem

$$\min_{\eta_k \in \mathbb{R}} \to \tau_k(\eta_k), \qquad (12)$$

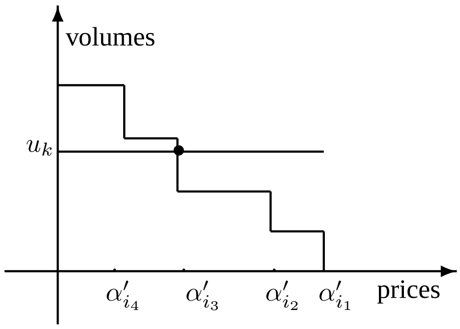

Fig. 1. The ordering method for the k-th zonal problem, $l_k = 4$. Solution: $y_{i_1} = a_{i_1}$, $y_{i_2} = a_{i_2}$, $y_{i_3} = u_k - a_{i_1} - a_{i_2}$, $y_{i_4} = 0$.

where

$$\begin{aligned}\tau_k(\eta_k) &= \max_{\substack{0\le x_k\le b_k,\\ 0\le z_k\le c_k, u_k\in\mathbb{R}}} M_k(u_k, x_k, z_k, \eta_k)\\ &= \max_{u_k\in\mathbb{R}}[\mu_k(u_k) + \eta_k u_k] - \min_{0\le x_k\le b_k}[f_k(x_k) + x_k(\lambda+\eta_k)]\\ &- \min_{0\le z_k\le c_k}[h_k(z_k) + \eta_k z_k].\end{aligned}$$

Thus, the initial problem (1)–(4) (or (8)–(10)) is replaced by its one-dimensional dual (11) with the cost function $\psi(\lambda)$, such that calculation of its value reduces to solution of n one-dimensional problems of form (12), whose calculation again reduces to solution of three one-dimensional problems.

Moreover, each function μ_k will be also given algorithmically, i.e. via solution of rather simple problem (6)–(7), and we can apply gradient or dual type methods to find the value of $\mu_k(u_k)$. As above, the dual method first requires to introduce the the Lagrange function

$$\tilde{M}_k(y, \theta_k) = \sum_{i\in I_k}\varphi_i(y_i) - \theta_k\left[\sum_{i\in I_k} y_i - u_k\right],$$

and then to solve the one-dimensional dual:

$$\min_{\theta_k\ge 0} \to \zeta_k(y), \tag{13}$$

where

$$\zeta_k(y) = \theta_k u_k + \sum_{i\in I_k}\max_{0\le y_i\le a_i}[\varphi_i(y_i) - \theta_k y_i].$$

However, in the affine case (6)–(7) is a linear programming problem and its solution can be found by the simple ordering algorithm. Let $|I_k| = l_k$, i.e. let the index set I_k contain l_k elements. Rearrange the indices of I_k into a sequence $\{i_1, \dots, i_{l_k}\}$ such that $\alpha'_{i_s} \ge \alpha'_{i_{s+1}}$, this requires $O(\ln(l_k))$ operations. Then we should assign sequentially the maximal feasible value for each resource amount

$$y_{i_s} = \min\{a_{i_s}, u_k - \sum_{p<s} y_{i_p}\},\ s = 1, \dots, l_k;$$

which yields the desired solution; see Figure 1. Note that the rearrangement of index sets of each zone should be made only one time before the starting the iteration process. This clearly gives an alternative to the previous dual method.

Therefore, we should in fact use only algorithms for a set of hierarchical one-dimensional problems. At the same time, this requires additional concordance rules for accuracies of all these problems, which can not be solved exactly. Indeed, each marginal function $\mu_k(u_k)$ is nonlinear and non-smooth.

IV. A TWO-LEVEL DUAL SOLUTION METHOD

We now present some other way to enhance the performance of the previous dual method. It consists in utilization of the Lagrangian multipliers only for the total resource bound, which yields a two-level iterative procedure with a single-dimensional dual optimization problem.

