Numerical simulation of the disturbances development in a supersonic boundary layer

S. A. Gaponov, A. N. Semenov

***Abstract*—**For solution of hydrodynamic stability problems the evolutionary method is offered. The essence of this method consists that an arbitrary initial disturbance is described by one wave with the greatest increment on large times, which varies according to the law $exp(-i\omega\tau)$. In order to verify the new method and to work out the numerical scheme the stability calculations were carried out also on the base of the classical theory. The evolutionary method is used to study the effects of the gas injection direction through a porous surface on stability of a supersonic boundary layer at the Mach number M=2. Results of the evolutionary method coincide with data of the classical theory very well.

The boundary layer receptivity process due to the interaction of three-dimensional slow acoustic disturbances is numerically investigated at a free stream Mach number of 2.0. Problem is solved in the linear approximation relatively excited disturbances by an acoustic wave. Numerical simulations were conducted with using the program complex Ansys. In general, matching the results of the approximate method (based on stability equations for low-frequency fluctuations) with direct numerical simulation data is satisfactory. Normalized solutions on the corresponding maxima of the velocity perturbations amplitudes are coincided well enough about a wall. The greatest discrepancy occurs in the area of the boundary layer edge where the approximation theory is inapplicable.

***Keywords*—** supersonic boundary layer, establishing method, acoustic waves, interaction, receptivity, numerical simulations

I. INTRODUCTION

This paper is the extended version of the presentation at the 12th International conference on applied and theoretical mechanics (Prague, March 18-20, 2016) which was published in [1].

The problems on the hydrodynamic stability and the interaction of a supersonic boundary layer with acoustic waves which are considered in the present paper were raised mainly in connection with the problem on the turbulence formation. Transition from laminar to turbulent state in shear flows occurs due to evolution of different disturbances inside the shear layer. Though there are several mechanisms and routes to go from a laminar to a turbulent state, most of them generally follow these fundamental processes: receptivity, linear instability, nonlinear instability and breakdown to turbulence. Here the first two problems will be considered: the linear instability, and the receptivity of a supersonic boundary layer to the external acoustic waves.

As for the linear instability of a supersonic boundary layer it was solved by many authors. A detailed review of their works can be found in [2-4]. It is necessary to notice that, as a rule, all authors use the standard method of elementary waves leading to the solution of the eigenvalue problem of the homogeneous system of ordinary differential equations with homogeneous boundary conditions. The lack of this method is in the difficulty to find the waves with the highest increment. Its search comes to the end successfully under a condition if its approximate value is known. Therefore, growing in time waves given the front direction and wave number, depending on the incoming values of main flow, such as Mach number, Reynolds number and others, are calculated on the base of small changes of determinative parameters. However, the wave with the maximum increment for some basic terms will not be the determinative one (with a maximum growth factor), for the other flow parameters. Therefore, it is desirable to have such a calculation method which would guarantee uniquely obtaining of the wave with the highest increment. For linear problems, this can be achieved by an evolutionary method by the integration over time of partial differential equations. Because any disturbance, satisfing uniform boundary conditions can be decomposed into the sum of the waves with different increments, the wave with the largest increment will dominate at large times. This method can be called by the usual term - the establishing method. In contrast to the generally accepted method of establishing when the solution goes to the constant, in our case the solution goes to the exponential dependence on time. In the hydrodynamic stability theory there are the temporary instability (wave number on uniform spatial coordinates are real) and the spatial instability (when perturbations with real frequencies growth in the space). At low amplification rate in the space and time, which is characteristic for the boundary layers, temporal and spatial increments are associated with the simple approximate relation: the amplification rate in space equals the negative temporary divided by the wave group velocity [2]. If necessary, the more precise value of the spatial amplification rate can be obtained by the classical method. In this paper

This work was supported in part by by Russian Foundation for Basic Research under Grant No.15-01-00866a.

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theoretically investigates the influence of the direction of the blowing gas through a porous surface on the stability of a supersonic boundary layer using the classical method of elementary waves and evolutionary method.

At present the most complex problem on the prediction of the transition position in the boundary layer flows is related to the receptivity of these flows to the external effects. This problem was discussed in detail for the first time in [5]. A review of the early works of the acoustic field effect on the transition from a laminar supersonic boundary layer to the turbulent one has been given in [2]. The problems on the supersonic flow aeroacoustics were studied mainly within the framework of the investigations on the conditions for the onset of auto-oscillations and sound generation by the supersonic shear flows in the jets and mixing layers. The advanced approach based on the idea of the possibility of the mutual influence of the acoustic and hydrodynamic waves has been demonstrated in [6].

The first attempts to investigate the interaction of the sound waves and supersonic boundary layer on the basis of the stability theory were undertaken in [7,8]. The problem on the excitation of unstable waves by sound was considered in [9]. The interaction of sound with a supersonic boundary layer experimentally was studied in [10–11] where the main results of theory [8] were confirmed.

In [12] the theory of interaction of external high-frequency acoustics of small amplitudes with a supersonic boundary layer on the basis of the linear equations of hydrodynamic stability is stated in detail. Its essence consists in the assumption that an acoustic wave length differs from a boundary layer thickness a little. Then in the first approximation the main flow current in a stationary boundary layer can be accepted as plane-parallel flow. The problem is reduced to integration of the ordinary differential equations which coefficients depend only on the longitudinal speed and temperature in a stationary flow. At finite incidence angle from the solution of stability equations it is possible to receive a distribution of the disturbance amplitude in a boundary layer excited by an acoustic wave and a reflection coefficient. At an approach of the incidence angle to zero the wave vector of an external acoustic wave becomes parallel to a streamline surface. This acoustic wave was called a longitudinal wave. For a longitudinal wave in approach of a parallel flow it isn't possible to construct the limited solution outside the boundary layer excluding the case of the low frequency, when the pressure disturbance inside the boundary layer practically does not depend on the normal coordinate. For more information on this subject can be found in [13].

Due to that within parallel main flow it isn't possible to solve the problem on an excitation of disturbances in a boundary layer by a longitudinal sound wave, it is necessary to simulate this process on the basis of the full Navier-Stokes equations. This method allows obtain detailed information about the perturbation that is difficult in experimental studies. Now the direct numerical modeling of stationary and non-stationary flows is successfully applied. The detailed review of existing methods of Computational Fluid Dynamics is available in [14].

In [15] the direct numerical simulation (DNS) was used for a study of resonant interactions of waves at Mach number M=4.5. In [16] other types of disturbances in the running stream (slow acoustic, entropy and vorticity waves) are considered. The numerical simulation of a hypersonic boundary layer receptivity to fast and slow acoustic waves at M = 6 is carried out in [17]. In [18] calculations of the boundary layer receptivity to fast and slow acoustic waves with M=4.5 carried out. Calculation and experimental studies of a hypersonic shock layer receptivity to acoustic disturbance at M = 21 have been conducted in [19]. All this studies were carried out only for the interaction of boundary layer with 2D acoustic waves. Apparently, there is a single work [20] which solved the interaction problem of 3D acoustic monochromatic waves with boundary layer. However in it only the case with the fixed sliding angle relative to the front edge of the plate was considered.

