Sensitivity Analysis of the Accident Rate of a Plant by the Generalized Perturbation Theory

E. F. Lima, D. G. Teixeira, P. F. Frutuoso e Melo, F. C. Silva, and A. C. M. Alvim

Abstract—we discuss the application of the Generalized Perturbation Theory (GPT) to the reliability of a system of three equal protection channels of an industrial plant. The influence of parameters such as the demand rate and the failure rate over the plant accident rate is discussed. Traditional methods have been used to study the influence of these parameters on the plant accident rate in which the system of differential equations derived from the Markov approach adopted is solved for each value of the demand rate. From the solution of this system of equations, curves for the accident frequency depending on the demand rate (direct calculation) are obtained. However, it is possible to obtain these curves by GPT in a faster way, wherein the calculation effort may be reduced by a factor of up to 10. It was found that for demand rates lower than 1000 / year, GPT calculations with 3rd and 5th orders of approximations of gave better results than those with 1st order approximation when compared to direct calculation. However, for demand rates equal or greater than 1000 / year, the 1st order approach presented better results than the 3rd and 5th orders.

***Keywords*—**Demand rate, Generalized Perturbation Theory, Markovian reliability analysis, Plant accident rate.

I. Introduction

INDUSTRIAL facilities are equipped with systems whose sole function is to protect the public, their personnel and equipment against destructive effects caused by accidents in which radioactive, toxic or flammable substances may be released into the environment.

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Typically, protective systems are periodically tested standby safety systems whose reliability figure of merit is their mean unavailability. This depends on the failure and repair rates of its constituent channels, on the test and maintenance policies imposed as well as on the system logic configuration.

However, from the point of view of safety, the facilities parameter that really matters is the accident (or hazard) rate.

It has been common practice to evaluate the plant accident rate as the product of the frequency of occurrence of the initiating event (also known as demand rate) by the protective system mean unavailability, where one assumes that the latter is independent of the former. This is a valid assumption only if the demand rate is low (typically less than 10/year) as happens to be for most initiating events in nuclear power plants.

Nevertheless, a significant effect of the demand rate on the plant accident rate may be found whenever the former assumes higher values. This influence has already been detected and discussed for some special cases [1]–[3].

As practical experience has shown, we consider that protective systems may have up to five identical channels for then we will be covering 90% of actual systems under current usage.

Markovian models have been used in order to model the plant accident rate by taking into account the interdependence between the demand rate and the unavailability of the protective system.

Markovian models consider the possibility of performing repair on the protective system both with the plant online as well as offline. In many instances, the first policy is not allowed. Besides, for those situations where an accident has occurred and the protective system did not perform properly, repair is not performed on it for the whole plant is already under damage. This situation is also modeled here.

As demand rates may be as high as 10,000/year, sensitivity analyses on the plant accident rate may require extensive calculations. For this reason, the generalized perturbation theory has been considered as an interesting option for facing the problem [4]. One of the advantages of GPT is the fact that it requires a reference solution for the plant accident rate and it may generate results by perturbing one or more parameters at the same time, thus considerably reducing the computer effort.

GPT is a heuristic method [5] widely used by the nuclear engineering community, as, for example, in reactor physics [6] and thermal hydraulics analysis [7]–[8].

The possibility of applying GPT to reliability analysis based on Markov models was discussed in [9]. The behavior of plant accident frequency as a function of failure rates and system demand was analyzed in [4].

Several studies published in the reliability area confirm the importance of determining the accident frequency of a plant on the basis of failure and demand rates [1]–[3]. Seeking to extend the application of GPT to this problem, this work makes a sensitivity analysis of the accident frequency of a plant equipped with a system of three protection channels.

As we are discussing a trip channel system with redundancy, it is necessary to consider the possibility of occurrence of common-cause failures. There are some models that treat common-cause failures, such as the basic parameter model, the multiple Greek letter model, and the α factor model [10]. In this work, we used the α factor model. The reason for choosing this model is that it is relatively simple to obtain its parameter values in practice.

II. The Three-channel problem

The reliability attribute of interest for protective systems is their average unavailability (U), which depends on the component (channel) failure rate (λ), repair rate (μ), human failure probability during maintenance activities (γ) and also on the number of repairmen available. From the point of view of plant safety analysis, the parameter that matters is the accident frequency (η), given by the product between the frequency of the initiating event, also called demand rate (ν), and the average unavailability of the protective system, $U(\lambda,\mu)$, where it is tacitly assumed that the latter is independent of the first. However, a significant effect of the demand rate on the average availability of the protective system can be found whenever the first assumes higher values (>10/year, in general [2]). This influence has been analyzed in practice (e.g., systems with up to two redundant channels [3]). Thus, in such cases, one should write:

$$\eta = \nu . U(\lambda,\mu,\nu) \tag{1}$$

We assume a three-channel protective system subject to a 2 – 3 : F, which means that the protective system failure occurs whenever at least 2 channels fail. The system unavailability will be modeled by means of a Markov chain because we need to model system repair and also unrevealed channel failures.