For the sake of clarity, we re-write problem (1)–(4) as follows:

$$\max_{(x,y,z)\in W, \sum_{k=1}^{n} x_k\le B} \to \mu(x, y, z) \tag{14}$$

where

$$\mu(x, y, z) = \sum_{k=1}^{n}\left[\sum_{i\in I_k}\varphi_i(y_i) - f_k(x_k) - h_k(z_k)\right], \tag{15}$$

and

$$W = \left\{(x, y, z)\ \middle|\ \begin{array}{c}\sum_{i\in I_k} y_i = x_k + z_k,\ 0\le y_i\le a_i,\ i\in I_k,\\ 0\le x_k\le b_k,\ 0\le z_k\le c_k,\ k = 1, \dots, n\end{array}\right\}. \tag{16}$$

Let us define the Lagrange function of problem (14)–(16) with respect to the total resource constraint:

$$L(x, u, z, \lambda) = \mu(x, y, z) - \lambda\left(\sum_{k=1}^{n} x_k - B\right),$$

where λ is the corresponding Lagrange multiplier. We can now replace problem (14)–(16) with its one-dimensional dual:

$$\min_{\lambda\ge 0} \to \psi(\lambda), \tag{17}$$

where

$$\begin{aligned}\psi(\lambda) &= \max_{(x,y,z)\ \in\ W} L(x, y, z, \lambda) = \lambda B\\ &+ \max_{(x,y,z)\ \in\ W}\sum_{k=1}^{n}\left[\sum_{i\in I_k}\varphi_i(y_i) - (f_k(x_k) + \lambda x_k) - h_k(z_k)\right]\end{aligned}$$

Its solution can be found by one of well-known single-dimensional optimization problem.

In order to calculate the value of $\psi(\lambda)$ we have to solve the inner problem:

$$\max \to \sum_{k=1}^{n}\left[\sum_{i\in I_k}\varphi_i(y_i) - (f_k(x_k) + \lambda x_k) - h_k(z_k)\right]$$

subject to

$$\sum_{i\in I_k} y_i = x_k + z_k,\ 0\le y_i\le a_i,\ i\in I_k,$$
$$0\le x_k\le b_k,\ 0\le z_k\le c_k,\ k = 1, \dots, n.$$

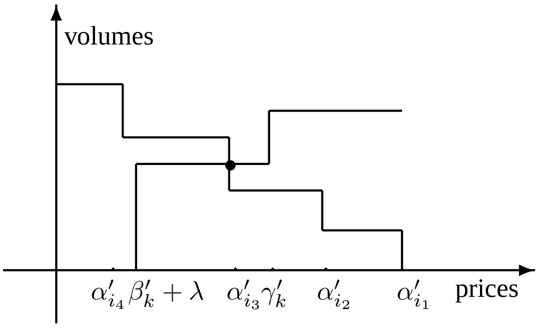

Fig. 2. The ordering method for the k-th zonal problem, case $l_k = 4$, $\beta'_k + \lambda < \gamma'_k$. Solution: $x_k = b_k$, $z_k = 0$, $y_{i_1} = a_{i_1}$, $y_{i_2} = a_{i_2}$, $y_{i_3} = b_k - a_{i_1} - a_{i_2}$, $y_{i_4} = 0$.

Obviously, this problem decomposes into n independent zonal convex optimization problems

$$\max \to \left[\sum_{i\in I_k} \varphi_i(y_i) - (f_k(x_k) + \lambda x_k) - h_k(z_k)\right] \quad (18)$$

$$\sum_{i\in I_k} y_i = x_k + z_k,\ 0 \le y_i \le a_i,\ i \in I_k, \quad (19)$$

$$0 \le x_k \le b_k,\ 0 \le z_k \le c_k, \quad (20)$$

for $k = 1, \dots, n$. Note that in the affine case (5) the cost function in (18) is rewritten as

$$\sum_{i\in I_k} \alpha'_i y_i - (\beta'_k + \lambda)x_k - \gamma'_k z_k,$$

i.e. (18)–(20) is a linear programming problem. It follows that we can find very easily an exact solution of each of these problems in a finite number of iterations by a simple ordering algorithm, similar to that applied to the problem(6)–(7); see Figure 2.

Inserting these procedures into a suitable single-dimensional optimization method for solving (17), we obtain an efficient method for the initial problem (14)–(16) as an alternative to the method of the previous section.

V. Numerical experiments

In order to evaluate the performance of all the methods we made several series of computational experiments for the affine case (5). The methods of Section III with solving of auxiliary problem (6)–(7) by the dual algorithm (see (13)) and by the ordering algorithm are denoted by (DML) and (DMLS), respectively. The method of Section IV is denoted as (SDM).

We utilized the golden section method for solving the single-dimensional optimization problems. The methods were implemented in C++ with a PC with the following facilities: Intel(R) Core(TM) i7-4500, CPU 1.80 GHz, RAM 6 Gb.

