Zero-Pole and Padé Finite-Dimensional Reduction of Delayed Controllers: A Comparative Study

Libor Pekař and Roman Prokop

Abstract**—**A comparative study of two approaches for simplification and rationalization of controllers with internal delays is presented in this paper. Delayed (or so-called anisochronic) controllers result from the use of compensatory or predictive control methods for time-delay systems. In this study - where the aim is to match the original infinite-dimensional controller with a simple finite-dimensional one - the Padé approximation of the whole transfer function and the methodology preserving dominant poles and zeros and asymptotic behavior are introduced and compared. Control of a circuit laboratory heating plant with delays in a special ring or proper and stable meromorphic functions is taken as an example for the comparison. The simple negative control feedback system and a control structure with two feedback controllers are implemented. Besides simulation results, real measurements on the laboratory appliance are presented as well.

Keywords**—**Rational approximation, Controller simplification, Time-delay systems, Delayed controllers, Algebraic control methods, Padé approximation, Two feedback controllers, Matlab-Simulink, Circuit heating plant.

I. Introduction

In recent decades a number of papers and works have been focused on model reduction or rational approximation of time-delay systems (TDS), see e.g. [1], [2] or a fair overview published in [3]. Reduction ideas and approaches can naturally be extended from plant models to their controllers. An overwhelming majority of these methods, however, deals with input-output delays only ignoring internal or state delays on the left-hand side of differential equations, i.e. those transfer functions with exponential terms in the denominator. Moreover, the obtained plant model or controller structures would be of a very high order, as it is presented in [4] for some particular cases.

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There are many control theory areas where the appropriate controller design leads to unordinary controller structures, such as in optimal control methods, compensatory ideas, control of high-order systems etc. For TDS, many approaches, especially compensatory or predictive ones, see e.g. [4]-[8], yield controllers with delays in the dynamics – so-called anisochronic controllers, which belong to the class of infinite-dimensional controllers (systems).

As well known, see e.g. [9], more than 95 % control loops in industry are equipped by proportional-integral-derivative (PID) controllers. In the light of this fact, many more sophisticated analytically derived control laws are hence disqualified from being used in practice. A possible way how to overcome this obstacle is to transform of simplify these controllers into the PID or any simple finite-dimensional form.

The ring of proper and stable quasipolynomial meromorphic functions (RMS), utilized in this paper, is a representative of the method giving rise to anisochronic controllers. The ring was originally defined and introduced in [7], and revised and extended in [10]. It is very effective in control of TDS, especially in case of a sufficiently good plant model.

Thermal or heating systems are typical examples of delayed systems [6], [11]. The presented comparative study is performed on control of a laboratory circuit heating plant in the RMS ring. The appliance was described in [12] and its detailed mathematical model was derived and introduced in [13]. Control design in the ring for the well known simple feedback loop was the aim of the paper [14]. A rather more complex control system with Two Feedback Controllers (TFC), the description of which is the matter of a section in this contribution, was used in [15]. Both the structures with corresponding derived controllers are utilized in this paper. These controllers are simplified and rationalized via two simple distinctive model reduction ideas. The first one, natural and intuitive, idea rests in matching dominant (right-most) controller zeros and poles and asymptotic behavior of the controller and a finite-dimensional model, say a PID one. The second methodology is based on the idea of the Padé expansion where the whole controller transfer function is rationalized instead of a separate delay elements, which enables to select the order of the resulting finite-dimensional model.

Results of rational approximations are finally verified and benchmarked by means of the comparison of original, unapproximated, control responses with those using reduced controllers, both, by simulations as well as real measurements on the laboratory appliance. They proof a high usability of the proposed methods, mainly the Padé-based one, especially with respect to the above mentioned note on using of PID controllers in industry.

The paper is organized as follows. Basic ideas and methodology of algebraic control design in the R_{MS} ring is concisely overviewed and summarized in Section II. General forms of anisochronic (delayed) controllers and linear finite-dimensional, particularly PID, ones and ideas of both the rationalization methodologies are introduced in Section III. In Section IV, a brief description, a model of the laboratory appliance and corresponding eventual controllers for both control system structures are briefly introduced. Approximating controllers' structures and simulated control responses comparison and analysis is the matter of Section V. Finally, Section VI provides real experimental results.

II. Controller Design in R_{MS} – An Overview

Modern control theory has been adopting algebraic approaches and parlance, which are based on system description in a suitable field, ring or module and the subsequent operation in the algebraic structure, for decades. The R_{MS} ring is a representative of this class of control design tools for linear time-invariant TDS. The definition of the R_{MS} ring was introduced in [7] and revised and extended in [10]. Since the aim of this paper is to deal with these definitions, the reader is referred to the mentioned papers. Yet, let us concisely introduce and summarize controller design procedures in the ring for both control systems, for the simple feedback control structure and the TFC one. Again, for further details, the reader is referred e.g. to [8], [14], [15].

A. Control design for the simple feedback loop

To let the reader know especially the notation of signals, the well known simple negative feedback loop is pictured in Fig. 1. Note that all the presented signals are assumed to be ratios of elements from R_{MS}.

