A finite element formulation for the elastodynamic analysis of mobile mechanical systems

N. Dumitru, R. Malciu, and M. Calbureanu

***Abstract*—**This paper has as main purpose to identify the dynamic components of the matrices in the motion equation, combining the motion of rigid body with the one of elastic body. The finite element modeling requires the identification of a proper procedure in order to establish accurate boundary conditions which assure continuous motion in the nodes. Finally, an application on a plane mechanism, a four-bar linkage, with theoretical results for the mathematical models validation is presented. These models are useful for the optimization of robot design and to implement active vibration control for real-time applications.

***Keywords*—**finite element, elastodynamic analysis, four-bar mechanism, elastic displacement

I. Introduction

RESEARCH regarding the finite element analysis of the mobile mechanical systems reached important stages, especially in the mathematical modeling. Some ways to develop the equations of motion are presented using Newton-Euler approach, using a virtual power formulation, or using Hamilton's principle with a Lagrangian formulation.

Remarkable results were obtained by some authors as: Y.Wang and J.K.Mills [1], G. Piras, W. L. Cleghorn [2], Gravouil, Elguedj and Maigre [3].

The Lagrange finite element formulation was used to derive such a dynamic model for the flexible planar linkage with two translational and one rotational degree of freedom, and then the dynamic model was applied to the flexible link planar parallel manipulator based on standard kineto-elastodynamic assumptions [1].

The dynamic finite element analysis of the flexible planar parallel manipulator was presented in [2] including the convergence analysis of the natural frequencies and the mapping of the first-order natural frequency with respect to the robot configuration. Elastic behavior was implemented by using FEM in order to obtain a finite dimensional model. It had also been found that the geometric stiffness and the dynamic terms have a negligible effect on the response for this particular manipulator.

A systematic procedure based on the Finite Element Method (FEM) and the Lagrange principle was presented in [4]. Links and joints were considered flexible and generic equations of motion expressed according to the angles of the actuated joints and the independent elastic degrees of freedom were obtained. This procedure was inspired by papers as [5-7] and is applicable for the elastodynamic modeling of industrial robots, either serial or parallel manipulators.

Papers like [8-10] showed that the major difficulty of using FEM was the nonlinearity of the motion equations. The coefficients that appear in equations are position (time) dependent and, in some practical application like mechanisms with a periodical motion, they may be period. To solve this problem the motion had to be considered "frozen" for a very short interval of time. In this case the obtained equations might be considered linear.

The two difficult and major problems when finite element method was used: one consisted in the fact that the equations contained more terms as in the classical procedures and the second was that the equations were only incremental valid, for a very short time interval; after this interval new coefficient for the motions equations had to be generate and the solutions previously obtained were the initial conditions for the new equations. The incremental motion equations were established for a general multi-bodies system with elastic elements being in a three-dimensional motion and the problems involved by using FEM procedures were analyzed.

The unknowns in the elasto-dynamic analysis of a mechanical system with liaisons were the nodal displacements and the liaison forces. By assembling the motion equations written for each finite element the liaisons forces were eliminated and only nodal displacements were contained as unknowns in the motion equations.

The liaisons between finite elements were realized by the nodes where the displacements might be equal or might be other type of functional relations between these. When two finite elements belonged to two different elements (bodies) the liaison realized by node might determine relations more complicated between nodal displacement and their derivatives.

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An example of using FEM is offered by [11] where it was presented the finite elements analysis of the mechanical behavior for three main solar collector tracking systems: for plate, for dish and for trough solar collectors. The modeling algorithm by using FEM, the characteristics of the loads and of the restrictions were presented and, finally, the aim was to find out the critical position of the tracking systems, when the equivalent stresses and the displacements had a maximum value and to identify the free oscillations characteristics (modes, frequencies, accelerations).

According to the conclusions, the analyzed structure of the tracking systems might be compared, as mechanical behavior. The solar collectors are used to transform the energy from the sun in heat used for domestic heat water or for buildings heating. It was necessary to find solution on the way to orient the solar collectors' surface normal to the solar radiation during a day light period and during one year, also. The solution was given by the tracking systems. There are two types of tracking systems, mainly: tracking systems with one independent motion (according to the diurnal motion) and tracking systems with two independent motions (according to the diurnal and seasonal motions). In the design process of the tracking system it was important to find out the critical position of this, in order to identify the position when the equivalent stresses and the displacements values were maxim. The stresses and displacements fields were identified by using the finite elements method. The analysis of the vibration frequencies and shapes was useful to avoid the resonance phenomenon due to the action of the external dynamic loads, as wind or earthquake.

Both Kineto-ElastoDynamic and Kineto-Elastostatic analyses of a four bar linkage mechanism were carried out in [12] for obtaining the elastic deformation responses. In order to provide a practical structural damping model able to deal with the frequency-dependent damping, the standard three parameters model of visco-elastic theory was introduced here to approximate the structural damping model for the dynamic analysis of flexible mechanism containing damping metal parts. Based on the experimental data of energy storage module and loss factor for a specific kind of damping alloy in a given frequency span, the three parameters were fitted using an optimization algorithm. The differential equations of beam element were derived through the established three parameters constitution in integral form and the virtual work principle. For the convenience of computation, the established finite element equations containing convolution integration were changed into three order ordinary differential equations. By means of the Kineto- Elastodynamics theory, the element dynamic equations were assembled into the system equations of flexible linkage mechanism, which were then transformed into a standard state variable model with time-varying coefficients. In order to solve the system equations efficiently, a closed form numerical algorithm was built by using the periodicity condition of mechanism. The solution of system's state differential equations was transformed into solving a large-scale linear algebraic equation group through time discretion.

Our paper presents a method for dynamic analysis of mechanisms where the kinematic elements were considered deformable solids. The method was tested on a four-bar mechanism.

The elastodinamic analysis was possible by coupling the motion as solid rigid body and the motion as deformable solid considering the links as finite elements. The equations of motion are decoupled by taking into account static and dynamic components for nodal forces matrix, stiffness matrix and damping matrix. The motion equations solving was possible by taking proper kinematic and geometric constraints imposed by the connections between the kinematic elements. The dynamic modeling is accomplished by developing a finite element formulation. The results of a numerical processing of mathematical models developed for a four-bar mechanism are presented in the second part of the paper. Understanding and controlling structural elastodynamic response are of great importance, due to their practical applications, especially for impact, contact and penetration problems.