The state transition diagram for the protective system with three identical channels and revealed failures under a 2 – 3: F logic may be seen in Figure 1.

The parameters in the triplets < i, j, k > shown in Figure 1 represent: i = number of operating channels, j = number of failed channels whose failures are unrevealed, and k = number of failed channels whose failures are revealed.

In Fig. 1 λ_k represents the failure rate of k channels that have failed due to common causes (common-cause failures). The transition from state 1 to state 7 means that all channels have failed due to that cause. The transition rate from state 1 to state 2 means that only one channel failure has occurred but there are 3 different ways because there are 3 channels on in state 1. As the number of repairmen is equal to the number of channels, the transition from state 10 to state 6 is equal to 3(1-γ)μ, meaning that each failed channel is assigned a repairman. For the case of the transition from state 10 to state 8 an unrevealed channel failure is assumed as a result of maintenance but again 3 repairmen are available.

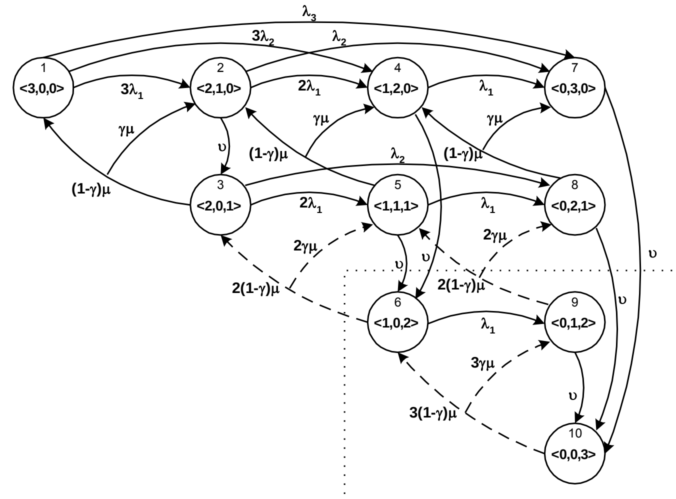

Fig. 1 State transition diagram for the 3-channel protective system

Due to the assumption of performing maintenance only when the plant is online, states 6, 9 and 10 will not be taken into account when evaluating the plant accident rate, as will be discussed later.

The approach for treating common-cause failures is based on the α model [10].

The α model [10] is based on multi-parameter generalized parameters that are related to the events known with the purpose of estimating in a direct way, the basic event common-cause probabilities. α can be defined as the fraction of events involving the failure of a particular component due to a common cause.

The probability of simultaneous failure of k and only k components due to a common cause is given by [10]:

$$\lambda_k = \frac{m}{\binom{m}{k}} \frac{\alpha_k}{\alpha_t} \lambda \tag{2}$$

where

$$\alpha_t = \sum_{k=1}^{m} k . \alpha_k \tag{3}$$

and m stands for the number of equal protective system channels.

The parameter α_k is subject to the following condition:

$$\sum_{k=1}^{m} \alpha_k = 1 \tag{4}$$

As there are three equal protective channels, then m = 3 and Eq. (2) is written as:

$$\lambda_k = \frac{3}{\binom{3}{k}} \frac{\alpha_k}{(\alpha_1 + 2\alpha_2 + 3\alpha_3)} \lambda \text{ , for } k = 1, 2, \text{ and } 3 \qquad (5)$$

From Eq. (5), one can write:

$$\alpha_3 = 1 - \alpha_1 - \alpha_2 \qquad (6)$$

Putting Eq. (6) in Eq. (5) with k = 1, 2, and 3 one has:

$$\lambda_1 = \frac{\alpha_1}{[3 - (2\alpha_1 + \alpha_2)]} \lambda \qquad (7)$$

$$\lambda_2 = \frac{\alpha_2}{[3 - (2\alpha_1 + \alpha_2)]} \lambda \qquad (8)$$

$$\lambda_3 = \frac{3(1 - \alpha_1 - \alpha_2)}{[3 - (2\alpha_1 + \alpha_2)]} \lambda \qquad (9)$$

which are the system failure rates as a function of parameters α_1 e α_2.

