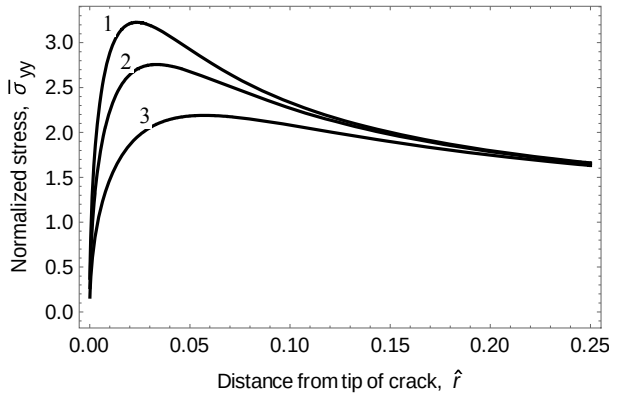


Fig. 1. Distribution of the normalized stress from normalized distance of tip of crack (1. - $\lambda = 70$ , 2- $\lambda = 50$ , 3.- $\lambda = 30$ )

The distribution $\sigma_{yy} / \sigma = \bar{\sigma}_{yy}(\hat{r}\lambda)$, is a positive bounded and continuous function of the parameter $\hat{r}$ over the entire determination interval $0 < \hat{r} < \infty$, which becomes to zero at $\hat{r} = 0$ typical for stress concentration. Strength can be estimated by the level of ultimate stresses $\sigma_{yy}$ . The typical distribution of the nonsingular normalized stresses near tip of crack Mode I for different value of the normalized scale parameter $\lambda$ is shown on the Fig1.

Instead of $\sigma^m = \sigma_{yy}^{\max}$ the magnitude of the stress intensity

$$\sigma^m = \underbrace{\max}_{0<\hat{r}<\infty} \sigma_i = \underbrace{\max}_{0<\hat{r}<\infty} \sqrt{\sigma_{xx}^2 + \sigma_{yy}^2 - \sigma_{xx}\sigma_{yy}}$$

can be used when it comes to plastic materials. The found regular stress solutions allow us to calculate the stress concentration coefficient $k_\sigma = \sigma^m / \sigma$, where $\sigma^m$ is the maximum value of stresses $\sigma_{yy}$ (or stress intensity $\sigma_i$ ). The calculated stress concentration coefficient depends on the scale parameter $\lambda$ . The obtained calculated strength estimates are compared with experimental ones.

An experimental study was carried out on plates of aluminum alloy, brass and steel. A crack was defined as a side cut in a strip that was loaded according to a three-point bending test.

As an example, we briefly discuss the experiment and calculation for aluminum alloy plates. The theoretical curve of the proportionality limit constructed from the stress intensity is shown in Fig 2.

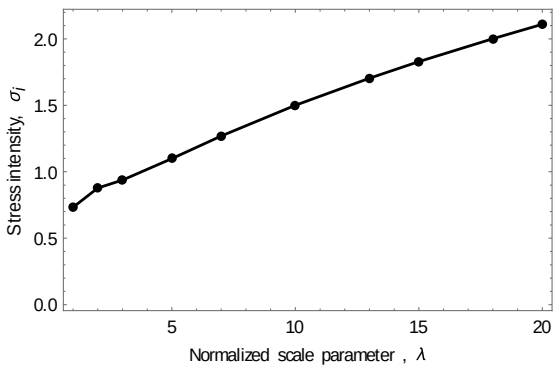


Fig. 2. Theoretical depenence of normalized stress intensity $\sigma_i / \sigma$ from scale parameter $\lambda$

For a plate with a crack 5mm long, the experimentally established proportionality limit is $\sigma = 65$ MPa. Thus, the stress concentration coefficient $k_\sigma = \sigma_u / \sigma = 1.154$ and according to Fig. 2 we have $\lambda = 5.5$ mm and then $s = l / \lambda = 0.91$. Further, assuming that the value of $s = 0.91$ mm is unchanged for the material under consideration, we give a forecast of the proportionality limit for the strip with different long cracks. For a plate with a crack of length $l = 10$ mm at mm, we have $\lambda = 11$, $k_\sigma = 1.56$ and predict using Fig.2 ultimate stresses (proportionality limit) $\sigma = 48.1$ MPa. For $l = 15$ mm, we get $\lambda = 16.5$, $k_\sigma = 1.93$ and $\sigma = 38.9$ MPa. Finally, for $l = 20$ mm, we also find $\lambda = 22$, $k_\sigma = 2.22$ and $\sigma = 33.8$ MPa. As a result, it turned out that these values of the ultimate stresses obtained theoretically coincide with experimental accuracy with high accuracy (the correlation coefficient is less than 6%).

As a result, parameter $\lambda = l / s$ is determined and scale parameter $s$ is found, which, along with the tensile strength, is a fracture characteristic. The experiments were carried out both for brittle and plastic materials. It is shown that the parameter $s$ for a particular material is a constant value and, along with the tensile strength, can be considered as a fracture parameter, giving a high accuracy forecast for the strength of materials. Moreover, the concept of stress concentration made it possible to predict theoretically the effect of increasing the proportionality limit in the vicinity of the crack tip for plastic materials, which was also confirmed by experiment. It is noted that for a plastic material this parameter is approximately an order of magnitude larger than the value obtained above for a brittle material.

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## REFERENCES

[1] A.Carpinteri, M. Paggi, "Asymptotic analysis in Linear Elasticity: From the pioneering studies by Wieghardt and Irwin until today," Eng. Fract. Mech., vol. 76, pp. 1771–1784, 2009.

[2] G.C. Sih, X.S.Tang, "Scaling of volume energy density function reflecting damage by singularities at macro-, meso- and microscopic level," Theor. Appl. Fract. Mech., vol. 43, pp. 211–231, 2005.

[3] M. Gutkin, E. Aifantis, "Dislocations in the theory of gradient elasticity," Scripta mater. vol. 40, no. 5, pp. 559‑566, 1999.

[4] S. Lurie, P. Belov, "Gradient effects in fracture mechanics for nano-structured materials," Eng. Fract. Mech., vol.130, pp. 3-11, August 2014.

[5] V.Vasiliev, S. Lurie, "Nonlocal Solutions to Singular Problems of Mathematical Physics and Mechanics," Mech. Solids, vol.53, pp.135–144, 2018.

[6] V. Vasiliev, S. Lurie, "Generalized theory of elasticity," Mech. Solids vol. 50, pp.379–388, 2015.

[7] S. Lurie, D. Volkov-Bogorodskiy, E. Moiseev, A. Kholomeeva, "Radial multipliers in solutions of the Helmholtz equations," Integral Transforms Special Funct. Vol. 30, pp. 254–263, 2019.

[8] V.Vasiliev , S. Lurie , V. Salov, "Estimation of the Strength of Plates with Cracks Based on the Maximum Stress Criterion in a Scale-Dependent Generalized Theory of Elasticity". Physical Mesomechanics, vol 22, no. 6, pp. 456-46, 2019.