II. Determination of Elastokinematic Parameters

A kinematic linkage made by n rigid bodies, connected through n-1 kinematic pairs was considered as represented in Fig. 1.

The following notations were used:

$T_i\left(\vec{x}_i, \vec{y}_i, \vec{z}_i\right)$ - the reference frame attached to the element "i", with the set of three mutually perpendicular (orthogonal) unit vectors: $\overline{W}_i\left(\bar{i}_i, \bar{j}_i, \bar{k}_i\right), i=\overline{1,n}$.

$T_0\left(\vec{x}_0, \vec{y}_0, \vec{z}_0\right)$ - the global reference frame the set of three orthogonal unit vectors: $\overline{W}_0\left(\bar{i}_0 . \bar{j}_0 . \bar{k}_0\right), i=\overline{1,n}$.

$\vec{\delta}_i$, $i=\overline{1,n}$ - the vector of relative translational displacement between elements i-1 and i, relative to the reference frame T_{i-1} if there is a translational pair between elements i and i-1

$$\overline{\delta_i} = \left\{\delta_i^x, \delta_i^y, \delta_i^z\right\}_{i-1} = \left\{\delta_i\right\}^T \left\{\overrightarrow{W}_{i-1}\right\} \tag{1}$$

$\overline{r_i}$, $i=\overline{1,n}$ - the position vector relative to the reference frame T_{i-1} attached to the point O'_i from where the relative translational displacement started.

$$\overline{r_i} = \left\{r_i^x, r_i^y, r_i^z\right\}_{i-1} = \left\{r_i\right\}^T \left\{\overrightarrow{W}_{i-1}\right\} \tag{2}$$

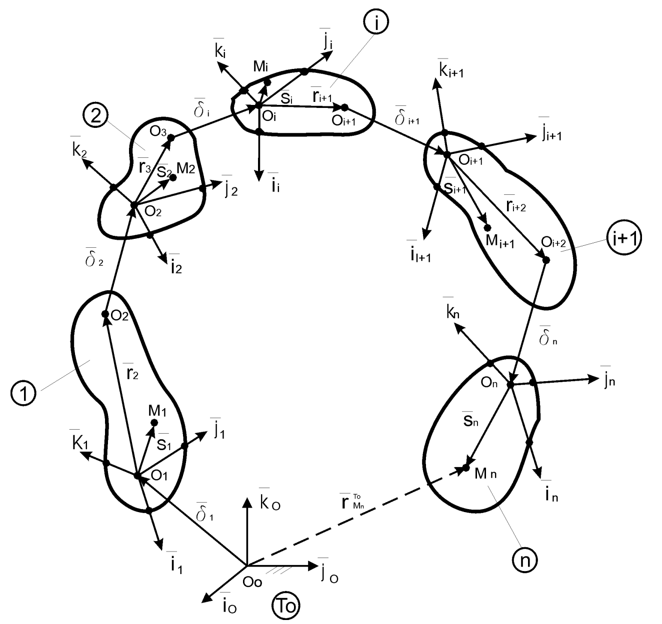

Fig. 1 kinematic model

Knowing the coordinate transformation matrix from a reference frame to another:

$$\{\overleftarrow{W}_{i\text{-}1}\}=[A_{oi\text{-}1}]\{\overleftarrow{W}_o\} \tag{3}$$

the vectors $\vec{\delta}_i$ and $\bar{r}_i$ became:

$$\overrightarrow{\delta_i}=\{\delta_i\}^T[A_{0i\text{-}1}]\{\overleftarrow{W}_o\} \tag{4}$$

$$\overline{r_i}=\{r_i\}^T[A_{oi\text{-}1}]\{\overleftarrow{W}_o\} \tag{5}$$

In Fig.2 it is represented the point M_n on the body E_n and $u(M,t)$ the elastic displacement of point M_n. The elastic displacement vector is:

$$\vec{u}=\{u\}^T\left\{\vec{w}_n\right\} \tag{6}$$

Or:

$$\{u\}=[N]\{d\} \tag{7}$$

where: - $[N]$ - the form functions matrix or the matrix of interpolation polynomials;

- $\{d\}$ - the nodal displacements vector.

The following equations may be written:

$$\vec{r}=\vec{r}_{n-1}+\vec{r}_u\ , \tag{8}$$

$$\vec{r}_u=\vec{r}_n+\vec{u} \tag{9}$$

where:

$$\vec{r}_{n-1}=\sum_{i=1}^{n}\left(\{r_i\}^T+\{\delta_i\}^T\right)[A_{0i-1}]\left\{\vec{W}_0\right\} \tag{10}$$

$$\vec{r}_n=\{r_n\}^T\left\{\vec{w}_n\right\}=\{r_n\}^T[A_{0,n}]\left\{\vec{w}_0\right\} \tag{11}$$

$$\vec{u}=\{d\}^T[N]^T[A_{0,n}]\left\{\vec{w}_0\right\} \tag{12}$$

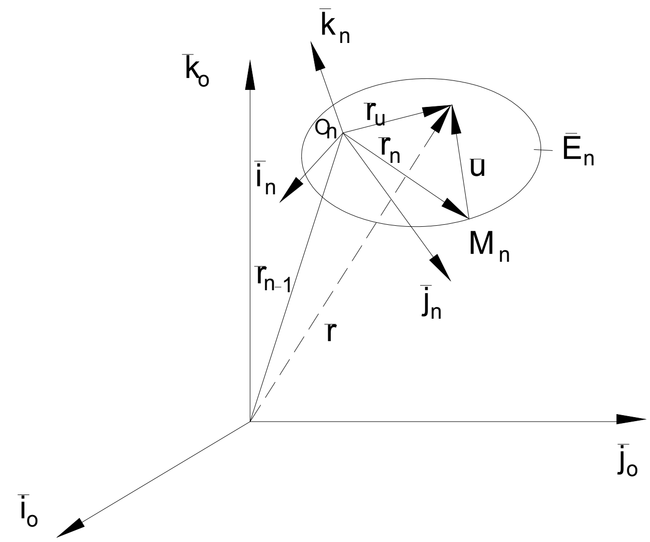

Fig. 2 the geometrical model of the elastic body

So:

$$\vec{r}_u=\{r_n\}^T[A_{0,n}]\left\{\vec{w}_0\right\}+\{d\}^T[N]^T[A_{0,n}]\left\{\vec{w}_0\right\} \tag{13}$$

$$\begin{aligned}\vec{r}&=\sum_{i=1}^{n}\left(\{r_i\}^T+\{\delta_i\}^T\right)[A_{0i-1}]\left\{\vec{W}_0\right\}+\\&+\{r_n\}^T[A_{0,n}]\left\{\vec{w}_0\right\}+\{d\}^T[N]^T[A_{0,n}]\left\{\vec{w}_0\right\}\end{aligned} \tag{14}$$

Considering the generalized coordinates:

$$\{q_i\}^T=\left\{\{r_i\}^T\ \{\delta_i\}^T\right\}=\{\{r_i\}\{\delta_i\}\}^T \tag{15}$$

the position vector of the point M_n became:

$$\vec{r}=\left(\sum_{i=1}^{n}\{q_i\}^T[A_{0i-1}]+\left(\{r_n\}^T+\{d\}^T[N]^T\right)[A_{0,n}]\right)\left\{\vec{w}_0\right\} \tag{16}$$

The equations for velocities and accelerations:

$$\vec{v}=\frac{d\vec{r}}{dt}=\left(\sum_{i=1}^{n}\left\{\dot{q}_i\right\}^T\left[A_{0,i-1}\right]+\left\{q_i\right\}^T\left[\tilde{\omega}_{0,i-1}\right]\left[A_{0,i-1}\right]+\right.$$
$$+\left\{\dot{d}\right\}^T[N]^T\left[A_{0,n}\right]+ \tag{17}$$
$$\left.+\left(\{r_n\}^T+\{d\}^T[N]^T\right)\left[\tilde{\omega}_{0n}\right]\left[A_{0n}\right]\right)\left\{\vec{w}_0\right\}$$

$$\vec{a}=\frac{d\vec{v}}{dt}=\left(\sum_{i=1}^{n}\left\{\ddot{q}_i\right\}^T\left[A_{0,i-1}\right]+2\sum_{i=1}^{n}\left\{\dot{q}_i\right\}^T\left[\tilde{\omega}_{0,i-1}\right]\left[A_{0,i-1}\right]+\right.$$
$$+\sum_{i=1}^{n}\{q_i\}^T\left[\tilde{\varepsilon}_{0,i-1}\right]\left[A_{0,i-1}\right]+\sum_{i=1}^{n}\{q_i\}^T\left[\tilde{\omega}_{0,i-1}\right]\left[\tilde{\omega}_{0,i-1}\right]\left[A_{0,i-1}\right]+$$
$$+\sum_{i=1}^{n}\left\{\ddot{d}\right\}^T[N]^T\left[A_{o,n}\right]+2\sum_{i=1}^{n}\left\{\dot{d}\right\}^T[N]^T\left[\tilde{\omega}_{o,n}\right]\left[A_{o,n}\right]+$$
$$+\sum_{i=1}^{n}\left(\{r_n\}^T+\{d\}^T[N]^T\right)\left[\tilde{\varepsilon}_{0,n}\right]\left[A_{0,n}\right]+$$
$$\left.+\sum_{i=1}^{n}\left(\{r_n\}^T+\{d\}^T[N]^T\right)\left[\tilde{\omega}_{0,n}\right]\left[\tilde{\omega}_{0,n}\right]\left[A_{0,n}\right]\right)\left\{\vec{w}_0\right\} \tag{18}$$

The motion of a linear system is described by the following equations system, [13-16]:

$$\left\{\ddot{d}\right\}^T[M]+\left\{\dot{d}\right\}^T[C]+\{d\}^T[K]=[F]^T \tag{19}$$

where: - $[M]$ is the masses matrix assembled for the whole mechanism;

- $[C]$ is the damping matrix assembled for the whole mechanism;
- $[K]$ is the rigidity matrix assembled for the whole mechanism;
- $[F]$ is the nodal forces vector assembled for the whole mechanism.

For the link "e" the equation (13) became:

$$\left\{\ddot{d}\right\}^T[M_e]+\left\{\dot{d}\right\}^T[C_e]+\{d\}^T[K_e]=[F_e]^T \tag{20}$$

where:

$$\left[M^{(e)}\right]=\iiint_{V^{(e)}}\rho_e m_e[N]^T[N]dV \tag{21}$$

$$\left[C^{(e)}\right]=\iiint_{V^{(e)}}[N]^T[\mu][N]dV+$$
$$+2\iiint_{V^{(e)}}\rho_e[N]^T\left(\left[\tilde{\omega}_{0n}\right]\left[\tilde{\omega}_{0n}\right]+\left[\tilde{\varepsilon}_{0n}\right]\right)[N]dV= \tag{22}$$
$$=\left[C_s^{(e)}\right]+\left[C_d^{(e)}\right]$$

$$\left[K^{(e)}\right]=\iiint_{V^{(e)}}[B]^T[D][B]dV+$$
$$+\iiint_{V^{(e)}}\rho_e[N]^T\left(\left[\tilde{\omega}_{0n}\right]\left[\tilde{\omega}_{0n}\right]+\left[\varepsilon_{0,n}\right]\right)[N]dV= \tag{23}$$
$$=\left[K_s^{(e)}\right]+\left[K_d^{(e)}\right]$$

The elementary nodal force may be determined with the relationship:

$$\left\{F^{(e)}\right\}=\iiint_{V^{(e)}}\{N\}^T[F]dV-\iiint_{V^{(e)}}\rho_e[N]^T\left\{\left(\left[\tilde{\varepsilon}_{0,n}\right]+\left[\tilde{\omega}_{0,n}\right]\left[\tilde{\omega}_{0,n}\right]\right)\{r_n\}+\right.$$
$$+\sum_{i=1}^{n}\left[A_{n-1}\right]\left(\left\{\ddot{q}_i\right\}+2\left[\tilde{\omega}_{0,i-1}\right]\left\{\dot{q}_i\right\}+\left[\tilde{\varepsilon}_{0,i-1}\right]\{q_i\}+\right.$$
$$\left.\left.+\left[\tilde{\omega}_{0,i-1}\right]\left[\tilde{\omega}_{0,i-1}\right]\{q_i\}\right)\right\}dV \tag{24}$$

The dynamic response of the system consists of two parts, one due to the initial conditions, which are rapidly amortized and one due to disturbing forces.