Therefore in this paper along with the solution of a stability problem researches on interaction of acoustic waves of different sliding angles with the supersonic boundary layer at Mach number M=2.0 are conducted.

II. BASIC EQUATIONS

The gas flow is described by the known Navier - Stokes, continuity, energy and state equations [9]:

$$\rho^* \frac{\mathrm{d}\mathbf{v}^*}{\mathrm{dt}} = -\,\mathrm{grad}\left(p^*\right) - \frac{2}{3}\mathrm{grad}\left(\mu^*\mathrm{div}\left(\mathbf{v}^*\right)\right) + 2\mathrm{Div}(\mu^*\dot{\mathrm{S}}),$$

$$c_p \rho^* \frac{dT^*}{dt} - \frac{dp^*}{dt} = 2\mu^* \dot{S}^2 - \frac{2}{3}\mu^*\left(\mathrm{div}\left(\mathbf{v}^*\right)\right)^2 +$$

$$+\mathrm{div}\left(\mu^*\mathrm{grad}\left(\frac{c_p T^*}{Pr}\right)\right),$$

$$\frac{d\rho^*}{dt} + \rho^* div(\mathbf{v}^*) = 0, \quad p^* = \rho^* R T^*$$

Here $\mathbf{v}^*$ –velocity with components $\left(u^*, v^*, w^*\right)$ in x, y, z − directions, p^*, ρ^*, T^* − pressure, density and temperature, c_P – specific heat at constant pressure, R − gas constant, $\dot{S}$ − velocity tensor, $Pr = c_p \mu^* / \lambda^*$, λ^* − thermal conductivity, μ^* − dynamic viscosity.

III. THE BOUNDARY LAYER STABILITY WITH VECTORED INJECTION

A. *The stability theory of plane-parallel flows*

In this paper we will explore the disturbance in supersonic boundary layers on a flat plate at high Reynolds numbers $Re_x = u_\infty^* x \rho_\infty^* / \mu_\infty^*$, where $u_\infty^*, \rho_\infty^*, \mu_\infty^*$ - velocity, density and dynamic viscosity in a free flow , x - distance from the front edge of the plate to researched area. In this case, the main flow is independent on the transverse, z-coordinate, weakly dependent from x-coordinate and velocity in y - direction is low. Therefore, the main (stationary) flow can be considered for a plane-parallel, depending only on y, only the velocity in the x-direction $u^*(y)$ is unequal to zero.

If to introduce the dimensionless coordinates, time, and parameters of flow in the form: $dX = dx/\delta$, $dY = dy/\delta$, $dZ = dz/\delta$, $d\tau = u_\infty^* dt/\delta$, $v = v^*/u_\infty^*$, $p = p^*/p_\infty^*$, $T = T^*/T_\infty^*$, $\rho = \rho^*/\rho_\infty^*$ where $\delta = \sqrt{x\mu_\infty^*/\rho_\infty^* u_\infty^*}$ - the thickness of the boundary layer, the velocity, density, pressure and temperature of the compressible gas in the boundary layer can be represented in the form:

$u = U(Y) + \varepsilon u'$, $v = \varepsilon v'$, $w = \varepsilon w'$, $p = P(Y) + \varepsilon p'$,

$T = T_0(Y) + \varepsilon\theta'$, $\rho = 1/T_0(Y) + \varepsilon\zeta'$,

$U(Y), P, T_0(Y)$ – velocity, pressure, and temperature in the unperturbed laminar boundary layer. Flow parameters disturbances which depend on X, Y, Z and τ are marked by the prime.

Equations for linear disturbances in the approximation of Dana-Lin [3] and Alekseev [23] in the two-dimensional boundary layer have the form:

$$\frac{1}{T_0(Y)}\left(\frac{\partial u'}{\partial\tau} + U\frac{\partial u'}{\partial X} + \frac{\partial U}{\partial Y}v'\right) = -\frac{1}{\gamma M_\infty^2}\frac{\partial p'}{\partial X} + \frac{\mu}{Re}\frac{\partial^2 u'}{\partial Y^2},$$

$$\frac{1}{T_0(Y)}\left(\frac{\partial v'}{\partial\tau} + U\frac{\partial v'}{\partial X}\right) = -\frac{1}{\gamma M_\infty^2}\frac{\partial p'}{\partial Y},$$

$$\frac{1}{T_0(Y)}\left(\frac{\partial w'}{\partial\tau} + U\frac{\partial w'}{\partial X}\right) = -\frac{1}{\gamma M_\infty^2}\frac{\partial p'}{\partial Z} + \frac{\mu}{Re}\frac{\partial^2 w'}{\partial Y^2}, \qquad (1)$$

$$\frac{\partial\zeta'}{\partial\tau} + U\frac{\partial\zeta'}{\partial X} + \frac{d}{dY}\left(\frac{1}{T_0}\right)v' + \frac{1}{T_0(Y)}\left(\frac{\partial u'}{\partial X} + \frac{\partial v'}{\partial Y} + \frac{\partial w'}{\partial Z}\right) = 0,$$

$$\frac{1}{T_0(Y)}\left(\frac{\partial\Theta'}{\partial\tau} + U\frac{\partial\Theta'}{\partial X} + \frac{dT_0}{dY}v'\right) = \frac{\gamma-1}{\gamma}\left(\frac{\partial p'}{\partial\tau} + U\frac{\partial p'}{\partial X}\right) +$$

$$+\frac{\mu}{PrRe}\frac{\partial^2\Theta'}{\partial Y^2}, \quad p'/P = T_0\zeta' + \theta'/T_0.$$

The system (1) is solved with the boundary conditions, [2]:

$$u' = v' = w' = \theta' = 0, \text{ at } Y = 0, \infty. \qquad (2)$$

B. The classical stability theory

The classical stability theory is founded on the method of elementary waves $\boldsymbol{a}', p' = (\boldsymbol{a}(Y), \pi)\exp\left(i\left(\bar\alpha X + \bar\beta Z - \bar\omega\tau\right)\right)$. In this case, vector components $\boldsymbol{a} = (f, \varphi, h, \theta, \zeta)$ – amplitudes of perturbations $u', v', w', \theta', \zeta'$. Equations (1) are given to system of the linear ordinary differential equations:

$$\frac{1}{T_0}\left[i\bar\alpha(U-c)f + \varphi\frac{dU}{dY}\right] + \frac{i\bar\alpha\pi}{\gamma M_\infty^2} = \frac{\mu}{Re}\frac{d^2 f}{dY^2},$$

$$\frac{1}{T_0}i\bar\alpha(U-c)h + \frac{i\bar\beta\pi}{\gamma M_\infty^2} = \frac{\mu}{Re}\frac{d^2 h}{dY^2},$$

$$\frac{1}{T_0(Y)}i\bar\alpha(U-c)\varphi + \frac{1}{\gamma M_\infty^2}\frac{d\pi}{dY} = 0, \qquad (3)$$