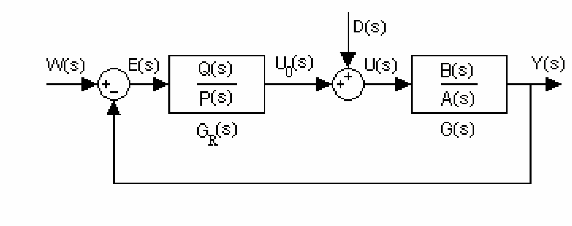


Fig. 1 Simple control feedback loop

External inputs, reference and load disturbance signals, respectively, have forms

$$W(s)=\frac{H_W(s)}{F_W(s)},\ D(s)=\frac{H_D(s)}{F_D(s)} \tag{1}$$

where $H_W(s)$, $H_D(s)$, $F_W(s)$, $F_D(s)\in R_{MS}$.

If a pair $A(s), B(s)\in R_{MS}$ is (Bézout) coprime [16], the closed-loop system is stable (in R_{MS} sense) if and only if there exists a coprime pair $P(s), Q(s)\in R_{MS}$ satisfying the Bézout identity

$$A(s)P(s)+B(s)Q(s)=1 \tag{2}$$

a particular stabilizing solution of which, $P_0(s), Q_0(s)$, can be further parameterized as

$$P(s)=P_0(s)\pm B(s)Z(s)\neq 0,\ Q(s)=Q_0(s)\mp A(s)Z(s) \tag{3}$$

where $Z(s)\in R_{MS}$.

The reference signal $w(t)=L^{-1}\{W(s)\}$ is asymptotically tracked if and only if $F_W(s)$ divides the product $A(s)P(s)$ in R_{MS} [17].

The load disturbance $d(t)=L^{-1}\{D(s)\}$ is asymptotically rejected if and only if $F_D(s)$ divides $B(s)P(s)$ in R_{MS}.

B. Control Design for the TFC Control System

As in the previous subsection, basic control requirements conditions for the TFC control loop (see Fig. 2) are provided.

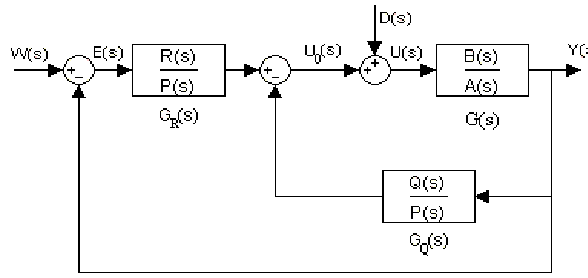


Fig. 2 TFC control feedback loop

The stability condition for the TFC structure can be formulated analogously to (2), yet with $T(s)=Q(s)+R(s)$ instead of $Q(s)$.

Load disturbance rejection is satisfied if $F_D(s)\,|\,(B(s)P(s))$; however, reference tracking requires the simultaneous fulfillment of two conditions: $F_W(s)\,|\,(A(s)P(s))$ and $F_W(s)\,|\,(B(s)Q(s))$. To guarantee that, $T(s)$ has to be decomposed to $Q(s)$ and $R(s)$ by putting weights to appropriate s-powers in numerators of both terms. These weighting coefficients bring additional degrees of freedom.

III. CONTROLLERS FORMS AND REDUCTION METHODOLOGIES

A. Delayed Controllers

Linear delayed (anisochronic) controllers belong to a wide family of infinite-dimensional systems. They are characterized by an infinite spectrum which results from the existence of delayed (exponential) terms in the transfer function. These delayed elements in the dynamics appear because of the endeavour to compensate non-approximated delays in the controlled model of TDS (or simply a system with input-output delay). For instance, controllers obtained from the algebraic design described in Section II lead to the anisochronic structure which can be described via the transfer function as

$$G_Q(s)=\frac{Q(s)}{P(s)} \tag{4}$$

where $Q(s), P(s)$ are quasipolynomials of the form

$$s^n+\sum_{i=0}^{n}\sum_{j=1}^{h_i} x_{ij} s^i \exp(-\vartheta_{ij} s), \vartheta_{ij}\geq 0 \tag{5}$$

where the degree (i.e. the highest s-power) of the numerator is less or (usually) equal to that of the denominator. If $x_{nj}=0$ for all j with $\vartheta_{ij}>0$, the controller is of a retarded structure; otherwise, it has a neutral form.

Let us relabel the controller in Fig. 1 in accordance to (4) for the further text.

B. Finite-Dimensional Controllers

The general transfer function form of linear finite-dimensional controllers is a basic technical knowledge; nevertheless, let us introduce it to be referenced below. Hence, consider a ratio $\overline{G}_Q(s)=\overline{Q}(s)/\overline{P}(s)$ with

$$\overline{Q}(s)=\sum_{i=0}^{n_Q} q_i s^i,\ \overline{P}(s)=s^{n_P}+\sum_{i=1}^{n_P} p_i s^i \tag{6}$$

Let us denote, in addition

$$\overline{R}(s)=\sum_{i=0}^{n_R} r_i s^i \tag{7}$$

for the controller numerator in the TFC structure.