The differential equation that governs the system behavior is given by:

$$\frac{d\,\underline{p}(t)}{dt} = \underline{\underline{M}}\,\underline{p}(t) \qquad (10)$$

where $\underline{p}(t)$ is a vector defined as follows:

$$\underline{p}(t) \equiv [p_1(t), ..., p_{10}(t)]^T \qquad (11)$$

and $p_i(t)$; i = 1,2,...,10, represents the probability that the system is in the i-th state and $\underline{\underline{M}}$ is the transition rate matrix, given by:

$$\underline{\underline{M}} = \begin{bmatrix} M_{1,1} & 0 & \bar{\gamma}\mu & 0 & 0 & 0 & \lambda_3 & 0 & 0 & 0 \\ 3\lambda_1 & M_{2,2} & \gamma\mu & 0 & \bar{\gamma}\mu & 0 & \lambda_2 & 0 & 0 & 0 \\ 0 & \nu & M_{3,3} & 0 & 0 & 2\bar{\gamma}\mu & 0 & 0 & 0 & 0 \\ 3\lambda_2 & 2\lambda_1 & 0 & M_{4,4} & \gamma\mu & 0 & \lambda_1 & \bar{\gamma}\mu & 0 & 0 \\ 0 & 0 & 2\lambda_1 & 0 & M_{5,5} & 2\gamma\mu & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \nu & \nu & M_{6,6} & 0 & 0 & \lambda_1 & 0 \\ \lambda_3 & \lambda_2 & 0 & \lambda_1 & 0 & 0 & M_{7,7} & \gamma\mu & 0 & 0 \\ 0 & 0 & \lambda_2 & 0 & \lambda_1 & 0 & \gamma\mu & M_{8,8} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda_1 & 0 & 0 & M_{9,9} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \nu & 3\gamma\mu & M_{10,10} \end{bmatrix}, \qquad (12)$$

where:

$$M_{1,1} = -(3\lambda_1 + 3\lambda_2 + \lambda_3)$$
$$M_{2,2} = -(2\lambda_1 + \lambda_2 + \nu)$$
$$M_{3,3} = -(2\lambda_1 + \lambda_2 + \mu)$$
$$M_{4,4} = -(\lambda_1 + \nu)$$
$$M_{5,5} = -(\lambda_1 + \nu + \mu)$$
$$M_{6,6} \equiv -(\lambda_1 + 2\mu)$$
$$M_{7,7} \equiv -\nu$$
$$M_{8,8} \equiv -(\nu + \mu)$$
$$M_{9,9} \equiv -(\nu + 2\mu)$$
$$M_{10,10} \equiv -3\mu$$

As this is an initial value problem, an initial condition must be specified. It is considered that all channels are initially on, so that:

$$\underline{p}(0) = [1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^T \qquad (13)$$

As the failure logic is given by 2 – 3:F, so that there must occur at least two channel failures for the system to fail, the plant accident rate, η, is given by [2]:

$$\eta = \frac{\nu}{\tau_p} \int_0^{\tau_p} [p_4(t) + p_5(t) + p_6(t) + p_7(t) + p_8(t) + p_9(t) + p_{10}(t)]\, dt \qquad (14)$$

Eq. (14) takes into account that on-line repair is feasible. As we are not going to take this policy into account (as discussed in [4]), then only the offline repair policy will be taken into account, so that Eq. (14) may be rewritten as:

$$\eta = \frac{\nu}{\tau_p} \int_0^{\tau_p} [p_4(t) + p_5(t) + p_7(t) + p(t)_8]\, dt \qquad (15)$$

which may be recast into:

$$\eta = \int_0^{\tau_p} \underline{h}^{+^T}\, \underline{p}(t) dt \qquad (16)$$

where

$$\underline{h}^{+^T} = \frac{\nu}{\tau_p} \underline{p}_T \qquad (17)$$

and

$$\underline{p}_T(t) = [0 \quad 0 \quad 0 \quad 1 \quad 1 \quad 0 \quad 1 \quad 1 \quad 0 \quad 0]^T \qquad (18)$$