$$i\bar\alpha(U-c)\zeta + \frac{d}{dY}\left(\frac{1}{T_0}\right)\varphi + \frac{1}{T_0}\left(i\bar\alpha f + i\bar\beta h + \frac{d\varphi}{dY}\right) = 0,$$

$$\frac{1}{T_0}\left[i\bar\alpha(U-c)\theta + \varphi\frac{dT}{dY}\right] + (\gamma-1)\left(i\bar\alpha f + i\bar\beta h + \frac{d\varphi}{dY}\right) =$$

$$= \frac{\gamma\mu}{PrRe}\frac{d^2\theta}{dY^2}, \quad \pi/\mathrm{P} = \mathrm{T}_0\zeta + \theta/\mathrm{T}_0, \text{ where } c = \bar\omega/\bar\alpha,$$

and the conditions (2) to the view:

$$f = \varphi = \theta = 0 \text{ at } Y = 0, \infty. \qquad (4)$$

Wave numbers $\bar\alpha$ and $\bar\beta$ are real at the temporary instability and a frequency ω is complex-valued, which is a result of solving the eigenvalue problem of stability of homogeneous equations with homogeneous boundary conditions. The flow in the boundary layer is unstable for positive values of the imaginary part of $\bar\omega = \bar\omega_r + i\bar\omega_i$.

In general, the number of eigenvalues is infinite, or at least large. However, we are interested primarily frequency with the highest values of the imaginary part. The search such frequencies is a challenge.

C. The evolutionary method

Basic equations. In this work for the first time for finding of such frequencies the evolutionary method is offered and realized. The essence of this approach is that the random assignment of initial data but sufficiently large τ, perturbations with the largest increments will prevail, which vary according to the law $exp(-i\bar\omega\tau)$.

For disturbances of the type $\boldsymbol{a}' = \tilde a(\tau, X, Y)\exp\left(i\bar\beta Z\right)$ equations (1) and boundary conditions (2) take the form:

$$\frac{\partial f}{\partial\tau} = -U\frac{\partial f}{\partial X} - \varphi\frac{\partial U}{\partial Y} - \frac{T_0}{\gamma M_\infty^2}\frac{\partial\pi}{\partial X} + \frac{\mu T_0}{Re}\frac{\partial^2 f}{\partial Y^2},$$

$$\frac{\partial h}{\partial\tau} = -U\frac{\partial h}{\partial X} - \frac{T_0}{\gamma M_\infty^2}i\bar\beta\pi + \frac{\mu T_0}{Re}\frac{\partial^2 h}{\partial Y^2},$$

$$\frac{\partial\varphi}{\partial\tau} = -U\frac{\partial\varphi}{\partial X} - \frac{T_0}{\gamma M_\infty^2}\frac{\partial\pi}{\partial Y}, \quad \pi = \frac{r}{\rho} + \frac{\theta}{T_0},$$

$$\frac{\partial\zeta}{\partial\tau} = -U\frac{\partial\zeta}{\partial X} - \varphi\frac{\partial r}{\partial Y} - \frac{d}{dY}\left(\frac{1}{T_0}\right)\left(\frac{\partial f}{\partial X} + i\bar\beta h + \frac{\partial\varphi}{\partial Y}\right), \qquad (5)$$

$$\frac{\partial\theta}{\partial\tau} = -\left[U\frac{\partial\theta}{\partial X} + \varphi\frac{\partial T}{\partial Y}\right] -$$

$$-(\gamma-1)T_0\left(\frac{\partial f}{\partial X} + i\bar{\beta}h + \frac{\partial\varphi}{\partial Y}\right) + \frac{\gamma\mu T_0}{PrRe}\frac{\partial^2\theta}{\partial Y^2}.$$

$$f(0,\infty) = \varphi(0,\infty) = h(0,\infty) = \theta(0,\infty) = 0.$$

The computational domain and the numerical scheme. The problem was solved for the periodic perturbation in the coordinate x, i.e. $\boldsymbol{a}(Y,X,\tau) = \boldsymbol{a}(Y,X+L,\tau)$. The region of an integration in the normal direction was enclosed interval $0<Y<Y^*$. We took into account the conditions of equality to zero disturbances at Y^*. Value Y * was accepted rather large that its additional increase did not lead to essential change of disturbances increments.

For the integration of the system (5) we used 2-step finite-difference scheme [24]:

$$\frac{f^{n+\frac{1}{2}} - f^n}{\Delta} = -U'\varphi^n + \frac{\mu T_0}{Re}\frac{f_{i+1}^{n+\frac{1}{2}} - 2f_i^{n+\frac{1}{2}} + f_{i-1}^{n+\frac{1}{2}}}{h_y^2} -$$

$$-\frac{T_0}{\gamma M^2}\frac{\left(\pi_{j+1}^n - \pi_{j-1}^n\right)}{2h_x},$$

$$\frac{h^{n+\frac{1}{2}} - h^n}{\Delta} = -\frac{i\bar{\beta}T_0\pi^n}{\gamma M_\infty^2} + \frac{\mu T_0}{Re}\frac{h_{i+1}^{n+\frac{1}{2}} - 2h_i^{n+\frac{1}{2}} + h_{i-1}^{n+\frac{1}{2}}}{h_y^2},$$

$$\frac{\varphi^{n+\frac{1}{2}} - \varphi^n}{\Delta} = -\frac{T_0}{\gamma M_\infty^2}\frac{\left(\pi_{i+1}^n - \pi_{i-1}^n\right)}{2h_y}, \qquad (6a)$$

$$\frac{\zeta^{n+\frac{1}{2}} - \zeta^n}{\Delta} = -\frac{d}{dY}\left(\frac{1}{T_0}\right)\varphi^n -$$

$$-\frac{1}{T_0}\left(\frac{f_{j+1}^n - f_{j-1}^n}{2h_x} + i\bar{\beta}h^n + \frac{\left(\varphi_{i+1}^n - \varphi_{i-1}^n\right)}{2h_y}\right),$$

$$\frac{\theta^{n+\frac{1}{2}} - \theta^n}{\Delta} = -(\gamma-1)T_0\left(\frac{f_{j+1}^n - f_{j-1}^n}{2h_x} + i\bar{\beta}h^n + \frac{\varphi_{i+1}^n - \varphi_{i-1}^n}{2h_y}\right) +$$

$$+\frac{\gamma\mu T_0}{PrRe}\frac{\theta_{i+1}^{n+\frac{1}{2}} - 2\theta_i^{n+\frac{1}{2}} + \theta_{i-1}^{n+\frac{1}{2}}}{h_y^2} - T_0'\varphi^n,$$

$$\pi^{n+\frac{1}{2}}/\mathrm{P} = \mathrm{r}^{n+\frac{1}{2}}/\rho + \theta^{n+\frac{1}{2}}/\mathrm{T}_0.$$

The second step:

$$\frac{\boldsymbol{a}^{n+1} - \boldsymbol{a}^{n+\frac{1}{2}}}{\Delta} = -U\frac{\boldsymbol{a}_{j+1}^{n+1} - \boldsymbol{a}_{j-1}^{n+1}}{2h_x}, \quad \frac{\pi^{n+1}}{P} = \frac{r^{n+1}}{\rho} + \frac{\theta^{n+1}}{T_0}. \quad (6б)$$

The scheme is absolutely stable, the approximation order is $O\left(\tau, h_x^2, h_y^2\right)$. Values f,φ,r,θ,π,h on the ($n+1$) layer were obtained from the each equation in the appropriate order. Unknown values at the boundary were obtained by interpolating on three neighboring points.