Note that it holds for a standard PID controller that $n_P=n_Q=2$, $p_0=0$.

The task is to match (4) and (5) with (6) (and/or (7)); hence, two simple ideas are described below.

C. Zeros, Poles and Asymptotic Behavior Matching

One can, naturally, try to preserve dominant (i.e. right-most) controller zeros and poles and asymptotic controller behavior, i.e. the slope of a step response direction at infinity for an integral controller and/or an initial point of a step response for a derivative one. Hence, for the pole-matching, the aim is to find $\{s_1, s_2, ..., s_{n_P}\}$ such that

$$\begin{aligned}&\operatorname{Re} s_1 \geq \operatorname{Re} s_2 \geq ... \geq \operatorname{Re} s_{n_P}\\ &P(s_1)=P(s_2)=...=P(s_{n_P})\\ &=\overline{P}(s_1)=\overline{P}(s_2)=...=\overline{P}(s_{n_P})=0\end{aligned} \tag{8}$$

and for every other s_i with $P(s_i)=0$ it holds that $\operatorname{Re} s_{n_P} \geq \operatorname{Re} s_i$.

Similarly, for the zero-matching, we have

$$\begin{aligned}&\{z_{Q1}, z_{Q2}, ... z_{Qn_Q}\}\\ &:=\left\{\begin{aligned}&z_1, z_2, ... z_{n_Q} : \operatorname{Re} z_1 \geq \operatorname{Re} z_2 \geq ... \geq \operatorname{Re} z_{n_Q},\\ &Q(z_1)=Q(z_2)=...=Q(z_{n_Q})\\ &=\overline{Q}(z_1)=\overline{Q}(z_2)=...=\overline{Q}(z_{n_Q})=0;\\ &\operatorname{Re} z_i \geq \operatorname{Re} z_{n_Q}, Q(z_i)=0\end{aligned}\right\}\end{aligned} \tag{9}$$

The set $\{z_{R1}, z_{R2}, ... z_{Rn_R}\}$ of zeros of $R(s)$ for the TFC structure can be defined analogously to (9).

Asymptotic conditions can be expressed as

$$\begin{aligned}&\lim_{s\to 0} s^k G_{.}(s)=\lim_{s\to 0} s^k \overline{G}_{.}(s)=\pm\infty, k=0,1,...k_{\max}\\ &\lim_{s\to 0} s^{k_{\max}+1} G_{.}(s)=\lim_{s\to 0} s^{k_{\max}+1} \overline{G}_{.}(s)\neq\pm\infty\end{aligned} \tag{10}$$

for controllers with integral behavior (or for those tracking or rejecting harmonic signals) and

$$\lim_{s\to\infty} G_{.}(s)=\lim_{s\to\infty} \overline{G}_{.}(s) \tag{11}$$

for derivative-like controllers. The value of $k_{\max}$ agrees with the so called order of the astatism of the controller and the lower index ” . ” means either R or Q.

D. Matching Based on the Padé Approximation

The transfer function rationalization via the Padé approximation is usually performed in such a way that the approximation is applied to exponential terms only. This technique leads to high-order approximation models. A different approach, used here, rests in the approximation of the whole transfer function based on the Taylor (Maclaurin) series expansion of the approximated and approximating model and matching of some (low-degree) coefficients, which agrees with conditions

$$\begin{aligned}&\left[G_{.}(s)\right]_{s=0}=\left[\overline{G}_{.}(s)\right]_{s=0},\\&\left[\frac{\mathrm{d}}{\mathrm{d}s}G_{.}(s)\right]_{s=0}=\left[\frac{\mathrm{d}}{\mathrm{d}s}\overline{G}_{.}(s)\right]_{s=0},\\&\left[\frac{\mathrm{d}^2}{\mathrm{d}s^2}G_{.}(s)\right]_{s=0}=\left[\frac{\mathrm{d}^2}{\mathrm{d}s^2}\overline{G}_{.}(s)\right]_{s=0},\\&\ldots\\&\left[\frac{\mathrm{d}^{l-1}}{\mathrm{d}s^{l-1}}G_{.}(s)\right]_{s=0}=\left[\frac{\mathrm{d}^{l-1}}{\mathrm{d}s^{l-1}}\overline{G}_{.}(s)\right]_{s=0}\end{aligned}\tag{12}$$

Notice that it is possible to calculate identities (12) at a different point from $s=0$, e.g. in the neighborhood of a frequency where a good approximation is desired. If any of derivatives does not exist, substitute $G_{.}(s)$ by $1/G_{.}(s)$, which is the case of controllers derived above as well. The value of l (usually) equals the number of approximating model parameters. However, in some cases, equations in (12) are not independent or do not include any variable; thus, l ought to be higher than the number of unknowns here.

IV. Circuit Heating Plant Model and its Controllers

Let us very concisely describe the circuit heating plant serving as a testing controlled device.