The perturbation in parameters α_1 and ν will give us a new accident rate η', which can be obtained by a Taylor series expansion:

$$\eta' = \eta + \sum_{n=1}^{\infty} \frac{1}{n!} \left\{ \frac{\partial^n \eta}{\partial \alpha_1^n} (\delta\alpha_1)^n + \frac{\partial^n \eta}{\partial \nu^n} (\delta\nu)^n \right\} + \sum_{n=2}^{\infty} \frac{1}{n!} \left[\sum_{m=1}^{n-1} \binom{n}{m} \left\{ \frac{\partial^n \eta}{\partial \alpha_1^{n-m} \partial \nu^m} (\delta\alpha_1)^{n-m} + \frac{\partial^n \eta}{\partial \nu^n} (\delta\nu)^m \right\} \right] \tag{19}$$

where one obtains from Eq. (16):

$$\frac{\partial^n \eta}{\partial \alpha_1^n} = \int_0^{\tau_p} \underline{h}^{+^T} \frac{\partial^n \underline{p}(t)}{\partial \alpha_1^n} dt \tag{20}$$

$$\frac{\partial^n \eta}{\partial \nu^n} = \int_0^{\tau_p} \underline{h}^{+^T} \frac{\partial^n \underline{p}(t)}{\partial \nu^n} dt + \begin{cases} \frac{1}{\nu} \eta \,; & if \quad n = 1 \\ \frac{\eta}{\nu} \int_0^{\tau_p} \underline{h}^{+^T} \frac{\partial^{n-1} \underline{p}(t)}{\partial \nu^{n-1}} dt \,; & if \quad n > 1 \end{cases} \tag{21}$$

and

$$\frac{\partial^n \eta}{\partial \alpha_i^{n-m} \partial \nu^m} = \int_0^{\tau_p} \underline{h}^{+^T} \frac{\partial^n \underline{p}(t)}{\partial \alpha_1^{n-m} \partial \nu^m} dt + \begin{cases} \frac{1}{\nu} \int_0^{\tau_p} \underline{h}^{+^T} \frac{\partial^{n-1} \underline{p}(t)}{\partial \alpha_1^{n-1}} dt \,; & if \quad m = 1 \\ \frac{m}{\nu} \int_0^{\tau_p} \underline{h}^{+^T} \frac{\partial^{n-1} \underline{p}(t)}{\partial \alpha_1^{n-m} \partial \nu^{m-1}} dt \,; & if \quad m > 1 \end{cases} \tag{22}$$

The derivatives of $\underline{p}(t)$ with respect to α_1 and ν parameters are the solutions of the following equations:

$$\frac{d}{dt}\left(\frac{\partial^n}{\partial \alpha_1^n} \underline{p}(t) \right) = M \left(\frac{\partial^n}{\partial \alpha_1^n} \underline{p}(t) \right) + \underline{S}_1^n(t) \tag{23}$$

$$\frac{d}{dt}\left(\frac{\partial^n}{\partial \nu^n} \underline{p}(t) \right) = M \left(\frac{\partial^n}{\partial \nu^n} \underline{p}(t) \right) + \underline{S}_\nu^n(t) \tag{24}$$

$$\frac{d}{dt}\left(\frac{\partial^n}{\partial \alpha_1^{n-m} \partial \nu^m} \underline{p}(t) \right) = M \left(\frac{\partial^n}{\partial \alpha_1^{n-m} \partial \nu^m} \underline{p}(t) \right) + \underline{S}_{1\nu}^{(n-m,m)}(t) \tag{25}$$

whose source terms are as follows:

$$S_1^{(n)}(t) = M_1 \left\{ a_n \underline{p}(t) + \sum_{i=1}^{n-1} \binom{n}{i} a_i \left(\frac{\partial^{n-i}}{\partial \alpha_1^{n-1}} \underline{p}(t) \right) \right\} \tag{26}$$

$$S_\nu^{(n)}(t) = M_\nu \left\{ \underline{p}(t)) \sum_{i=1}^{n-1} \binom{n}{i} \frac{\partial^{n-i}}{\partial \nu^{n-i}} \underline{p}(t) \right\} \tag{27}$$

$$S_{1\nu}^{(n-m,m)} = M_1 \left\{ \sum_{i=1}^{n-1} \binom{n-m}{l} a_i \left(\frac{\partial^{n-l}}{\partial \alpha_1^{n-m-l} \partial \nu^m} \underline{p}(t) \right) + a_i \frac{\partial^m}{\partial \nu^m} \underline{p}(t) \right\} + m.M_\nu \left(\frac{\partial^{n-1}}{\partial \alpha_1^{n-m} \partial \nu^{m-1}} \underline{p}(t) \right) \tag{28}$$

where

$$a_i = \frac{i!\, 2^{i-1}}{\left(3 - [2\alpha_1 + \alpha_2]\right)^i} \text{ , with } \quad i = 1, 2, \ldots, n \,,$$