The value $\bar{\omega}$ was determined by the formula: $\bar{\omega} = -\frac{1}{iN2\Delta}ln\left(\frac{\pi^{n+N}}{\pi^n}\right)$. Calculations were performed as long until it was constant with the acceptable accuracy. In this case the real or imaginary part $q\left(Y_c, X, \tau_c\right)$ were changed as the $q_{r,i}\left(Y,X,\tau\right) = a_{r,i}\sin\left(\alpha X + \psi_{r,i}\right)$. The value of $\bar{\alpha} = 2\pi n/L$, where n- number of periods stacked on the calculating range L.

IV. BOUNDARY LAYER EQUATIONS

In self-similar variables boundary layer equations have the form [25]:

$$\frac{d}{dY}\left(\mu\frac{dU}{dY}\right) + g\frac{dU}{dY} = 0, \quad \frac{dg}{dY} = \frac{U}{2T_0}$$

$$\frac{d}{dY}\left(\frac{\mu}{Pr}\frac{dT_0}{dY}\right) + g\frac{dT_0}{dY} + (\gamma-1)M_\infty^2\mu\left(\frac{dU}{dY}\right)^2 = 0,$$

Here $\gamma = C_p/C_V$ - ratio of specific teats, $M_\infty = u_\infty^*/a_\infty$ - Mach number and a_∞ - sound velocity at the boundary layer edge. At a uniform gas blowing through a wall at an angle λ to the main flow direction the velocity components on a wall are defined as follows: $V(0) = Gsin\lambda$, $U(0) = Gcos\lambda$. Because $g(0) = -V(0)Re/T_w$, [2], $g(0) = -GResin\lambda/T_w$. Introducing the parameter $C_q = -GRe/T_w$, characterizing the intensity of the blowing or suction through the surface, the boundary conditions on thermally insulated surface can be written as:

$$g(0) = C_q sin\lambda, \; U(0) = T_w C_q cos\lambda/Re, \; \frac{dT_0}{dY}(0) = 0;$$

$$T_0(\infty) = U(\infty) = 1;$$

The dependence of μ^* on temperature was adopted in accordance with the Sutherland's law, which in dimensionless form can be written as follows:

$$\mu^* = \mu^*(T_\infty^*)\left(\frac{T_0^*}{T_\infty^*}\right)^{3/2}\frac{T_\infty^* + T_s}{T_0^* + T_s},$$

where T_s - Sutherland's constant. In wind tunnels without heating at a constant stagnation temperature T_{st}^*, $T_\infty^* = T_{st}^*/\left(1+(\gamma-1)M_\infty^2\right)$.

V. The boundary layer interaction with acoustic waves

Scheme of an interaction of the sliding along the surface of monochromatic sound waves with the boundary layer of the streamline surface (plate) is shown in Fig. 1. Wave front section which is parallel to normal coordinate y is represented by the letter *S*. Wave vector of the sound wave $\tilde{\alpha}$ with projections α and β on x and z respectively is parallel to the plane (x, z). Velocity of the running flow is parallel to an axis x and is perpendicular the planes (y, z). The figure shows a conditional boundary layer, its thickness δ and the sliding angle $hi = arctg(\beta/\alpha)$. The acoustic wave is periodic on z with period $\lambda_z = 2\pi/\beta$ and on x with period $\lambda_x = 2\pi/\alpha$. Thus, the external parameters of the acoustic wave is changed by law: $q_i = q_i^0 \cos(\alpha x + \beta z - \omega t)$, were q_i^0 - oscillation amplitude, ω – angular frequency. Problem is solved in the linear approximation relatively excited disturbances by an acoustic wave.

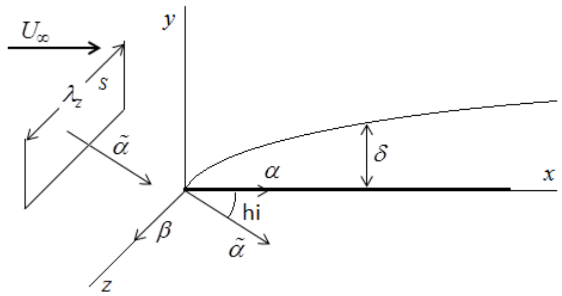

Fig. 1

A. The approximate theory

Parameters of external sliding acoustic waves have the view:

$$(u', w', p', T') = (f_\infty, h_\infty, \pi_\infty, \theta_\infty)\exp\left(i\left(\bar{\alpha}X + \bar{\beta}Z - \bar{\omega}\tau\right)\right),\ v' = 0$$

Here $\bar{\alpha}, \bar{\beta}, \bar{\omega}$ - wave numbers and frequency. In the two-dimensional boundary layer on a solid plate, in a parallel flow approximation and small wave numbers $\bar{\alpha}$ (low frequency waves) the perturbation amplitudes in a boundary layer is described by a system of ordinary differential equations, [2]:

$$\frac{1}{T_0}\left[i\bar{\alpha}(U-c)f + \varphi\frac{dU}{dY}\right] + \frac{i\bar{\alpha}\pi}{\gamma M_\infty^2} = \frac{\mu}{Re}\frac{d^2 f}{dY^2},$$

$$\frac{1}{T_0} i\bar{\alpha}(U-c)h + \frac{i\bar{\beta}\pi}{\gamma M_\infty^2} = \frac{\mu}{Re}\frac{d^2 h}{dY^2},$$

$$i\bar{\alpha}(U-c)\zeta + \frac{d}{dY}\left(\frac{1}{T_0}\right)\varphi + \frac{1}{T_0}\left(i\bar{\alpha}f + i\bar{\beta}h + \frac{d\varphi}{dY}\right) = 0,$$

$$\frac{1}{T_0}\left[i\bar{\alpha}(U-c)\theta + \varphi\frac{dT}{dY}\right] +$$

$$(\gamma-1)\left(i\bar{\alpha}f + i\bar{\beta}h + \frac{d\varphi}{dY}\right) = \frac{\gamma\mu}{PrRe}\frac{d^2\theta}{dY^2},$$

$$dp'/dy = d\pi/dY = 0, \quad \pi/\mathrm{P} = \mathrm{T}_0\zeta + \theta/\mathrm{T}_0.$$

For sliding waves $c = 1 - \sqrt{(\bar{\alpha}^2 + \bar{\beta}^2)}/(M_\infty\alpha)$.