A. Plant Description

A photo of the appliance is depicted in Fig. 3.

Fig. 3 Circuit heating plant photo

In principle, the heat transferring fluid is transported using a continuously controllable pump into a flow heater. Warmed liquid then goes through a long insulated coiled pipeline which causes the significant internal delay in the system. The air-water heat exchanger (cooler) with two cooling fans represents a heat-consuming appliance. The expansion tank compensates for the expansion effect of the water.

B. Mathematical Model

Since the modeling and identification of the laboratory plant was thoroughly derived and introduced in [13], only a final results used in this paper is going to be presented.

We choose here to control cooler output temperature, $\vartheta_{CO}(t)$, by means of the heater power, $P_H(t)$. For this relation, the following transfer function was derived

$$G(s)=\frac{\vartheta_{CO}(s)}{P_H(s)}=\frac{\left[b_{0D}\exp(-\tau_0 s)+b_0\right]\exp(-\tau s)}{s^3+a_2s^2+a_1s+a_0+a_{0D}\exp(-\vartheta s)}\tag{13}$$

where $a_2,a_1,a_0,a_{0D},b_{0D},b_0$ are (non-delay part) plant parameters, τ,τ_0,ϑ stand for plant delays (input-output and internal ones, respectively). It was determined that for the operating point

$$\begin{aligned}&\left[u_p,u_c,P_H,\vartheta_{HO},\vartheta_{CI},\vartheta_{CO},\vartheta_A\right]\\&=\left[5\,\mathrm{V},3\,\mathrm{V},300\,\mathrm{W},44.1\,°\mathrm{C},43.8\,°\mathrm{C},36\,°\mathrm{C},24°\mathrm{C}\right]\end{aligned}\tag{14}$$

that the parameters in (13) are

$$\begin{aligned}&b_{0D}=2.334\cdot10^{-6},b_0=-2.146\cdot10^{-7},a_2=0.1767,a_1=0.009,\\&a_0=1.413\cdot10^{-4},a_{0D}=-7.624\cdot10^{-5},\tau_0=1.5,\tau=131,\vartheta=143\end{aligned}\tag{15}$$

C. Derived Delayed Controllers

To demonstrate the complexity of eventual controllers resulting from the algebraic control design in the R_{MS} ring, their transfer functions are introduced. Thus, the use of the simple feedback loop according to Fig. 1 gives

$$G_Q(s)=\frac{m_0^3\left(s^3+a_2s^2+a_1s+a_0+a_{0D}\exp(-\vartheta s)\right)}{\left(b_{0D}+b_0\right)\left(s+m_0\right)^3-m_0^3\left(b_{0D}\exp(-\tau_0 s)+b_0\right)\exp(-\tau s)}\tag{16}$$

where $m_0>0$ is a tunable parameter. Note that the controller (16) was calculated for step-wise reference and load-disturbance signals.

Controllers' laws for the TFC control systems are even more complex

$$\begin{aligned}&G_Q(s)=m_0^3\frac{\left(s^3+a_2s^2+a_1s+a_0+a_{0D}\exp(-\vartheta s)\right)(1-\gamma)t_1s^2}{\left(p_4s^4+p_3s^3+p_2s^2+p_1(s)s+p_0(s)\right)\left(s+m_1\right)}\\&G_R(s)=m_0^3\\&\frac{\left(s^3+a_2s^2+a_1s+a_0+a_{0D}\exp(-\vartheta s)\right)\left(\gamma t_1s^2+\left(t_1m_1+t_0\right)s+t_0m_1\right)}{\left(p_4s^4+p_3s^3+p_2s^2+p_1(s)s+p_0(s)\right)\left(s+m_1\right)}\end{aligned}\tag{17}$$

where $m_0,m_1>0,\gamma\in[0,1]$ are selectable parameters, for real coefficients p_2,p_3,p_4 holds that

$$p_2 = 6m_0^2(b_0 + b_{0D})^2,\; p_3 = 4m_0(b_0 + b_{0D})^2,\; p_4 = (b_0 + b_{0D})^2 \tag{18}$$

and quasipolynomials $p_0(s), p_1(s)$ read

$$\begin{aligned} p_0(s) &= m_0^4(b_0 + b_{0D})\begin{pmatrix} b_0(1-\exp(-\tau s)) \\ + b_{0D}(1-\exp(-(\tau+\tau_0)s)) \end{pmatrix} \\ p_1(s) &= m_0^3 \\ &\begin{pmatrix} 4(b_0 + b_{0D})^2 - (b_0 + b_{0D}\exp(-\tau_0 s)) \\ (b_0(m_0\tau + 4) + b_{0D}(m_0(\tau+\tau_0)+4))\exp(-\tau s) \end{pmatrix} \end{aligned} \tag{19}$$

The complexity is partially caused by the fact that the controllers above were derived for a linear-wise reference.

V. Approximating Controllers and Simulation Results

Let us derive and benchmark finite-dimensional controllers obtained by methodologies introduced in Section III. PID and P (i.e. proportional) controllers with $n_P = n_Q = 2$, $p_0 = 0$, and $n_P = n_Q = 0$, respectively, are selected as "initial" approximating structures (according to (6), (7)). Resulting controllers are compared with the original ones by means of simulated control responses characteristics.