Applying the finite element method, the continuous system was replaced with a discrete system with a finite number of degrees of freedom. Unknowns of the problem are no longer the displacement functions *u(x,y,z,t)*, but the nodal displacements *d(t)*.

The system of partial differential equations (Lame's equations from the elasticity theory) turned into a system of differential equations. Matrices involved in the general equation of motion were identified based on the idea of overlapping rigid body motion over the deformable body motion. Rigid body motion was introduced by a reference set of coordinates that define the location and orientation of the local reference of every cinematic element.

If the rigid body motion is eliminated, a system of linear differential equations is obtained, obviously because of the admitted assumptions, such as:

- Assumption of small deformations;
- Assumption of small displacements;
- Assumption of linear material (Hooke Law).

Also, in the event that the damping force is proportional relative to velocity, a system of linear differential equations with constant coefficients ($[M]$, $[K]$] and $[C]$) is got. The composition of the two motions leaded to a complete dynamic analysis with multiple applications in practice.

The motion of the mechanical system is described by the general equation (19), where the coefficients ($[M]$, $[K]$ and $[C]$) may vary over time. There were developed many numerical methods in order to solve differential equations at high speed, some of these methods considering that the dynamic response may be obtained with satisfactory accuracy by superimposing only the first eigenvectors.

III. Elastodynamic Analysis of a Four- Bar Mechanism

During the past two decades, considerable attention was paid to the investigation of the dynamic analysis and vibration control of flexible mechanisms in order to achieve high- speed and lightweight machines with accurate performance. Most of the area of mechanism deformation analysis was based on linear theory, whereby the effect of elastic deformations on the gross-body motion was assumed to be negligible.

Various methods including finite element method, lump mass method, substructure method and continuum mechanics method have been discussed by various researchers. Among other methods, the finite element models have been employed in more general to flexible mechanisms. Flexible links in a mechanism are commonly modeled as elastic beams with and without consideration of the effects of large deformations, shear deformations, rotary inertia and axial deformations.

Once modeling of an unconstrained link is completed, the Lagrange multiplier method or the augmented Lagrange equations may be used to formulate the equations of motion for the entire mechanism by enforcing continuity conditions across the interfaces. These differential equations governing the kineto-elastodynamic behaviors of a mechanism are solved directly using numerical or analytical methods to study modal analysis, deflections and stresses in a planar mechanism using a cubic polynomial mode shape.

The four-bar mechanism presented by Fig. 3 was considered, with the known data as follows:

- the elements' length [mm];
- the areas of the cross sections [mm^2];
- the external forces system: gravity forces [N], inertia tensors, technological torque[N·mm].

The following steps were completed:

1. The dynamic analysis of the mechanism considering its kinematic elements to be rigid.

For this analysis the dynamic models` method has been used, obtaining de variation laws for the angular velocity and acceleration of the driving element and links from the analyzed mechanism (Fig 4 - Fig. 9).

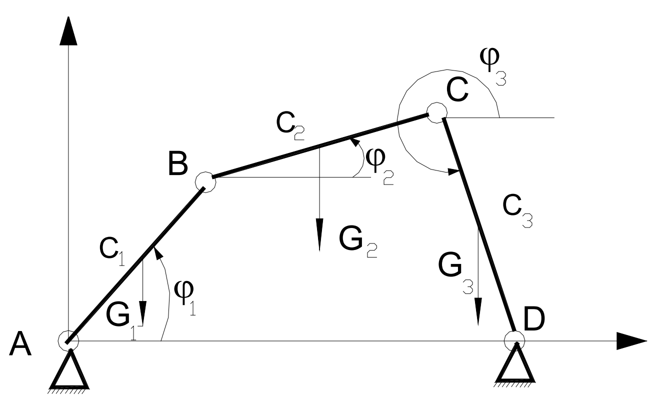

Fig. 3 four-bar mechanism

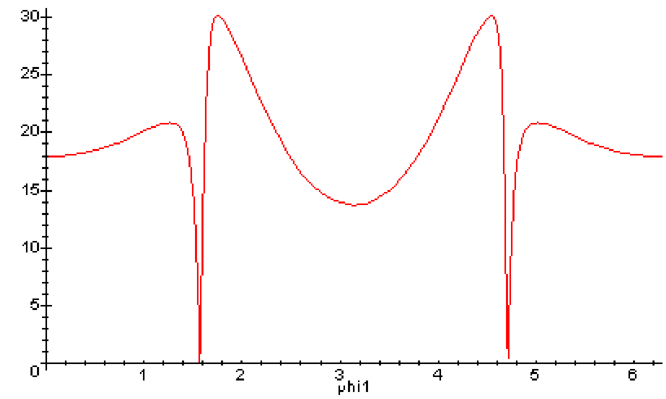

Fig. 4 the variation law of the crank angular velocity $\omega_1(\varphi_1)$

Fig. 5 the variation law of the crank angular acceleration $\varepsilon_1(\varphi_1)$