$$M_\nu = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix} \tag{29}$$

and

$$M_1 = 2M' + \lambda M_\lambda^{(1)} \tag{30}$$

where

$$M' = \begin{bmatrix} M'_{1,1} & & & & & & & & & \\ 3\lambda_1 & M'_{2,2} & & & & & & & & \\ 0 & 0 & M'_{3,3} & & & & & & 0 & \\ 3\lambda_2 & 2\lambda_1 & 0 & -\lambda_1 & & & & & & \\ 0 & 0 & 2\lambda_1 & 0 & -\lambda_1 & & & & & \\ 0 & 0 & 0 & 0 & 0 & -\lambda_1 & & & & \\ \lambda_3 & \lambda_2 & 0 & \lambda_1 & 0 & 0 & 0 & & & \\ 0 & 0 & \lambda_2 & 0 & \lambda_1 & 0 & 0 & 0 & & \\ 0 & 0 & 0 & 0 & 0 & \lambda_1 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{31}$$

with

$$M'_{1,1} = -(3\lambda_1 + 3\lambda_2 + \lambda_3)$$
$$M'_{2,2} = -(2\lambda_1 + \lambda_2)$$
$$M'_{3,3} = -(2\lambda_1 + \lambda_2)$$

and

$$M_{\lambda}^{(1)} = \left[\begin{array}{cccccccccc} 0 & & & & & & & & & \\ 3 & -2 & & & & & & & & \\ 0 & 0 & -2 & & & & & & & \\ 0 & 2 & 0 & -1 & & & & 0 & & \\ 0 & 0 & 2 & 0 & -1 & & & & & \\ 0 & 0 & 0 & 0 & 0 & -1 & & & & \\ -3 & 0 & 0 & 1 & 0 & 0 & 0 & & & \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & & \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \tag{32}$$

However, according to the source reciprocity relationship [11], Eqs. (20)– (22) may be recast into:

$$\int_0^{\tau_p} \underline{h}^{+^T} \frac{\partial^n \underline{p}(t)}{\partial \alpha_1^n} dt = \int_0^{\tau_p} \underline{p}^{*^T} \underline{S}_1^{(n)}(t) dt \tag{33}$$

$$\int_0^{\tau_p} \underline{h}^{+^T} \frac{\partial^n \underline{p}(t)}{\partial \nu^n} dt = \int_0^{\tau_p} \underline{p}^{*^T}(t) \underline{S}_\nu^{(n)}(t) dt \tag{34}$$

and

$$\int_0^{\tau_p} \underline{h}^{+^T} \frac{\partial^n \underline{p}(t)}{\partial \alpha_1^{n-m} \partial \nu^m} dt = \int_0^{\tau_p} \underline{p}^{*^T}(t) \underline{S}_{1\nu}^{(n-m,m)}(t) dt \tag{35}$$

where $\underline{p}^*(t)$, the importance function associated to the integral quantity η is the solution of the following equation:

$$-\frac{d}{dt} \underline{p}^*(t) = M^T \underline{p}^{*^T}(t) + h^{+^T} \tag{36}$$

Putting Eqs. (33)–(35) into Eqs. (20)–(22), respectively, and the resulting equations in Eq. (19), one obtains:

$$\begin{aligned} \eta' = \eta &+ \sum_{n=1}^{\infty} \frac{1}{n!} \left\{ \left(\int_0^{\tau_p} \underline{p}^{*^T}(t) \underline{S}_1^{(n)}(t) dt \right) (\delta\alpha_1)^n + \left(\int_0^{\tau_p} \underline{p}^{*^T}(t) \underline{S}_\nu^{(n)}(t) dt \right) (\delta\nu)^n \right\} + \\ &+ \sum_{n=2}^{\infty} \frac{1}{n!} \left[\binom{n}{m} \left\{ \left(\int_0^{\tau_p} \underline{p}^{*^T}(t) \underline{S}_{1\nu}^{(n-m,m)}(t) dt \right) (\delta\alpha_1)^{n-m} (\delta\nu)^m \right\} \right] + \\ &+ \frac{1}{\nu} [\eta(\delta\nu) + \sum_{n=2}^{\infty} \frac{1}{n!} \left\{ n \left(\int_0^{\tau_p} \underline{p}^{*^T}(t) \underline{S}_\nu^{(n-1)}(t) dt \right) (\delta\nu)^n + n \left(\int_0^{\tau_p} \underline{p}^{*^T}(t) \underline{S}_1^{(n-1)}(t) dt \right) (\delta\alpha_1)^{n-1} (\delta\nu) \right\} + \\ &+ \sum_{n=3}^{\infty} \frac{1}{n!} \left\{ \sum_{m=2}^{n-1} \binom{n}{m} m \left(\int_0^{\tau_p} \underline{p}^{*^T}(t) \underline{S}_{1\nu}^{(n-m,m-1)}(t) dt \right) (\delta\alpha_1)^{n-m} (\delta\nu)^m \right\}] \end{aligned} \tag{37}$$

where the source terms $\underline{S}_1$, $\underline{S}_\nu$ *and* $\underline{S}_{1\nu}$ where defined in Eqs. (26)–(28).

