$$\tau_{l,\alpha} = \frac{dU_l(z)}{dz}\Big|_{z=1} = \frac{1-\left(\frac{p_\alpha}{1-\varepsilon q_\alpha}\right)^l}{(1-p_\alpha-\varepsilon q_\alpha)\left(\frac{p_\alpha}{1-\varepsilon q_\alpha}\right)^l} < \infty,$$

$$\alpha = \overline{1;k}.$$

Now let m be fixed and $l \to \infty$ $(n = l + m - 1 \to \infty)$. Then, renumbering the state of the automaton in the reverse order, it is easy to verify that the generating function of the probability of changing the action is a solution of the boundary value problem (9), if in it we replace l by m, p by r and r by p.

The solution obtained has the following form

$$U_m(z) = \frac{(1-r_\alpha z-\varepsilon q_\alpha z)\left(\frac{r_\alpha z}{1-\varepsilon q_\alpha z}\right)^m}{1-z+(1-r_\alpha-\varepsilon q_\alpha)z\left(\frac{r_\alpha z}{1-\varepsilon q_\alpha z}\right)^m}$$

and

$$\sigma_{m,\alpha} = U_m(1) = 1, \qquad (11)$$

$$\tau_{m,\alpha} = \frac{dU_m(z)}{dz}\Big|_{z=1} = \frac{1-\left(\frac{p_\alpha}{1-\varepsilon q_\alpha}\right)^m}{(1-p_\alpha-\varepsilon q_\alpha)\left(\frac{p_\alpha}{1-\varepsilon q_\alpha}\right)^m} < \infty,$$

$$\alpha = \overline{1;k}.$$

If $l \to \infty$ and $m \to \infty$, then the infinite automaton remains forever in that subset of states in which it was at the initial instant of time. In this case $U_l(z) = 0$ and

$$\sigma_{l,\alpha} = 0, \qquad \tau_{l,\alpha} = \infty.$$

Passing to the limit in (7), we obtain that

$$\lim_{m\to\infty} U_l^{(n)}(z) = U_l(z), \qquad \lim_{l\to\infty} U_l^{(n)}(z) = U_m(z),$$

$$\lim_{\substack{l\to\infty \\ m\to\infty}} U_l^{(n)}(z) = U_l(z) = 0.$$

Thus, by the continuity theorem [7], the sequence of finite automata $\{T_{kn,k}(l,m;\boldsymbol{\varepsilon})\}_{l=1}^{\infty}$, $\{T_{kn,k}(l,m;\boldsymbol{\varepsilon})\}_{m=1}^{\infty}$ and $\{T_{kn,k}(l,m;\boldsymbol{\varepsilon})\}_{l,m=1}^{\infty}$ converges to the corresponding limit automata $T_k(\infty,m;\boldsymbol{\varepsilon})$, $T_k(l,\infty;\boldsymbol{\varepsilon})$ and $T_k(\infty,\infty;\boldsymbol{\varepsilon})$ of the same structure and, according to [4], the asymptotic behavior of the finite automaton $T_{kn,k}(l,m;\boldsymbol{\varepsilon})$ in a stationary random environment $C(a_1,r_1;a_2,r_2;\ldots;a_k,r_k)$ is determined by the behavior of the corresponding limit automaton $T_k(l,m;\boldsymbol{\varepsilon})$.

IV. CONCLUSION

Analyzing formulas (8), (10), (11) and taking into account Definition 2, with respect to the behavior of the automata $T_{kn,k}(l,m;\varepsilon)$ and $T_k(l,m;\varepsilon)$ in a stationary random environment $C(a_1,r_1;a_2,r_2;\ldots;a_k,r_k)$, we can draw the following conclusion.

□. The behavior of a finite stochastic automaton $T_{kn,k}(l,m;\varepsilon)$ in a stationary random environment $C(a_1,r_1;a_2,r_2;\ldots;a_k,r_k)$ is:

1. expedient if

$$\begin{Bmatrix} p_1 - p_\alpha \le \varepsilon(q_\alpha p_1 - q_1 p_\alpha) \\ r_1 - r_\alpha \le \varepsilon(q_\alpha r_1 - q_1 r_\alpha) \end{Bmatrix}, \quad \alpha = \overline{2;k}. \qquad (12)$$

at any finite integer values of the quantities l and m.

2. inexpedient if

$$\begin{Bmatrix} p_1 - p_\alpha \ge \varepsilon(q_\alpha p_1 - q_1 p_\alpha) \\ r_1 - r_\alpha \ge \varepsilon(q_\alpha r_1 - q_1 r_\alpha) \end{Bmatrix}, \quad \alpha = \overline{2;k}. \qquad (13)$$

at any finite integer values of the quantities l and m.

It should be noted that in expressions (13) and (14) both inequalities are not weak at the same time.

3. indifferent if

$$\begin{Bmatrix} p_1 - p_\alpha = \varepsilon(q_\alpha p_1 - q_1 p_\alpha) \\ r_1 - r_\alpha = \varepsilon(q_\alpha r_1 - q_1 r_\alpha) \end{Bmatrix}, \quad \alpha = \overline{2;k}. \qquad (14)$$

4. If (12) or (13) are not satisfied, then the behavior of the automaton $T_{kn,k}(l,m;\varepsilon)$ in a stationary random environment $C(a_1,r_1;a_2,r_2;\ldots;a_k,r_k)$ can be either expedient either inexpedient.

□. The behavior of an infinite stochastic automaton $T_k(l,\infty;\boldsymbol{\varepsilon})$ in a stationary random environment $C(a_1,r_1;a_2,r_2;\ldots;a_k,r_k)$ is expedient if

$$p_1 - p_\alpha < \varepsilon(q_\alpha p_1 - q_1 p_\alpha), \ \alpha = \overline{2;k} \qquad (15)$$

at any finite integer values of the quantity l.

□. The behavior of an infinite stochastic automaton $T_k(\infty,m;\boldsymbol{\varepsilon})$ in a stationary random environment $C(a_1,r_1;a_2,r_2;\ldots;a_k,r_k)$ is expedient if

$$r_1 - r_\alpha < \varepsilon(q_\alpha r_1 - q_1 r_\alpha), \ \alpha = \overline{2;k} \qquad (16)$$

at any finite integer values of the quantity m.

□V. The behavior of an infinite stochastic automaton $T_k(\infty,\infty;\boldsymbol{\varepsilon})$ in any stationary random environment $C(a_1,r_1;a_2,r_2;\ldots;a_k,r_k)$ is indifferent.

According to [4], the asymptotic behavior of the finite stochastic automaton $T_{kn,k}(l,m;\varepsilon)$ is completely determined by the behavior of the corresponding infinite automaton $T_k(l,m;\boldsymbol{\varepsilon})$.

In conclusion, we note that the stochastic automaton $T_{kn,k}(l,m;\varepsilon)$, at integer of the values of the parameter ε ($\varepsilon = 0$ or $\varepsilon = 1$) is a deterministic automaton. Consequently, from (12) - (16) we obtain the condition for the expedient behavior of these automata.

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