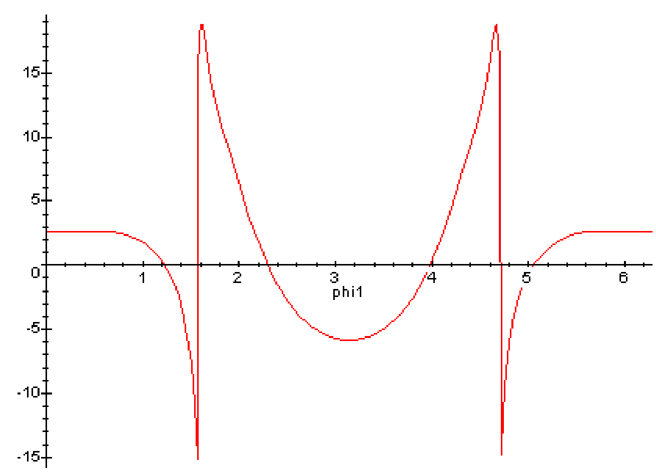

Fig. 6 the variation law of the coupler angular velocity $\omega_2(\varphi_1)$

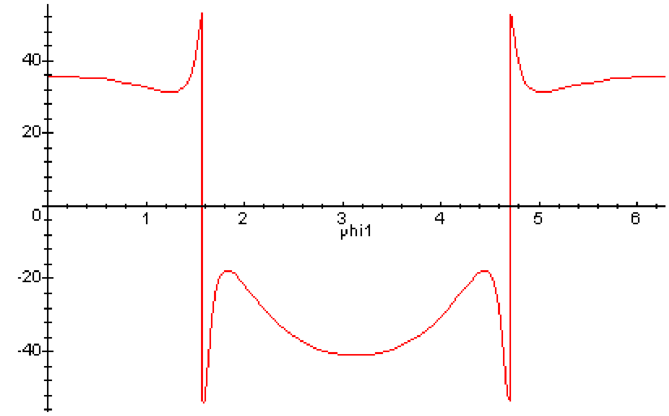

Fig. 7 the variation law of the rocker angular velocity $\omega_3(\varphi_1)$

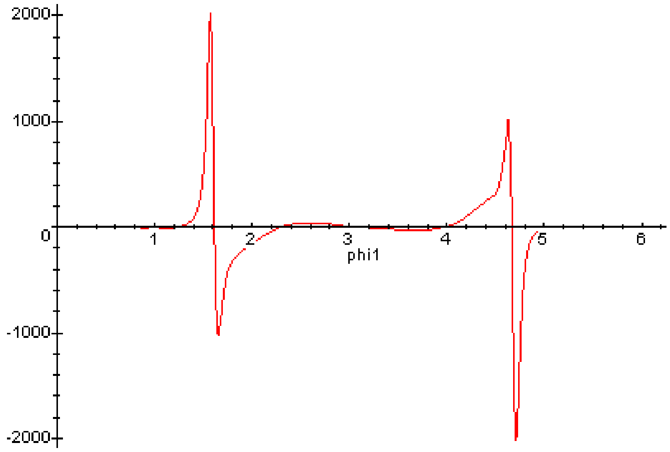

Fig. 8 the variation law of the coupler angular acceleration $\varepsilon_2(\varphi_1)$

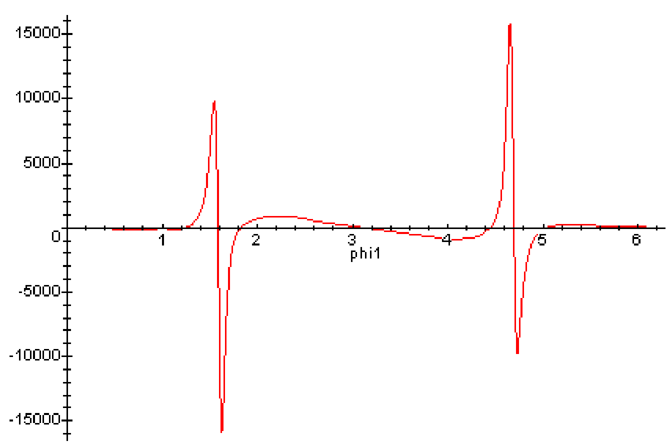

Fig. 9 the variation law of the rocker angular acceleration $\varepsilon_3(\varphi_1)$

2. The elastodynamic analysis of the mechanism

The dynamics general equation that rules the mechanism motion has the following form:

$$[M]\left\{\ddot{q}\right\}+[C]\left\{\dot{q}\right\}+[K]\{q\}=\{F(t)\} \tag{25}$$

where: $[M]$- the masses matrix assembled for the whole mechanism; $[K]$ - the rigidity matrix assembled for the whole mechanism; $[C]$ - the damping matrix assembled for the whole mechanism; $[F]$ - the nodal forces vector assembled for the whole mechanism; $\{q\}$- the vector of the nodal displacements.

The kinematic elements of the mechanism were considered as bar-type elements with three freedom degrees on each node. Based on the mathematical models presented before, there were identified and calculated the static and dynamic components of the matrices which define the motion equation of the mechanism in the local reference frame, as, the rigidity matrix, the damping matrix, the nodal forces matrix.

2.1 The mechanism links were considered as bar-type elements, with three degrees of freedom per node.

2.2 The nodal displacements for a complete description of the mechanism motion in dynamic regime were defined and identified.

2.3 Based on the mathematical models presented in the paragraph 2, the static and dynamic components of the matrices that defined the equation of motion of the mechanism in the local system of axes were identified and calculated: Stiffness matrix; Damping matrix; Matrix of nodal forces.

2.4 The coordinate transformation matrices from the local reference frame to the global reference frame were defined.

2.5 The matrices from the equation of motion in the global reference frame were evaluated.

2.6 The matrices noted above were assembled for the entire mechanism.

2.7 The boundary conditions and the limit conditions were defined.

2.8 A computer program for results numerical processing was developed.

Exemplifications of numerical processing results are presented below. So, the variation laws of longitudinal and transverse elastic displacement are presented in Fig. 10 and Fig. 11, for the entire length of the crank, in Fig. 12 and Fig. 13 for the entire length of the coupler, and in Fig. 14 and Fig. 15 for the entire length of the rocker.

In Fig. 16 and Fig. 17 there are represented the time variation laws of longitudinal and transverse nodal velocities for the node between the crank and the coupler.

In Fig. 18 it is represented the angular nodal velocity, $v_{\theta 21}$, and in Fig 19 it is represented the angular nodal acceleration, $a_{\theta 21}$, for the node between the crank and the coupler.