A. Zeros, Poles and Asymptotic Behavior Matching

The rightmost zeros of controller (16) with values (15) and $m_0 = 0.012$ are

$$\begin{aligned} &\{z_{Q1}, z_{Q2}, z_{Q3}\} \\ &= \{-2.97619\cdot 10^{-3}, -6.43535\cdot 10^{-3} \pm 3.077622\cdot 10^{-2}\,\mathrm{j}\} \end{aligned} \tag{20}$$

It is clear that the use of a PID controller is impossible since it can not have such a distribution of dominant zeros. Therefore, a consider a more general approximating linear finite-dimensional controller structure with $n_P = n_Q = 3$, $p_0 = 0$. Because of

$$\begin{aligned} &\{s_1, s_2, s_3\} \\ &= \{0, -6.74644\cdot 10^{-3} \pm 2.346829\cdot 10^{-2}\,\mathrm{j}\}, \\ &\lim_{s\to 0} sG_R(s) = 0.10978 \end{aligned} \tag{21}$$

the approximating controller reads

$$\overline{G}_Q(s) = \frac{22.2481s^3 + 0.35256s^2 + 2.2846\cdot 10^{-2}s + 6.546\cdot 10^{-5}}{s\left(s^2 + 1.3493\cdot 10^{-2}s + 5.96275\cdot 10^{-4}\right)} \tag{22}$$

However, this controller causes a very high control action, as can be seen in Fig. 4.

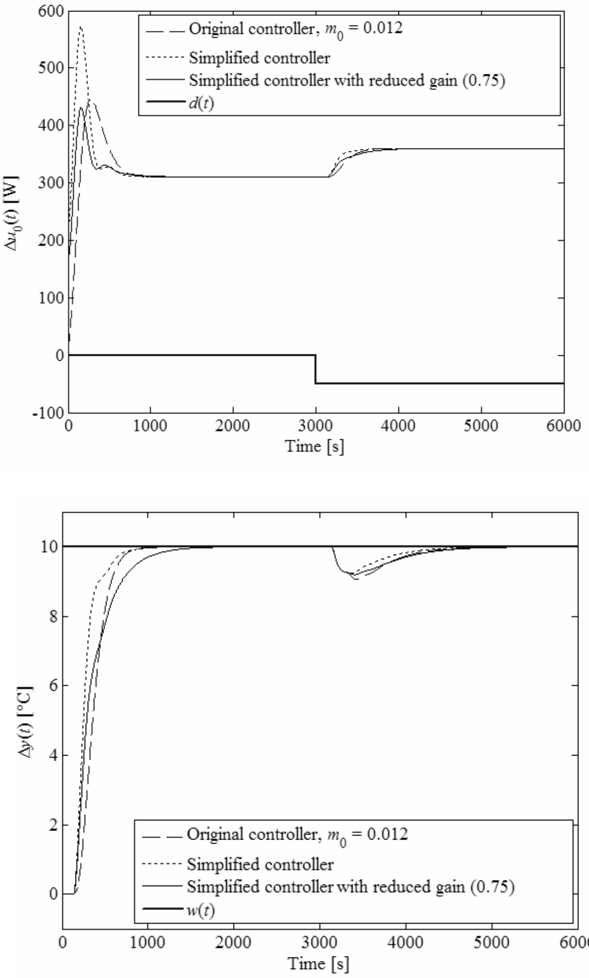


Fig. 4 Simulation control responses of $\Delta u_0(t)$ (left) and $\Delta y(t)$ (right) for the simple feedback structure with controllers (16) vs. (22) (and that with a reduced gain)

In the figure, where $\Delta u_0(t)$ stands for the manipulated input (i.e. heater electricity input power) and $\Delta y(t)$ means the controlled output (i.e. temperature behind a heat exchanger), $w(t)$ is the reference signal and $d(t)$ stands for a load disturbance. Notice that the maximum manipulated input deviation is $\Delta u_{0,\max} = 450\,\mathrm{W}$. Therefore, a reduced controller gain as $\overline{G}_{Q2}(s) = 0.75\overline{G}_Q(s)$ has been set eventually.