The conditions on the solid surface are $f = h = \varphi = \theta = 0$.

At $Y=\infty$ $f_\infty = 1$, $h_\infty = \bar{\beta}/\bar{\alpha}$, $\pi_\infty = \gamma M_\infty\sqrt{(\bar{\alpha}^2 + \bar{\beta}^2)}/\bar{\alpha}$,

$$\theta_\infty = -\frac{M_e(\gamma-1)\bar{\alpha}\left(1 + \bar{\beta}^2/\bar{\alpha}^2\right)}{\sqrt{(\bar{\alpha}^2 + \bar{\beta}^2)}},\ \varphi_\infty = const.$$

Distributions of the perturbations amplitude and constant φ_∞ are determined from the solution of the differential equations with given boundary conditions.

B. The direct numerical simulation

The evolution of disturbances in the layer was simulated numerically using the ANSYS Fluent software package.

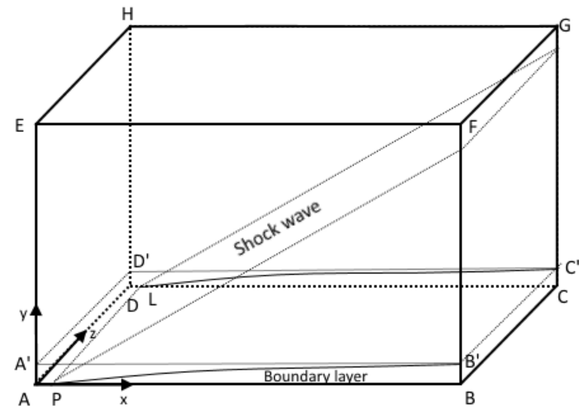

Fig. 2 Diagram of the computational domain

In Fig. 2 the computational domain is shown. The flow irection is shown in fig. 1. The height of the parallelogram AE was selected to avoid of the interaction of shock waves, which forms in the leading edge vicinity of the plate due to viscous-inviscid interaction, with the top side (EHGF). The width AD is taken equal to the wavelength in z-direction $\lambda_z = 2\pi/\beta$.

On an entrance (side AEHD) and on the top side flow parameters are set which are composed of the stationary part and parameters of a sound wave:

$$p^* = p_\infty^* + Ap_\infty^*\cos(\alpha x + \beta z - \omega t)$$

$$T^* = T_\infty^* + AT_\infty^*\frac{(\gamma-1)}{\gamma}\cos(\alpha x + \beta z - \omega t)$$

$$u^* = u_\infty^* + A\sqrt{\frac{RT_\infty^*}{\gamma}}\cos(hi)\cos(\alpha x + \beta z - \omega t)$$

$$v^* = A\sqrt{\frac{RT_\infty^*}{\gamma}}\sin(hi)\cos(\alpha x + \beta z - \omega t),$$

where A is a dimensionless pressure amplitude.

The Mach number $M_\infty = \sqrt{(u^{*2} + v^{*2})/\gamma RT^*}$, wave vectors:

$$\alpha = \tilde{\alpha}\cos(hi),\ \beta = \tilde{\alpha}\sin(hi),\ \tilde{\alpha} = \omega/[(M_\infty\cos(hi) - 1)\sqrt{\gamma RT_\infty^*}].$$

The nonslip boundary conditions (**$\mathbf{v}^*$=0**) are imposed on the plate surface (PLCB) and the plate temperature corresponds to the adiabatic condition ($\partial T^*/\partial y = 0$). Flow parameters on the surface ADLP were taken from a symmetry condition. Flow parameters on sides AEFB and DHGC were taken from a periodicity condition of a solution in z-direction. On the outflow boundary, the unknown variables are extrapolated from an internal domain.

The computational domain consisted of two subdomains, which are divided from each other by the plane A'B'C'D'. The solution in a wall area (bottom subdomain) strongly changes along the coordinate y in comparison with a change in the top subdomain. Therefore in the top subdomain, which height is A'E, the computational grid step on coordinate y (h_y) may exceed the corresponding step in the bottom subdomain at several times. The computational steps in z, x-directions may be identical in the both domains.

It is supposed that free flow disturbances are rather small ($A<<1$) that the development of disturbances in all domains satisfies to linear laws. Disturbances are remained monochromatic on a time and a lateral coordinate.

VI. Results

A. Stationary flow parameters in the boundary layer

The calculations results of longitudinal velocity profiles at Mach number $M_\infty = 2, T_s = 110^\circ K, T_0^* = 300^\circ K$ for different values of the normal injection parameter C_q are presented in Fig 3. Note, that approaching to value of velocity to unit is slowed with increasing injection rates. Thus one can clearly see that normal blowing leads to increasing of boundary layer thickens. Furthermore, an inflection point is appeared in the velocity profile which can contribute to destabilization of the boundary layer.

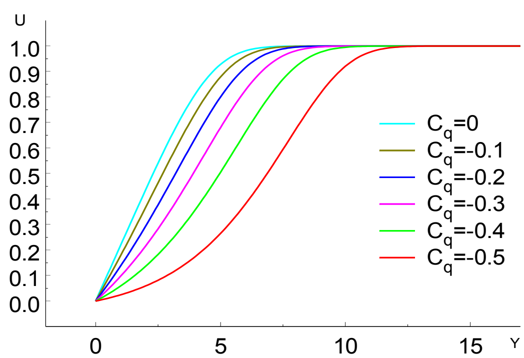

Fig. 3 Distribution of longitudinal velocities for various values of the parameter Cq

Influence of blowing direction in the distribution of longitudinal velocity is shown in Fig. 4. The velocity distribution without blowing is marked by symbols. From these data it follows that the stationary flow parameters are dependent on the tangential injection weakly. The normal velocity component plays a decisive role in this respect.

B. Results on the stability theory

Main results were obtained on the basis of equations (5). Stability calculations were carried out by the classical theory (3)-(4) for processing of the settlement scheme (6). The rectangular mesh with 240 points in the X- coodrinate and 400 point in Y- coordinate with the time step $\Delta = 0.001$ was used.

As already mentioned, at large times the solution of equations (5) are described by an exponential dependence on will not dwell on the initial data which were set arbitrarily. the time, regardless of the initial data. Therefore, we

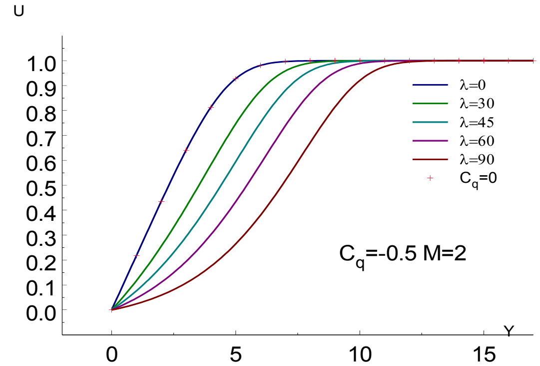

Fig. 4 Dependences of longitudinal velocities on the normal coordinate for the different λ.