The time variation laws of longitudinal and transverse nodal accelerations for the same node, between the crank and the coupler, are shown in Fig. 20 and Fig. 21,

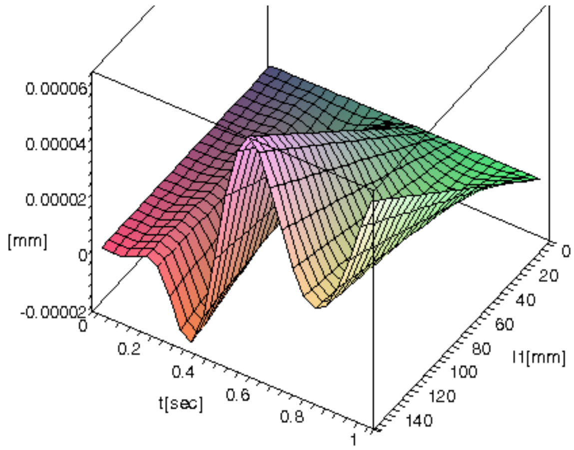

Fig. 10 the variation law of longitudinal elastic displacement of the crank 1 (t =0...1 sec.)

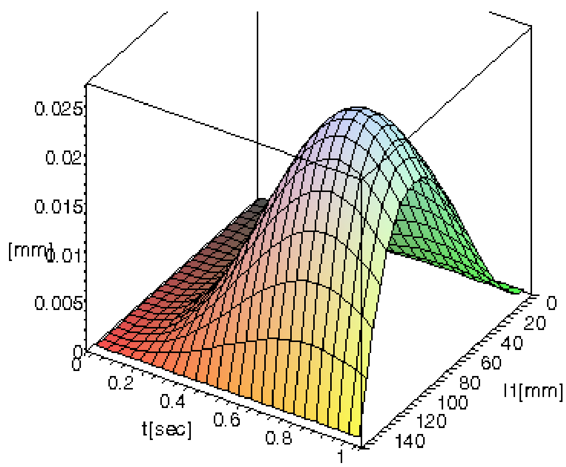

Fig. 11 the variation law of transverse elastic displacement of the crank 1 (t =0...1 sec.)

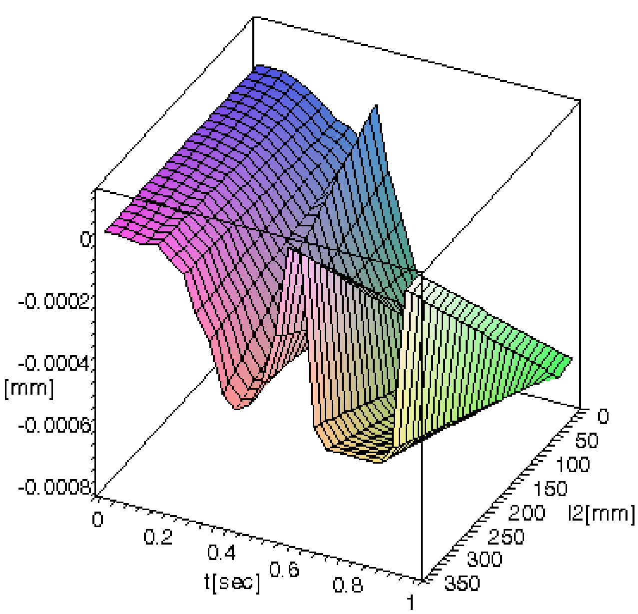

Fig. 12 the variation law of longitudinal elastic displacement of the coupler 2 (t =0...1 sec.)

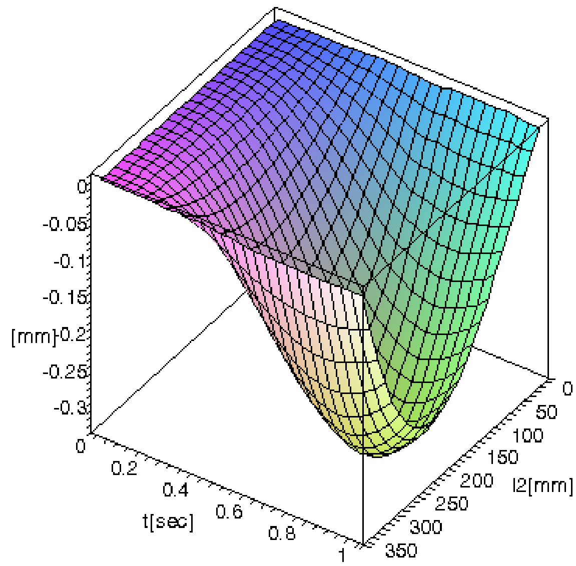

Fig. 13 the variation law of transverse elastic displacement of the coupler 2 (t =0...1 sec.)

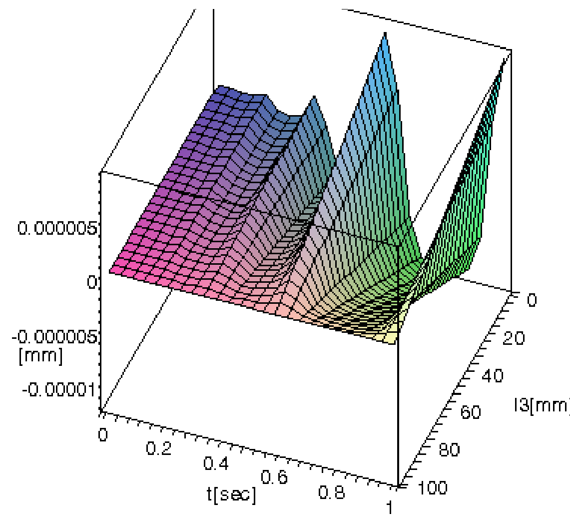

Fig. 14 the variation law of longitudinal elastic displacement of the rocker 3 (t =0...1 sec.)