Regarding the TFC case, robust analysis has yielded the settings: $m_0 = 0.02,\ m_1 = 0.005, \gamma = 0.4$. Controller $G_Q(s)$ can be approximated by a proportional model where the identity $\lim_{s\to 0} G_Q(s) = \lim_{s\to 0} \overline{G}_Q(s)$ is taken as an objective, hence

$$\overline{\overline{G}}_Q(s) = \overline{\overline{G}}_Q = 24.6114 \tag{23}$$

Dominant poles and zeros of $G_R(s)$ in (17) are the following

$$\{s_1, s_2, s_3\} = \{0, 0, -1.5421755 \cdot 10^{-4} \pm 2.917941 \cdot 10^{-2}\,\mathrm{j}\}$$

$$\{z_{R1}, z_{R2}, z_{R3}, z_{R4}\} = \begin{Bmatrix} -2.0971272 \cdot 10^{-3}, -2.9761945 \cdot 10^{-3}, \\ -6.43535 \cdot 10^{-3} \pm 3.077622 \cdot 10^{-2}\,\mathrm{j} \end{Bmatrix} \tag{24}$$

which implies that a PID model can be used here as

$$\overline{G}_R(s) = \frac{98.7822 s^2 + 0.50115395 s + 6.16545 \cdot 10^{-4}}{s^2} \tag{25}$$

where the condition $\lim_{s \to 0} s^2 G_R(s) = \lim_{s \to 0} s^2 \overline{G}_R(s)$ $= 6.16545 \cdot 10^{-4}$ is considered in addition.

However, control process with controllers (23) and (25) is unstable (and hence not displayed here); therefore, we have changed controllers‘ gains to $\overline{G}_{Q2} = 0.5\overline{G}_Q$, $\overline{G}_{R2}(s) = 0.5\overline{G}_R(s)$ - the corresponding control responses are depicted in Fig. 5.

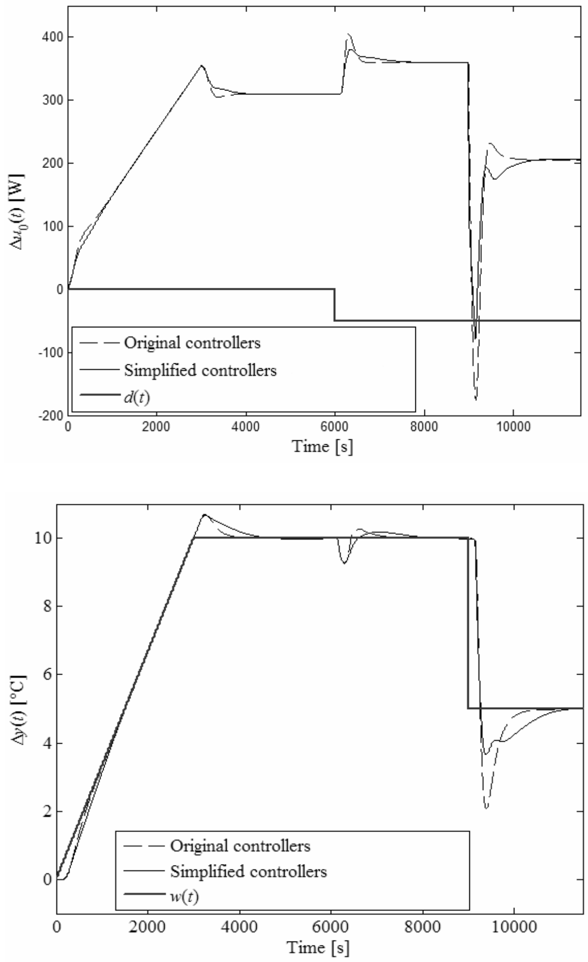


Fig. 5 Simulation control responses of $\Delta u_0(t)$ (left) and $\Delta y(t)$ (right) for the TFC structure with controllers (17) vs. (23) and (25) (with a reduced gain)

Approximating controllers derived on the basis of a requirement of equality of dominant controllers' poles and zeros provide satisfactory control responses that are a bit slower then the ones obtained by the algebraic approach in the R_{MS} ring, yet with reduced overshoots. Nevertheless, controllers' gains had to be adjusted.

B. *Matching Based on the Padé Approximation*

Consider now the procedure based on the Padé approximation. For the simple control system, since $G_Q(0) \to \infty$, the derivatives in (12) are calculated for $1/G_Q(s)$. In addition, condition $\left[1/G_Q(s)\right]_{s=0} = \left[1/\overline{G}_Q(s)\right]_{s=0}$ leads to identity $0 = 0$ directly, which implies no useful result. Therefore, let $l = 4$ rather than $l = 3$. The resulting controller reads

$$\overline{G}_Q(s) = \frac{-36.873 s^2 + 0.871844 s + 2.91172 \cdot 10^{-3}}{s\left(s + 3.59065 \cdot 10^{-2}\right)} \tag{26}$$

As second, for TFC, controller $G_Q(s)$ in (17) has no pole at zero, hence, $k = 1$ and a proportional model is considered here. Finally, $G_R(s)$ owns a double zero pole and $\left[1/G_R(s)\right]_{s=0} = \left[1/\overline{G}_R(s)\right]_{s=0} \Rightarrow 0 = 0$ again, therefore take $l = 4$ and a PID model.

The resulting model reduction is expresses by the eventual approximating controllers' transfer functions as

$$\begin{aligned} \overline{G}_R(s) &= \frac{47.2676 s^2 + 0.416887 s + 6.16545 \cdot 10^{-4}}{s^2} \\ \overline{G}_Q(s) &= 24.6114 \end{aligned} \tag{27}$$

The corresponding simulation control responses are pictured in Figs. 6 and 7, for the simple feedback loop and the TFC structure, respectively.