Eq. (37) is used for performing the sensitivity analysis for the plant accident rate. Note that the use of the importance concept and of source terms characterizes the use of the generalized perturbation theory.

III. Case studies

To use GPT for performing the sensitivity analysis on the accident frequency of a plant η (using a system of three equal protection channels), the system demand rate ν and α_1 (fraction of events involving the failure of a particular component due to a common cause) were perturbed.

The input data used is presented in Table 1 [2]–[3].

Table 1 Input parameters

Parameter	Symbol	Value
Proof test interval	τ_p	1 yr
Channel failure rate	λ	10/yr
Channel repair rate	μ	365/yr
Human failure probability during channel repair	γ	0.01
Probability of two channel failures due to common-cause failure	α_2	0.1

Moreover, parameter α_1 was varied as follows:

$$\alpha_1 = 0{,}7 + 0{,}05(n-1), \quad n = 1,\ldots,5 \tag{38}$$

The reference value for parameter $\alpha_1$was assumed as 0.8. The value assumed for α_2 is presented in Table 1. This means that we are assuming a 10% probability of two simultaneous channel failures due to common causes.

In the sensitivity studies for each α_1 value, variations of ν according to Table 2 were adopted. That is, for each case shown, were a different interval for ν is defined, the ν values in these ranges are given according to equation in the last column of the table. Also, a reference value for the demand rate is set, as also shown in the table.

Table 2 Perturbed demand rates

Case	Range (yr^{-1})	ν_{Ref} (yr^{-1})	Perturbed parameter value (yr^{-1})
1	$1 \le \nu \le 9$	5	1+1(n-1); n=1,...,9
2	$10 \le \nu \le 50$	30	10+5(n-1); n=1,...,9
3	$55 \le \nu \le 95$	75	55+5(n-1); n=1,...,9
4	$100 \le \nu \le 300$	200	100+25(n-1); n=1,...,9
5	$300 \le \nu \le 500$	400	300+25(n-1); n=1,...,9
6	$500 \le \nu \le 750$	625	500+25(n-1); n=1,...,11
7	$750 \le \nu \le 1000$	875	750+25(n-1); n=1,...,11
8	$1000 \le \nu \le 2000$	1500	1000+250(n-1); n=1,...,5
9	$2000 \le \nu \le 3000$	2500	3000+250(n-1); n=1,...,5
10	$3000 \le \nu \le 4000$	3500	3000+250(n-1); n=1,...,5
11	$4000 \le \nu \le 6000$	5000	4000 + 250(n-1); n=1,...,9
12	$6000 \le \nu \le 10000$	8000	6000 + 500(n-1); n=1,...,9

Sensitivity calculations using GPT mean solving the system of differential equations of Eq. (2) 60 times since there are 12 reference values for the demand rate and 5 values for the α_1 parameter [Eq. (38)] were adopted (see Table 2). Comparatively, the results presented in [12] required the solution of the same system more than 600 times.

IV. Results and Discussion

Figure 2 shows the sensitivity analysis performed on the plant accident rate for α_1 = 0.75. The curves for the direct solution and for the perturbations considering the first, third and fifth orders expansions terms are displayed [Eq. (37)].

Figs. 2 – 5 display the sensitivity analysis for α_1 = 0.75, 0.8, 0.85, and 0.95, respectively. It can be seen that the results involve the presentation of the direct solution and of the perturbed solutions with different Taylor expansion orders.