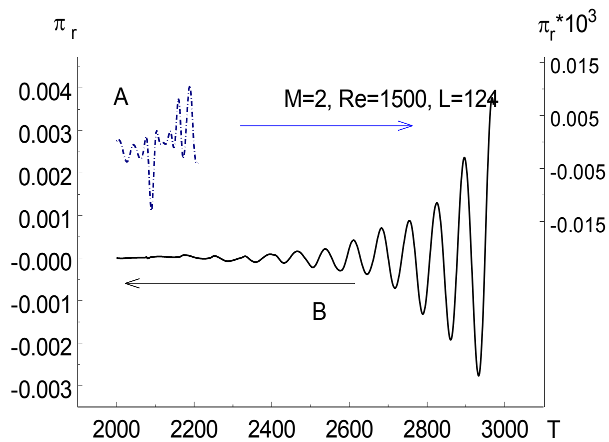

Fig. 5 The dependence of the real part of the pressure perturbation near the walls over time

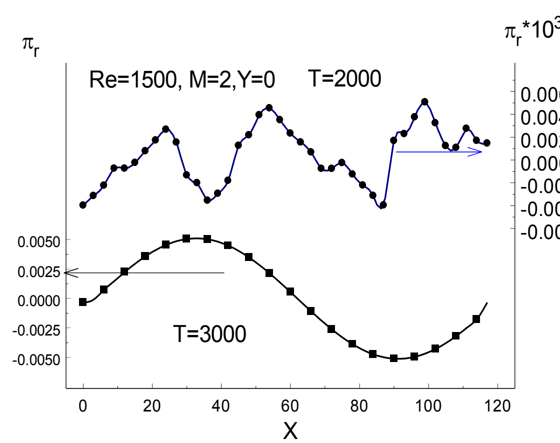

Fig. 6 The dependence of the real part of the pressure perturbation on the coordinate X

Fig. 5 shows the time variation of the real part of the pressure amplitude near the wall, $Y=0$, when $Y^*=40$. In the graph B the result is shown in the time interval $2000<\tau<3000$. Initial values of π_r of the graph B increased in one thousand times are shown on the top graph A, from which one can see that in the initial time moments there are several frequencies. However, over time the most growing frequency is allocated which changes under the law $\cos(\omega_r\tau+\psi_0)exp(\omega_i\tau)$.

Fig. 6 shows the dependence of the real part of the pressure amplitude on space coordinate X at the two times analogously to Fig. 5. It is seen that at large times the spatial dependence is described by a harmonic dependence with the wave number $\alpha = 2\pi / L$ rather well.

The necessary computational domain is determined empirically by comparing of the growth rates for different values of Y^* with the data of the classical theory. From Fig. 7 it is clearly visible that at the thickness $Y^* = 40$ results of numerical modeling differ from data of the classical theory a little.

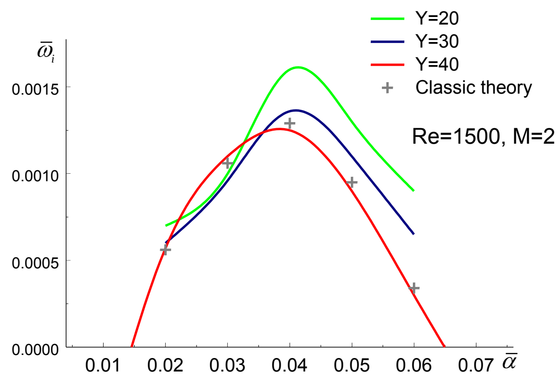

Fig. 7 Dependences of the increments on the wave number for different thicknesses Y^*

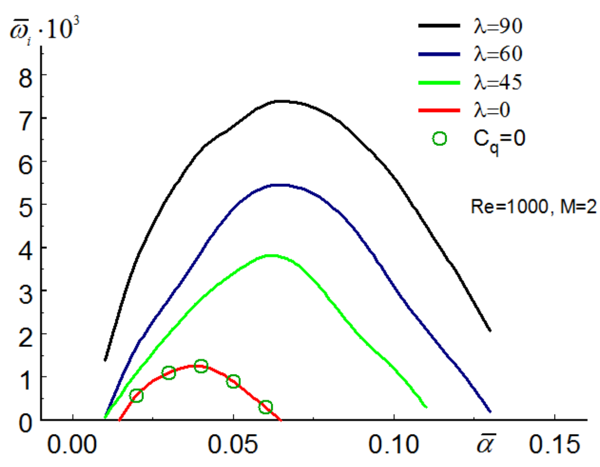

Fig. 8 Dependences of the grow rates on the parameter α for $C_q = -0.5$ and $C_q = 0$

Fig. 8 shows a change of grow rates depending on the wave number α for various injection directions of at $C_q = -0.5$. The red line corresponds to tangential blowing and green circles marks represent the results without blowing. It is seen that the boundary layer stability increases with decreasing of the angle λ, and tangential blowing ($\lambda = 0$) does not affect the boundary layer stability. At the same time normal blowing can increase the rate amplification in several times.

C. The numerical simulation interaction of a boundary layer with acoustic waves

The results of the numerical simulation of the boundary layer interaction with acoustic waves were conducted for the Mach number M_∞ = 2.0, Prandtl number *Pr*= 0.72, ratio of specific heats γ=1.4. The dependence of the dynamic viscosity μ^* on temperature was adopted in accordance with the Sutherland's law.

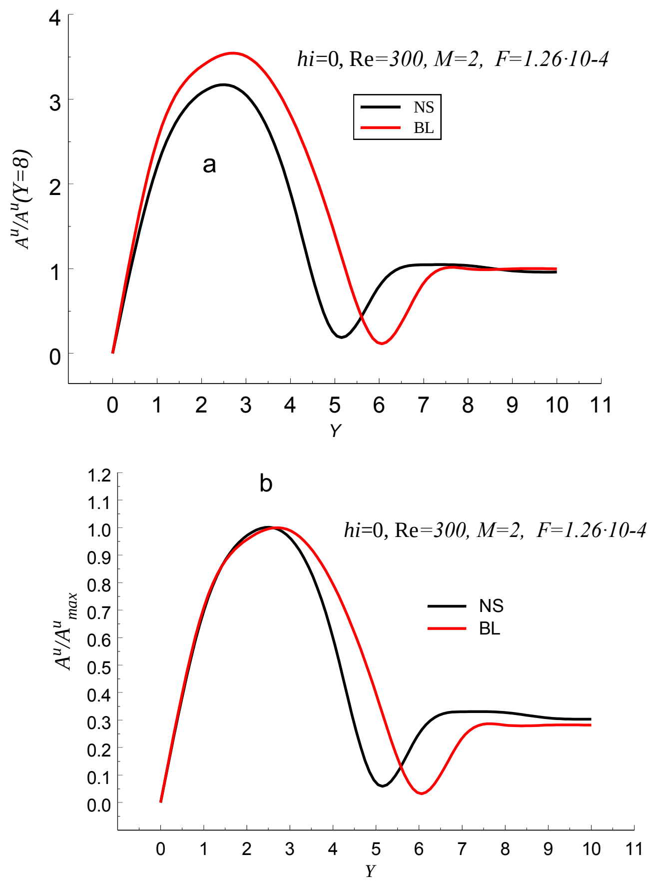

Fig. 9 Distribution of the fluctuations amplitude of the longitudinal velocity related to its value at Y^* = 10 (a) and maxima (b)

The dimensions of the computational domain and its subdomains (Fig.2) were the following: AB=65mm, AA'=3mm, AD=2π/β. The plate PLCB was located at a distance of 5mm from the entrance side (AEHD) to the computational domain. So, the plate length was equal to 60mm.