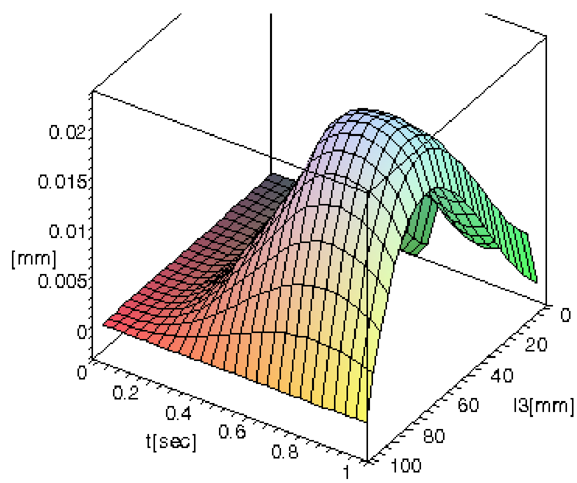

Fig. 15 the variation law of transverse elastic displacement of the rocker 3 (t =0...1 sec.)

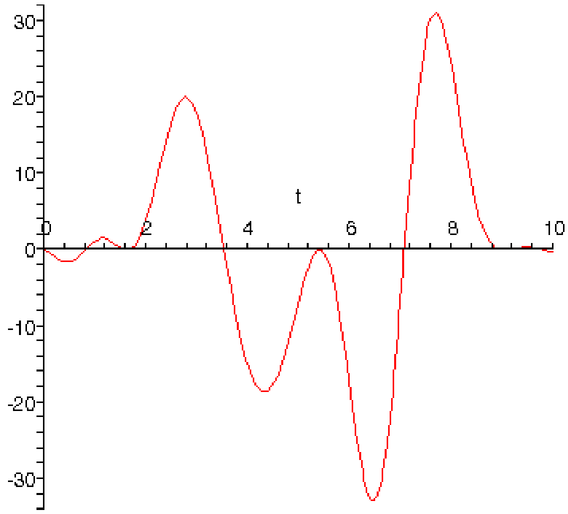

Fig. 16 the time variation law of longitudinal nodal velocity for the node between the crank and the coupler, [mm/s]

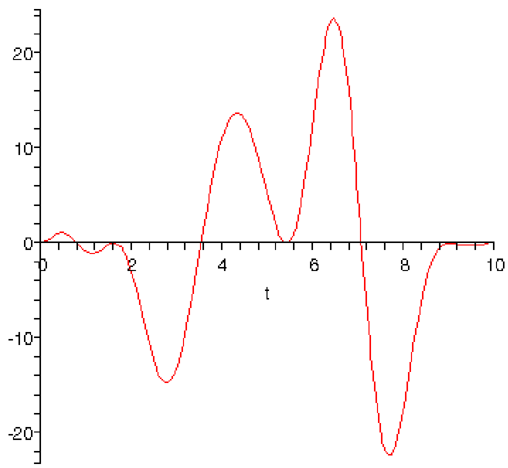

Fig. 17 the time variation law of transverse nodal velocity for the node between the crank and the coupler, [mm/s]

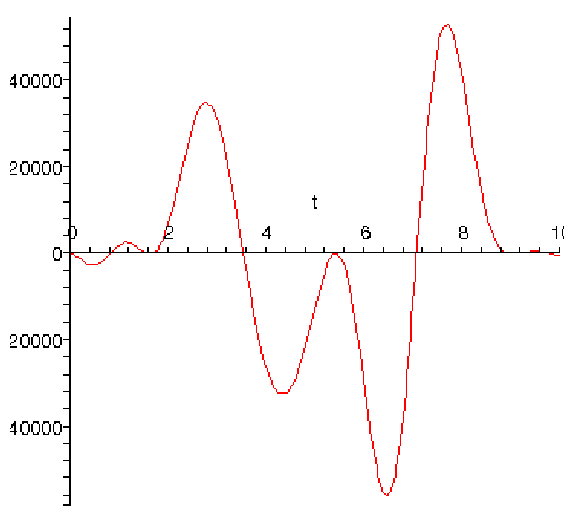

Fig. 18 the time variation law of angular nodal velocity for the node between the crank and the coupler, $v_{\theta 21}$, [rad/s]

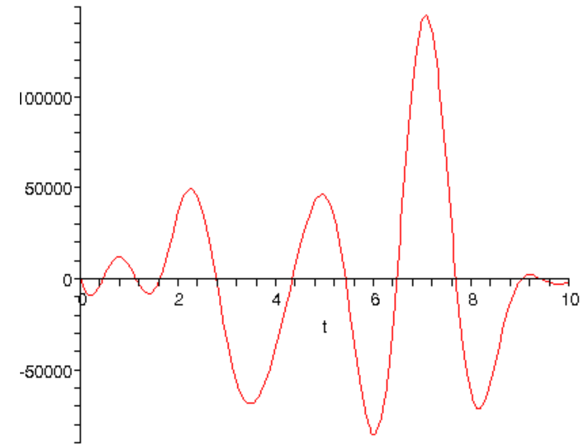

Fig. 19 the time variation law of angular nodal acceleration for the node between the crank and the coupler, $a_{\theta 21}$ [rad/s^2]

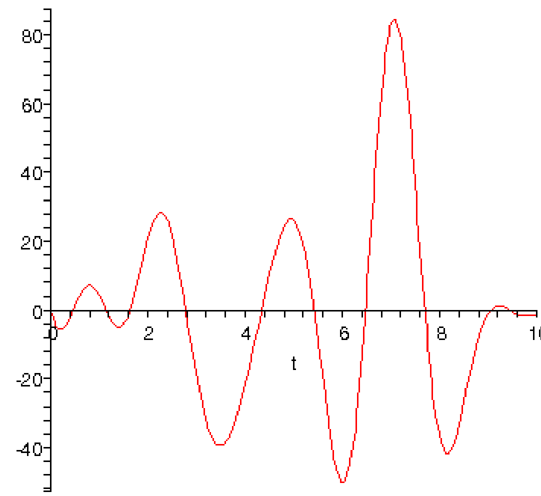

Fig. 20 the time variation law of longitudinal nodal acceleration for the node between the crank and the coupler [mm/s^2]