The initial intervals for choosing the dual variable λ (and the additional dual variables in (DML) and (DMLS)) were taken as [0,1000]. The initial intervals for choosing the zonal allocation shares u_k in (DML) were taken as $[0, R]$

ε	N_ε	T_ε (DML)	T_ε (DMLS)	T_ε (SDM)
10^{-1}	20	3.3907	0.0447	0.0050
10^{-2}	24	3.9427	0.0520	0.0038
10^{-3}	29	4.9633	0.0613	0.0043
10^{-4}	34	5.7347	0.0713	0.0057

TABLE I
RESULTS OF TESTING WITH $J = 510$, $n = 70$, $\delta = 10^{-2}$

		(DML)	(DMLA)	(DMLS)	(DMLAS)	(SDM)
J	N_ε	T_ε	T_ε	T_ε	T_ε	T_ε
210	24	1.7453	1.2240	0.0266	0.0187	0.0009
310	24	2.4480	1.7967	0.0337	0.0238	0.0025
410	24	3.1980	2.3910	0.0403	0.0303	0.0028
510	24	3.9427	2.9007	0.0520	0.0363	0.0038
610	24	4.6097	3.4167	0.0588	0.0431	0.0038
710	24	5.3070	3.9220	0.0659	0.0487	0.0040
810	24	6.0260	4.4427	0.0754	0.0540	0.0031
910	24	6.9170	4.9533	0.0910	0.0665	0.0047
1010	24	7.4843	5.4797	0.0988	0.0735	0.0047

TABLE II
RESULTS OF TESTING WITH $n = 70$, $\varepsilon = 10^{-2}$, $\delta = 10^{-2}$

with $R = B + \sum_{k=1}^{n} c_k$, B was chosen to be 1000. Values of b_k and c_k were chosen by trigonometric functions in $[1, 11]$, values of a_i were chosen by trigonometric functions in $[1, 2]$, as well as the coefficients of all the functions f_k, h_k, and φ_i in (5). The number of zones was varied from 5 to 105, the number of users was varied from 210 to 1010. Users were distributed in zones either uniformly or according to the normal distribution. The processor time and number of iterations, which were necessary to find an approximate solution of problem (6) within the same accuracy, were not significantly different for these two cases of distributions.

Further we report the results of tests, which include the time and number of iterations needed to find a solution of problem (6) within some accuracies. Let ε and δ denote the desired accuracy of finding a solution of problem (6) and solutions of auxiliary inner problems in (DML) and (DMLS). Let J denote the total number of users, N_ε the number of upper iterations in λ, T_ε the total processor time in seconds. For the same accuracy, both the methods gave the same numbers of upper iterations, so that the main difference was in the processor time. The results of computations are given in Tables I–III. We inserted also the results for (DML) and (DMLS) with adaptive strategy of choosing the inner accuracies. We named by (DMLA) and (DMLAS), respectively, this version of (DML) and (DMLS). In Table I, we vary the accuracy ε, in Tables II and III we vary the total number of users and the number of zones, respectively.

From the results we can conclude that (DMLS) has the significant preference over (DML), and that the adaptive versions enhance the performance of these methods. At the same time, (SDM) has the significant preference over (DMLS) and (DMLAS), i.e. it showed the best results for all the test problems. This enables us to apply (SDM) for online solution of these resource allocation problems.

		(DML)	(DMLA)	(DMLS)	(DMLAS)	(SDM)
n	N_ε	T_ε	T_ε	T_ε	T_ε	T_ε
15	24	3.6200	2.6877	0.0425	0.0302	0.0019
25	24	3.6927	2.7240	0.0460	0.0321	0.0016
35	24	3.7500	2.7917	0.0476	0.0337	0.0013
45	24	3.7970	2.7970	0.0488	0.0366	0.0034
55	24	3.8487	2.8383	0.0497	0.0388	0.0034
65	24	3.9480	2.8857	0.0519	0.0378	0.0044
75	24	3.9740	2.9167	0.0512	0.0384	0.0047
85	24	4.0210	2.9530	0.0506	0.0397	0.0038
95	24	4.1720	3.0260	0.0535	0.0416	0.0035
105	24	4.2187	3.0467	0.0564	0.0429	0.0053

TABLE III
RESULTS OF TESTING WITH $J = 510$, $\varepsilon = 10^{-2}$, $\delta = 10^{-2}$

VI. CONCLUSIONS

In this work, we considered a problem of managing limited resources in a zonal wireless communication network and gave its constrained convex optimization problem formulation. We proposed several dual based methods and analyzed their performance. We suggested procedures for solving subproblems in the case of the affine cost (utility) functions, which are based on simple ordering rules. The results of the numerical experiments confirmed the rapid convergence of these methods.

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