At a fine-tuning of the calculated scheme the comparison of numerical results (NS) with data of the parallel flow theory (BL) at a low-frequency approach was carried out. Such comparison for the fluctuations amplitude of the longitudinal velocity related to its value on the boundary layer edge, $Y^*=y/\delta=8$, at $hi = 0$, Re=300, $F=\omega\rho_\infty^* u_\infty^{*2}/\mu_\infty^* = 1.26\cdot 10^{-4}$ is given in fig. 9a. Data of the numerical simulation was obtained at free stream parameters: $M_\infty = 2.0, v_\infty^* = 0$, $p_\infty^* = 5600 Pa, T_\infty^* = 164^\circ K$. In general, matching the results of the approximate method (based on stability equations for low-frequency fluctuations) with direct numerical simulation data is satisfactory. If to normalize solutions on the corresponding maxima (Fig. 9b), it is possible to notice very good coincidence of results near a wall. From this comparison it is possible to conclude, that the computational grid used in numerical simulation is good enough. The greatest discrepancy occurs in the area of the boundary layer edge where the approximation theory is inapplicable.

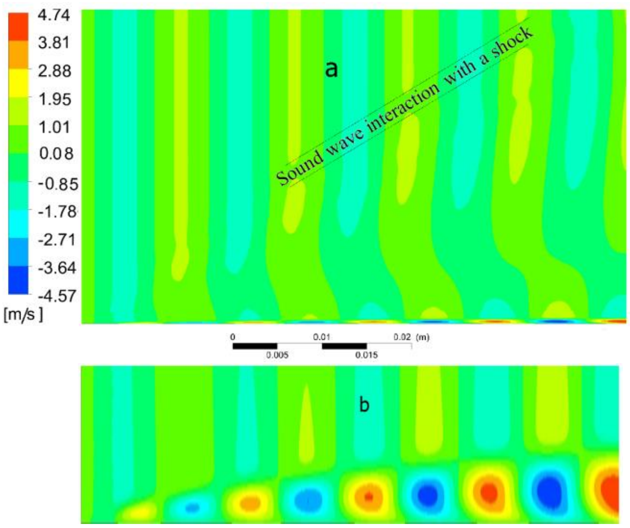

Fig. 10 Instantaneous velocity perturbation contours induced by planar free-stream acoustic wave ($F = 10^{-4}$ and $hi = 0^\circ$)

The main results were obtained with the following computational subdomain grid. Both in the bottom subdomain and in the top subdomain (Fig.2) the number of nodes was 1500 on *x*-coordinate, 300 on *y*-coordinate and 40 on z-coordinate.

In Fig. 10a the instantaneous contour of the full velocity perturbation induced by planar free-stream acoustic wave ($F = 10^{-4}$ and $hi = 0^\circ$) is shown in the area between the surface plate and the top side of computational domain. This contour is shown more detail near the plate in Fig. 10b. It can be seen that at the some distance from the plate edge a periodic structure under shock wave coincides with the structure above the shock wave practically. It tells about a weak intensity of the jump created by a leading edge area of the thin plate.

Within the boundary layer (Fig.10b) almost periodic structure on longitudinal coordinate with the period of a little biger in the comparison with the period near a jump is visible. It follows from this that the phase velocity of the perturbation exceeds value of $u_e^*(1-1/M_e)$, and it corresponds to eigen fluctuations ("Tollmien-Schlichting" waves) of a boundary layer [2]. Along with the strong change of the velocity perturbation intensity in the boundary layer the weak dependence of the perturbations amplitude in the area between the shock wave and the boundary layer edge is observed. Perturbations amplitude changing within this area is explained by the interference of an external acoustics with the acoustics which was created by the non-stationary boundary layer.

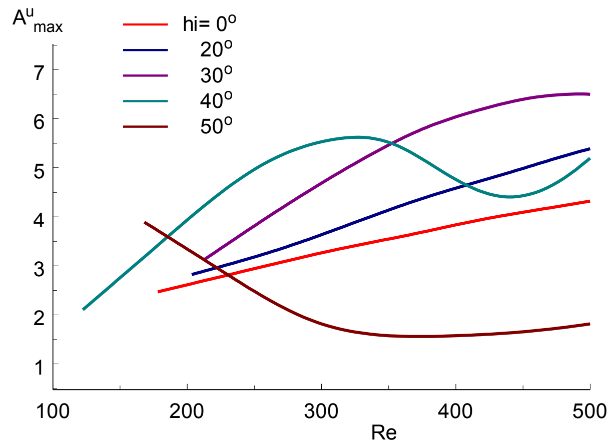

Fig. 11 Amplitude maximum of the full velocity in depending on a Reynolds number at different sliding angle

The dependence of the maximum disturbance amplitudes of a full velocity ($\tilde{u}'^* = u'^* \cos(hi) + w'^* \sin(hi)$) in the boundary layer, A^U_{max} on the Reynolds number at the different sliding angles is shown in Fig. 11 ($F = 0.445\times 10^{-4}$). At the small sliding angles perturbations amplitudes monotonously increases, at least up to Re=500. With an increase of a sliding angle (hi=30°) there is a maximum in this dependence. It is displaced to a leading edge of a plate with the increase of *hi*. The amplitude increase in the field of Reynolds numbers Re > 450 is connected with the boundary layer instability.

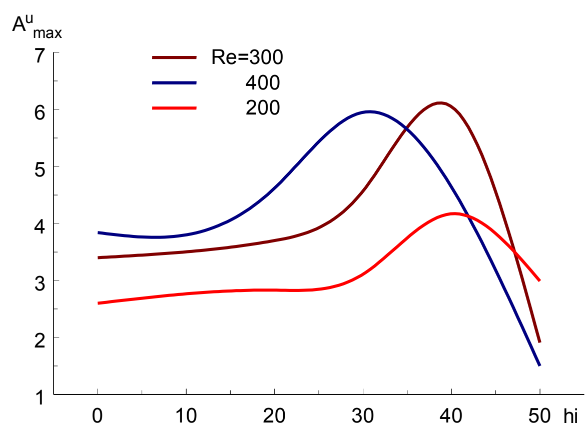

Fig. 12 Amplitudes maximum of the full velocity in depending on the orientation angle.