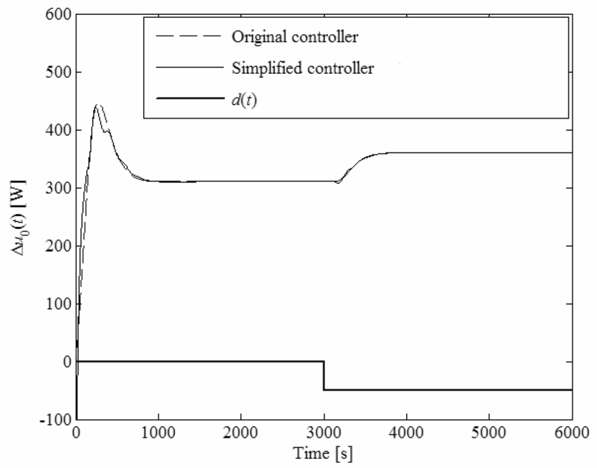
Obviously, this type of approximation provides a very good simulation result closely matching the original control responses curves without changing of controllers' settings. Its disadvantage can be viewed in rather complex calculations for the solution of set (12).

C. *Conclusion*

Since the latter method gives much better simulation results for this particular example, eventual control laws do not need to be modified, and the P and PID structures are industrial standards and of a simpler form than controllers derived by the former method, the Padé approximation based methodology is chosen for real-life verification measurements.

VI. Real Measurements Results

Finally, verify the usability of controllers (26) and (27) that has arisen from the simplification of controllers (16) and (17) using the Padé approximation. The corresponding comparison of simulated and measured control responses are displayed in Figs. 8 and 9.



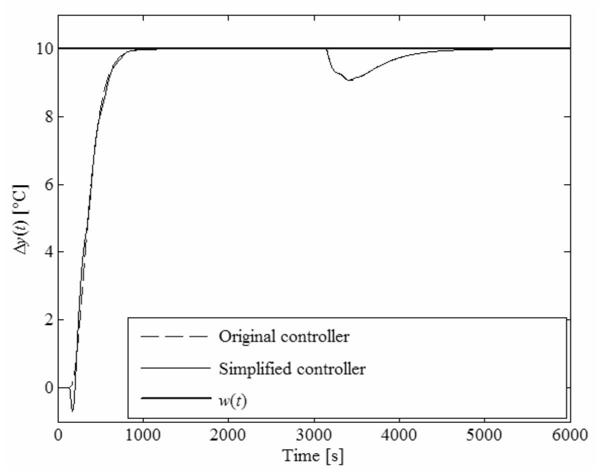
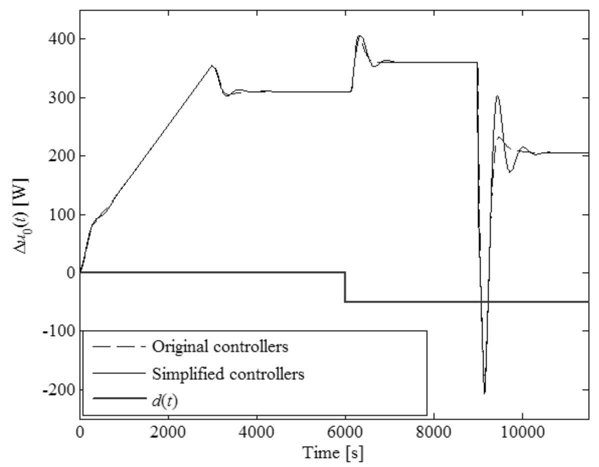


Fig. 6 Simulation control responses of $\Delta u_0(t)$ (left) and $\Delta y(t)$ (right) for the simple feedback structure with controllers (16) vs. (26)



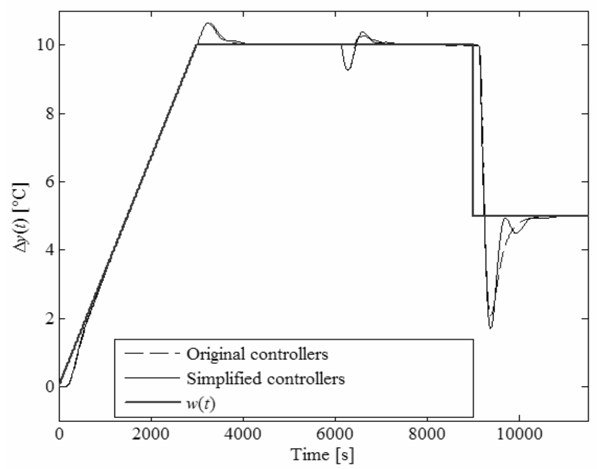
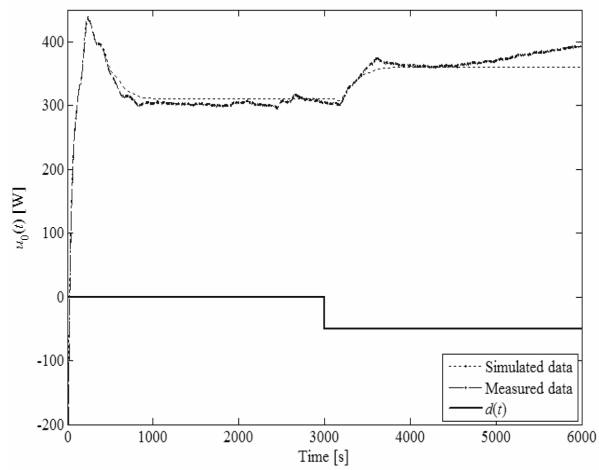