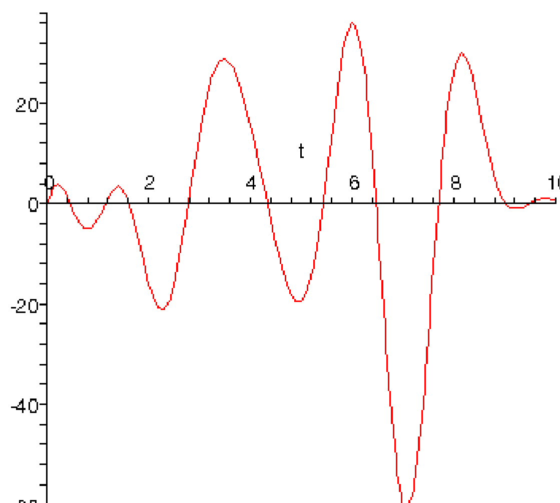

Fig. 21 the time variation law of transverse nodal acceleration for the node between the crank and the coupler [mm/s^2]

IV. Conclusion

Mechanism systems are now being required to run at increasingly higher speeds while maintaining their positioning accuracy. The higher operating speeds mean that the mechanisms need to be made as light as possible to reduce the inertial forces and thus the driving torque requirements. However, the lighter members are more likely to vibrate elastically due to inertial and external forces. It thus becomes necessary to include in the dynamic analysis of mechanisms not only the effect of the rigid body motion, but also the flexibility of the linkages.

Most of the area of mechanism deformation analysis is based on linear theory, whereby the effect of elastic deformations on the gross-body motion is assumed to be negligible.

As general conclusion of our paper it may be noticed that the variation diagrams of the elasto-kinematic parameters (displacements, velocities and accelerations) are strongly influenced in form and values by the variation of the rigid body motion kinematic parameters (angular velocity and angular acceleration), for every analyzed kinematic element.

The future research purpose is to use experimental equipment with high performance diagnosis apparatus, in order to determine the kinematic parameters which characterize the vibrations of the four-bar mechanism, for different working conditions, and experimentally verify the mathematical models and the numerical results presented in this paper. We hope to prove that the elastodynamic analysis of the four-bar mechanism considered as an assembly of finite elements coupled in nodes which materialized the kinematic pairs leads to results more accurate and closer to those of experimental models.

References

[1] X. Wang, J. K. Mills, "FEM dynamic model for active vibration control of flexible linkages and its application to a planar parallel manipulator", *Applied Acoustics*, vol. 66, no. 10, 2005, pp. 1151–1161.

[2] G. Piras, W. L. Cleghorn, J. K. Mills, "Dynamic finite-element analysis of a planar high-speed, high-precision parallel manipulator with flexible links", *Mechanism and Machine Theory*, vol. 40, no. 7, 2005, pp. 849–862.

[3] Gravouil, T. Elguedj, H. Maigre, "An explicit dynamics extended finite element method. Part 2: Element-by-element stable-explicit/explicit dynamic scheme", *Computer Methods in Applied Mechanics and Engineering*, 2009, pp 2318-2328.

[4] M. Rognant, E. Courteille, P. Maurine, "A systematic procedure for the elasto-dynamic modeling and identification of robot manipulators", *IEEE Transaction on Robotics* 26, 6, 2010, pp. 1085 - 1093.

[5] S. Marie, P. Maurine, "Elasto-geometrical modelling of closed-loop industrial robots used for machining applications", in *Proceedings IEEE International Conference on Robotics and Automation*, 2008, pp. 1294– 1300.

[6] A. A. Shabana, *Dynamics of Multibody Systems*. Cambridge University Press, 2005.

[7] S.K. Dwivedy, P.Eberhard, "Dynamic analysis of flexible manipulators, a literature review", *Mechanism and Machine Theory*, 2006; pp.749-777.

[8] S. Vlase, H. Teodorescu-Draghicescu, "Finite Element Method Used in the Multibody Systems With Elastic Elements", in *Proceedings of the 4th IASME/WSEAS International Conference on CONTINUUM MECHANICS (CM'09)*, ISSN: 1790-2769; ISBN: 978-960-474-056-7

[9] L. Scutaru, S. Vlase, H. Teodorescu, "Study of vibration suppresion of mechanical systems with elastic elements", in *Proceedings of the 3rd WSEAS/IASME International Conference on Dynamical Systems and Control*, ISSN: 1790-5117; ISBN: 978-960-6766-08-4, Arcachon, France, October 13-15, 2007

[10] S.Vlase, H. Teodorescu, L. Scutaru, "On the form of the motion equations of the multibody systems with elastic elements", in *Proceedings of the 5th WSEAS Int. Conf. on Non-Linear Analysis, Non-Linear Systems and Chaos*, ISSN: 1790-5117, ISBN: 960-8457-54-8, Bucharest, Romania, October 16-18, 2006

[11] M.T.Lates, "Mechanical Behavior Analysis with the Finite Elements Method of Solar Collector's Tracking Systems", *WSEAS TRANSACTIONS on APPLIED and THEORETICAL MECHANICS*, ISSN: 1991-8747 Issue 7, Volume 3, July 2008

[12] D. Yuan, J. Liu, Y. Jiang, "Dynamic Analysis of Flexible Mechanism Using Visco-elastic Damping Model", in *Proceedings of the 3rd IASME / WSEAS International Conference on CONTINUUM MECHANICS (CM'08)*, ISBN: 978-960-6766-38-1, ISSN: 1790-2769

[13] M. Cherciu, N. Dumitru, "Computer aided kinematic modeling of the mobile mechanical systems", *The Ninth IFToMM International Symposium on Theory of Machines and Mechanisms, SYROM 2005*, Bucharest 2005, pag.861.

[14] F. Amirouche, *Fundamentals of Multibody Dynamics. Theory and Applications*, Birkhauser, Boston, 2006.

[15] N. Dumitru, M. Cherciu, A. Zuhair, "The Finite Element Modeling of The Dynamic Response of The Mechanisms", *The 13th International Congress on Sound and Acoustics*, Vienna, 2006.

[16] N. Dumitru, M. Cherciu, A. Zuhair, "Theoretical and experimental modeling of the dynamic response of the mechanisms with deformable kinematic elements", *12th IFToMM World Congress*, Besancon, France, 2007.