Finally, the dependence of maximum amplitude on the sliding angle at fixed positions x is shown in Fig. 12.

VII. CONCLUSION

In the paper disturbances inside the boundary layer are investigated at a free stream Mach number of 2.0.

The new method of stability problem solving of the boundary layer is proposed, which is based on an evolutionary perturbations development in time. This method can be called by the usual term - the establishing method. In contrast to the generally accepted method of establishing when the solution goes to the constant, in our case the solution goes to the exponential dependence on time. This method allows determine parameters of a disturbance with the greatest increment.

Influence of the gas blowing direction through a porous surface on the supersonic boundary layer stability was studied for the first time. In the contrast to the strong influence of normal blowing on the boundary layer stability, tangential blowing has a little effect on it.

The boundary layer receptivity process due to the interaction of three-dimensional acoustic waves with a flat plate and the evolution of disturbances inside the boundary layer are investigated numerically. Both the steady and unsteady solutions are obtained by solving the full Navier-Stokes equations using the ANSYS Fluent software package

The simulation data for low-frequency disturbances near the plate surface well agree with the approximate results which were obtained on a base of the equations of the classical stability theory. The greatest discrepancy occurs in the area of the boundary layer edge where the approximation theory is inapplicable.

It has been found that interaction of sound with the boundary layer lead to an increase of the amplitude of disturbances inside the boundary. It has been shown that the amplitude of fluctuations inside the boundary layer may exceed the amplitude of the external acoustic field in many times and it depends on the parameters of the wave and the main flow. At a given position (Reynolds's number) there is an optimum sliding angle at which the maximum fluctuations of a boundary layer are excited. With increasing of the Reynolds number a critical value of a sliding angle decreases and the fluctuations intensity increases in a boundary layer.

The simulation of interaction of acoustic waves with the flat plate shows that "Tollmien-Schlichting" waves are generated in the boundary layer.

REFERENCES

[1] S. A. Gaponov, A. N. Semenov, "Solution evolutionary method of compressible boundary layers stability problems," *International Journal of Mathematical and Computational Methods,* vol. 1, pp. 175-181, 2016.

[2] S.A. Gaponov and A.A. Maslov, *Development of Disturbances in Compressible Flow*, Novosibirsk, Science, 1980 (in Russian).

[3] C.C. Lin, *The Theory of Hydrodynamic Stability*, Cambridge, University press, 1955.

[4] L.M. Mack, *Boundary Layer Stability Theory*, Document 900-277, Rev. A, Pasadena, California: JPL, 1969.

[5] M.V. Morkovin, "On transition experiments at moderate supersonic speeds," *J. Aeronaut. Sci*., vol. 24, pp. 480-486, 1957.

[6] C.K.W. Tam and D.E. Burton, "Sound generated by instability waves of supersonic flows. 1. Two-dimensional mixing layers," *J. Fluid. Mech*., vol. 138, pp.249-273, 1984.

[7] L.M. Mack, "Linear stability theory and the problem of supersonic boundary layer transition," *AIAA Journal*, vol. 13, pp.278-289, 1975.

[8] S.A. Gaponov, "Interaction between a supersonic boundary layer and acoustic disturbances," *Fluid Dynamic*, vol. 12 no.6, pp. 858-862, 1977.

[9] S.A. Gaponov , "Excitation by sound of Tollmien-Schlichting waves in a supersonic boundary layer," *Fluid Dynamics*, vol. 18, no.3, pp. 384-390, 1983.

[10] S.A. Gaponov, V.A. Lebiga, A.A. Maslov and V.G. Pridanov, "Development of disturbances caused by external sound field in a supersonic boundary layer," in *Proc. IXth All-Union Acoustic Conf*., Moscow, Acoustic Institute, 1977, 49-52 (in Russian).

[11] V. A. Lebiga, A. A. Maslov, V. G. Pridanov, "Experimental investigation of the stability of a supersonic boundary layer on a plate with blunting of the leading edge," *Fluid Dynamics*, vol.12 no.4, pp. 541-545.

[12] S. A. Gaponov, B. V. Smorodsky. "Acoustics and instability of high-speed boundary layers," *International Journal of Mechanics*, vol. 6, no. 1, pp. 9-16, 2012.

[13] S.A. Gaponov , B.V. Smorodsky, "Distirbance excitation in supersonic boundary layer by acoustics ," in *Proc. International Conference on the Methods of Aerophysical Research,* Novosibirsk, 1998, part 3, pp.103-108.

[14] E. S. G. Maciel, E. M. Ferreira, A. P. Pimenta, N. E. Mastorakis, "TVD and ENO Applications to Supersonic Flows in 2D – Initial Study," *International journal of mathematical models and methods in applied sciences,* vol. 8, 441-463, 2014

[15] Y.Ma, X. Zhong, "Receptivity of a supersonic boundary layer over a flat plate. Part 2. Receptivity to free-stream sound," *J. Fluid. Mech., vol*. 488, pp. 79—121, 2003.

[16] Y.Ma, X. Zhong, "Receptivity of a supersonic boundary layer over a flat plate. Part 3. Effects of different types of free-stream disturbances," *J. Fluid. Mech.*, vol. 532, pp. 63—109, 2005.

[17] I. V. Egorov, V. G. Sudakov, A. V. Fedorov, " Numerical modeling of the receptivity of a supersonic boundary layer to acoustic disturbances," *Fluid Dynamics,* vol. 41, no. 1, 37-48, 2006.

[18] M. R. Malik, P. Balakumar, Receptivity of Supersonic Boundary Layers to Acoustic Disturbances, *AIAA Paper 2005-5027*, 2005.

[19] A. N. Kudryavtsev, S. G. Mironov, UDC 532.526 T. V. Poplavskaya, and I. S. Tsyryul'nikov, "Experimental study and direct numerical simulation of the evolution of disturbances in a viscous shock layer on a flat plate," *Journal of Applied Mechanics and Technical Physics*, vol. 47, no. 5, pp. 617–627, 2006

[20] P. Balakumar, Transition in a Supersonic Boundary Layer Due to Acoustic Disturbances, *AIAA Paper 2005-0096,* 2005.

[21] L.G. Loitsjansky, *Mechanics of Liquid and Gas*, Moscow, Science, 1973 (in Russian).

[22] D.W. Dunn and C.C. Lin, "On the stability of the laminar boundary layer in a compressible fluid," *J. Aeronaut. Sci*., vol. 1, pp. 455-477, 1955.

[23] M. A. Alexeev, On Asymptotic Approximations in the Laminar Boundary Layer Stability at Supersonic Speeds, *Proceeding of TsAGI No. 1420*, Moscow, TsAGI, 1972 (in Russian).

[24] V. M. Kovenja, *Differential Methods of the Solution of Multidimensional Tasks*. Novosibirsk, NGU, 2004, (in Russian).

[25] L. M. Mack, "Computation of the stability of the laminar compressible boundary layer," *Methods in Computational Physics*, New York, Academic Press, vol. 4, pp. 247-299, 1965.