Fig. 7 Simulation control responses of $\Delta u_0(t)$ (left) and $\Delta y(t)$ (right) for the TFC structure with controllers (17) vs. (27)



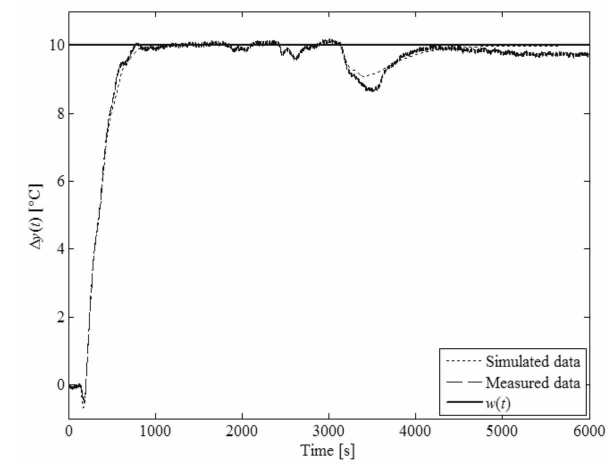


Fig. 8 Measured vs. simulated control responses of $\Delta u_0(t)$ (left) and $\Delta y(t)$ (right) for the simple feedback structure with controller (26)

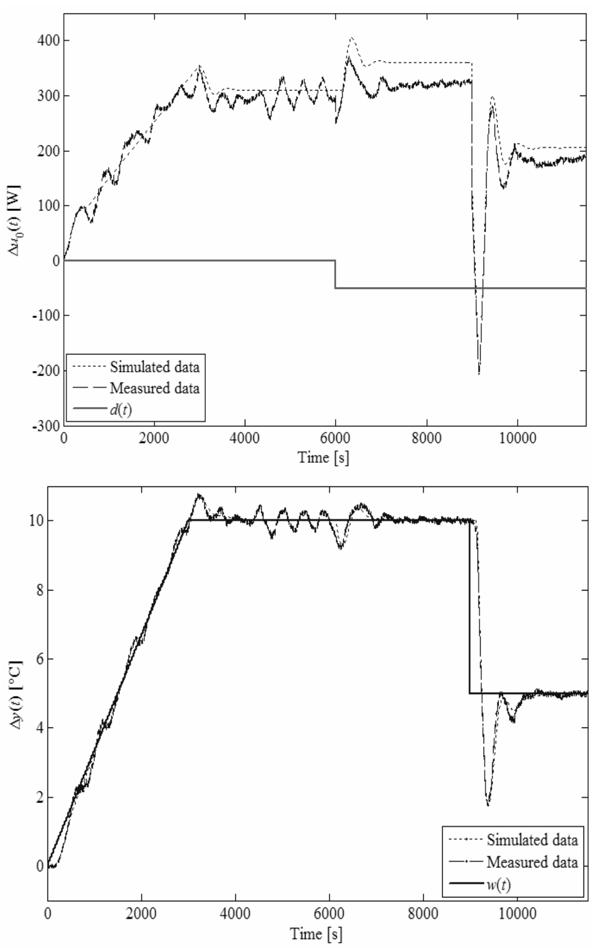


Fig. 9 Measured vs. simulated control responses of $\Delta u_0(t)$ (left) and $\Delta y(t)$ (right) for the TFC structure with controllers (27)

To avoid the abrupt change in the control action at the beginning of Fig. 8, we would suggest using a low-pass filter on the reference signal. Other undesirable effect can be seen near the end of the measurement where due to rapidly decreasing ambient temperature the control action increases whereas controlled temperature can not reach the reference value.

Although real control responses are not as satisfactory as in the case of original controllers derived using the R_{MS} ring, figures above prove the usability and applicability of simplified finite-dimensional (PID) controllers as well. Particularly, the reaction to the load disturbance when using the TFC structure is surprisingly good.

VII. Conclusion

Both the theoretical and practical aspects of two simple delayed model reduction and rationalization methodologies have been the aim of this paper. The presented approaches represent two possible easy-to-implement ways how to transform controller models with delays to the most used PID (or similar linear finite-dimensional) form. Namely, zero-poles-asymptotic matching and Padé approximation based procedures have been introduced. Matlab-Simulink simulation and real-measurements result on a laboratory circuit heating plant with internal delays have proofed, particularly, the high performance of the letter idea. All the primal infinite-dimensional controllers have been derived in the algebraic way using the R_{MS} ring which also approves the usability of the whole complex control design methodology.

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