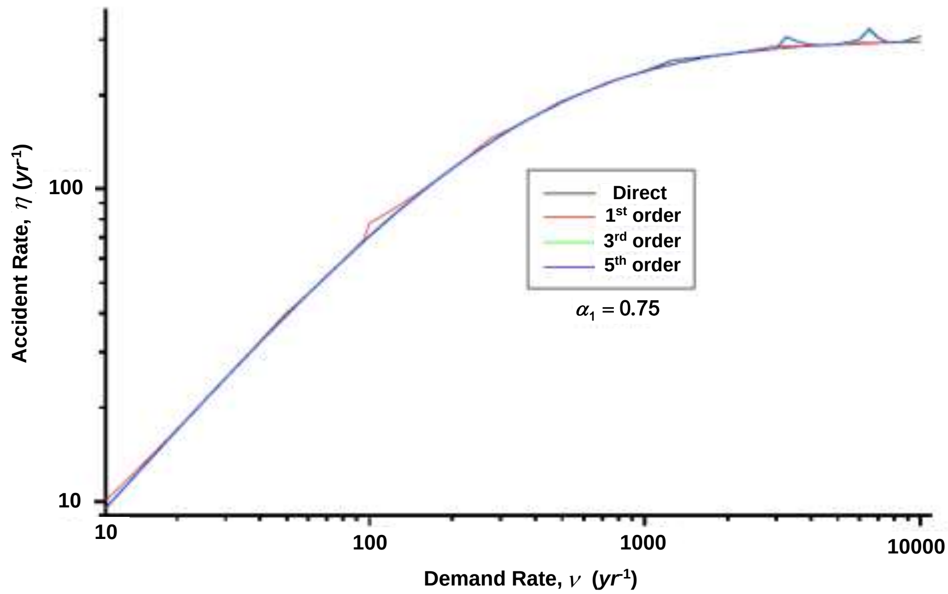

Fig. 2 Sensitivity analysis for α_1 = 0.75

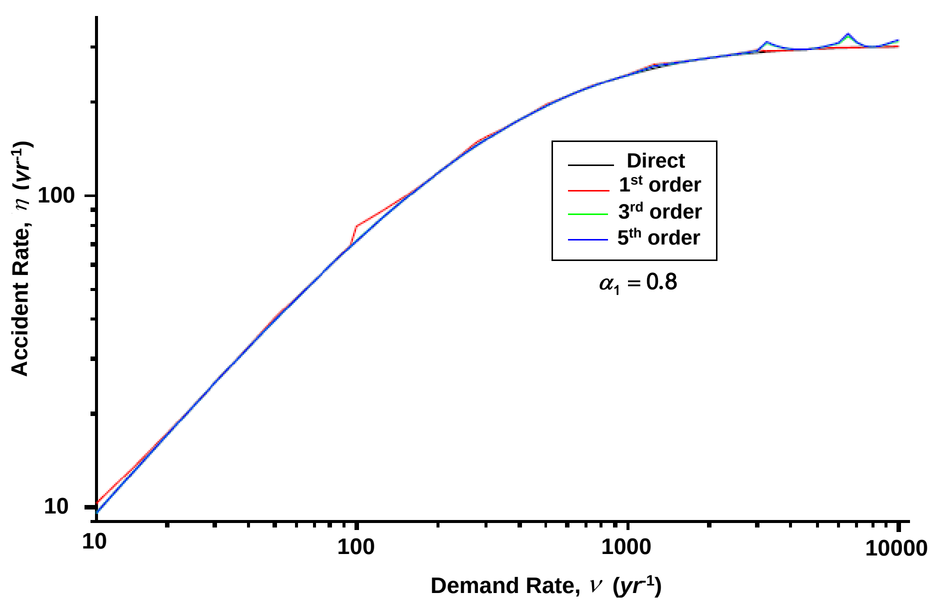

Fig. 3 Sensitivity analysis for α_1 = 0.8

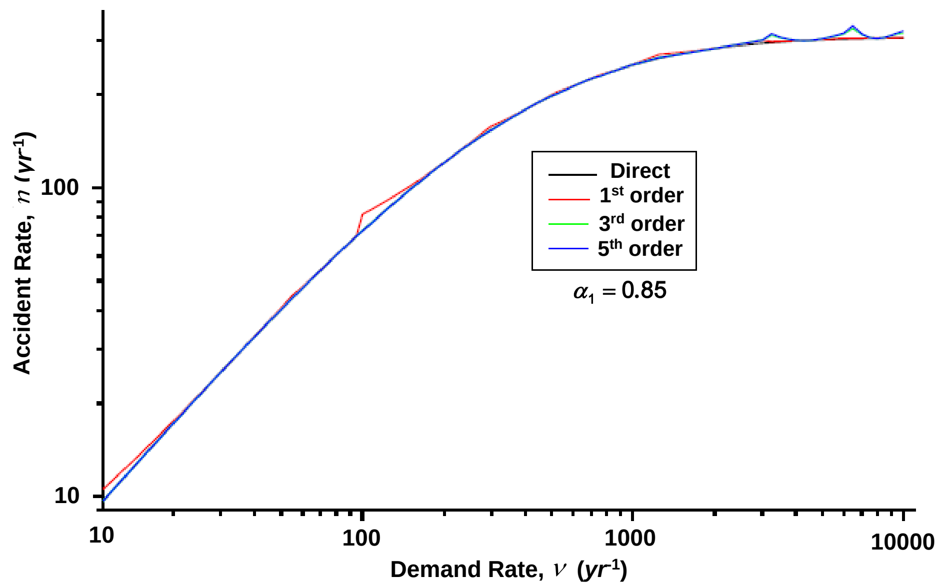

Fig. 4 Sensitivity analysis for $\alpha_1 = 0.85$

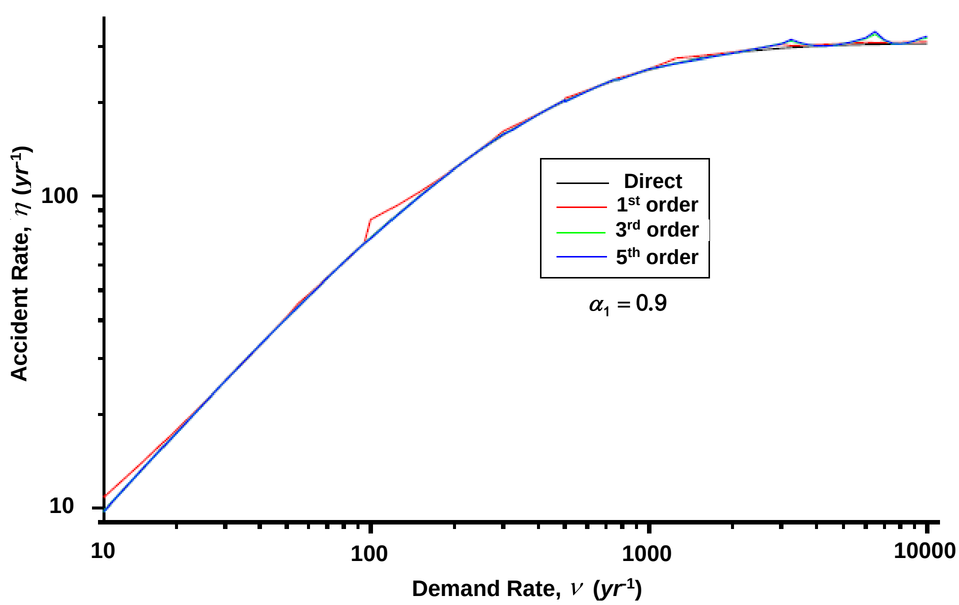

Fig. 5 Sensitivity analysis for $\alpha_1 = 0.9$

It was found that for all values of α_1, the 3rd and 5th approximation orders were better for ν <1,000/year, whereas for ν > 1,000/year, the 1st order approximation is better.

For ν < 1,000/year, the accident frequency as a function of the demand rate increases rapidly, with the need to use higher-order Taylor series to represent the function in this range. Therefore, the approaches of 3rd and 5th orders were better, while the approach of 1st order came to have a deviation of up to 8.8% for ν = 100 / year, Figure 5.

For ν > 1,000 / year, the accident rate is almost asymptotic and its derivatives of higher orders are close to zero. As the approach of 1st order does not use these derivatives, their results are better in this range. The approaches of 3rd and 5th orders use these derivatives and therefore tend to increase the deviation, reaching 12.6% for ν = 6,500/year for the 5th order approach.

I. CONCLUSIONS

The purpose of this paper was to analyze the sensitivity of a three-channel protective system, considering as an integral quantity of interest, the plant accident rate. The system sensitivity analysis was quite satisfactory and GPT is recommended GPT to perform sensitivity analyses because the computational effort is reduced by a factor of 10.

It can be seen from the results shown that the 1st, 3rd and 5th approximations orders showed good results in relation to the direct calculation at well-defined intervals as discussed. This behavior holds true for the other $\alpha_1$values. Therefore, we recommend the use of GPT in this type of approach and other problems. For future work, such as the use of up to 5 protection channels, we recommend using the GPT. However, one must analyze what approximation to use for the demand rate intervals.

Another important feature is the consideration of channel aging.

It is important to discuss the influence of channel aging on the protective system unavailability and also plant maintenance policies that significantly affect the plant accident rate. This consideration is justified by the fact that the assumption of channel useful life (that is, exponential failure times) may be too restrictive due to plant stressing conditions. An initial discussion on this may be found elsewhere [13].

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